The Foerder Institute for Economic Research
at Tel Aviv University

מכון למחקר כלכלי על שם
ד"ר ישעיהו פורדר
על יד אוניברסיטת תל אביב

The Eitan Berglas School of Economics

בית-הספר לכלכלה ע"ש איתן ברגלס

עמותה רשומה

Clerks

Kfir Eliaz, Daniel Fershtman, Alexander Frug

Discussion Paper No. 3-2024

The Foerder Institute for Economic Research
and
The Sackler Institute of Economic Studies

Clerks*

Kfir Eliaz[†] Daniel Fershtman[‡] Alexander Frug[§]

July 3, 2024

Abstract

We study the optimal dynamic scheduling of workers to tasks when task-completion is privately observed (hence, workers can delay the release of completed tasks), and when idle time is the only means of providing incentives. Our main result characterizes a scheduling rule, and the equilibrium it induces, maximizing the expected discounted output subject to workers' incentive constraints. When workers are inherently slow, a simple rotation scheme suffices to attain first-best output, but when they are more productive, optimal scheduling alternates between phases with and without delay. Our analysis highlights a trade-off between the quality and size of workforce.

Keywords: Strategic servers, non-monetary incentives, optimal scheduling, moral hazard, idle time, multi-server systems.

*Financial support from the Foerder Institute and the Sapir Center is gratefully acknowledged.
[†]Tel-Aviv University and NHH; email: kfire@tauex.tau.ac.il
[‡]Tel-Aviv University, Emory University, and CEPR; email: danfershtman@gmail.com
[§]Universitat Pompeu Fabra and Barcelona School of Economics; email: alexander.frug@upf.edu

1 Introduction

*"This job would be great if it wasn't for the ******* customers."* Randal in *Clerks*

In 2018, bed occupancy in the internal wards of most public hospitals in Israel surpassed 100%. This meant that either patients were left in the Emergency Room (ER), or they were put in temporary beds in the halls of the internal wards.[1] There was mounting pressure from the public and the media to resolve this crisis. While most hospitals argued that a resolution demanded higher budgets, one hospital ("Haemek") was able to cut down the occupancy in its internal wards to less than 75% within one year and without any additional expenses! What this hospital administration realized was that the root of the problem was the wards' response to the patient routing rule used by the ER. This involved assigning patients to whichever ward had a vacant bed. Administration suspected the wards were reacting to this by deliberately slowing down their release of patients to keep a high bed occupancy, many of which consisted of "cured" patients which required less treatment and care.

In light of this, the hospital administration decided to change the patient assignment rule to the following: each patient was assigned to a ward according to a fixed order (the first patient to ward 1, the second patient to ward 2, and so on until it was ward 1's turn again) *independently* of the bed occupancy in the wards. If a patient was assigned to ward i which did not have any vacant bed while ward j did, then the patient was put in ward j but was cared for by the staff of ward i. Within two months the bed occupancy across all wards fell to 75% (Linder, 2019).[2]

This anecdote highlights a broader phenomenon: service providers respond to the process of organizing work (e.g., the rules for allocating tasks), and their response may undermine the original objective of the designed work process (which was designed for a fixed behavior of workers). This is especially true in settings with the following characteristics: (i) workers face an inflow of tasks and are not compensated according to their performance, and (ii) workers are the experts on the scene in the sense that

[1]Internal wards are responsible for catering to a wide range of internal disorders, providing inpatient medical care to thousands of patients each year. In most hospitals in Israel, patients are routed from the emergency room to one of several internal wards that are similar in structure but have separate medical and administrative staff.

[2]See also Dagan, Lichtman-Sadot, Shurtz, and Zeltzer (2024) for an empirical analysis of this routing reform.

they are the ones who know whether a task was successfully completed or whether they put sufficient effort into it. One prominent setting with these features is the public sector where clerks handle public reception or cases/applications/appeals/complaints filed by the public. Examples include departments of motor vehicles, social services, unemployment insurance offices, postal offices, and public healthcare services. Even within the private sector, many customer service representatives are paid fixed wages, as is often the case in retail banking.

When wages are fixed, one of the main ways to incentivize workers is by designing the *work process*, which consists of several components. One component is a rule for determining when new tasks are assigned to a worker and what type of tasks. Some tasks are more work intensive or tedious than others (e.g., operating a cashier or a service window may be more laborious than ordering office supplies or filing paperwork). In addition, spacing new task assignments can also reward workers with some idle time. A second component of work-process design determines the relation between the number of servers (e.g., doctors, car mechanics, etc.) and the "capital" they use for work, such as beds in the case of hospitals, mechanic work stations in garages, or "windows" in the case of DMVs, social services, etc.

The workers, in turn, may respond strategically to the work-process design. For example, a worker who anticipates an unpleasant task to be allocated soon might be strategically slow on his current tasks - hoping to see the next task assigned to someone else; redundant, easy paperwork might keep service providers busy and reduce the number of daily appointments; and a physician's motivation to release a patient may depend on whether she expects to be responsible for fewer patients for some duration.

In addition, the designer of the work process often faces an "appearance constraint": the system should have the appearance of being constantly at work. For instance, if there are patients waiting in the ER, there should not be empty beds in any of the wards; if there are people waiting in line in the DMV/social security office/bank, there should not be any service window with a sign "next window please". Consequently, when tasks are waiting to be processed, all the capital that is used for processing them (beds, windows, work-stations) should be in use. This restricts the relation between workers and capital and imposes constraints on how workers can be incentivized with idle time or more rewarding tasks. This paper takes a first step at understanding this design problem and the economic implications of its solution.

We propose a simple stylized model that captures the main forces at work. There are n workers referred to as "servers". Servers work on tasks sequentially: only when they complete one task can they start working on a new one. To complete a task a server needs to work at a "work station", which can accommodate at most one server. A work station can represent a desk, a public reception window, a patient's bed, a phone in a call center or a machine in a factory. There are $m < n$ work stations. Time is continuous and the completion of a task is modeled as a Poisson process with arrival rate of λ, which represents the servers' productivity. There is an infinite supply of tasks, which are routed to the various servers according to a rule designed by a principal. Once a server completes a task, he decides at each subsequent time whether to release the task or to withhold it. Only the server working on a task observes when it is finished, but the release of a task is observed by all (the principal and all other servers).

The servers' incentives to release completed tasks stem from the following considerations. When a server is assigned a new task he incurs an immediate *set-up cost* of K and he incurs a flow *effort cost* of c at every period in which he works on the task. Idle time is costless (and hence valuable) while withholding a completed task bears a flow cost of $d < c$. The idea is that dwelling on a finished task does not require much effort, yet the server is not completely free to engage in other activities as during idle time (e.g., he cannot make personal calls or surf the web). The principal gets a normalized payoff of one from every released task.

The principal and the servers apply the same discount factor to future payoffs. Our goal is to highlight key features of optimal task allocation rules - ones that attain the highest discounted expected payoff for the principal in (perfect Bayesian) equilibrium - and to characterize equilibria that achieve these payoffs.

When servers are sufficiently slow - i.e., their productivity is below some threshold - the first-best output is achieved by a simple rotation scheme, where servers release tasks immediately upon completion and a server who releases a task is replaced by the non-working server who was idle for the longest time. This is because the relatively long period it takes a server to complete a task provides ample idle time to incentivize a server to release a finished task. However, when servers are fast enough so that their productivity rate exceeds the threshold, there is no equilibrium that attains the first-best: The idle time provided by working servers is just too short to incentivize servers to release a task and incur the future costs of working.

We characterize the second-best, i.e., derive the minimal amount of inevitable delay (inefficiency) in such systems, and find a task allocation scheme and an induced equilibrium that attains the minimal inefficiency. In this equilibrium servers voluntarily sit on finished tasks and delay their release. To prove that the above scheme and its induced equilibrium are optimal, we show that they attain a naive upper bound on the principal's discounted expected payoff. One important message of this result is that some level of inefficiency is necessary for the organization to operate (constrained) optimally. In our model this takes the form of shirking by sitting on completed task. The equilibrium we identify has the feature that the inherent delay is self-enforcing: it is a best response to the delay tactic of the servers.

The equilibrium described above highlights a trade-off that a principal faces with regards to the productivity of its servers. On the one hand, more productive workers can produce more output per unit of time; but on the other hand, faster production requires inefficient delay in order to incentivize workers to release completed tasks. This suggests a natural trade-off between quantity and quality of servers. We analyze this trade-off in our model. First, holding the number of servers and work stations fixed, we find that the total output indeed increases with the servers' productivity rate. However, when the servers' productivity becomes sufficiently high (so that delay becomes inevitable), the increase in output becomes rather slow. Building on this observation, we illustrate that a group of arbitrarily productive servers can be outperformed by a slightly larger group of mediocre servers (i.e., servers who are so slow that simple rotation without delay is incentive compatible). This hints at a tendency to prioritize quantity over quality in organizations with the features described earlier in the introduction.

We conclude our analysis by considering three extensions of our benchmark model. The first extension enriches the discretion of each server by allowing him to shirk and sit on a task even before it is finished. Specifically, at any moment of time in which the task has yet to be completed, a server decides whether to exert effort and incur a flow cost of c or shirk and incur the lower cost of d by sitting on the task. The productivity rate is λ if and only if effort is exerted; otherwise, it is zero. Put differently, once assigned to a task, a server decides between two levels of productivity - zero or λ - such that positive productivity is more costly.

A consequence of this new moral hazard dimension of effort exertion is that the allocation scheme, which was optimal in the benchmark model, will now lead to shutdown. The reason is that if a server is willing to put in effort at the time of receiving a task, then he will strictly prefer to immediately release it upon completion. But this means that servers that are productive enough will not receive ample idle time, and hence, will not be motivated to put in effort in the first place. Thus, incentivizing effort is inconsistent with self-enforcing voluntary delays. In particular, an optimal task-assignment scheme cannot solely rely on the servers and requires some form of intervention by the principal. We illustrate one such scheme when there are two servers, a single work station and no set-up costs.

The second extension we consider is that of diverse productivity rates. The presence of this heterogeneity introduces a new complication: to maximize output it is now important to give more tasks to the more productive servers. We illustrate this in the simple case of two servers and a single work station. Here, the first-best is not incentive compatible since it requires the more productive server to constantly work. We characterize the second-best in the case when simple rotation (i.e., servers taking turns working at the work station without any delays) is incentive-compatible. We show that the optimum is attained by a stochastic mechanism in which there is a probability that the more productive server works again after completing a task.

Our third extension introduces the possibility of releasing an unfinished task, which gives zero payoff to the principal. This means that a server now decides at each point in time - both before and after finishing the task - whether to release it. We assume that when an unfinished task is released, there is a positive probability p that it returns (immediately) to be re-processed, and that it is publicly known who released it.

Focusing on the case with two servers and one work station we show that there is an interval of p values such that (i) for values above this interval, the principal can attain the first-best by assigning a returning task and all future tasks to the server who released the unfinished task; (ii) for values below this interval, the principal cannot prevent premature releases; and (iii) for values inside the interval, the principal can prevent premature releases but at a cost of delay.

Our paper makes a methodological contribution by proposing a framework for analyzing dynamic task allocation in organizations where wages are not responsive to output, and workers are privately informed about task completion. As our extensions illustrate, our framework can easily be adapted to capture a variety of frictions and can be applied to a broad range of settings. In addition, our analysis offers an explanation of perceived inefficiencies that are commonly observed in organizations with the above mentioned features, and also suggests possible interventions for improving work processes. For instance, in many cases what may seem as wasteful idleness of workers is actually a necessary feature for sustaining second-best production. Similarly, reducing the amount of capital available to workers can increase total output.

The remainder of the paper is organized as follows. Related literature is discussed below. Section 2 presents the benchmark model. The optimal allocation scheme is described and proven to be optimal in Section 3. Section 4 analyzes the trade-off between quantity and quality of servers. Section 5 considers the three extensions discussed above. Concluding remarks are given in Section 6.

Related literature

The paper is related to several strands of literature. First, it is related to a number of papers studying "strategic service provision." Gavazza and Lizzeri (2007) studies a problem involving servers who face a queue of customers. They consider a game between servers who choose their speed of service taking into account the cost of quicker service, the benefit of free time after all customers have been served, and the demand for service that increases with quicker service (such that more customers means less idle time). The main insight is that disclosing the service speeds of servers lowers the incentives to invest in quicker service and can lead to an equilibrium where customers are worse off relative to a regime without disclosure. In addition to having a very different framework than theirs (more below), slower service in our model is actually *necessary* to avoid shutdown (when servers are sufficiently productive).

Geng, Huh, and Nagarajan (2015) considers a game between two servers who simultaneously choose their production rates in response to a routing policy. The servers face a Poisson arrival of customers, and each server's payoff is a sum of two functions: one that is *inversely* related to the amount of its steady-state idle time, and another that decreases with the difference between the servers' workload (measured as the ratio of idle time to production rate). The authors explore a given set of routing policies, and for each policy in this set, they characterize properties of the Nash equilibrium in the game between the servers. Gopalakrishnan, Doroudi, Ward, and Wierman (2016) embeds into a canonical $M/M/N$ queuing framework servers who choose their productivity rate in order to maximize the difference between idle time and the cost associated with their productivity. Focusing on a random task allocation rule, the authors give necessary and sufficient conditions for the existence of a solution to the first-order conditions of a symmetric Nash equilibrium.

Armony, Roels, and Song (2021) also studies a simultaneous game between two servers who choose their production rate. The innovation in that paper is that the cost of each server is the sum of three components: the cost of its production rate, the expected time it takes to service a customer and the expected time that each of its customers needs to wait until they are serviced. The main result of the paper characterizes properties of the Nash equilibrium for two configurations of queues: (i) two independent $M/M/1$ queues, where each server has her own infinite-buffer first-come-first-served (FCFS) queue and customers are routed randomly and uniformly between the two queues, and (ii) a single $M/M/2$ with a single infinite-buffer FCFS queue.

There are two key differences between all of these papers and ours. First, all these papers consider static complete-information games where workers simultaneously choose their production rate. In contrast, we analyze a dynamic model where the production rates are fixed (though we relax this in one of our extensions) but completion times are private information and workers decide when to release tasks. Second, all of the above papers compare the output of *exogenously* given allocation rules. We, however, characterize the maximal expected discounted output that can be attained.

A second related strand of literature considers the problem of scheduling tasks or "work-design." Coviello, Ichino, and Persico (2014, 2015) study a decision-maker facing a growing queue of tasks that arrive at an exogenous rate. Their 2014 paper characterizes the production function, which relates the output rate to the effort rate (that governs completion time) and the activation rate (at which tasks are started). Their 2015 paper applies this production function to a dataset of judges' handling of court cases to estimate the effect of increased case load. Bray, Coviello, Ichino, and Persico (2016) models case-scheduling by judges as a classic multi-armed bandit problem, and argues that prioritizing the oldest hearing (case) is optimal when the case completion hazard rate function is decreasing (increasing). In contrast to our work, these papers do not consider the moral hazard problem inherent in a worker's discretion of when to release a completed task. In Eliaz, Fershtman, and Frug (2022), we study a scheduling problem in which a decision-maker sequentially chooses among alternatives – which can be interpreted as tasks being assigned to workers – when the resulting periodic payoffs depend on both chosen and unchosen alternatives in that period. Besides considering different type of scheduling problem, agents are not strategic in that paper.

Mylovanov and Schmitz (2008) studies a model of task scheduling in the presence of moral hazard. They study a principal who needs three tasks to be completed within two periods. The principal can assign at most two tasks to one of many identical agents, who each chooses whether to privately exert costly effort. High effort contributes to higher probability of completing a task, which is publicly observed. Unlike in our framework, the principal has a rich set of instruments that include payments, which can be made contingent on task completion, and a task assignment scheme that depend on whether agents completed the tasks assigned to them.

Finally, the paper is also part of a growing literature studying dynamic delegation without monetary transfers; See, e.g., Frankel (2016), Guo (2016), Li, Matouschek, and Powell (2017), Bird and Frug (2019), Forand and Zápal (2020), Guo and Hörner (2020), and Lipnowski and Ramos (2020). All these papers consider a principal and a single agent, whereas we study a dynamic scheduling problems where multiple agents are assigned to multiple tasks over time.

A recent paper that does consider a dynamic assignment problem among multiple agents is De Clippel, Eliaz, Fershtman, and Rozen (2021). However, the incentive problem in that paper stems from *adverse selection* rather than moral hazard. More specifically, each period a principal wishes to assign a task to one of two agents, depending on who is most qualified to complete the task. Since agents privately learn whether they are qualified for the current period's task, and since they both want to be selected regardless of their qualification, the principal's problem is to design an optimal assignment rule that depends on the public history of task completion.

2 Model

We consider a continuous-time model with one principal, n "servers" and m "work-stations". The servers face an infinite inflow of tasks. The principal assigns these tasks to the servers. Completing a task requires one server to work at one of the work-stations for a stochastic amount of time. Specifically, the completion rate of server $i \in \{1, 2, ..., n\}$ is $\lambda > 0$. At any moment of time, one and only one server occupies each work-station.

Only the server working on a task observes when it is completed, and the only decision he faces is when to release a completed task. That is, a server cannot reject a task that is allocated to him and cannot abort an incomplete task. However, once the task is completed, he decides at any moment of time whether to release it or to "sit on it," delaying its release. When a task is released, its release is publicly observed.

When a server receives a task, he incurs an immediate set-up cost of $K > 0$. While working on an (unfinished) task, he incurs a flow effort cost of $c > 0$. While sitting on a completed task, the server incurs a flow delay cost of $d \in (0, c)$. The principal receives a payoff of 1 for each completed task when the task is released. There is a common discount rate of $r > 0$.

An allocation rule selects m servers at time zero and specifies a history-dependent probability distribution over available servers (i.e., servers that are not busy working on a task) that determines which server will work on the next task, whenever some work-station becomes vacant. An allocation rule induces an extensive game of imperfect information among the servers. The principal's payoff from an allocation rule is defined as the principal's highest expected discounted payoff over all perfect Bayesian equilibria (PBEs) of the induced game. We say that an allocation rule is optimal if there is no other allocation rule that attains a higher payoff for the principal.

3 Optimal allocation and delay

The first-best expected discounted payoff for the principal is attained when all stations are occupied at all times and tasks are released immediately upon their completion. Hence, the principal's expected payoff under the non-strategic, first-best benchmark is given by $m\lambda/r$.

When servers are sufficiently slow (low enough λ), the first-best can be attained by *simple rotation without delay*: servers take turns occupying stations, each server immediately releases a task once it is finished and joins the end of line of idle servers. The reason is that the servers' slow production rate gives ample idle time to compensate for the future cost of working. However, when the servers are sufficiently productive (high enough λ), the idle time that each server would get may not be enough. Consequently, some delay must be injected into the system.

In the baseline case with $n = 2$ and $m = 1$, a simple allocation and delay scheme can be employed: Servers take turns working on tasks, and from the second task onwards, they withhold the release of each task for a fixed amount of time, say τ (the release of the first task is not delayed). When τ is set such that upon completing a task, a server is indifferent between withholding the task and releasing it, this scheme attains the second-best output.[3]

This result, which will follow as a special case of Proposition 1 below, can be understood intuitively. Server i is willing to release a completed task because the expected value of idle time he will enjoy – while server j is working and delaying a release of a task – is just long enough to compensate for the cost of working on the next task assignment. Delaying the very first task is not needed since the role of such (costly) delays is only to incentivize previous task releases.

To generalize the above scheme to any $n \geq 2$ and $1 \leq m < n$, we begin by making several preliminary observations. First, as in the case of $n = 2$ and $m = 1$, it is suboptimal to delay the release of any task after which a "new" server is called to work on his very first task. Second, with many servers and work stations, the order in which tasks are allocated and completed need not coincide. Hence, whenever a work station is vacated, a server with the longest current break is assigned a task. Third, when delays begin to take place, it is suboptimal to have each server delay the release of a task by the same amount of time. To see this, let i be the first server to be assigned a task for the second time. Any delay that is added to incentivize server i before he is called to work again also provides *the same amount* of idle time for *all* the servers who are idle at that time. Thus, the next server to be called to work has already received enough idle time so any additional delay will over-compensate some servers.

To build some intuition for the general delay strategy our scheme will use, note that in the baseline case of $n = 2$ and $m = 1$, delaying a release of a completed task is similar to temporarily freezing the production process of the system. When $m > 1$ and task completion is privately observed, delaying a release of a specific task does not stop work on other work stations. We now turn to present a general scheme of allocation and delay which we later prove to be optimal for any number of servers and work stations.

[3]We emphasize that this scheme is not *uniquely* optimal. For example, instead of fixed-time delays, delay times may be random. Also, for some parameters, delays may not be uniform over time and begin at a later stage.

Rotation with Delay Scheme

The *Rotation with Delay Scheme* (RDS) is described by several steps, where each release of a completed task brings the allocation scheme to the next step. Each step can be characterized by the *state* of the system, where a state is a pair: (i) a set I of servers that are *in* work stations, and (ii) a vector O that specifies the queue of servers that are *out* of work stations, where earlier vector entries correspond to servers at earlier positions in the queue.

Initially, the first m servers are "in", and whenever a working server releases a task, he exits and is replaced by the earliest server to have entered the "out"group.

Formally, these steps can be described as follows.

- *Step 1*: Let the initial state be $(I_1, O_1) = (\{1, ..., m\}, (m+1, ..., n))$, and let $k \in I_1$ be the first server that releases a task in Step 1. The allocation rule then transitions to Step 2, where the state is

$$(I_2, O_2) = (\{1, ..., m, m+1\} - \{k\}, (m+2, ..., n, k)) .$$

- *Step ℓ*: Given the state $(I_\ell, O_\ell) = (\{\ell_1, ..., \ell_m\}, (\ell_{m+1}, ..., \ell_n))$, let $\ell_k \in I_\ell$ be the first server that releases a task in this step. The allocation rule then transitions to Step $(\ell + 1)$, in which the state is

$$(I_{\ell+1}, O_{\ell+1}) = (\{\ell_1, ..., \ell_m, \ell_{m+1}\} - \{\ell_k\}, (\ell_{m+2}, ..., \ell_n, \ell_k)) .$$

Under *RDS*, servers sit on finished tasks and delay their release. These delays occur in cycles: they begin in some step σ and then recur every $n - m$ steps. Furthermore, the delay strategy has the feature that a server starts by sitting on a task for τ units of time, unless before reaching this deadline, some other working server happens to release a completed task. In the latter case, the delay switches to being a Poisson process with rate λ.

Formally, let $\Sigma = \{\sigma_j\}_{j \in \mathbb{N}}$ denote the subset of steps that initiate delay, where $\sigma_j = 1 + j(n - m)$. At each moment, each server is in one of four possible phases:

- *Idle phase*: The server does not work, pays no cost, and enters the productive phase (described below) according to the transition rule specified above.

- *Productive phase*: The server works on a task, pays a flow cost of c, and completes the task according to a Poisson process with rate λ. Upon completing a task, in every step $\sigma \notin \Sigma$, the server then releases the task and enters his idle phase, whereas in every step $\sigma \in \Sigma$, he enters a τ-delay phase (see below).
- *τ-delay phase*: The server sits on a finished task, pays a flow cost of $d < c$, and exits this phase once the earliest of the following events occur: (i) τ units of time have elapsed, in which case the server releases the task and enters his idle phase, or (ii) some other working server releases a task, in which case the server enters the λ-delay phase (see below).
- *λ-delay phase*: The server sits on a finished task, pays a flow cost of $d < c$, and exits this phase according to a Poisson process with rate λ. In every step $\sigma \notin \Sigma$, the server then releases the task and enters the idle phase, whereas in every step $\sigma \in \Sigma$, he enters the τ-delay phase.

The role of two delay phases – Defining two separate delay phases makes the analysis tractable when $m \geq 2$ by simplifying and equalizing all the incentive constraints. To see this, consider a server who has just completed a task. To decide whether to release the task or not, that server needs to estimate the time until he will be reassigned to a task. This will depend on the history if the expected release time of other servers depends on whether they are still working or delaying the release of a completed task. When we also have the λ-delay phase that begins upon the allocation of a task to another server (which hence requires task releases to be publicly observed), the expected time between a release of a completed task and the next assignment to that same server will be the same regardless of the history.

Moreover, to see how this delay structure generalizes the fixed-time delay in the simple case of $n = 2$ and $m = 1$, note the following. From some point onwards, *RDS* induces a cycle, where the system freezes production every $n - m$ task releases for τ units of time. Whenever a step in Σ is reached, servers who finish a task start sitting on the finishes task for τ units of time. The first server to reach the τ limit (i.e, the first server to finish a task in the relevant stage), releases the task. This will cause all other servers who were in the midst of their τ delay phase to immediately switch to the λ delay phase. Thus, within each step in Σ there is only a single stretch of τ delay. It follows that in terms of productivity over time, the scheme can be viewed as if, every

$n - m$ completed tasks, the system freezes for τ units of time to provide incentives to servers who are currently idle; at all other times, the system operates in full productivity (with instantaneous completion rate of $m\lambda$).

3.1 Illustration of the RDS

Figure 1 below illustrates possible dynamics under RDS for the case of four servers and two work stations.

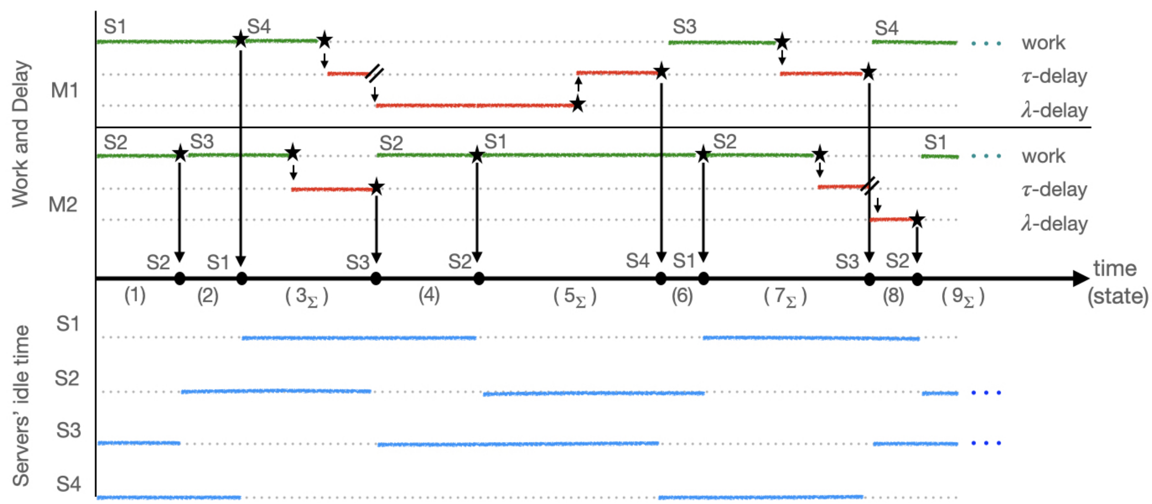

Figure 1: Illustration of dynamics under the rotation-with-delay scheme.

The bold horizontal time-axis in the middle of Figure 1 divides it into two interrelated panels; the lower panel indicates for each server ($S1$, $S2$, $S3$, and $S4$) the times at which he is idle, and the upper panel specifies, for each point in time, whether the server currently assigned to each of the work stations ($M1$ and $M2$) works on a task or delays the release of a completed task according to either $\tau-$delay or $\lambda-$delay.

At the start, $S1$ and $S2$ are assigned to $M1$ and $M2$, respectively, while $S3$ and $S4$ are idle. Until the first release of a task, the system is at state $1 \notin \Sigma$ as denoted below the time-axis. The system moves to state 2 when the first task is released. Under the specified realization, $S2$ completes a task first. This is indicated by the left-most $\star$

symbol at the work-line of $M2$. The arrow from that $\star$ down to the time-axis indicates the immediate release of the completed task. At that point, $S2$ and the currently idle $S3$ switch positions. State 2 of the system is similar to state 1. The next server to complete a task (in this case, $S1$) releases it without delay since $2 \notin \Sigma$. $S1$ is then replaced by $S4$ – the idle server with the longest current break, and the system moves to state 3.

Since $3 \in \Sigma$, the first working server (among $S3$ and $S4$) to complete a task will not release it immediately but only after τ units of time. In the figure, this is reflected in the short down-pointing arrow from the left-most $\star$ symbol at stage 3 (task completion by $S3$ on $M2$) down to the $\tau-$delay line of $M2$. In the depicted realization, before the τ units of time elapsed, server $S4$ on the other work station also completed a task, which puts both servers in the τ-delay phase simultaneously. When a task is finally released by $S3$ (who completed the task first within the current state of the system), $S3$ will be replaced by $S2$ (the idle server with the longest current break) and $S4$ will move from the incomplete $\tau-$delay phase into the $\lambda-$delay phase (in the figure, the unsuccessful completion of the $\tau-$delay phase is marked by the rotated double-slash symbol).

This marks the beginning of state 4 of the system and since $4 \notin \Sigma$, there will be no $\tau-$delays at that state. Note that both servers occupying the work stations are waiting for an event that arrives with Poisson rate λ to release a task (whether it being exiting the $\lambda-$delay phase for $S4$ or completing a task for $S2$.) A released task will move the system to state $5 \in \Sigma$. As can be seen in the figure, the current realization induces a transition of $S4$ from $\lambda-$delay back to a new $\tau-$delay phase after which $S4$ finally releases a task, advancing the system to state 6, and so on.

3.2 Optimal delay

Suppose a server has completed a task. Set τ (corresponding to the $\tau-$delay phase described above) such that from the perspective of the server, the expected value of idle time until his next task exactly compensates the additional cost of working on that future task, relative to the flow cost of sitting on it until it is completed:

$$\frac{d}{r}\left(1 - e^{-r\tau}\left(\frac{m\lambda}{m\lambda + r}\right)^{n-m}\frac{\lambda}{\lambda + r}\right) = e^{-r\tau}\left(\frac{m\lambda}{m\lambda + r}\right)^{n-m}\left(K + \frac{c}{\lambda + r}\right). \quad (1)$$

Note that both expressions on the right and left hand sides of the equality consider periods of length ($\tau +$ *one cycle of others' work without delay* $+$ *own work on a single task*). Specifically, on the right hand side, the future costs of working on a single task, $K + \frac{c}{\lambda+r}$, are discounted based on two factors: the expected discounted time between completed tasks under rotation without delay,

$$\Delta(\lambda) \equiv \left(\frac{m\lambda}{m\lambda + r}\right)^{n-m},$$

plus the delay of τ units of time. When a server considers releasing a task, the RHS of (1) therefore represents the expected costs until the server will finish working on the *next* task assigned under the RDS. Alternatively, the server can sit on the completed task and incur a flow cost of d for the same duration. Both courses of behavior will put the server at the end of that time frame at a work station with a completed task at hand which the server can then again decide whether to release or withhold.

The time τ^* that solves (1) guarantees that a server who follows the allocation scheme described above expects to receive a continuation value of $\frac{d}{r}$ at the time of releasing a completed task. That is,

$$\frac{d}{r} = e^{-r\tau^*}\Delta(\lambda)\left(K + \frac{c}{\lambda + r} + \frac{\lambda}{\lambda + r}\frac{d}{r}\right). \tag{2}$$

The strategies defined by *RDS* with τ^* defined by (2), therefore constitute an equilibrium. We denote

$$D(\lambda) \equiv e^{-r\tau^*}\Delta(\lambda) = \frac{\frac{d}{r}}{K + \frac{c}{\lambda+r} + \frac{\lambda}{\lambda+r}\frac{d}{r}}.$$

When $\tau^* = 0$, *RDS* collapses to simple rotation without delay. Note that simple rotation without delay is an equilibrium if the RHS of (2) at $\tau^* = 0$ is no greater than the LHS, i.e., $D(\lambda) \geq \Delta(\lambda)$.

Lemma 1. *The equation $\Delta(\lambda) = D(\lambda)$ has a unique solution, $\lambda^* > 0$, and for all $\lambda < \lambda^*$, simple rotation without delay constitutes an equilibrium.*

Proof. We first show that there is at most one $\lambda > 0$ that solves the equation $\Delta(\lambda) =$

$D(\lambda)$, or equivalently,

$$\left(\frac{m\lambda}{r+m\lambda}\right)^{n-m}=\frac{\frac{d}{r}}{K+\frac{c}{\lambda+r}+\frac{\lambda}{\lambda+r}\cdot\frac{d}{r}}. \tag{3}$$

To do so, we rewrite this equation as a polynomial in λ that equals zero:

$$\begin{aligned}0=&-\left(m^{n-m}K\right)\lambda^{n-m+1}+m^{n-m}\left(\frac{n}{m}d-Kr-c\right)\lambda^{n-m}\\&+\sum_{i=1}^{n-m-1}\left[d\binom{n-m}{i}m^{i}r^{n-m-i}+\frac{d}{r}\binom{n-m}{i-1}m^{i-1}r^{n-m-i+1}\right]\lambda^{i}+d.\end{aligned}$$

Note that the coefficient on the largest exponent $\left(\lambda^{n-m+1}\right)$ is negative and the coefficients on all the positive exponents from $n-m-1$ to 1 are positive. The coefficient on λ^{n-m}, the second largest exponent, can be either negative or positive. Descartes' rule of signs states that if the nonzero terms of a single-variable polynomial with real coefficients are ordered by descending variable exponent, then the number of positive roots of the polynomial is at most the number of sign changes between consecutive (nonzero) coefficients. Since the sign changes in the above polynomial are either $(-,-,+,...,+)$ or $(-,+,...,+)$, there is at most one positive root (i.e., at most one positive solution for λ).

We now show that there must exist a positive solution to (3). Note that both sides are continuous in λ. When $\lambda=0$, the LHS is zero whereas the RHS is positive. When $\lambda\to\infty$, the LHS tends to one while the RHS tends to a constant below one. Hence, there must be some positive λ where both sides equate.

Finally, note that $\lambda<\lambda^*$ implies $D(\lambda)>\Delta(\lambda)$, which guarantees that simple rotation without delay is incentive compatible whenever $\lambda<\lambda^*$. ■

We refer to servers with productivity rate $\lambda\leq\lambda^*$ as *mediocre servers*.[4] While for mediocre servers simple rotation without delay is incentive compatible, it is no longer the case for teams of servers with $\lambda>\lambda^*$.

Our next result establishes that, when simple rotation without delay is not incentive compatible (and thus, the first best cannot be attained), the principal's discounted expected payoff under *RDS* is equal to his first-best payoff scaled down by the following ratio: One minus the expected discounted time between tasks under simple rotation,

[4] Recall that the value of λ^* depends on other parameters.

to one minus the expected discounted time between tasks under incentive-compatible delay.

Proposition 1. *For any $\lambda > \lambda^*$, the principal's expected discounted payoff from the RDS is given by*

$$\frac{m\lambda}{r}\left(\frac{1-\Delta(\lambda)}{1-D(\lambda)}\right). \tag{4}$$

Proof. Under *RDS* the first $n-m$ tasks are released immediately upon completion, which yields the expected discounted payoff

$$\sum_{j=1}^{n-m}\left(\frac{m\lambda}{m\lambda+r}\right)^j = \frac{\frac{m\lambda}{m\lambda+r}-\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m+1}}{1-\frac{m\lambda}{m\lambda+r}} = \frac{m\lambda}{r}\left(1-\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m}\right).$$

As we show below, the subsequent output (discounted to time zero), after the first $n-m$ releases have occurred, is equal to[5]

$$\frac{D}{1-D}\frac{m\lambda}{m\lambda+r}\frac{1-\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m}}{\frac{r}{m\lambda+r}}.$$

We derive this subsequent output as the discounted sum of $n-m$ components, S_k, with $k \in \{1, ..., n-m\}$, each of which corresponds to a sum over outputs at different times. For each $k \in \{1, ..., n-m\}$, denote by S_k the sum of outputs obtained at the end of steps $(k+(n-m)j)_{j\in\mathbb{N}}$. By construction, since $k \leq n-m$, for every k there is only a single element of Σ until the allocation scheme reaches the end of step $k+n-m$. Hence, the value of the output (discounted to time zero) obtained at the end of step $k+n-m$ is

$$\left(\frac{m\lambda}{m\lambda+r}\right)^k D.$$

Since there is a single element of Σ between any two consecutive elements of S_k, the discounting between every two consecutive elements of S_k is D. Thus, the time-zero

[5] In what follows, we explicitly write the expression for $\Delta(\lambda)$, and suppress the dependence of D on λ.

value of S_k is given by

$$\frac{D}{1-D}\left(\frac{m\lambda}{m\lambda+r}\right)^k.$$

Aggregating over all S_k, each discounted to time zero, yields

$$\frac{D}{1-D}\sum_{k=1}^{n-m}\left(\frac{m\lambda}{m\lambda+r}\right)^k=\frac{D}{1-D}\frac{m\lambda}{m\lambda+r}\frac{1-\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m}}{1-\frac{m\lambda}{m\lambda+r}}.$$

The total expected discounted output under *RDS* is therefore given by

$$\frac{m\lambda}{r}\left(1-\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m}\right)+\frac{D}{1-D}\frac{m\lambda}{m\lambda+r}\frac{1-\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m}}{1-\frac{m\lambda}{m\lambda+r}},$$

which simplifies to (4). ■

We now show that *RDS* maximizes the principal's expected discounted payoff.

Proposition 2. *The RDS maximizes the principal's expected discounted payoff.*

Proof. Since $\min\Sigma = 1+n-m$, the first $n-m$ tasks are released immediately upon completion. Hence, the principal's total expected discounted payoff from the first $n-m$ completed tasks is the same as in the first-best output.

Let t_{n-m} denote the (stochastic) point in time at which the $(n-m)$-th task is released. We now establish an upper bound on the discounted expected output from t_{n-m} onward. To do this, we first derive the total available budget of idle time that can be used to incentivize servers to release all the future completed tasks starting from the $(n-m+1)$-th task.

As there are m work stations and n workers, $n-m$ workers will be idle at each instant. Since the alternative to releasing a task is to sit on it and pay a flow cost of d, it follows that the total value of idle time that can be allocated to servers is equal to

$$(n-m)\frac{d}{r}.$$

However, not all of this budget can be used in order to provide incentives. The reason is that some servers will necessarily enjoy idle time before being assigned to their first task. In order to minimize the wasted budget of idle time, a server must be assigned to its second task only after all servers have been assigned their first task. To understand the total amount of the budget that is wasted, note that up until the release of the first task, $n - m$ servers consume idle time from the budget. This idle time, which amounts to

$$(n-m)\,\frac{d}{m\lambda + r},$$

cannot incentivize them to release their future assigned tasks.

Similarly, between the first and second release of a completed task, there will be $n - m - 1$ idle servers that have never worked, and hence the idle time that is wasted in the time interval between the first and second release is

$$(n-m-1)\left(\frac{m\lambda}{m\lambda + r}\right)\frac{d}{m\lambda + r},$$

where the new component $\frac{m\lambda}{m\lambda+r}$ arises due to discounting to time zero (since the production rate without any delays is given by $m\lambda$, starting from any point in time, the discounted output until the next completion is given by $\frac{m\lambda}{m\lambda+r}$). Continuing in this manner, in step k we must subtract the idle time wasted between the $(n - m - (k - 1))$-th release and the $(n - m - k)$-th release, which is equal to

$$(n-m-k)\left(\frac{m\lambda}{m\lambda + r}\right)^{k}\frac{d}{m\lambda + r}.$$

Thus, the maximal expected *usable* budget of idle time, discounted to time zero, is

$$\mathcal{B} = (n-m)\frac{d}{r} - \frac{d}{m\lambda + r}\sum_{j=1}^{n-m} j\left(\frac{m\lambda}{m\lambda + r}\right)^{n-m-j} = \frac{dm\lambda}{r^2}\left(1 - \left(\frac{m\lambda}{m\lambda + r}\right)^{n-m}\right), \quad (5)$$

where the last equality follows from simple but tedious algebra.

Next, we compute the difference between the total discounted expected costs that are accrued when m work stations are constantly staffed by servers who work (first-best production), and the total costs that are accrued when the servers at the stations just sit on completed tasks (indefinite delay). In order for the servers to agree to release completed tasks from the $(n - m + 1)$-th task onward, this difference in costs must be covered by the total budget of idle time that is available when the $(n - m)$-th task is released. This difference in costs is given by the expression

$$\underbrace{K + m\left(\frac{c+\lambda K}{r}\right)}_{\text{costs of first-best production}} - \underbrace{m\frac{d}{r}}_{\text{cost of indefinite delay}} = K + m\left(\frac{c-d+\lambda k}{r}\right). \tag{6}$$

The expression $(c + \lambda K)/r$ reflects the average production cost for each work station. Note that the station where the $(n - m)$-th task is released requires an immediate (and hence discrete) cost of K (while the servers on the other $m - 1$ work stations are still working on their tasks).

The amount in (6) represents the entire difference in the costs incurred between first-best production and indefinite delay of completed tasks. However, since servers cannot reject task assignments, they do not need to be compensated for the costs associated with the production of their first assignment, starting from time t_{n-m} (for each server, the first decision after time t_{n-m} arrives only upon completion of his first assignment).

To calculate the total costs that require compensation to incentivize first-best production, we subtract from (6) the costs associated with the servers' first assigned tasks, *starting from* t_{n-m}. These costs involve: (i) a cost of

$$(m-1)\frac{c-d}{\lambda+r},$$

reflecting the total expected cost of completing a task for the $m - 1$ servers who are already working on a task at time t_{n-m}; (ii) the expected cost of producing a task, for the server who receives his first task at time t_{n-m}, as well as the future expected costs of those $n - m$ workers who are idle after the allocation of the task at time t_{n-m},

$$\left(K + \frac{c-d}{\lambda+r}\right)\left(1 + \frac{m\lambda}{m\lambda+r} + ... + \left(\frac{m\lambda}{m\lambda+r}\right)^{n-m}\right).$$

Hence, the total net expected discounted cost $\mathcal{C}$ of first-best production requiring compensation, starting from t_{n-m}, is given by:

$$\mathcal{C} = K + m\left(\frac{c-d+\lambda k}{r}\right) - \left((m-1)\frac{c-d}{\lambda+r} + \left(K + \frac{c-d}{\lambda+r}\right)\sum_{j=0}^{n-m}\left(\frac{m\lambda}{m\lambda+r}\right)^{j}\right)$$

$$= \frac{m\lambda}{r}\left(\frac{c-d}{r+\lambda}+K\right)\left(\frac{m\lambda}{r+m\lambda}\right)^{n-m}.$$

Recall that, starting from any point in time, the first-best expected discounted continuation output is given by $m\lambda/r$. As a result, we obtain a naive upper bound of

$$\frac{\mathcal{B}}{\mathcal{C}}\cdot\frac{m\lambda}{r} \tag{7}$$

on the attainable (i.e., incentive compatible) expected discounted output starting from t_{n-m}. That is, the principal cannot hope to obtain a fraction of the first-best payoff greater than $\mathcal{B}/\mathcal{C}$.

Substituting into (7) the expression for $\mathcal{B}$ and $\mathcal{C}$, we have the following upper bound on future discounted production starting from t_{n-m} relative to time t_{n-m}:

$$\frac{\mathcal{B}}{\mathcal{C}}\cdot\frac{m\lambda}{r} = \frac{d}{r}\left(\frac{1}{\frac{c-d}{r+\lambda}+K}\right)\left(\frac{1-\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m}}{\left(\frac{m\lambda}{r+m\lambda}\right)^{n-m}}\right)\frac{m\lambda}{r}. \tag{8}$$

We now show that *RDS* attains this upper bound. That is, we show that the payoff under this scheme, starting from time t_{n-m}, is equal to (8). Recall that the expected discounted output under *RDS*, as derived in the proof of Proposition 1, consists of two components. The first corresponds to the output from the first $n-m$ released task, which are released without delay, whereas the second component,

$$\frac{D}{1-D}\frac{m\lambda}{m\lambda+r}\frac{1-\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m}}{1-\frac{m\lambda}{m\lambda+r}},$$

corresponds to the remaining continuation output.

To complete the proof, it therefore remains to show that the latter continuation output, adjusted to time t_{n-m}, is equal to (8). That is, it remains to show that

$$\frac{\frac{D}{1-D}\frac{m\lambda}{m\lambda+r}\frac{1-\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m}}{1-\frac{m\lambda}{m\lambda+r}}}{\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m}}$$

is equal to (8). Plugging in

$$D = \frac{\frac{d}{r}}{K + \frac{c}{\lambda+r} + \frac{\lambda}{\lambda+r}\frac{d}{r}}$$

into the expression above and simplifying yields the result. ■

Can fewer work stations increase output?

Productivity in our setting is constrained by two upper bounds: (i) a non-strategic "technological" productivity bound, and (ii) a bound reflecting the "incentive budget" that arises due to moral hazard. The former is *increasing* in the number of work stations since in the absence of strategic considerations, additional work stations enable more servers to work in parallel. This is the effective bound when $\lambda < \lambda^*$.

When $\lambda > \lambda^*$, as Propositions 1 and 2 show, the technological upper bound cannot be achieved. However, in the proof of Proposition 2 we show that RDS in fact utilizes *all* of the usable incentive budget $\mathcal{B}$ (see equation (5)). This means that when $\lambda > \lambda^*$, the effective upper bound on productivity (i.e., the above mentioned incentive-budget bound) is attainable.

At the beginning of the interaction, some idle time must be allocated to the servers before they work. With a higher number of work stations, the amount of unusable incentives at the beginning of the interaction decreases since the first completions arrive faster.[6] If the servers are very impatient, this more than compensates the additional long-term incentive budget that arises when there are fewer work stations. The next result shows that when the players are not too impatient, reducing the amount of work stations increases the available incentive budget. Thus, as long as there is delay in the optimal allocation rule, adding work stations is strictly worse for the principal.

Proposition 3. *Fix the model parameters $(c, d, K, \lambda, r, n, m)$ such that there is positive delay in RDS. Then, provided that the total expected output is positive, increasing the number of work stations reduces the principal's expected payoff, for sufficiently small r.*

[6]For instance, in the above example, when $m = 2$ the expected time of the first completion is $1/2$, while with $m = 1$ the expected time of the first completion is 1 and that of the second completion is 2.

Proof. We begin with two simple observations. First, note that if the number of work stations is weakly greater than the number of servers, total output is zero. Hence, to examine the effects of adding a work station, suppose $m < n - 1$.

Second, the assumption that RDS features positive delay implies that, from the proof of Proposition 2, the upper bound representing the total incentive budget $\mathcal{B}$ (given by the RHS of equation (5)) is attainable. Hence, the proof consists of showing that the RHS of (5) decreases with m when r is sufficiently small.

To establish this, it is sufficient to show that, for r sufficiently small,

$$\frac{\mathcal{B}(m)}{\mathcal{B}(m+1)} > 1,$$

where $\mathcal{B}(m)$ denotes the total incentive budget with m work stations (holding the other model parameters fixed). Substituting the expression from the RHS of equation (5) for m and $m+1$, the above inequality is equivalent to

$$\frac{m}{m+1} \frac{\left(\frac{m\lambda}{m\lambda+r}\right)^{n-m} - 1}{\left(\frac{(m+1)\lambda}{(m+1)\lambda+r}\right)^{n-(m+1)} - 1} > 1.$$

Taking the limit as r approaches zero. we obtain the condition

$$\frac{m}{m+1}\left(\frac{m+1}{m}\frac{m-n}{m-n+1}\right) > 1.$$

This condition indeed holds since $\frac{m-n}{m-n+1} > 1$ if and only if $m < n-1$, where the latter holds by assumption. ■

3.3 Discussion and interpretation

In our stylized model, a server is in one of only three states: *work* (efficiently occupying a work-station), *delay* (inefficiently occupying a work-station), or *idle*. More generally, work and inefficiencies in production processes can come in varying intensities and forms.

The idle state in our model represents various alternatives to reward the worker through lowering his work intensity or assigning him more pleasant tasks. Our assumption that only a given number of servers can be idle at each point in time captures the idea that altering the workload of a team of servers is subject to organizational constraints (e.g., the relation between the amount of servers and the capital available to them, and possibly the need to appear constantly at work).

Delays in our model represent various inefficiencies that organizations may inject into work processes. Thus, our model proposes a new perspective on seemingly inefficient project choices or inefficient work processes by interpreting them as possible means to provide incentives. We now illustrate this point in a simple example where instead of delaying task releases, an inefficient production protocol is selected in optimum.

Suppose that there are two servers and one work station, and assume that when a server is called to work on a task he can follow the "**efficient protocol**" – do only the efficient component of the job, or to follow an "**extended protocol**" where he starts with an inefficient component, and upon completion, begins working on the efficient one. The efficient component is completed, as before, according to a Poisson process with arrival rate λ (which does not depend on whether the server worked on the inefficient component first). The inefficient component is completed according to a Poisson process with parameter μ.

When a server is allocated a task, he pays a set-up cost of K and a flow cost of c until the efficient component is completed - regardless of the protocol. When the server releases a completed task after the efficient protocol, the principal gets 1. When a completed task is released after the extended protocol, the principal gets $1+\beta$.

Consider $\lambda > \lambda^*$. That is, the first-best outcome (the non-strategic benchmark) - where the servers only work on the efficient component and release completed tasks without delays - is not incentive compatible. For computational simplicity, suppose that μ solves:

$$\frac{\lambda^*}{\lambda^*+r} = \frac{\mu}{\mu+r}\frac{\lambda}{\lambda+r} \;\Rightarrow\; \mu = \frac{(\lambda+r)\lambda^*}{\lambda-\lambda^*}.$$

This implies that μ is set such that the server's considerations under the extended protocol are identical to those in our original model (where only the efficient protocol exists) when the productivity rate of all servers is λ^*. Therefore, simple rotation without delay – provided that the servers follow the extended protocol, is now incentive compatible. Thus, while the principal's payoff from the non-strategic benchmark is $\frac{\lambda}{r}$, her payoff from the extended protocol is $(1+\beta)\frac{\lambda^*}{r}$. Hence, under the assumption that

$$(1+\beta)\frac{\lambda^*}{r} < \frac{\lambda}{r},$$

the extended protocol is indeed less efficient.

Since $\lambda > \lambda^*$, the payoff from RDS (if the servers follow the efficient protocol) is given by (4). Hence, for values of β such that

$$\frac{\lambda}{r}\left(1 - \frac{\Delta(\lambda)}{D(\lambda)}\right) < (1+\beta)\frac{\lambda^*}{r},$$

the principal prefers to introduce inefficiency in the work process by adopting the seemingly inefficient – extended protocol.

4 More vs. better servers

A natural question that arises is how does the total output of the system change with the servers' productivity. By Lemma 1, the output coincides with the first-best $m\lambda/r$ for $\lambda < \lambda^*$. For higher levels of productivity, the total output is a fraction $\left(\frac{1-\Delta(\lambda)}{1-D(\lambda)}\right)$ of the first-best. To see that the total output increases with λ consider two production rates $\lambda^* < \lambda_1 < \lambda_2$, and note that upon receiving a task assignment, the more productive server (the one with λ_2) incurs lower expected costs ($K + \frac{c}{\lambda_2} < K + \frac{c}{\lambda_1}$). Hence, this server requires a shorter break upon releasing a completed task. The total output of the system with λ_2 is therefore strictly higher than that with λ_1. Therefore, we can derive a uniform upper bound on the total output of the system with n servers and m work stations, by taking the servers' production rate to infinity.

Corollary 1. *The discounted expected output of n servers and $m < n$ work stations at the limit when $\lambda \to \infty$ is equal to*

$$(n-m)\left(1 + \frac{d}{rK}\right). \tag{9}$$

This follows from Propositions 1 and 2, taking the limit of (4) as $\lambda \to \infty$, noting that $\lim_{\lambda\to\infty} D(\cdot) = \frac{\frac{d}{r}}{K+\frac{d}{r}}$ and that $\lim_{\lambda\to\infty}\left(\frac{m\lambda}{r}(1-(\frac{m\lambda}{m\lambda+r})^{n-m})\right) = n-m$.

While more productive servers complete more tasks per unit of time, they also shorten the idle time of servers who are not at work stations. Indeed, at an optimal equilibrium, as λ exceeds the threshold λ^*, servers increase the delay until they release completed tasks. This points at a trade-off between quantity and quality of servers. Note that this trade-off exists only because servers are strategic in their decision of when to release completed task. Put differently, higher quality servers amplify the moral hazard problem. In light of this, we ask the following question: Given the number of work stations m, how many mediocre servers (i.e., sufficiently slow servers so that simple rotation without delay is incentive compatible) are needed to outperform a group of $n > m$ unboundedly productive servers?

Recall that the production rate under *RDS* fluctuates over time (see the discussion at the beginning of Section 3). As an intermediate step, it is useful to consider a "uniform-rate system" that outputs a completed task every μ units of time. The uniform rate μ that generates a total output equal to (9) is given by

$$\mu = (n - m)(r + \frac{d}{K}).$$

Consider a system of m work stations and sufficiently many non-strategic servers (i.e., servers who release tasks immediately upon completion). Clearly, if the completion rate of every server at a work station is μ/m, the output equals (9), independently of the exact number of servers. Let λ^*_{n+j} be the value of λ that solves $D(\lambda) = \Delta(\lambda)$ when there are $n + j$ servers. Since λ^*_{n+j} increases in j, we need to find the minimal j for which $\mu/m \leq \lambda^*_{n+j}$.

For the following Proposition, denote

$$D_{\lambda=\frac{\mu}{m}} \equiv \frac{\frac{d}{r}}{K + \frac{c}{\frac{\mu}{m}+r} + \frac{\frac{\mu}{m}}{\frac{\mu}{m}+r} \cdot \frac{d}{r}}.$$

Proposition 4. *Let*

$$j = \left\lceil \frac{\log D_{\lambda=\frac{\mu}{m}}}{\log(\frac{\mu}{\mu+r})} \right\rceil + m - n. \tag{10}$$

A system with m stations and $n + j$ servers with productivity rate μ/m generates the same discounted expected output as a system with m stations and n unboundedly productive servers.

Proof. Recall that *RDS* reduces to simple rotation (without delay) when $\lambda = \lambda_n^*$, i.e., when servers are exactly indifferent between releasing a completed task and sitting on it indefinitely when using a simple rotation scheme. In light of this, we proceed by finding the minimal integer j that satisfies $\lambda_{n+j}^* \geq \mu/m$. Recall that λ_{n+j}^* is the solution to

$$\frac{\frac{d}{r}}{K + \frac{c}{\lambda_{n+j}^*+r} + \frac{\lambda_{n+j}^*}{\lambda_{n+j}^*+r} \cdot \frac{d}{r}} = \left(\frac{m\lambda_{n+j}^*}{m\lambda_{n+j}^*+r}\right)^{n+j-m},$$

where the LHS captures the idle time between tasks required to make a server indifferent between releasing or sitting on a completed task (it is the discount factor that multiplies the server's expected future costs at the time of task completion), while the RHS captures the idle time between tasks (via the discount factor that multiplies future expected costs) under simple rotation. Hence, $\lambda_{n+j}^* \geq \mu/m$ if $j = \lceil x \rceil$, where x is the solution to

$$\frac{\frac{d}{r}}{K + \frac{c}{\mu/m+r} + \frac{\mu/m}{\mu/m+r} \cdot \frac{d}{r}} = D_{\lambda=\frac{\mu}{m}} = \left(\frac{\mu}{\mu+r}\right)^{n+x-m}. \tag{11}$$

Taking the logarithm of both sides of the equation yields the desired result. ■

When r is sufficiently close to zero, the expression in (10) becomes significantly simpler:

Corollary 2. *At the $r \to 0$ limit, a system with m stations and $n + m(\lceil c/d \rceil - 1)$ servers with productivity rate equal to μ/m generates the same discounted expected output as a system with m stations and n unboundedly productive servers.*

Proof. From (11) it follows that at the $r \to 0$ limit,

$$j = \left\lceil \lim_{r\to 0} \frac{D_{\lambda=\frac{\mu}{m}}}{\frac{\mu}{\mu+r}} \right\rceil + m - n.$$

Since

$$\lim_{r\to 0} D_{\lambda=\frac{\mu}{m}} = \lim_{r\to 0} \frac{\log\left(\frac{d(n-m)+Knr}{d(n-m)+\left(\frac{K^2nr2}{d^2}\right)+\frac{c}{d}Kmr+2Kr(n-m)}\right)}{\log\left((n-m)\frac{d+Kr}{Kr+d(n-m)+Kr(n-m)}\right)} = m\frac{c}{d} + (n-2m),$$

we obtain the desired result. ■

One implication of Corollary 2 is that when r is sufficiently low and c is close to d, even $n+1$ mediocre servers can generate the same discounted expected output as n unboundedly productive ones. The following example illustrates this.

Example. Let $r = 0.01, c = 2.5, d = 2, K = 1, m = 3$ and $n = 5$. By Corollary 1, the discounted expected output when $\lambda \to \infty$ is equal to $(5-3)(1+2/(0.01\cdot 1)) = 402$. Let us derive the productivity rate that delivers this output when there are 6 servers who immediately release completed tasks under simple rotation. This is given by multiplying 402 by $r = 0.01$ yielding $\mu = 4.02$. Dividing by $m = 3$ we obtain $\lambda = 1.34$, which is the required production rate of each server needed to attain a total uniform production rate of μ for the entire system.

Next, we derive $\lambda^*_{n=6}$ for the above parameters by solving:

$$\frac{\frac{2}{0.01}}{1+\frac{2.5}{\lambda^*_{n=6}+0.01}+\frac{\lambda^*_{n=6}}{\lambda^*_{n=6}+0.01}\cdot\frac{2}{0.01}} = \left(\frac{3\lambda^*_{n=6}}{3\lambda^*_{n=6}+0.01}\right)^{6-3}.$$

The solution is $\lambda^*_{n=6} \approx 1.5$. It follows that it is indeed optimal for each of the six servers to immediately release completed tasks under simple rotation. Hence, six servers with $\lambda \approx 1.34$ generate the same discounted expected output as five unboundedly productive servers. □

5 Extensions

5.1 Endogenous effort

In the previous sections we assumed that a server could not affect his rate of production λ and had only one decision to make: release a finished task or just sit on it. The motivation for this assumption was to isolate the effect of this new form of moral hazard on the server's behavior and the overall productivity of a team of servers. In this section we consider an extension of our basic model where the server's rate of production is endogenous. Specifically, a server now has two decisions to make: (1) an *effort decision* – whenever the task is unfinished, the server decides whether to work or to shirk, and (2) a *release decision* – whenever a task is finished, the server decides whether to release it or not (just as in our benchmark model). Working has a completion rate of λ and flow cost c (as before), while shirking has a completion rate of 0 and flow cost of d (like that of sitting on a completed task). To give the most transparent illustration of the additional hidden effort decision and its implications, we focus on the case with two servers, one work station, and $K = 0$.

The introduction of an effort decision has two important implications: (1) it affects the conditions under which the first-best is attainable, and (2) it affects the second-best level of output and the means of achieving it (when the first-best is unattainable). We begin by analyzing the first implication.

First best – Recall that in our benchmark model, we proved (Lemma 1) the existence of a unique cutoff productivity rate $0 < \lambda^* < \infty$ such that simple rotation without delay is incentive compatible (and hence, the first-best is attainable) if and only if the servers' productivity is $\lambda \leq \lambda^*$. This result was shown for $K > 0$. When $K = 0$, it is still true that if simple rotation without delay is incentive compatible for a given productivity rate λ, then it is also incentive compatible for $\lambda' < \lambda$.[7]

Adding an effort decision effectively reverses this conclusion. Specifically, now there is a *lower* bound $0 < \underline{\lambda} \leq \infty$, which depends on the other parameters, such that simple rotation without delay is incentive compatible whenever $\lambda \geq \underline{\lambda}$. The lower bound is strictly positive and $\underline{\lambda} = \infty$ represents the possibility that simple rotation is never

[7] In contrast to the $K > 0$ case, when $K = 0$ the threshold productivity level below which simple rotation without delay is incentive-compatible can also be zero or infinity for some parameter values.

incentive compatible.

To see why, consider the *effort incentive constraint* under simple rotation without delay:

$$\frac{c}{\lambda + r} + \frac{\lambda}{\lambda + r} \cdot \frac{0}{\lambda + r} \leq \frac{d}{r}\left(1 - \left(\frac{\lambda}{\lambda + r}\right)^2\right). \tag{12}$$

On the left-hand-side we have the expected cost of exerting effort until a task is completed plus the expected cost of idle time (which is zero) during the period in which the other server works on his task. On the right-hand-side we have the alternative option of shirking and paying a flow cost of d for the same duration of time (i.e., the expected time it takes to complete two tasks). If $c \geq 2d$, the above constraint is violated for all λ implying that simple rotation without delay is never incentive compatible. However, if $c < 2d$, the constraint holds whenever

$$\lambda \geq \frac{(c - d)r}{2d - c}.$$

Next, consider the *release incentive constraint*:

$$\frac{0}{\lambda + r} + \frac{\lambda}{\lambda + r} \cdot \frac{c}{\lambda + r} \leq \frac{d}{r}\left(1 - \left(\frac{\lambda}{\lambda + r}\right)^2\right). \tag{13}$$

The left-hand-side is the expected cost that is incurred when a server releases a completed task: He enjoys idle time while the other server works and then incurs the cost of effort on his next task. The right-hand-side is the flow cost of sitting on a completed task for the same duration.

From inequalities (12) and (13), it follows that the effort incentive constraint is the more demanding one: While the right-hand-side of both constraints is equal, the left-hand-side in the effort incentive constraint is strictly higher.

If simple rotation without delay is incentive compatible for some λ, why is it not incentive compatible for lower productivity rates when servers also make an effort decision (even though it is incentive compatible if servers only make a release decision)? The reason for this is the relative timing of the cost of effort and the benefit of idle time in the incentive constraints.

As is common in many dynamic economic models, in the effort constraint, the cost of effort *precedes* the reward. When servers are slow (low λ), having a long interval of working time and then an equally long interval of idle time may not be as attractive because of discounting (since the reward comes in the relatively distant future). When the productivity rate increases, the time intervals of work and being idle shrink identically, but the effect of discounting becomes weaker; hence, simple rotation without delay may become incentive compatible. However, in the release constraint, the order is reversed: First, the servers enjoys idle time, and only later they incur the cost of effort (on the next task). Since as shown above, the effort constraint implies the release constraint, effectively, the latter constraint can be ignored. By contrast, in our benchmark model, the release incentive constraint is the only constraint and hence, the binding one.

Second best — We now turn to analyze the second-best outcome when simple rotation without delay is not incentive compatible. A naive upper bound on the total expected output (discounted to the time of the first task assignment) can be obtained by multiplying the first-best output, $\frac{\lambda}{r}$ by the ratio of the available budget for incentive provision to the total expected costs that must be covered under the first best. The available budget is the value of idle time starting from the end of the first task (the first task wastes idle time for the server who has not worked yet): $\frac{\lambda}{r+\lambda}\frac{d}{r}$. The costs that need to be covered consist of a constant flow cost of effort c net of the inevitable cost of d due to being allocated a task, i.e., $\frac{c-d}{r}$. Hence, the upper bound is given by,

$$\frac{\lambda}{r} \times \left\{ \left(\frac{\lambda}{\lambda + r} \frac{d}{r} \right) \bigg/ \left(\frac{c-d}{r} \right) \right\},$$

which simplifies to

$$\frac{d}{r} \frac{\lambda^2}{(r+\lambda)(c-d)}. \tag{14}$$

This upper bound completely ignores the constraints imposed by the dynamic structure of the environment and simply identifies which fraction of the required budget to incentivize first-best is available, and assumes that all of it can be utilized efficiently to incentivize productive effort.

Below, we show that the upper bound in (14) is in fact attainable so it characterizes the expected output in optimum, whenever the first-best cannot be attained. It turns out, however, that unlike in our benchmark model, we can no longer rely on allocation schemes akin to RDS where the principal only specifies a time-invariant task assignment rule and the servers *voluntary* delay task releases to slow down production just enough. In a nutshell, the reason behind this is similar to the observation given above about the incentive compatibility of simple rotation without delay: *the effort incentive constraint is more demanding than the release incentive constraint.* [8]

In light of this, to show that the upper bound in (14) can be attained we introduce the following task allocation scheme, which we refer to as the *stochastic shutdown mechanism* (SSM). The first task is allocated to server 1. When this server releases a task, the second task is allocated to server 2. Beginning from the allocation of the second task, a public lottery will be performed whenever a task is assigned, provided that there were no earlier "shutdowns" (defined next). The public lottery is as follows: with probability p, all future tasks will be assigned to the same server who is currently with the task, and with probability $1-p$, the next task will go to the server who is currently free. Note that if the outcome of the lottery is that a server will get all future tasks, then it is optimal for this server to not exert effort and never release the task. Consequently, we refer to this outcome as "shutdown".

Suppose no shutdown occurred and both servers exert effort. The payoff from exerting effort and immediately releasing completed tasks increases in p, and the minimal p for which a server prefers to exert effort and immediately release a finished task satisfies:

$$\frac{d}{r} = \frac{c}{\lambda + r} + (1-p)\left(\frac{\lambda}{\lambda + r}\right)^2 \frac{d}{r}. \tag{15}$$

This implies the following observation, which is related to earlier discussions on the relation between the effort and release incentive constraints.

Lemma 2. *If (15) holds, then each server will strictly prefer to immediately release a finished task.*

[8] In more detail, suppose we use RDS. Since delaying as well as releasing a completed task are both incentive compatible, a server must be indifferent between the two. Denote the continuation payoff from any of these actions (followed by equilibrium play) by X. Now suppose that the task is unfinished. Shirking forever yields a continuation payoff of X while the payoff from working on the task and receiving a continuation payoff of X only upon completion is strictly lower than X since $c > d$. That is, if voluntary delays are incentive compatible, exerting effort cannot be incentive compatible.

Proof. Since $\frac{d}{r} = \frac{d}{\lambda+r} + \frac{\lambda}{\lambda+r}\frac{d}{r}$ and $\frac{d}{\lambda+r} < \frac{c}{\lambda+r}$, it follows that

$$\frac{\lambda}{\lambda+r}\frac{d}{r} > (1-p)\left(\frac{\lambda}{\lambda+r}\right)^2 \frac{d}{r},$$

which implies that

$$\frac{d}{r} > (1-p)\left(\frac{\lambda}{\lambda+r}\right)\frac{d}{r}.$$

Thus, the cost of not releasing a completed task is greater than that of releasing it. ■

Lemma 2 means that the minimal p that incentivizes effort will also ensure that finished tasks will be released immediately. The next result is that SSM with the minimal p that incentivizes effort attains the second-best.

Proposition 5. *SSM with p that satisfies (15) attains the upper bound in (14).*

Proof. The value of p that satisfies (15) is given by

$$p^* = \frac{r}{\lambda}\frac{r(c-d)+\lambda(c-2d)}{d\lambda}.$$

Note that for a given p, SSM generates the expected output (discounted to the time at which the first task is assigned) of

$$\sum_{j=0}^{\infty}(1-p)^j(\frac{\lambda}{\lambda+r})^{j+1} = \frac{\lambda}{p\lambda+r}.$$

Plugging p^* into this expression and rearranging yields (14). ■

5.2 Heterogeneous production rates

Up until now we maintained the assumption of symmetric production rates among servers. Under this assumption, inefficiency is fully captured by the overall amount of delay. However, with heterogeneous production rates, overall performance depends not only on whether the working servers are delaying completed tasks, but also on which servers are working. Hence, with heterogeneous production rates the goal of finding optimal allocation mechanisms and characterizing the servers' equilibrium behavior is much more challenging. However, achieving this goal - even in simple cases - leads to new insights and novel comparative statics.

To illustrate this, we focus on a simple but important scenario: There is one work station and two servers, and simple rotation without delay is incentive-compatible. When servers are symmetric, this implies that the first best is trivially attained since all finished tasks are released immediately. On the other hand, when the servers have heterogeneous production rates, say $\lambda > \mu > 0$, the first best requires that only the most productive server works at all times, which is clearly not incentive compatible. Our question is, what is the second best in this case?

Proposition 6. *Assume that simple rotation without delay is incentive-compatible. Then the principal's optimal (second-best) expected discounted payoff is given by*

$$\frac{\lambda}{\lambda + r}\left(1 + \frac{\varphi\lambda + (1-\varphi)\mu}{r}\right),$$

where

$$\varphi = \frac{d}{c + K(\lambda + r)}.$$

The expression for the second-best output can be understood as follows. The first task is allocated to the most productive server (say server 1), who releases it without delay (this generates a payoff of $\lambda/(\lambda + r)$). From that point onwards, the system's production rate is the weighted average of the two production rates, where the weights are determined by the parameters as stated in the proposition.

The proof of the proposition proceeds as follows. First, we identify a naive upper bound on the fraction of first-best output that can be produced by server 1 in any incentive compatible scheme, beginning from the first task release. As in previous sections, the idea behind this upper bound is to ignore the constraints arising from the dynamic nature of the problem. This upper bound is equal to the value of φ in the proposition.

Next, we show that this upper bound can be attained via a specific allocation rule. Moreover, under the allocation rule that we propose, the equilibrium behavior prescribes no delays to any of the servers. Hence, at every instant, either server 1 or server 2 will put active effort (no delays), which explains the weighted average form of the servers' productivity rates as stated in the proposition.

To prove that the naive upper bound is attainable we propose the *stochastic repetition scheme* under which: Server 1 is assigned the initial task; whenever server 1 releases a task, the next task is also assigned to server 1 with probability p and it is assigned to server 2 with probability $1-p$; and whenever server 2 releases a completed task, the next task is assigned to server 1 with certainty.

We now turn to the formal proof of the Proposition.

Proof. We first derive a naive upper bound on the output that can be attained from server 1. Suppose that at the time in which server 1 decides whether to release a completed task, we conduct the following lottery: with probability φ server 1 will continue working forever (releasing all completed tasks upon completion), and with the complementary probability server 1 will get idle time indefinitely. Consider the value of φ that makes server 1 indifferent between accepting the lottery and sitting on the task indefinitely, i.e., the value of φ that solves the equation:

$$\varphi\left(K+\int_0^\infty e^{-rt}\left(c+\lambda K\right)dt\right)=\frac{d}{r}.$$

The solution is given by

$$\varphi=\frac{d}{c+K(\lambda+r)}.$$

Hence, an upper bound on the expected future output, discounted to the time of the first release (i.e., the time when the lottery is performed) is $\varphi\lambda/r$.

Let us now compute the total expected discounted output of server 1 following the release of the first task (discounted to this point in time) in the stochastic repetition scheme where p is chosen to make server 1 indifferent between releasing a finished task or not. To do this, we first derive the expected discount factor between the time at which a task was assigned to server 1 and the earliest subsequent time at which a task was assigned to server 2. This is given by the following expression:

$$(1-p)\frac{\lambda}{\lambda+r}+p(1-p)\left(\frac{\lambda}{\lambda+r}\right)^2+\quad ...\quad=\frac{\lambda(1-p)}{r+\lambda(1-p)}.$$

Next, we compute the expected discount factor from the time at which a task was assigned to server 2 and the earliest subsequent time at which a task is assigned to server 1. This is given by $\mu/(r+\mu)$. Finally, the discounted expected output generated by server 1 from the time at which he was assigned a task until the earliest time at which a task is assigned to server 2 is given by:

$$\frac{\lambda}{\lambda+r} + p\left(\frac{\lambda}{\lambda+r}\right)^2 + p^2\left(\frac{\lambda}{\lambda+r}\right)^3 + \quad ... \quad = \frac{\lambda}{r+\lambda(1-p)}.$$

It follows that the total expected output generated by server 1, discounted to the time at which the first task is released, is given by:

$$p\frac{\lambda}{r+\lambda(1-p)}\left(1+\frac{\mu}{r+\mu}\frac{\lambda(1-p)}{r+\lambda(1-p)}+\left(\frac{\mu}{r+\mu}\frac{\lambda(1-p)}{r+\lambda(1-p)}\right)^2+\ ...\right)$$
$$+(1-p)\frac{\mu}{r+\mu}\frac{\lambda}{r+\lambda(1-p)}\left(1+\frac{\mu}{r+\mu}\frac{\lambda(1-p)}{r+\lambda(1-p)}+\left(\frac{\mu}{r+\mu}\frac{\lambda(1-p)}{r+\lambda(1-p)}\right)^2+\ ...\right),$$

which reduces to

$$\frac{\lambda}{r}\frac{\mu+pr}{\lambda(1-p)+\mu+r}. \tag{16}$$

The value of p that makes server 1 indifferent between releasing a finished task and not doing so solves the equation:

$$\frac{d}{r} = p(K+\frac{c}{\lambda+r}+\frac{\lambda}{\lambda+r}\frac{d}{r}) + (1-p)\frac{\mu}{\mu+r}(K+\frac{c}{\lambda+r}+\frac{\lambda}{\lambda+r}\frac{d}{r}),$$

for which the solution is

$$p = \frac{d(\lambda+r)-\mu(c-d)-K\mu(\lambda+r)}{\lambda r(\frac{d}{r}+K)+r(c+Kr)}.$$

Plugging this into (16) yields the desired upper bound $\varphi\lambda/r$.

Finally, note that our assumption that simple rotation is incentive compatible implies that, in optimum, $p > 0$, which in turn implies that it is strictly optimal for server 2 to release tasks immediately upon completion under the stochastic repetition scheme, completing the proof. ■

The above result offers several interesting comparative statics observations. First, if the servers become less patient, or if the costs related to working on a task (c, K) increase, then the amount of tasks that will be allocated to the most productive server will *decrease* – which will decrease the total productivity of the system. Perhaps more interesting is that two systems that are identical in all parameters except for d will perform differently at the second-best even when neither system exhibits any delay (specifically, the system with a higher d will be more productive).

Recall that when simple rotation without delay is incentive compatible, $p > 0$ and so server 1 is assigned more tasks than server 2. Even when simple rotation is not incentive compatible, there are some situations in which the above stochastic-repetition scheme still induces both servers to immediately release finished tasks. To see this, suppose that the idle time that server 1 gets in simple rotation without delay is enough to incentivize him to release completed tasks immediately (i.e., server 2 is sufficiently slow). If this is the case, we can find $p \geq 0$ that makes server 1 indifferent between releasing a finished task or not. If given this p, the idle time that server 2 gets is enough to incentivize him to immediately release finished tasks, we are done (i.e., the scheme induces immediate releases of finished tasks and gives more tasks to server 1). If the idle time for server 2 is not enough, we need to inject delay into the scheme. This raises the following questions: Is it optimal only for server 2 to delay? Should server 1 be the only one delaying? It is optimal for both servers to delay? These questions are left for future research.

5.3 Premature release

In this subsection we consider the possibility that servers can release unfinished tasks. For example, in the hospital setting, a patient may be discharged before sufficient time has elapsed during which his symptoms did not recur; in a setting of a public sector office, an application may proceed to the next step before all forms were filled and signed. Such premature releases are costly to the principal if only finished tasks contribute to his payoff. In addition, unfinished tasks may return to be properly finished, causing delays in the processing of new awaiting tasks. Continuing the above examples, a patient who is discharged prematurely is more likely to become sick again (perhaps even more severely than before), and may return to the ER or to another hospital for further treatment; incomplete applications by individuals who are eligible for the service they request may lead to wrongful rejections, or may be sent back to the department that released it.

The possibility to release unfinished tasks means that upon receiving a task, a server now decides at each point in time (both before and after finishing a task) whether to release it. For a server, releasing an unfinished task is tempting as it saves the flow cost of working on a task and allows the server to enjoy the idle time sooner. For the principal, unfinished tasks do not contribute to this payoff and she would therefore like to prevent their release. A key insight in this section is that even when the principal can use threats of punishments (within the allocation rule) to make premature releases an off-path phenomenon, such threats may require inefficient delays that occur on-path.

To illustrate this we focus on the following environment. There are two servers and a single work station ($n = 2, m = 1$). After a server is assigned a task (and pays the cost of K) he decides at each point in time whether to release it. If an unfinished task is released, then with probability p it returns immediately and it is publicly known which server released it. The principal gets a payoff of 1 whenever a *finished* task is released and a payoff of 0 if a task is released prematurely (suppose that these payoffs are not immediately observed so the principal cannot learn from them). We focus on the case where both servers have productivity $\lambda \leq \lambda^*$ so that in the benchmark where premature releases are ruled out, simple rotation without delay is incentive compatible and achieves the first-best.

To deter premature releases, the harshest punishment that the principal can commit to is to allocate all future task to the server who was found to release an unfinished task (which happens when an unfinished task returns). The severity of this punishment is limited for two reasons: (i) a server can avoid the costs associated with all future tasks by sitting indefinitely on the returning unfinished task (which a server must accept), and (ii) premature releases are caught only with probability p. Still, for a sufficiently high p, the threat of this punishment can incentivize the servers to attain the first-best by following a simple rotation without delay.

Proposition 7. *The principal can attain the first-best if and only if p is at least*

$$p^* = \frac{\frac{c}{\lambda+r} + \frac{r}{\lambda+r}u}{K + \frac{c}{\lambda+r} + \frac{\lambda}{\lambda+r}\frac{d}{r} - u},$$

where

$$u = \frac{\lambda\left(c + K\left(\lambda + r\right)\right)}{r(2\lambda + r)}.$$

Proof. Assume that whenever a server i releases an unfinished task and that task returns, server i is assigned all future tasks. Let s be the strategy that calls for a server to work on each task until completion and then release it immediately. The first-best output is achieved when both servers follow s. Let s' be a strategy with the following features: when a server is first assigned a task, he releases it immediately; if the task returns (in which case, all subsequent tasks will be assigned to this server), he sits on the task indefinitely; if the task does not return, the server follows strategy s. Let h be some history in which server i is assigned a task after both servers played according to s. The profile s is an equilibrium if and only if there is no history h after which a server gains by deviating to s'. I.e., if and only if

$$p\left(K+\frac{c}{\lambda+r}+\frac{\lambda}{\lambda+r}\frac{d}{r}\right)+(1-p)u\geq\frac{c}{\lambda+r}+\frac{\lambda}{\lambda+r}u, \tag{17}$$

where u is the continuation cost from following s evaluated at the point of releasing a task; i.e., u solves

$$u=(\frac{\lambda}{\lambda+r})(K+\frac{c}{\lambda+r}+\frac{\lambda}{\lambda+r}u),$$

which yields the expression for u given in the statement of the proposition.

It follows that the LHS of inequality (17) can be written as

$$p\left[\frac{r}{\lambda+r}\left(K+\frac{c}{\lambda+r}\right)+\frac{\lambda}{\lambda+r}\left(\frac{d}{r}-\frac{\lambda}{\lambda+r}u\right)\right]+u.$$

Since $\lambda\leq\lambda^*$ it follows that $\frac{d}{r}\geq u$, and hence, the LHS of inequality (17) increases in p, and is greater or equal than the RHS when $p=1$. It follows that there exists $p=p^*$ at which (17) holds with equality, which yields the expression in the statement of the proposition.

If $p<p^*$, then inequality (17) is violated. This implies that s is not an equilibrium and hence, the first-best cannot be attained. ■

While the exact characterization of the threshold p^* requires a formal argument, the existence of a range of p values sufficiently close to 1, for which the first best can be achieved, is quite intuitive. Similarly, since the threat of punishment for a returning task is limited, when p is sufficiently close to 0, premature releases cannot be deterred. Interestingly, there is a range of intermediate p values for which premature releases can be completely deterred. However, this requires deliberately reducing the system's productivity. Specifically, we show below that in addition to p^* there is another threshold p^{**} such that premature releases can be deterred only if $p \geq p^{**}$ and importantly, $p^{**} < p^*$. Thus, as can be seen in Figure (2), for intermediate values of p, premature releases can be made an off-equilibrium event but task releases will be inefficiently delayed. Hence, from the perspective of an outside observer who is aware of the workers' abilities, the system will appear inefficiently slow.

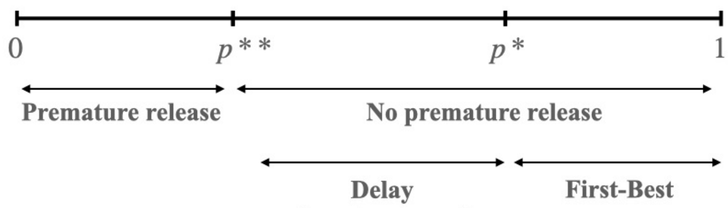

Figure 2: Release and Delay as a function of p.

To understand the intuition for the intermediate range, consider $p = p^*$ and suppose that a server is now working on an unfinished task. Since $p = p^*$, the server is indifferent between following simple rotation without delay and prematurely releasing the task. Consider a slightly lower value of p. Now, the expected payoff from premature release is higher. Since the principal is already using the most severe punishment for returning tasks, the only way to deter premature releases for this lower p is to raise the expected payoff from releasing a finished task. This can only be done if other servers produce less efficiently, or, strategically delay their task releases.

However, delay cannot be voluntary as in *RDS* because servers who are incentivized not to release early have a strict incentive to release finished tasks. One way to inject delay is with the following scheme, which we call *simple rotation with minimal release time*. In this scheme servers take turns working on tasks, but if a server releases a task before time T, or if the task returns, then that server is assigned all future tasks. Thus, the minimal release time T introduces delay into the system.

The minimal delay T that deters premature releases is derived as follows. Let s^* be a strategy that calls for a server to work on a task until completion. If it takes less than T units of time to finish the task, then the server sits on the finished task until T units of time have passed at which point he released it. Otherwise, he releases it immediately upon completion. Consider a server who did not finish a task after T units of time following a history in which both servers followed s^*. Let $\tilde{s}$ be a deviation from s^* which calls for this server to release the unfinished task, and to continue playing according to s^* if the task does not return. But if the task does return, then $\tilde{s}$ calls for the server to sit on the returning task indefinitely. The minimal delay time T satisfies that the server is indifferent between deviating to $\tilde{s}$ and continuing to play according to s^*.

To express this formally, suppose both servers follow s^*. Let u^* denote the continuation cost of a server at the point in time when a task is completed, and let v^* be the continuation cost at the point in time in which the task is allocated to the server. Then

$$v^* = K + \int_0^T \lambda e^{-\lambda t} \left(\int_0^t e^{-rs} c ds + \int_t^T e^{-rs} d \, ds \right) + \int_T^\infty \lambda e^{-\lambda t} \int_0^t e^{-rs} c ds dt$$
$$+ \left(\int_0^T \lambda e^{-\lambda t} e^{-rs} ds + \int_T^\infty \lambda e^{-\lambda t} \int_0^t e^{-rs} ds dt \right) u^*$$

and

$$u^* = \left(e^{-rT} - \frac{r}{\lambda + r} e^{-(\lambda + r)T} \right) v^*$$

It follows that the minimal delay T satisfies

$$p \left(K + \frac{c}{\lambda + r} + \frac{\lambda}{\lambda + r} \frac{d}{r} \right) + (1 - p) u^* = \frac{c}{\lambda + r} + \frac{\lambda}{\lambda + r} u^*. \tag{18}$$

There is a finite T that solves this equation if p is greater than

$$p^{**} = \frac{\frac{c}{\lambda + r}}{K + \frac{c}{\lambda + r} + \frac{\lambda}{\lambda + r} \frac{d}{r}}.$$

To see why, note that when $T \to \infty$, $u^* \to 0$ and p^{**} is the value of p that satisfies equation (18) in this case.

It follows that when $p \in (p^{**}, p^*)$ there is a minimal amount of delay for which simple rotation with minimal release time deters premature releases. An interesting feature of this mechanism (and any other mechanism that deters premature releases) is that when $p < p^*$ only finished tasks are released and yet there is inefficient delay even though $\lambda < \lambda^*$. Thus, for an outside observer who knows that $\lambda < \lambda^*$ the system will appear to be unnecessarily inefficient. Note that this variation of our model offers an additional rationale for delays as means to attain the second-best output. Specifically, delays are needed to generate incentives even when the servers are sufficiently slow. This suggests that delays (or, more generally, inefficient work practices) can be even more widespread than what they seem to be in our original model.

6 Concluding remarks

We opened the paper with the example of Haemek Hospital, where the administration realized that doctors at the internal wards were delaying the release of patients in response to the allocation rule that assigned a new patient to whichever ward had an empty bed. This meant that when wards were working at full capacity, the first ward to release a patient was then assigned a new patient from the ER. To address this problem, the administration severed the link between releasing a patient and being assigned a new one: the responsibility for new patients was assigned to wards using simple rotation regardless of the number of empty beds in that ward.

The insight from this story is that the inherent moral hazard of strategically delaying the release of finished tasks can be addressed by weakening the relation between which server is assigned a task and which server just finished working on one. Indeed, this is a key feature of the optimal allocation rules that we characterize. When servers are identical in their productivity, it is optimal for a server who releases a finished task to enter a queue, and he will be assigned a new task when he gets to the front of that queue. When servers have diverse productivity levels, optimality requires a balance between efficiency - which calls for the most efficient server to always work - and incentive compatibility - which calls to let servers have idle time once they finish working on a task. This means that compared with the slowest server, the fastest server is more likely to be immediately assigned a new task upon releasing a finished one, but this probability must be strictly lower than one.

Optimally addressing the moral hazard problem that we highlight in this paper gives rise to interesting (and somewhat unintuitive) features of service systems: when servers are sufficiently productive and patient, more work stations reduce total output, and in some cases it is better to have more servers of lower quality.

References

Armony, M., G. Roels, and H. Song (2021). Pooling queues with strategic servers: The effects of customer ownership. *Operations Research 69*(1), 13–29.

Bird, D. and A. Frug (2019). Dynamic non-monetary incentives. *American Economic Journal: Microeconomics 11*(4), 111–150.

Bray, R. L., D. Coviello, A. Ichino, and N. Persico (2016). Multitasking, multiarmed bandits, and the Italian judiciary. *Manufacturing & Service Operations Management 18*(4), 545–558.

Coviello, D., A. Ichino, and N. Persico (2014). Time allocation and task juggling. *American Economic Review 104*(2), 609–623.

Coviello, D., A. Ichino, and N. Persico (2015). The inefficiency of worker time use. *Journal of the European Economic Association 13*(5), 906–947.

Dagan, O., S. Lichtman-Sadot, I. Shurtz, and D. Zeltzer (2024). Strategic effort and congestion reduction: Evidence from an emergency department routing reform. Working paper.

De Clippel, G., K. Eliaz, D. Fershtman, and K. Rozen (2021). On selecting the right agent. *Theoretical Economics 16*(2), 381–402.

Eliaz, K., D. Fershtman, and A. Frug (2022). On optimal scheduling. *American Economic Journal: Microeconomics, Forthcoming*.

Forand, J. G. and J. Zápal (2020). Production priorities in dynamic relationships. *Theoretical Economics 15*(3), 861–889.

Frankel, A. (2016). Delegating multiple decisions. *American economic journal: Microeconomics 8*(4), 16–53.

Gavazza, A. and A. Lizzeri (2007). The perils of transparency in bureaucracies. *American Economic Review 97*(2), 300–305.

Geng, X., W. T. Huh, and M. Nagarajan (2015). Fairness among servers when capacity decisions are endogenous. *Production and operations management 24*(6), 961–974.

Gopalakrishnan, R., S. Doroudi, A. R. Ward, and A. Wierman (2016). Routing and staffing when servers are strategic. *Operations Research 64*(4), 1033–1050.

Guo, Y. (2016). Dynamic delegation of experimentation. *American Economic Review 106*(8), 1969–2008.

Guo, Y. and J. Hörner (2020). Dynamic allocation without money. Working paper.

Li, J., N. Matouschek, and M. Powell (2017). Power dynamics in organizations. *American Economic Journal: Microeconomics 9*(1), 217–241.

Linder, R. (2019). Without a single shekel: The secret of the hospital that managed to reduce the load in the ER and wards. *TheMarker*.

Lipnowski, E. and J. Ramos (2020). Repeated delegation. *Journal of Economic Theory 188*, 105040.

Mylovanov, T. and P. W. Schmitz (2008). Task scheduling and moral hazard. *Economic Theory 37*, 307–320.