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Matching Workers

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Matching Workers

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Abstract

This paper studies the matching of workers within the firm when the productivity of workers depends on how well they match with their co-workers. The firm acts as a coordinating device and derives value from this role. It is shown that a worker's contribution to firm value changes over time in a non-trivial way as co-workers are replaced by new workers.

The paper derives optimal hiring and replacement policies, including an optimal stopping rule, and characterizes the resulting equilibrium in terms of worker flows, firm output and the distribution of firm values. Simulations of the model reveal a rich pattern of worker turnover dynamics and their connections to the resulting firm values distribution.

The paper stresses the role of horizontal differences in worker productivity, which are different from vertical, assortative matching issues. It derives the rent from organizational capital, with worker complementarities playing a key role. We compare the model to match-specific productivity models and explore the essential differences, with the emphasis laid on worker interactions and complementarities.

Key words: worker interactions, firm value, complementarity, worker value, organizational capital, Salop circle, hiring, firing, match quality, optimal stopping.

JEL codes: E23,E24, D23, J24.

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Matching Workers[1]

1 Introduction

How does the value of the firm depend on the value of its workers? When one considers firms that have little physical capital – such as IT firms, software development firms, investment banks and the like – the neoclassical model does not seem to provide a reasonable answer. The firm has some value that is not manifest in physical capital. Rather, Prescott and Visscher's (1980) 'organization capital' may be a more relevant concept in this context. One aspect of the latter form of capital, discussed in that paper, is the formation of teams and this is the issue taken up in the current paper. We ask how workers affect each other in production and how this interaction affects firm value. Garicano and Wu (2012, p.1394) state that "organizational rent is the economic return to organizational capital...an important theme in organizational economics that is yet to be explored." The current paper offers such an exploration.

The paper studies the value of firms and their hiring and firing decisions in an environment where the productivity of the workers depends on how well they match with their co-workers and the firm acts as a coordinating device. This role of the firm is what generates value.

In the model, match quality derives from a production technology whereby workers are randomly located on the Salop (1979) circle and depends negatively on the distance between them. It is shown that a worker's contribution in a given firm changes over time in a nontrivial way as co-workers are replaced with new workers. The paper derives optimal hiring and replacement policies, including an optimal stopping rule, and characterizes the resulting equilibrium in terms of employment and the distribution of firm values.

A key result is the derivation of an optimal worker replacement strategy, based on a productivity threshold that is defined relative to the other workers. The derivation is non-trivial and underlines the importance of worker complementarities in productivity. Thus the model is not equivalent to one with idiosyncratic shocks to individual workers or to job-worker pairings.

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This replacement strategy, interacted with exogenous worker separation and firm exit shocks, generates rich turnover dynamics. The resulting firm values distribution are found to be – using illustrative simulations – non-normal, with negative skewness and negative excess kurtosis. This shape reflects the fact that, as firms mature, there is a process of forming good teams on the one hand and the effects of negative separation and exit shocks on the other hand.

The paper proceeds as follows: in Section 2 we outline the model. We describe the set up and delineate the interaction between workers. In Section 3 we derive optimal hiring and firing policy, including a stopping rule, and study the implications for firm value. In Section 4 we allow for exogenous worker separation. Section 5 places the model in the context of the literature. Section 6 discusses key assumptions in light of the results. Section 7 presents simulations of the model, exploring the mechanisms inherent in it. Section 8 concludes.

2 The Model

In this section we first describe the set-up of the firm and the production process (2.1). We then define worker interactions and the emerging state variables (2.2). We subsequently provide stylized facts supporting this way of modelling (2.3). We end the section (2.4) with a short discussion of optimal stopping, to prepare for the optimal replacement analysis in the next section.

2.1 The Set-Up

A firm enters the market by sinking an entry cost K. The firm starts off with three workers. In each period, a firm faces an exogenous exit probability. If the firm does not exit, it can replace at most one worker. It does so by first firing one of the existing workers without recall, and then sampling – from outside the firm – one worker. Thus, we do not allow the firm to compare the existing and the sampled worker and hire the more productive one. We rationalize this by assuming that it takes a period to learn a worker's productivity. Replacing a worker is costly. Wages and productivity distributions are time independent.

The main focus of the paper is horizontal worker heterogeneity. Thus, although workers are identical from an *ex ante* perspective, the value of a worker to a firm is random. More specifically, we assume that how well workers' team up depends on their personal characteristics, and that these characteristics are random at the stage at which the firm decides on whom to hire.

A common way to model worker heterogeneity, and which we use in this paper, is to attribute to each worker a location in a metric space, and apply a distance measure to capture the differences between the workers. In order to ensure that workers with different locations to be equally attractive in expected terms, we have to put restrictions on the space in which workers are located. A common way to obtain this is to assume that a worker has a location on a Salop (1979) circle and that workers are allocated uniformly on the circle.[2] In this case, the distribution of the distance from a worker to a co-worker randomly placed on the circle is independent of the worker's location. Note that this is not the case if the workers are uniformly allocated on a line segment, in which case a worker at the middle of the segment on average has a shorter distance to a randomly allocated co-worker than a worker close to the end point. More generally, in an n dimensional Euclidean space, an $n-1$ dimensional sphere will also have the property that the distribution of the distance to a randomly placed co-worker will be independent of a worker's location on the sphere. However, in this case the distribution of the distance to a randomly placed co-worker is no longer uniform. In the discussion section we argue that a higher-dimensional sphere may be a convenient location space if there are more than three workers.

In what follows we therefore attribute to all workers a position on a Salop circle, with their placement randomly and independently drawn from a uniform distribution. Any new worker placement will be drawn independently from the same distribution. Note that if two workers are close on the circle, a third worker will either be close to or far away from both of the workers. Hence the distances from the third, new worker, to each of the existing workers are workers are positively correlated. This seems reasonable. The productivity of a team of workers is assumed to depend negatively on the distance between the workers.

Let $\beta = \frac{1}{1+r}$ denote the discount factor and r the discount rate of the firm. In the simulations below we let r include a stationary probability of exiting the market, after which the value of the firm is zero.

[2] In a two-dimensional Euclidean space, one may equivalently locate the workers along the boundary of any simply connected set as long as distance is measured along the boundary.

2.2 Workers' Productivity and Interactions

We now turn to a formal description. The three workers are located on the unit circle. The one in the middle (out of the three) is the j worker who satisfies

$$\min_j \sum_{i=1}^{3} d_{ij} \tag{1}$$

where d_{ij} is the distance between worker i and j, and $d_{ii} = 0$. We shall define two state variables δ_1, δ_2 as follows:

$$\begin{aligned} \delta_1 &= \min_{i,j} d_{ij} && (2) \\ \delta_2 &= \min_{j} d_{kj} \quad , k \neq i^*, j^* \quad i^*, j^* = \arg\min_{i,j} d_{ij} && (3) \end{aligned}$$

The first state variable δ_1 expresses the distance between the two closest workers. The second state variable δ_2 expresses the distance between the third worker and the closest of the two others.

The following figure illustrates:

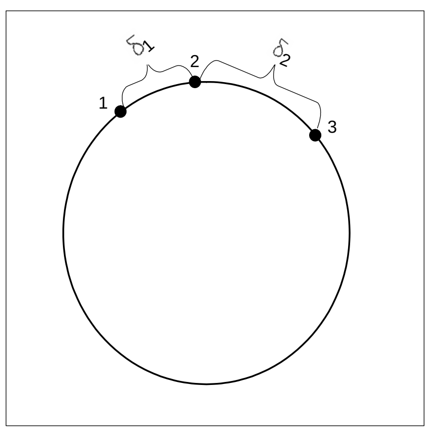

Figure 1: The State Variables

Every period, each worker works together with both co-workers to produce output. Output depends negatively on the distance between the workers. When measuring the distance between two peripheral workers, we assume that it is measured on the segment that goes through the middle man, not the other way around the circle (even if that is shorter). Partly this is meant to capture the structure of a team, that it needs a common ground.

Partly it is done for convenience, as it simplifies the algebraic expressions somewhat. It is not important for the results[3].

Every period, each worker works together with both co-workers to produce output. Production y_{ij} is negatively related to the distance d_{ij} :

$$y_{ij} = \frac{\widetilde{y}}{3} - d_{ij} \tag{4}$$

The firm's total output is then given by the linear additive function:

$$\begin{aligned} Y &= y_{12} + y_{13} + y_{23} \\ &= \widetilde{y} - \sum_{i=1}^{3} d_{ij} \end{aligned} \tag{5}$$

The firm's total output is written as a linear additive function:

$$Y = \widetilde{y} - 2(\delta_1 + \delta_2) \tag{6}$$

In the baseline case we assume that wages are independent of match quality. This is consistent with a competitive market where firms bid for *ex ante* identical workers prior to knowing the match quality. The profits (π) of the firm are then given by:

$$\begin{aligned} \pi &= Y - W \\ &= \widetilde{y} - 2(\delta_1 + \delta_2) - W \\ &= y - 2(\delta_1 + \delta_2) \end{aligned} \tag{7}$$

where W is the total wage bill and y is production net of wages ($\widetilde{y} - W$).

Within a period, the firm cannot fire the workers. Hence it will produce as long as output is positive. We will assume that this is always the case. Furthermore, the firm may want to exit the market endogenously if δ_1 is sufficiently high. In what follows we rule this out by assumption. Below we show that in equilibrium it will never be optimal to exit the market or halt production after a bad draw if $K > 4(1+r)/3r$. Allowing for firm exit after a bad draw is trivial, though cumbersome, and does not add interesting new results.

[3] In an earlier version of the paper, we assumed that the distance between the *peripheral* workers were always measured along the shortest line segments. All the results still prevail.

As already mentioned, the firm can replace up to one worker each period, at a cost c, incurred in the following period. It replaces the worker who is further away from the middle worker. The new values δ_1' and δ_2' are random draws from a distribution that depends on δ_1. We write $(\delta_1', \delta_2') = \Gamma\delta_1$. Figure 2 illustrates, how, without loss of generality, workers 1 and 2, who are not replaced, are situated symmetrically around the north pole:

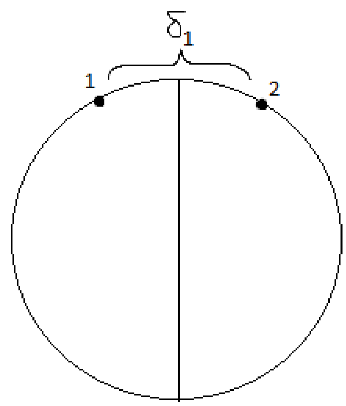

Figure 2: Incumbent Workers

From Figure 2 it follows that Γ can be characterized as follows:

1. With probability $1 - 3\delta_1$, $\delta_1' = \delta_1$ and $\delta_2' \sim unif[\delta_1, \frac{1-\delta_1}{2}]$
2. With probability $2\delta_1$, $\delta_1' \sim unif[0, \delta_1]$ and $\delta_2' = \delta_1$
3. With probability δ_1, $\delta_1' \sim unif[0, \delta_1/2]$ and $\delta_2' = \delta_1 - \delta_1'$

Note that the transition probabilities, and hence continuation values when replacing, are a function of δ_1 and thus are independent of δ_2. Hence only δ_2 influences continuation values in states where the firm is not replacing. That is, as follows from the definition of profits (equation 7), the continuation value of inaction is a function of $(\delta_1 + \delta_2)$.

2.3 Microeconomic Stylized Facts

The afore-going set-up aims at capturing properties that have been found in empirical micro-studies of team production and complementarities within firms. To note just a few examples: Hamilton, Nickerson and Owan (2003) find that teamwork benefits from collaborative skills involving communication, leadership, and flexibility to rotate through multiple jobs. Team production may expand production possibilities by utilizing collaborative skills. Turnover declined after the introduction of teams. Bresnahan, Brynjolfsson and Hitt (2002) study U.S. evidence and stress the importance of complementarities between workplace organization (and organizational changes) and computerization. Garicano and Wu (2012) discuss how performing complementary tasks leads to the formation of an homogenous team.

A recent study, undertaken by MIT's Human Dynamics Laboratory, collected data from electronic badges on individual communications behavior in teams from diverse industries. The study, reported in Pentland (2012), stresses the huge importance of communications between members for team productivity. In describing the results of how team members contribute to a team as a whole, the report actually uses a diagram of a circle (see Pentland (2012, page 64)), with the workers placed near each other contributing the most. The findings state that face to face interactions are the most valuable form of communications, much more than email and texting, thereby emphasizing the role of physical distance.

2.4 A Detour: One-Dimensional Optimal Stopping

Before we continue, we will briefly examine our model with only two workers. Our model then collapses to an optimal stopping model as in McCall (1970). It can also be viewed as a simplified version of the Jovanovic (1979 a,b) model, where the entrepreneur learns the worker type after one period.[4]

The owner of a firm needs two workers to produce. Analogous with the two-period case, we assume that per period output net of wages is given by $y - \varepsilon$. Let $V(\varepsilon)$ denote the value function of the firm. After each period, the firm decides whether it will replace one of the workers (which one is arbitrary). If no worker is replaced, the NPV pay-off from the next period and onwards is $(y-\varepsilon)\frac{1+r}{r}$. It follows that

$$V(\varepsilon) = y - \varepsilon + \max \beta[(y-\varepsilon)\frac{1+r}{r}, (EV(\varepsilon') - c)]$$

where the expectation is taken with respect to ε'. It is well known that the solution of this problem is an optimal stopping rule of the form "stop replacing if $\varepsilon \leq \bar{\varepsilon}$ for some $\bar{\varepsilon}$," where $\bar{\varepsilon}$ solves

$$\frac{y - \bar{\varepsilon}}{r} = \frac{EV(\varepsilon') - c}{1+r} \tag{8}$$

[4] Pissarides (2000, Chapter 6) studies a similar optimal stopping model.

At $\bar{\varepsilon}$, the firm is indifferent between replacing and keeping one of the workers. If the worker is replaced, the new worker will be within the stopping region with probability $2\bar{\varepsilon}$, and the expected distance is $\bar{\varepsilon}/2$. With the complementary probability, the distance exceeds $\bar{\varepsilon}$. The expected value of the distance is (conditioning on being outside the stopping region) is $1/4+\bar{\varepsilon}/2$. Inserting for $V(\varepsilon)$ and manipulating gives that $\bar{\varepsilon}$ solves [5]

$$\frac{\bar{\varepsilon}^2}{r} - (\frac{1}{4} - \bar{\varepsilon}) - c = 0 \tag{9}$$

The first term reflects the expected gain from replacing in terms of lower distances in all periods if the draw is good. The second term reflects the cost associated with a higher expected distance next period, and the last term the pocket cost of replacement. Solving the equation gives

$$\bar{\varepsilon} = \frac{1}{2}r\left(\sqrt{\frac{1}{r}\left(4c+r+1\right)} - 1\right)$$

In the next section we employ a similar logic in the more challenging and essentially different three worker case.

3 Optimal Hiring and Firing with Worker Complementarities

Our aim in this section is to derive an optimal stopping rule for worker replacement. With three workers, this problem is more complex than with two workers. The reason is that the replacement depends not only on the position of the middle man, but also on the distance between the two remaining workers, i.e., how good they are matched. In this section we first show that a firm's search rule can be characterized by an optimal stopping rule. Then we derive this stopping rule. Finally, we close the model by deriving the wage solution.

[5] Equation (8) thus reads

$$\frac{y-\bar{\varepsilon}}{r} = 2\bar{\varepsilon}\frac{y-\bar{\varepsilon}/2}{r} + (1-2\bar{\varepsilon})[\frac{y-(1/4+\bar{\varepsilon}/2)+(y-\bar{\varepsilon})/r}{1+r}] - \frac{c}{1+r}$$

where we again have inserted for (8) on the right-hand side. This expression simplifies to

$$\frac{\bar{\varepsilon}^2(1+r)}{r} - (1-2\bar{\varepsilon})(\frac{1}{4} - \frac{\bar{\varepsilon}}{2}) - c = 0$$

which simplifies to equation (9).

3.1 Optimal Stopping

In this subsection we show that the optimal stopping problem can be characterized by a stopping rule of the form "stop searching if $\delta_2 \leq \overline{\delta}_2(\delta_1)$." In the next subsection we characterize this stopping rule.

Note that the existence of a stopping rule of this form is not obvious. For example, suppose we formulate the stopping rule in terms of total distance $X = 2(\delta_1 + \delta_2)$ rather than in terms of δ_1 and δ_2, that is, stop if $X \leq \bar{X}$ for some $\bar{X} > 0$. Such a stopping rule cannot be optimal. To see this, note that (i) for a given X, the pay-off if stopping is independent of the decomposition of X into δ_1 and δ_2, and (ii) the pay-off if replacing for a given X is decreasing in δ_1 (see below). Hence it cannot be optimal to apply a stopping rule under which stopping depends only on total distance.

By the logic of equation (8), note that in the stopping region, we have that

$$V(\delta_1 + \delta_2) = (y - 2(\delta_1 + \delta_2))\frac{1+r}{r} \tag{10}$$

Outside the stopping region, the continuation value depends only on δ_1. Define $\overline{V}(\delta_1) \equiv EV(\delta_1', \delta_2')|\delta_1$ as the expected continuation value if the firm chooses to replace. The value function in the case of replacement can then be written as:

$$V(\delta_1, \delta_2) = y - 2(\delta_1 + \delta_2) + \beta\bar{V}(\delta_1) \tag{11}$$

We start by showing an important property of the value function.

Lemma 1 $\overline{V}(\delta_1 + \Delta) > \bar{V}(\delta_1) - 2\Delta\frac{1+r}{r}$

Proof. Consider replacement in two cases in which the distances between the remaining workers are δ_1 and $\delta_1 + \Delta$, respectively. We refer to the two cases as the δ_1-case and the $\delta_1 + \Delta$-case, respectively. The expected pay-offs only depend on the distances between the agents, and not on their exact location on the circle. Without loss of generality, we can therefore assume that in both cases, the two workers are located symmetrically around the north pole, and that the draw of the new worker is the same in the two cases. In what follows we assume that the firm in the $\delta_1 + \Delta$ case follows exactly the same replacement strategy as the firm in the δ_1 case (replaces the worker on the left hemisphere whenever the optimal strategy in the δ_1 case prescribes so, the same for the worker on the right hemisphere, and stops searching after the same draws of location). We refer to it as the replication strategy. This is clearly in the choice set of the firm. Hence if we can show that the replication strategy gives the firm in the $\delta_1 + \Delta$ case a profit that is strictly greater than $\bar{V}(\delta_1) - 2\Delta\frac{1+r}{r}$, the proof is complete.

Let δ_1^n and $\delta_{1\Delta}^n$ denote the state variable in the two cases after n periods, and let $\Delta^n \equiv \delta_1^n - \delta_{1\Delta}^n$. Define δ_2^n and $\delta_{2\Delta}^n$ correspondingly. Consider first the case with $n = 1$. Let $\Delta\delta_{tot}$ be defined as $\Delta\delta_{tot} \equiv \delta_{1\Delta}^1 + \delta_{2\Delta}^1 - \delta_1^1 - \delta_2^1$. It follows that the difference in output the first period after replacement is equal to $2\Delta\delta_{tot}$. There are three possibilities:

(i) The new worker is located below the workers in the $\delta_1 + \Delta$ case, as in area A of Figure 3. It follows that $\Delta\delta_{tot} = \Delta/2$, and hence that the difference in per period output is Δ.

(ii) The new worker is located between the workers in the δ_1 case, as in area C of the figure. Then $\Delta\delta_{tot} = \Delta$, and the difference in output is 2Δ.

(iii) The new worker is between a worker in the δ_1 and the $\delta_1 + \Delta$ case (on the same side), as in area B of the figure. Then $\Delta\delta_{tot} \in [\Delta/2, \Delta]$, and the difference in output is in the interval $[\Delta, 2\Delta]$.

Hence the difference in output the next period is at most 2Δ, and with strictly positive probability it is strictly less than 2Δ. It follows that the expected difference in output next period is strictly less than 2Δ. This is a general property of replacement. Hence if we can show that $\Delta^n \leq \Delta$ for all n with the replication strategy, it follows that the profit in the $\delta_1 + \Delta$ case under the replication strategy is strictly higher than $\bar{V}(\delta_1) - 2\Delta\frac{1+r}{r}$, in which case the proof is complete.

If the firm in the $\delta_1 + \Delta$ case follows the replication strategy, it will in all future periods have either two, one or zero workers in a different location than in the δ_1-case. The corresponding values for Δ^n are either Δ (if both workers are in different locations), $\pm\Delta/2$ (if only one of the workers is in a different location) or 0 (if none of the workers is in a different location). Hence $\Delta^n \leq \Delta$ for all n, and this completes the proof.

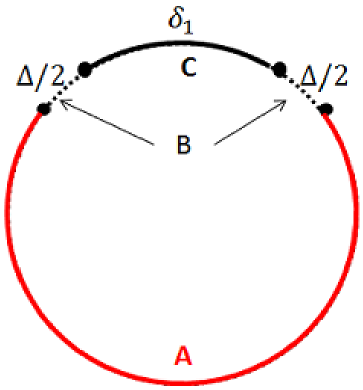

Figure 3: Lemma 1

■

The lemma captures the essence of replacement: it makes a bad draw less costly than without replacement, since the firm can always make a new draw. For any δ_1, δ_2, let $D(\delta_1, \delta_2)$ denote the value of replacing less the value of stopping, i.e., from equation (10) and (11),

$$\begin{aligned} D(\delta_1, \delta_2) &\equiv y - 2(\delta_1 + \delta_2) + \beta \bar{V}(\delta_1) - (y - 2(\delta_1 + \delta_2))\frac{1+r}{r} \\ &= \beta \bar{V}(\delta_1) + 2(\delta_1 + \delta_2)\frac{1}{r} - y\frac{1}{r} \end{aligned} \tag{12}$$

Lemma 2 *Consider the case in which $\delta_1 = \delta_2 = \delta'$. There exists a unique δ^* such that the firm does not replace if and only if $\delta' \leq \delta^*$.*

Proof. First, note that if δ' is sufficiently small, the firm will not replace. This follows from the fact that the gain from replacing is at most $2\delta'/r$, which is smaller than the direct cost c for sufficiently low values of δ'. Now from equation (12) we have that

$$D(\delta', \delta') = \beta \bar{V}(\delta') + 4\delta'\frac{1}{r} - y\frac{1}{r}$$

From Lemma 1 it follows that the right-hand side is strictly increasing in δ'. Hence the equation $D(\delta', \delta') = 0$ has at most one solution. The Lemma thus follows. ■

With these two lemmas in hand, we can easily prove the following proposition:

Proposition 3 ***Existence of an optimal stopping rule****: Let δ_1^* be determined as in Lemma 2. Then if $\delta_1 > \delta_1^*$, the firm replaces. For any $\delta_1 \leq \delta_1^*$ there exists a value $\bar{\delta}_2(\delta_1)$ such that the firm will stop replacing if and only if $\delta_2 \leq \bar{\delta}_2(\delta_1)$. Furthermore, $\bar{\delta}_2(\delta_1)$ is strictly decreasing in δ_1.*

Proof. Since $D(\delta_1, \delta_2)$ is strictly increasing in both arguments, it follows from Lemma 2 that the firm does not replace $\delta_1 \leq \delta_2 \leq \delta_1^*$, while it does replace if $\delta_1^* \leq \delta_1 \leq \delta_2$, with one of the inequalities being strict. Hence it is sufficient to show that for any $\delta_1 \leq \delta_1^*$, there exists a unique $\bar{\delta}_2(\delta_1)$ such that the firm stops replacing if and only if $\delta_2 \leq \bar{\delta}_2(\delta_1)$ (where $\bar{\delta}_2(\delta_1)$ may be equal to $\frac{1}{2} - \delta_1$ in which case the firm never replaces). However, this follows directly from the fact that D is increasing in δ_2.

The optimal stopping is implicitly defined by the equation $D(\delta_1, \delta_2) = 0$. Since D is strictly increasing in both argument, it follows that $\bar{\delta}_2(\delta_1)$ is strictly decreasing in δ_1. ■

The finding that $\bar{\delta}_2(\delta_1)$ is strictly decreasing in δ_1 deserves a comment. At $\delta_1 = \delta_1^*$, $\bar{\delta}_2(\delta^*) = \delta_1^*$. As δ_1 decreases below δ_1^*, $\bar{\delta}_2(\delta_1)$ increases above δ_1^*. This rules out the possibility of a non-monotonicity in stopping behaviour, in the sense that a good draw that reduces δ_1 makes the firm more choosy and induces it to replace more. Appendix A shows the full derivation of δ^*.

As will become clear below, a firm will replace for large values of δ_1 provided that r and c are not too big.

3.2 Characterizing the Stopping Rule

In this section we will characterize $\bar{\delta}_2(\delta_1)$. Now

$$\begin{aligned} V(\delta_1, \delta_2) &= \pi(\delta_1, \delta_2) + \beta \max[V(\delta_1, \delta_2), \overline{V}(\delta_1) - c] \\ &= y - 2(\delta_1 + \delta_2) + \max[\frac{y - 2(\delta_1 + \delta_2)}{r}, \frac{\overline{V}(\delta_1) - c}{1 + r}] \end{aligned} \tag{13}$$

It follows directly from proposition 4 in Stokey and Lucas (1989, p.522) that the value function exists. By definition the optimal stopping rule must satisfy

$$V(\delta_1, \bar{\delta}_2(\delta_1)) = \overline{V}(\delta_1) - c$$

Or (from equation (13))

$$\frac{y - 2(\delta_1 + \bar{\delta}_2(\delta_1))}{r} = \frac{\overline{V}(\delta_1) - c}{1 + r} \tag{14}$$

Let $E^{|x}$ denote the expectation conditional on x. Intuitively, the expected value of replacement, $\overline{V}(\delta_1)$, is given by:

$$\begin{aligned}\overline{V}(\delta_1) \quad = \quad & \underbrace{y - 2 \cdot E^{|\delta_1}\left(\delta_1' + \delta_2'\right)}_{\substack{(1)\text{ : expected flow output} \\ \text{after replacement}}} \\ & + \underbrace{\Pr(\delta_2' \leq \overline{\delta}_2(\delta_1'))}_{\substack{(2)\text{ : probability of} \\ \text{stopping}}} \cdot \underbrace{\frac{y - 2 \cdot E^{|\delta_1, \delta_2' \leq \overline{\delta}_2(\delta_1')}(\delta_1' + \delta_2')}{r}}_{\substack{(3)\text{ : expected discounted value} \\ \text{if stopped after replacement}}} + \\ & + \underbrace{\Pr(\delta_2' > \overline{\delta}_2(\delta_1))}_{\substack{(4)\text{ : probability of} \\ \text{replacing again}}} \cdot \underbrace{\frac{\overline{V}(\delta_1) - c}{1 + r}}_{\substack{(5)\text{ : expected discounted value} \\ \text{if replacing again}}}\end{aligned} \tag{15}$$

There are two important points about this equation:

(i) The probability of stopping (2) includes the possibility that the smallest distance δ_1 has changed to δ_1', and the expected value if stopped (3) takes this into account.

(ii) The probability of replacing again (4) and the expected discounted value if replacing again (5) build on the fact that repeated replacement can occur when the smallest distance between the workers remained the same (follows from Lemma 1 in the previous section).

We will show that equation (15) can be expressed as

$$\begin{aligned}\overline{V}(\delta_1) \quad = \quad & y - (\frac{1}{2} + \delta_1) \\ & + \frac{(\delta_1 + 2\overline{\delta}_2)y - 2\overline{\delta}_2(2\delta_1 + \overline{\delta}_2) - 2\delta_1^2}{r} \\ & + (1 - \delta_1 - 2\overline{\delta}_2)\frac{\overline{V}(\delta_1) - c}{1 + r}\end{aligned} \tag{16}$$

1. First we show that expected flow output (1) from equation 15 is $y - 2 \cdot E^{|\delta_1}\left(\delta_1' + \delta_2'\right) = y -$ $(\frac{1}{2} + \delta_1 + \frac{\delta_1^2}{2})$. Consider Figure 2. The following is true:

- With probability $2 \cdot \left(\frac{1}{2} - \frac{\delta_1}{2}\right)$ the new worker falls outside the arc between the two incumbents (to the left or to the right), and the expected sum of distances between all workers in this case will be $2\left(\delta_1 + \frac{1}{2} \cdot \left(\frac{1}{2} - \frac{\delta_1}{2}\right)\right)$

- With probability δ_1 the new worker will fall between the two incumbents, and the total sum of distances between all workers will be $2\delta_1$

Summing up, the total expected sum of distances between all workers after replacement is:

$$
\begin{aligned}
2 \cdot E^{|\delta_1}\left(\delta_1' + \delta_2'\right) &= 2 \cdot \left(\frac{1}{2} - \frac{\delta_1}{2}\right) \cdot 2 \cdot \left(\delta_1 + \frac{1}{2} \cdot \left(\frac{1}{2} - \frac{\delta_1}{2}\right)\right) + \delta_1 \cdot 2\delta_1 = \\
&= \frac{1}{2} + \delta_1 + \frac{\delta_1^2}{2}
\end{aligned}
$$

2. Then we show that the probability of stopping (2) and the expected discounted value if stopped (3) in equation 15 above is:

$$
\Pr(\delta_2' \leq \overline{\delta}_2(\delta_1')) \cdot \frac{y - 2 \cdot E^{|\delta_1, \delta_2' \leq \overline{\delta}_2(\delta_1')}(\delta_1' + \delta_2')}{r} = \frac{(\delta_1 + 2\overline{\delta}_2)y - 2\overline{\delta}_2(2\delta_1 + \overline{\delta}_2) - 2\delta_1^2}{r}
$$

- With probability δ_1 the new worker will fall between the two incumbents, in which case the firm will stop. The total sum of distances between the workers in this case will be $2\delta_1$. The expected discounted value in this case will be $\frac{y-2\delta_1}{r}$
- With probability $2\overline{\delta}_2$ the new worker falls outside the two incumbents and below the threshold, and the firm will stop. The expected distance between the new worker and the closest incumbent is $\frac{\overline{\delta}_2}{2}$, so that the expected total sum of distances between the workers in this case will be $2 \cdot \left(\delta_1 + \frac{\overline{\delta}_2}{2}\right)$ .The expected discounted value in this case will be $\frac{y-2\delta_1-\overline{\delta}_2}{r}$

Summing up:

$$
\begin{aligned}
\Pr(\delta_2' &\leq \overline{\delta}_2(\delta_1')) \cdot \frac{y - 2 \cdot E^{|\delta_1, \delta_2' \leq \overline{\delta}_2(\delta_1')}(\delta_1' + \delta_2')}{r} \\
&= \delta_1 \cdot \frac{y - 2\delta_1}{r} + 2\overline{\delta}_2 \cdot \frac{y - 2\delta_1 - \overline{\delta}_2}{r} \\
&= \frac{(\delta_1 + 2\overline{\delta}_2)y - 2\overline{\delta}_2(2\delta_1 + \overline{\delta}_2) - 2\delta_1^2}{r}
\end{aligned}
$$

3. Finally we show that

$$\Pr(\delta_2' > \overline{\delta}_2(\delta_1))\frac{\overline{V}(\delta_1) - c}{1+r} = (1 - \delta_1 - 2\overline{\delta}_2)\frac{\overline{V}(\delta_1) - c}{1+r}$$

This comes from the fact that with probability $(1 - \delta_1 - 2\overline{\delta}_2)$ the new worker is above the $\overline{\delta}_2$ threshold. The firm will keep replacing and pay the cost c again.
We have thus fully derived equation (16).

Let us write:

$$\begin{aligned} & (\delta_1 + 2\overline{\delta}_2)y - 2\overline{\delta}_2(2\delta_1 + \overline{\delta}_2) - 2\delta_1^2 \\ = \quad & (\delta_1 + 2\overline{\delta}_2)(y - 2(\delta_1 + \overline{\delta}_2)) + 2\overline{\delta}_2^2 + 2\delta_1\overline{\delta}_2 \end{aligned}$$

Hence we can re-write (16) as follows:

$$\begin{aligned} \overline{V}(\delta_1) \quad = \quad & y - (\frac{1}{2} + \delta_1 + \frac{\delta_1^2}{2}) \\ & + \frac{(\delta_1 + 2\overline{\delta}_2)(y - 2(\delta_1 + \overline{\delta_2})) + 2\overline{\delta}_2^2 + 2\delta_1\overline{\delta}_2}{r} \\ & + (1 - \delta_1 - 2\overline{\delta}_2)\frac{\overline{V}(\delta_1) - c}{1+r} \end{aligned} \tag{17}$$

Substituting out $\overline{V}(\delta_1)$ and using (14), gives the rule (see Appendix B for details):

$$c + \frac{1}{2} + \frac{\delta_1^2}{2} - \delta_1 - 2\overline{\delta}_2 = \frac{2\delta_1\overline{\delta}_2 + 2\overline{\delta}_2^2}{r} \tag{18}$$

This cut-off rule has a very intuitive interpretation:

The LHS of (18) represents net costs of replacing, evaluated at the threshold ($\overline{\delta}_2$). If not replacing the worker, the total distance is given by $2(\delta_1 + \ \overline{\delta}_2)$.When replacing the worker, the firm expects to have a distance of $\frac{1}{2} + \delta_1 + \frac{\delta_1^2}{2}$,(see derivation of equation 16 above). The firm pays c when replacing the worker. So the net costs are $c+$ the expected total distance with replacement less the total distance without replacement. The net costs are thus

$$c + \frac{1}{2} + \frac{\delta_1^2}{2} + \delta_1 - 2(\delta_1 + \overline{\delta}_2) = c + \frac{1}{2} + \frac{\delta_1^2}{2} - \delta_1 - 2\overline{\delta}_2$$

which is the LHS of (18).

The RHS of (18) represents the gains from replacement associated with lower costs in all future periods if the draw is good.

With probability δ_1 the new worker will be between the two existing workers who have a distance of δ_1 between them. The total distance between the three workers is $2\delta_1$. Existing total distance is $2(\delta_1+\overline{\delta}_2)$, and the savings in distance is thus $2\overline{\delta}_2$. Multiplying this with the probability of the event, δ_1, gives the first term in the nominator of the RHS of (18).

With probability $2\overline{\delta}_2$ the worker is not between the existing workers but within a distance of $\overline{\delta}_2$ from one of them. The expected distance of the new worker to the nearest existing worker is $\overline{\delta}_2/2$ and to the other existing worker it is $\delta_1 + \overline{\delta}_2/2$. The per period cost savings is thus

$$2(\delta_1 + \overline{\delta}_2) - [\delta_1 + \frac{\overline{\delta}_2}{2} + (\delta_1 + \frac{\overline{\delta}_2}{2})] = \overline{\delta}_2$$

Multiplying this with the probability of the event $2\overline{\delta}_2$ gives the second term of the RHS of (18).

We see from equation (18) that an increase in δ_1 reduces the net cost of replacing (reduces the left-hand side) and increases the gain of replacement (the right-hand side) This means that the higher is δ_1 the worse is the team and the more the firm is willing to replace. Thus $\overline{\delta}_2(\delta_1)$ is declining, as shown previously. The intuition for optimal behavior is simple. The gain from replacing is higher the higher is δ_1 (for a given δ_2), as the higher is the probability that an improvement will take place, and the higher is the expected gain given that an improvement takes place.

Not surprisingly, the optimal stopping rule is independent of the productivity level $\tilde{y}$, and hence also of the wage level W. For later reference we formulate this as a corollary

Corollary 4 *The optimal stopping rule is independent of the wage level and the overall productivity of the firm*

3.3 Turnover Dynamics With Optimal Stopping

The following figure illustrates this optimal behavior:

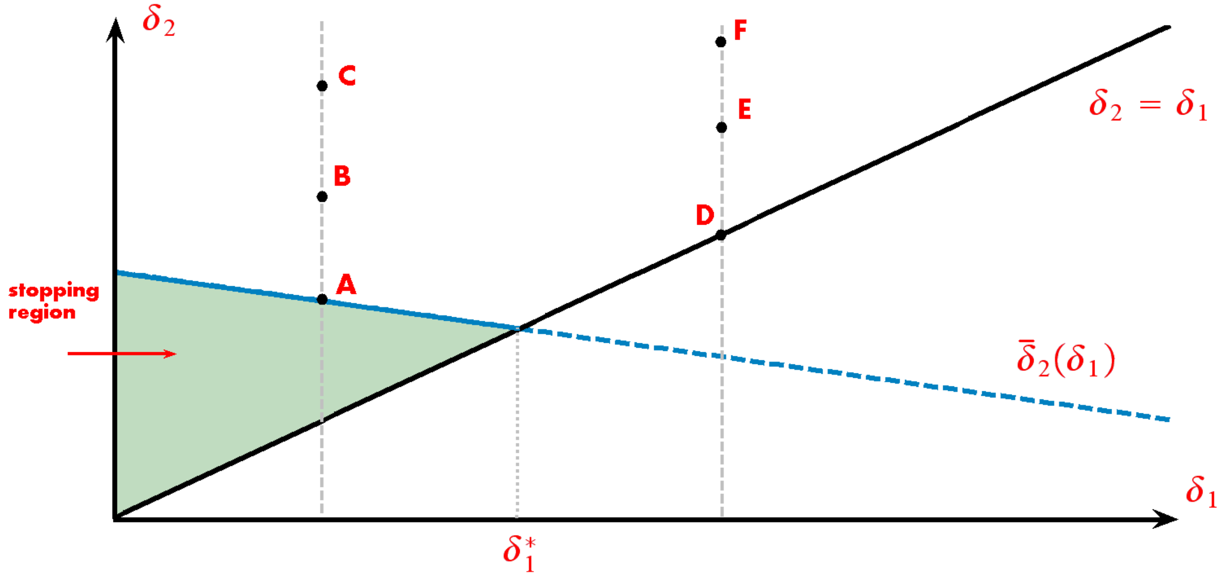

Figure 4: Optimal Policy

The space of the figure is that of the two state variables, δ_1 and δ_2. The feasible region is above the 45 degree as $\delta_2 \geq \delta_1$ by definition. The downward sloping line shows the optimal replacement threshold $\overline{\delta}_2$ as a function of δ_1.

With the replacement of a worker, the firm may move up and down a vertical line for any given value of δ_1 (such as movement between A, B and C or between D, E and F). If the replacement implies a lower value of δ_1, this vertical line moves to the left. This is what happens till the firm gets into the absorbing state of no further replacement in the shaded triangle formed by the $\delta_1^* = \overline{\delta}_2(\delta_1^*)$ point, the intersection of $\overline{\delta}_2(\delta_1)$ line with the vertical axis, and the origin ($\delta_1 = \delta_2 = 0$).

The following properties of turnover dynamics emerge from this figure and analysis:

(i) At the NE part of the $\delta_1 - \delta_2$ space, δ_1, δ_2 are relatively high, output is low, and the firm value is low. Hence the firm keeps replacing and there is high turnover. Note that some workers may stay for more than one period in the firm when in this region. The dynamics are leftwards, with δ_1 declining, but δ_2 may move up and down.

(ii) Above the $\overline{\delta}_2(\delta_1)$ threshold, left of δ_1^*, newcomers may still be replaced, but veteran workers are kept.

(iii) In the stopping region there is concentration at a location which is random, with a flavor of New Economic Geography agglomeration models. Thus firms specialize in the sense of having similar workers. There is no turnover, and output and firm values are high.

(iv) Policy may affect the regions in $\delta_1 - \delta_2$ space via its effect on c. The discount rate affects the regions as well.

(v) These replacement dynamics imply that the degree of complementarity between existing workers may change. This feature is unlike the contributions to the match of the agents in the assortative matching literature, where they are of fixed types.

Our main purpose in this paper is to study replacement, and this can be done in partial equilibrium. Still, for completeness we demonstrate in the appendix how the model can be closed by endogenizing the wage w (in the case where w is a competitive wage and not a bargained wage) and pin it down by a free entry condition.

4 Wage bargaining

In this section we assume that wages are determined by bargaining. In each period, after the worker type is observed, the agents bargain over wages. In order to make the bargaining game tractable, we make some simplifying assumptions. First, we assme that the bargaining games in different periods are independent. Hence, bargaining in a given period is essentially over the output in that period. We also assume that the replacement decision is determined unilaterally by the firm, like in the standard right-to-manage model of employment and wage bargaining. Since wages in future periods are independent of wages in the current period, there is no reason why the firm should let current wages influence the replacement decision. We assume that the middle man is chosen before the bargaining starts.

Suppose first that the output of the bargaining game coincide with the agents' Shapley values.[6] The value of a any subset of the three workers without the firm is zero. Given our production function, the value of one worker and a firm is zero. The value of having two workers ij is y_{ij}. With all workers present, the output is given by $y_{12} + y_{13} + y_{23}$. In the appendix

[6] Brugerman et al (2017) show that the Shapley values are the limit equilibrium pay-offs in the Rolodex game. In this game, the firm bargains with the workers sequencially. If a proposal is rejected, the worker is placed at the end of the queue of workers unless the match is destroyed for exogenous reasons. They show that as the probability of exogenous destruction goes to zero, the pay-offs of the Rolodex game converges to the Shapley values.

we show that the Shapley value of the firm is $(y_{12} + y_{13} + y_{23})/3 = Y/3$, while the Shapley value of worker i is $\sum_{j=1,j\neq i} y_{ij}/3$. The per period profit of the firm is thus (from 6)

$$\pi^S(\delta_1, \delta_2) = \frac{1}{3}[\tilde{y} - 2(\delta_1 + \delta_2)]$$

[7]

In addition to, or as a generalization of the Shapley values, we will analyze the following wage bargaining game. Suppose the firm has three agents that bargain which each of the workers separately, without getting information on the bargaining outcome in the other games. Hence in each game, the agent and the worker bargain under the presumption that the bargaining outcome in the other bargaining games will be the equilibrium outcome. If an agreement is not reached, the worker will not receive any income that period, while the replacement decision and the pay-off in the following periods will be unaffected. Without loss of generality, let worker 2 be the middle man, worker 1 the worker closest to the middle man, and worker 3 the worker most distant to the middle man. Define the marginal value of worker i, S_i, as the value of production with all workers present less the value of production with all workers but worker i present. The marginal contribution is thus $y_{12}+y_{13}$ for worker 1, and defined accordingly for worker 2 and 3. Hence the It follows that (recall that y is output net of wages and $\tilde{y}$ output not subtracting wages when distances are zero)

$$\begin{aligned}
S_1 &= [\tilde{y} - 2(\delta_1 + \delta_2)] - [y_{23} - \delta_2] = \tilde{y} - y_{23} - 2\delta_1 - \delta_2 = \frac{2}{3}\tilde{y} - 2\delta_1 - \delta_2 \\
S_2 &= [\tilde{y} - 2(\delta_1 + \delta_2)] - [y_{13} - \delta_1 - \delta_2] = \tilde{y} - y_{13} - \delta_1 - \delta_2 = \frac{2}{3}\tilde{y} - \delta_1 - \delta \\
S_3 &= [\tilde{y} - 2(\delta_1 + \delta_2)] - [y_{12} - \delta_1] = \tilde{y} - y_{12} - \delta_1 - 2\delta_2 = \frac{2}{3}\tilde{y} - \delta_1 - 2\delta_2
\end{aligned}$$

Denote by α the share of the surplus that accrue to the worker in each period. Hence the wage in each period is given by

$$w_i = \alpha S_i$$

It follows that wages are highest for the middle man, second highest for worker 1 and lowest for worker 3. Note also that the sum of the surpluses

[7] An interpretation is as follows: the firm has three agents that bargain which each of the workers separately, without getting information on the bargaining outcome in the other games. Hence in each game, the agent and the worker bargain under the presumption that the bargaining outcome in the other bargaining games will be the equilibrium outcome.

may be larger than total output. This is most clearly seen when $\delta_1 = \delta_2 = 0$. Then the sum of the surpluses is equal to $2\tilde{y}$, while total output is only $\tilde{y}$. We require $\alpha < 1/2$ to ensure that the firm gets a positive profit in total. Note that the pay-off to the firm is equal to the Shapley value when $\alpha = 1/3$

By summing up the surpluses, it follows that the per period profit of the firm now can be written as

$$\begin{aligned} \pi &= (1-2\alpha)\tilde{y} - (1-2\alpha)[2(\delta_1+\delta_2)] \\ &= (1-2\alpha)[\tilde{y} - 2(\delta_1+\delta_2)] \end{aligned}$$

It follows that the replacement model with bargaining is isomorphic to a repacement model without bargaining, but with the productivity $\tilde{y}$ instead of y and the cost of mismatch to be scaled down with a factor of $(1-2\alpha)$. The next proposition follows immediately:

Proposition 5 *Suppose wages are determined by bargaining as described above, with the workers' bargaining power is given by α. Then the firms' replacement behaviour is identical to the firm's replacement behaviour when the workers' bargaining power is 0 and the replacement cost is $c/(1-\alpha)$.*

To see this, note that the value function with bargaining reads (analogous to (13)), and with topscript α denoting bargaining power

$$\begin{aligned} V^{\alpha}(\delta_1,\delta_2) &= (1-\alpha)[y-2(\delta_1+\delta_2)] + \max[(1-\alpha)\frac{y-2(\delta_1+\delta_2)}{r}, \frac{\overline{V}(\delta_1)-c}{1+r}] \\ &= (1-\alpha)\left\{y-2(\delta_1+\delta_2)] + \max[\frac{y-2(\delta_1+\delta_2)}{r}, \frac{\overline{V}(\delta_1)-c/(1-\alpha)}{1+r}]\right\} \end{aligned}$$

which is identical to the value function (13) when the worker's bargaining power is zero and the replacement cost is $c/(1-\alpha)$ except for the scaling parameter $(1-\alpha)$, which does not influence the replacement decision. The proposition thus follows.

From equation (18) it follows that the replacement threshold $\bar{\delta}_2(\delta_1)$ is decreasing in c; the higher is the replacement cost, the less the firm replaces. The corollary follows imediately:

Corollary 6 *Suppose $c > 0$. Then an increase in the worker's bargaining power α reduces the replacement threshold $\bar{\delta}_2(\delta_1)$, and hence leads to less replacement. If $c = 0$, then $\bar{\delta}_2(\delta_1)$, and hence the firm's replacement behaviour, is independent of α.*

The intuition for the result is straight-forward. With wage bargaining, wages will increase if the match quality increases. Hence the employees (although not necessarily the current ones) get a share of the surplus if the match improves, while the firm pays the entire replacement cost c. This is akin to a hold-up problem, and the firm "underinvest" (replace too little) in the presence of bargaining. A conjecture of our model is thus that there will be less replacement in countries in which worker empowerment is strong.

If $c = 0$, there is no direct cost of replacement. In this case the cost of replacement is the expected lower match quality next period only. However, this cost is also scaled down by bargaining. Therefore the replacement strategy is not distorted in this case.

5 Exogenous Replacement

We now allow, with probability λ, for one worker to be thrown out of the relationship at the end of every period. If the worker is thrown out, the firm is forced to search in the next period.[8] Thus, if the replacement shock hits, one of the workers, chosen at random, has to be replaced. The firm can only hire one worker in any period, and hence will not voluntarily replace a second worker if hit by a replacement shock. If the shock does not hit, the firm may choose to replace one of its workers or not. We retain the assumption that wages are exogenous to the firm.

Suppose one worker is replaced by the firm as above. The transition probability for (δ_1, δ_2) was denoted by $\Gamma(\delta_1)$, and depends only on δ_1. We refer to this as the basic transition probability.

The forced transition probabilities are the transition probabilities which occur when one worker is forced to leave, to be denoted by $\Gamma^F(\delta_1, \delta_2)$. Which of the three incumbent workers leaves is random: with probability 1/3 the least well located worker leaves, in which case the transition probability is $\Gamma(\delta_1)$; with probability 1/3, the second best located worker leaves, in which case the transition probability is $\Gamma(\delta_2)$; with probability 1/3, the best located worker leaves, in which case the distance between the two remaining workers is $\min[\delta_1 + \delta_2, 1 - \delta_1 - \delta_2]$. It follows that the forced transition probabilities can be written as

$$\Gamma^F(\delta_1, \delta_2) = \frac{1}{3}\Gamma(\delta_1) + \frac{1}{3}\Gamma(\delta_2) + \frac{1}{3}\Gamma(\min[\delta_1 + \delta_2, 1 - \delta_1 - \delta_2]) \qquad (19)$$

[8] With minor adjustments of the model, replacement can be interpreted as a change of position on the circle of one worker, due to learning to work better with other workers or, the opposite, the “souring” of relations.

With exogenous replacement, the probability distributions for δ_1' and δ_2' depend on both δ_1 and δ_2, not just δ_1 as above. The Bellman equation reads:

$$\begin{aligned} V(\delta_1, \delta_2) &= \pi(\delta_1, \delta_2) + \beta[\lambda E^{\Gamma^F} V_1(\delta_1', \delta_2') - c] \\ &\quad +(1-\lambda)\beta \max[V(\delta_1, \delta_2), \bar{V}(\delta_1) - c] \end{aligned} \tag{20}$$

The first term in the bracket shows the expected NPV of the firm if the firm is hit by a replacement shock. The second term in the bracket shows the expected NPV if the firm is not hit by a replacement shock. It follows directly from Proposition 4 in Stokey and Lucas (1989, p. 522) that the value function exists. Furthermore, due to continuity, we know that the optimal replacement strategy can be characterized by an optimal stopping rule provided that λ is small.

6 The Model in the Context of the Literature

The paper bears (limited) similarity to Kremer's (1993) O-ring production function model. The similarity pertains to the importance attributed to the idea of workers working well together. In that model firms employ workers of the same skill and pay them the same wage. In this set-up quantity cannot substitute for quality. But the models differ in their treatment of the matching of workers: in Kremer (1993) there is a multiplicative production function in workers/tasks and this underlies their complementarity. In the current paper there is explicit modelling of the match between workers, formalized as random state variables, which realization elicits the firm's optimal worker replacement policy.

The paper stresses the role of horizontal differences in worker productivity, as opposed to vertical, assortative matching issues. The literature on the latter – see the prominent contributions by Eeckhout and Kircher (2010, 2011), Shimer and Smith (2000), and Teulings and Gautier (2004)), and the overview by Chade, Eeckhout, and Smith (2016) – deals with the matching of workers of different types. Key importance is given to the vertical or hierarchical ranking of types. These models are defined by assumptions on the information available to agents about types, the transfer of utility among workers (or other mating agents), and the particular specification of complementarity in production (such as supermodularity of the joint production function). In the current paper, workers are ex-ante homogenous, there is no prior knowledge about their complementarity with other workers before joining the firm, and there are no direct transfers between them. In similar vein, the models of Garicano and Rossi-Hansberg (2006) and Caliendo and Rossi-Hansberg (2012), whereby agents organize production by matching with others in knowledge hierarchies, stresses the vertical dimension of worker communication. In terms of those models, the current paper is relevant for the modelling of team formation at a particular hierarchical level. Thus these approaches are complementary to ours.

The paper has points of contact with papers in the search literature. We exploit the idea of optimal stopping, as in McCall (1970) and the rich strand of search literature which followed (see McCall and McCall (2008), in particular chapters 3 and 4, for a comprehensive treatment). The existing literature does not cater, however, for the key issue examined here, namely that of worker complementarities. Conceptually this is an important distinction, and it allows us to analyze team formation in detail. Technically it also gives rise to new challenges. Total match quality (or output) depends on two variables that are stochastic ex ante, the distances from the best placed worker to each of her two co-workers. At the same time the firm replaces only one worker at a time. This creates a new dimension to the optimal stopping problem, which, in contrast to most earlier studies, now depends on a state variable (the distance between the two closest workers who are not replaced in a given round). Furthermore, optimal stopping behaviour depends on this state variable in a non-trivial way, and it is not even obvious from the outset that a simple optimal stopping rule exists.

Our paper shares some features with the search model of Jovanovic (1979 a,b): there is heterogeneity in match productivity and imperfect information ex-ante (before match creation) about it; these features lead to worker turnover, with good matches lasting longer.[9]But it has some important differences: the Jovanovic model stresses the structural dependence of the separation probability on job tenure and market experience. There is growth of firm-specific capital and of the worker's wage over the life cycle. In the current model the workers do not search themselves and firms do not offer differential rewards to their workers. But the Jovanovic model does not cater for the key issue here, namely that of worker complementarities.

Burdett, Imai and Wright (2004) analyze models where agents search for partners to form relationships and may or may not continue searching for different partners while matched. Both unmatched and matched agents

[9]Pissarides (2000, Chapter 6) incorporates this kind of model into the standard DMP search and matching framework, keeping the matching function and Nash bargaining ingredients, and postulating a reservation wage and reservation productivity for the worker and for the firm,respectively.

have reservation match qualities. A crucial difference with respect to the current set-up is that they focus on the search decisions of both agents in a bi-lateral match and stress the idea that if one partner searches the relationship is less stable, so the other is more inclined to search, potentially making instability a self-fulfilling prophecy. They show that this set-up can generate multiple equilibria. In the current paper we do not allow for the workers themselves to search but rather focus on the main issue, which is optimal team formation through search by firms.

7 Discussion of the Model

Our model builds on several strong assumptions regarding technology, wage determination, search behaviour, etc. We turn now to a brief discussion of these assumptions in light of the analysis.

One important underlying assumption is that workers are horizontally but not vertically differentiated. From an *ex ante* perspective, workers are identical, while *ex post* the workers may work more or less well together. Our assumption reflects a view that an interesting part of team formation is related to horizontal differences, i.e., finding workers who work particularly well together. Of course finding the correct mix of workers with respect to productivity (ability, "types") is also important. As shown in the literature review, there exists a substantial literature on vertical worker heterogeneity and search. We view our contribution as complementary to this literature.

Our second assumption is the use of the Salop circle as the set of possible worker locations. The main reason why we use the Salop circle is that it conveniently allow the distances from a given worker to a randomly placed co-worker to be independent of the worker's location. Hence, this modelling technique readily implies that the workers' location, *ex ante*, does not influence his expected contribution to a team. As already indicated in the text, this property does not carry over to a location on a line segment. A worker located close to the middle of the line will on average be closer to randomly allocated co-workers than a worker located close to the an end point. In addition, the Salop circle easily captures the notion that if A works well with B and B with C, then A and C are also likely to work well together. There may exist other stochastic structures that capture the same type of regularities, but the Salop structure does so in a particularly nice and tractable way. Note that we could alternatively let output depend positively on the difference between the workers, in order to capture a love of variety. To some extent this may be a matter of interpretation of what a good match is.

As indicated in the text, another representation which qualitatively captures the same properties are $n-1$ dimensional spheres in n-dimensional Euclidean space. With this model formulation, the distribution of distances of a new worker will be non-linear. More importantly, it may be convenient to choose a higher-dimensional location space if the number of workers in the team exceeds 3. In a two-dimensional space, it is not clear which of four workers are more peripheral. On a two-dimensional sphere, there are ways to deal with this, for example by defining closeness as the area of a circle on the sphere that contain all three locations. However, it is beyond the scope of this paper to explore these issues further.

We assume that wages are independent of match quality. As mentioned above, this is consistent with a competitive market where firms bid for ex ante identical workers prior to knowing the match quality. An alternative formulation would be to allow for bargaining, in which case part of the surplus from a good match would be allocated to the worker. This will give rise to a hold-up problem, if the firm pays the entire cost of replacing the worker and only gets a fraction less than one of the return in terms of a better match. The effect will be equivalent to reducing the circumference with a fraction equal to the workers' bargaining power, and can hence be easily captured within our framework. The effect will, naturally, be less replacement. In addition, if the firm is unable to extract the rents going to workers *ex ante* through a lower fixed wage, this rent will have to be dissipated in some other way, for instance through unemployment as in Shapiro and Stiglitz (1984) and Moen and Rosen (2006). Hence our model in this case may link worker replacement to the unemployment level. Furthermore, in the present version of the model, workers have no incentives to do on-the-job search, as wages are the same across firms. With wage bargaining, workers may have an incentive to search for a new job, and bargaining may therefore lead to on-the-job search.

Throughout we have assumed that the efficiency of a given team stays constant over time. Although a natural assumption as a starting point, one may think that the quality of a team may develop over time. As the workers get to know each other better, their ability to communicate and collaborate may improve. On the other hand, good relationships may sour over time. Introducing dynamics of team quality may lead to interesting hiring patterns. For instance, a firm that has been passive for a while may start a replacement frenzy if the relationship suddenly sours. This is on our agenda for future research.

8 Illustrative Simulations: Exploring the Mechanisms

We undertake simulations in order to explore the mechanisms inherent in the model. This gives a sense of the model's implications for worker turnover, firm age, firm value and the connections between them, revealing rich patterns. In particular, we examine the properties of the resulting firm value distributions and relate them to replacement policy. The dynamic evolution of these variables is due to both the random draw of workers and the firm's optimal replacement policy. The interaction of worker draws, exogenous shocks and firm policy is not trivial and generates non-normal firm value distributions. We explain the properties of these distributions, as expressed by their first four moments, in terms of the mechanisms of the model.

When simulating we look at the full model, with both endogenous and exogenous replacement and allowing for exogenous firm exit. As in the previous section, the value function is given by (20). Let $\overline{\beta}$ denote the pure time preference factor, where $\beta = \overline{\beta}(1 - s)$. This value function can be found by a fixed point algorithm. Appendix D provides full details. When simulating firms over time, we use the value function formulated above. We simulate 1000 firms over 30 periods, and repeat it 100 times to eliminate run-specific effects. In the benchmark case, we set: $y = 1, c = 0.01, \overline{r} = 0.04$ (the pure discount rate), $\lambda = 0.1, s = 0.1$.

8.1 The Distribution of Firm Values

Plotting the simulated values of (V, δ_1, δ_2) space, as in Figure 4, one gets:

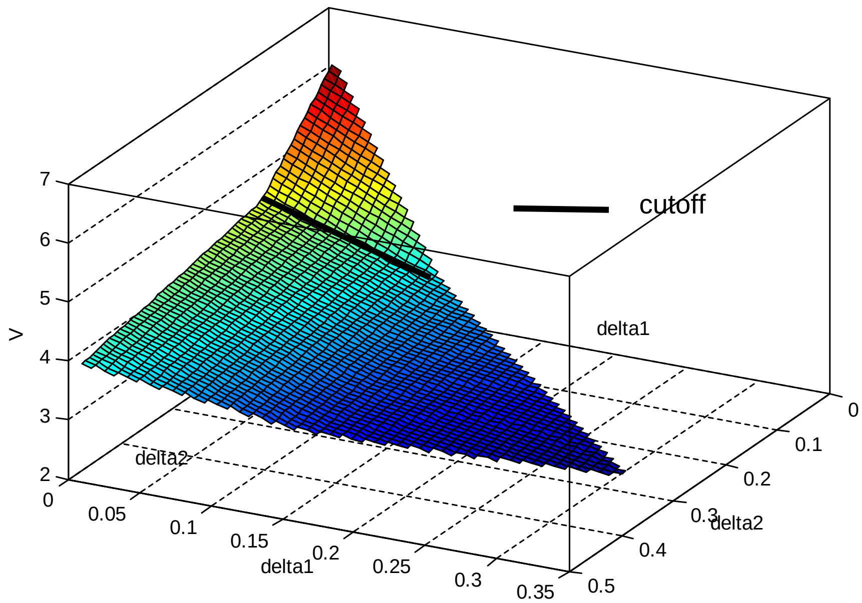

Figure 5: Simulated V, δ_1, δ_2

Figure 5 shows the results looking from the NE of Figure 4 towards the stopping region in the SW, beyond the black cutoff line of the optimal stopping rule $\overline{\delta}_2\ (\delta_1)$. The figure shows the concentration of high values in the stopping region, where the slope is quite steep and where maximum value is 6.21 with $\delta_1 = \delta_2 = 0$ and $V = \frac{y}{r}(1+r))$. It also shows the large dispersion in the low value region at the front of the figure, where the slope is relatively flat. Minimum value is computed numerically to be 2.51 with $\delta_1 = \delta_2 = 1/3$.In what follows, the latter region will show up as the long tail of the lower part of the cross-sectional value distribution

8.2 Firm Value and Age

Figures 6 show firm value distributions and their moments by firm age.[10]

[10] To construct the distributions of firm value by age we looked for all periods and all firms, when each particular age was observed. For example, due to the firm exit shock and the entry of new firms, age 1 will be observed not only for all firms in the first period, but also in all cases when a firm exogenously left and was replaced by a new entrant. In

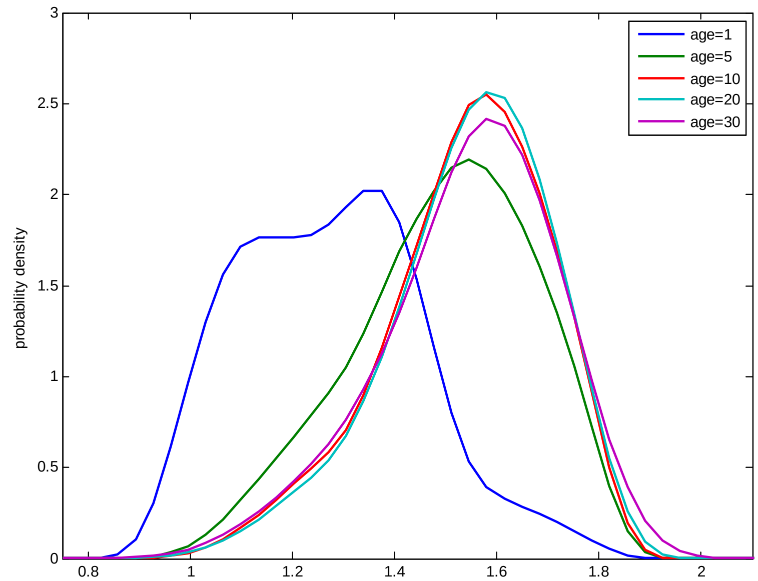

Figure 6a: Cross-sectional log firm values, by age

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this manner we gathered observations of values for all ages, from 1 to 30, and built the corresponding distributions.

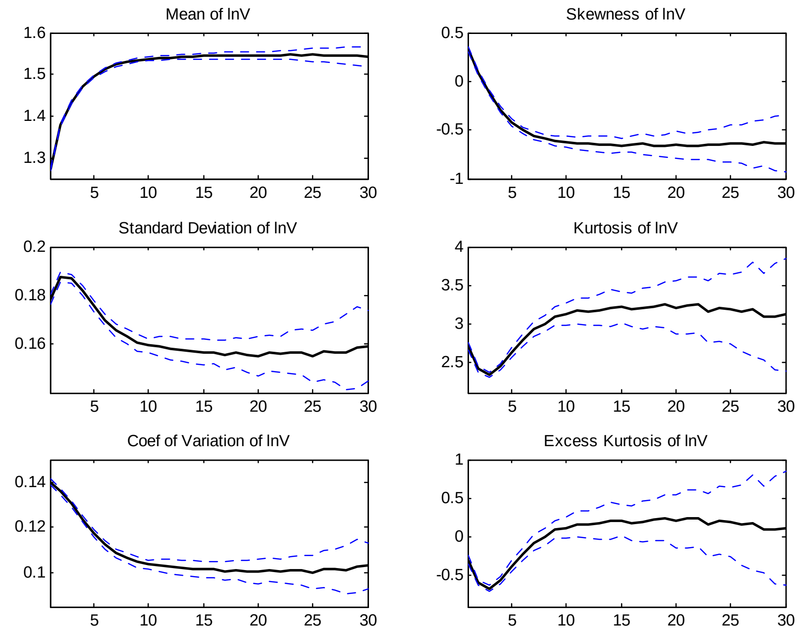

Figure 6b: Moments of cross-sectional log firms value, by age

The patterns reflect the pure process of convergence, disrupted from time to time by workers' exogenous exits, without the entry of new-born firms. The value of the firm grows with age as a result of team quality improvements, while the standard deviation is rather stable. As firms mature, more of them enter the absorbing state, with relatively high values, and at the same time there are always unlucky firms that do not manage to improve their teams sufficiently, or which have been hit by a forced separation shock. Therefore the distribution becomes more and more skewed over time. Excess kurtosis fluctuates.

These turnover dynamics of the model are very much in line with the findings in Haltiwanger, Jarmin and Miranda (2013), whereby, for U.S. firms, both job creation and job destruction are high for young firms and decline as firms mature.

We run a regression of the simulation data to further study the connection between firm value and firm age. Here we look only at a simulated subsample of firms which have survived until the 30th period. There have been 45 such firms in our simulation. The estimated equation is:

$$\overline{\ln(V)}_t = c_0 + c_1 * \ln(t) \tag{21}$$

where $\overline{\ln(V)}_t$ is the average logged value of firms at age t, $t = 1, 2, ..., 30$. The results are presented in Table 1:

Table 1

The Relation Between Firm Value and Age

Regression Results of Simulated Values

c_1	0.05
	(0.01)
c_0	1.37
	(0.02)
R^2	0.62

The coefficients are highly significant and imply a positive relation, illustrated below:

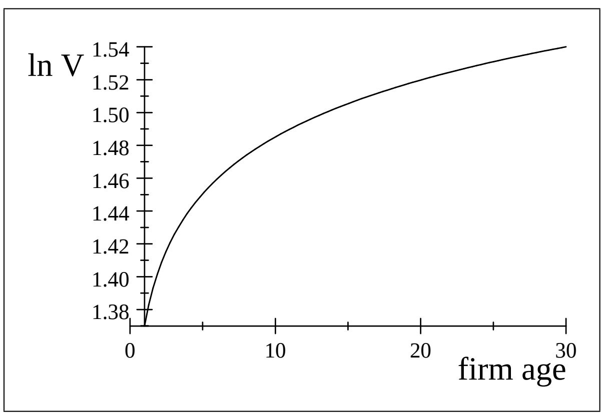

Figure 7: Predicted firm value (logs) and firm age

Figure 7 shows that overall, despite exogenous separation shocks, firms tend to increase in value as they mature, due to the improvement of their teams' quality. This is in line with the findings of Haltiwanger, Lane and Spletzer (1999) whereby productivity rises with age for U.S. firms in Census Bureau data, covering the period 1985-1996.

8.3 The Role of Model Parameters

The core parameters of the model at the benchmark are the worker replacement cost, $c = 0.01$, the annual rate of interest, $r = 0.04$, the exogenous worker replacement rate, $\lambda = 0.1$, and the exogenous firm destruction rate, $s = 0.1$. In addition, we set the flow output at $y = 1$. Changes in these parameters affect the values of the firms both directly, through the value function and exogenous random events, and indirectly, through adjustments in the optimal hiring decisions. In what follows we analyze changes in these core parameters.[11]

The following patterns emerge:

(i) Increases in the cost of replacement c or in the interest rate r are illustrated in Figure 8a (and reported in rows 2-6 of Appendix E Table E1).

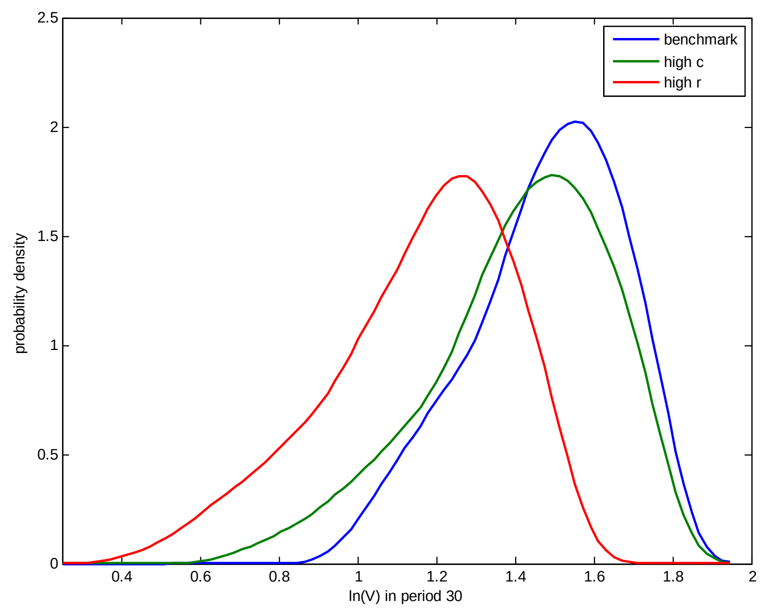

Figure 8a: effects of c and r

These two different increases affect the values distribution similarly: the mean value goes down, the coefficient of variation goes up, skewness becomes more negative and excess kurtosis goes up from negative to positive. Both higher costs of replacement and costs of time make the firms retain their teams rather than improve them; firms enter the stopping region more quickly, with worse teams than before and the mean value goes down.

[11] Table E1 in Appendix E presents the moments of the log firm value distributions for the changes in the parameter values analyzed here, relative to their benchmark values.

As firms tend to stay with their current, randomly-drawn, teams, firm values become more dispersed. Along the same lines, extreme values become relatively more frequent and excess kurtosis goes up. As inaction becomes optimal for so many firms, firms values become more concentrated above the mean. At the same time, in any period there are always unlucky firms, which have just obtained a very bad team as a result of the λ or s shock. Hence skewness becomes more negative. The sensitivity to the interest rate is higher than to changes in replacement costs. Thus, under higher c or higher r the distribution has a longer left tail, lower mean, and fatter and longer tails relative to the benchmark.

(ii) Increases in the exogenous worker separation rate λ are illustrated in Figure 8b (and reported in rows 7-9 of Appendix Table E1).

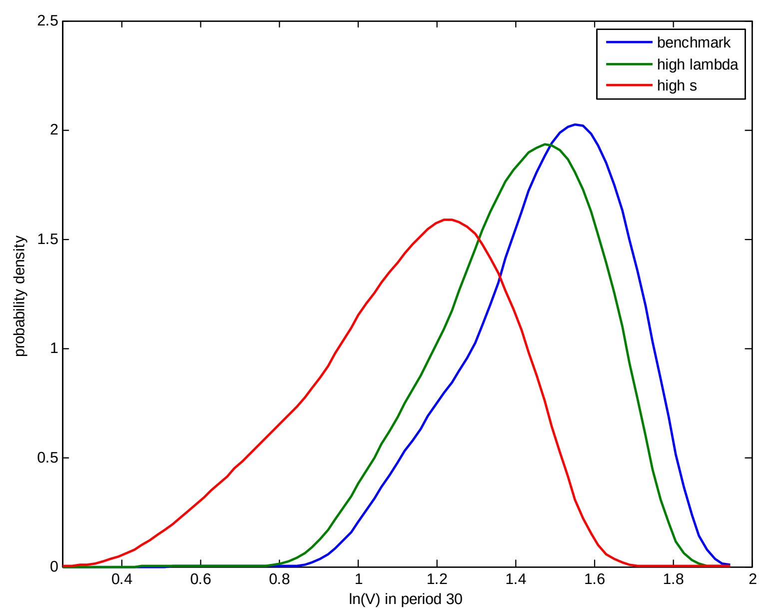

Figure 8b: effects of λ and s

Increased separation depresses the mean value, slightly increases the coefficient of variation, make the skewness less negative and kurtosis more negative. The possibility of a worker's exogenous exit is a burden on the firms, limiting their control over teams and the possibility to improve them. Hence the decrease in mean value. With optimization repeatedly disrupted by the shock, less firms are able to achieve the high-value steady state in each given period, there are less values concentrated above the mean, and skewness becomes less negative. Kurtosis becomes more negative as λ grows, implying that the bulk of the dispersion now comes from moderate deviations from the mean. Such a separation shock may hit any firm, occasionally throwing some firms out of the stopping region, or bringing other firms into it; the sample becomes more homogenous in terms of values, with extreme deviations from the mean less frequent, hence the negative excess kurtosis.

(iii) The simulated increases in the exogenous firm destruction rate s, also shown in Figure 8b, as well as in rows 10-12 in Appendix Table E1, brings the mean value down, raises the coefficient of variation, and skewness becomes more negative while kurtosis becomes less negative. As there is a positive probability for any firm of being closed down in the next period, and due to the constant inflow of new-born firms which have not yet started to improve their teams, the mean value in the simulated cross-section goes down as s goes up. The inflow of random worker triples increases dispersion drastically, so the coefficient of variation goes up. As there are less firms in the stopping region and extreme values become more frequent, excess kurtosis goes up. The inflow of new firms with all kinds of values, including extremely low ones, makes the left tail of the distribution longer and skewness more negative.

(iv) Going the other way and shutting down exogenous worker separation and firm destruction, $\lambda = s = 0$, presented in row 13 of Table E1, has firms just smoothly converge to the stopping region. Removing exogenous uncertainty improves the mean value drastically and it is higher than in any other specification. The coefficient of variation is low, as a result of massive convergence. Likewise, excess kurtosis is substantially negative. Skewness is slightly negative as there is no drag on value as a result of some unlucky firms being hit by a shock or replaced, with all the firms allowed to converge (and they do so by period 30).

To sum up, each of the parameters above has an impact on the process of convergence into the stopping region. The factors that facilitate stopping, such as high c and r or low λ produce higher concentration of firms in the stopping region and therefore make skewness more negative. The replacement of old firms by new ones does not impact the process of convergence directly. It adds new triples everywhere, thereby lengthening the left tail of the distribution and adding more extreme values – skewness becomes more negative and excess kurtosis goes up. The factors that impede firms, namely high c, high r, high λ or high s decrease mean firm value. The factors that make the firms stop quickly wherever they are (high c or r), or add new triples exogenously, such as high s, make values more dispersed, distribution tails fatter, and excess kurtosis higher.

9 Conclusions

The paper has characterized the firm in its role as a coordinating device. Thus, output depends on the interactions between workers, with complementarities playing a key role. The paper has derived optimal policy, using a threshold on a state variable and allowing for endogenous hiring and firing. Firm value emerges from optimal coordination done in this manner and fluctuates as the quality of the interaction between the workers changes. Simulations of the model generate non-normal firm value distributions, with negative skewness and negative excess kurtosis. These moments reflect worker turnover dynamics, whereby a large mass of firms is inactive in replacement, having attained good team formation, while exogenous replacement and firm exit induce dispersion of firms in the region of lower value. Future work will examine alternative production functions, learning and training processes, and wage-setting mechanisms within this set-up.

References

[1] Bresnahan, Timothy F., Erik Brynjolfsson and Lorin M.Hitt, 2002. "Information Technology, Workplace Organization, and the Demand of Skilled Labor: Firm-Level Evidence," **Quarterly Journal of Economics** 117,1, 339-376.

[2] Burdett, Kenneth, Imai Ryoichi, and Randall Wright, 2004. "Unstable Relationships," **Frontiers of Macroeconomics** 1,1,1-42.

[3] Caliendo, Lorenzo and Esteban Rossi-Hansberg, 2012. "The Impact of Trade on Organization and Productivity," **Quarterly Journal of Economics** 127,3,1393-1467.

[4] Chade, Hector, Jan Eeckhout and Lones Smith, 2016. "Sorting Through Search and Matching Models in Economics," **Journal of Economic Literature**, forthcoming.

[5] Eeckhout, Jan and Philipp Kircher, 2010. "Sorting and Decentralized Price Competition," **Econometrica**, 78, 539–574

[6] Eeckhout, Jan and Philipp Kircher, 2011. "Identifying Sorting – In Theory," **Review of Economic Studies**, 78 (3), 872-906.

[7] Garicano, Luis and Esteban Rossi-Hansberg, 2006. "Organization and Inequality in a Knowledge Economy," **Quarterly Journal of Economics** 121, 4, 1383-1435.

[8] Garicano, Luis and Yanhui Wu, 2012. "Knowledge, Communication and Organizational Capabilities," **Organization Science** 23, 5, 1382-1397.

[9] Jovanovic, Boyan, 1979a. "Job-Matching and the Theory of Turnover," **Journal of Political Economy** 87,5,1, 972-990.

[10] Jovanovic, Boyan, 1979b. "Firm-Specific Capital and Turnover," **Journal of Political Economy** 87, 6, 1246-1260.

[11] Haltiwanger, John C., Ron S. Jarmin and Javier Miranda, 2013. "Who Creates Jobs? Small vs. Large vs. Young, " **Review of Economics and Statistics** 95,2,347-361.

[12] Haltiwanger, John .C., Julia I. Lane, and James R. Spletzer, 1999. "Productivity differences across employers: The role of employer size, age, and human capital." **American Economic Review Papers and Proceeding**s 99, 94–98.

[13] Hamilton, Barton H., Jack A. Nickerson, and Hideo Owan, 2003. "Team Incentives and Worker Heterogeneity: An Empirical Analysis of the Impact of Teams on Productivity and Participation," **Journal of Political Economy** 111,3,465-497.

[14] Kremer, Michael, 1993. "The O-Ring Theory of Economic Development," **Quarterly Journal of Economics** 108, 3, 551-575.

[15] McCall, Brian P. and John J. McCall. 2008. **The Economics of Search.** Routledge, London and New York.

[16] McCall, John J., 1970. "Economics of Information and Job Search," **Quarterly Journal of Economics** 84(1), 113-126.

[17] Moen, Espen R. and Asa Rosen, 2006."Equilibrium Contracts and Efficiency Wages," **Journal of the European Economic Association** 4(6),1165–1192.

[18] Pentland, Alex, 2012. "The New Science of Building Great Teams," **Harvard Business Review** 90, 4, 60-71.

[19] Pissarides, Christopher A., 2000. **Equilbrium Unemployment Theory**, MIT Press, Cambridge, 2nd edition.

[20] Prescott, Edward C. and Michael Visscher, 1980. "Organization Capital," **Journal of Political Economy**, 88, 3, 446-461.

[21] Salop, Steven C., 1979. "Monopolistic Competition with Outside Goods," **Bell Journal of Economics** 10, 141-156.

[22] Shapiro, Carl and Joseph E. Stiglitz, 1984. "Equilibrium Unemployment as a Worker Discipline Device," **American Economic Review** 74, 3, 433-444.

[23] Shimer, Robert and Lones Smith, 2000. "Assortative Matching and Search," **Econometrica** 68, 2, 343-369.

[24] Stokey, Nancy L. and Robert E. Lucas Jr., 1989. **Recursive Methods in Economic Dynamics**, Harvard University Press, Cambridge, MA.

[25] Teulings, Coen N. and Pieter A. Gautier, 2004. "The Right Man for the Job," **Review of Economic Studies**, 71, 553-580.

10 Appendix A. Solution of the Cut-Off δ^*

In this Appendix we show how to derive δ^*. We repeat the cut-off equation for convenience

$$c + \frac{1}{2} + \frac{\delta_1^2}{2} - \delta_1 - 2\overline{\delta}_2 = \frac{2\delta_1\overline{\delta}_2 + 2\overline{\delta}_2^2}{r} \tag{22}$$

If $\overline{\delta}_2 = 0$, the left-hand side of (22) is strictly positive while the right-hand side is zero (since $\delta_1 \leq 1/3$ by construction). As $\overline{\delta}_2 \to \infty$, the left-hand side goes to minus infinity and the right-hand side to plus infinity. Hence we know that the equation has a solution. Since the left-hand side is strictly decreasing and the right-hand side strictly increasing in $\overline{\delta}_2$, we know that the solution is unique.

In the text we have already shown that $\overline{\delta}_2(\delta_1)$, if it exists, is decreasing in δ_1. It follows that δ^* can be obtained by inserting $\overline{\delta}_2 = \delta_1 = \delta^*$ in (22). This gives

$$c + \frac{1}{2} + \frac{\delta^{*2}}{2} - \delta^* - 2\delta^* = \frac{2\delta^*\delta^* + 2\delta^{*2}}{r} \tag{23}$$

Hence δ^* is the unique positive root to the second order equation

$$c + \frac{1}{2} - \delta^{*2}\frac{8-r}{2r} - 3\delta^* = 0 \tag{22}$$

11 Appendix B. Derivation of Equation (18)

Substituting (14) into (17) gives

$$\begin{aligned}\frac{y-2(\delta_1+\overline{\delta}_2(\delta_1))}{r}(1+r)+c &= y-(\frac{1}{2}+\delta_1+\frac{\delta_1^2}{2}) \\ &+\frac{(\delta_1+2\overline{\delta}_2)(y-2(\delta_1+\overline{\delta_2}))+2\overline{\delta}_2^2+2\delta_1\overline{\delta}_2}{r} \\ &+(1-\delta_1-2\overline{\delta}_2)\frac{y-2(\delta_1+\overline{\delta}_2(\delta_1))}{r}\end{aligned} \tag{25}$$

Collecting all terms containing $y-2(\delta_1+\overline{\delta}_2(\delta_1))$ on the left-hand side gives

$$\begin{aligned}&\frac{y-2(\delta_1+\overline{\delta}_2(\delta_1))}{r}[1+r-(\delta_1+2\overline{\delta}_2)-(1-(\delta_1+2\overline{\delta}_2))]+c-y \\ &= -(\frac{1}{2}+\delta_1+\frac{\delta_1^2}{2})+\frac{2\overline{\delta}_2^2+2\delta_1\overline{\delta}_2}{r}\end{aligned} \tag{26}$$

which simplifies to

$$-2(\delta_1+\overline{\delta}_2(\delta_1))+c=-(\frac{1}{2}+\delta_1+\frac{\delta_1^2}{2})+\frac{2\overline{\delta}_2^2+2\delta_1\overline{\delta}_2}{r} \tag{27}$$

Collecting terms gives

$$c+\frac{1}{2}+\frac{\delta_1^2}{2}-\delta_1-2\overline{\delta}_2(\delta_1)=\frac{2\overline{\delta}_2^2+2\delta_1\overline{\delta}_2}{r} \tag{28}$$

which is equation (18).

12 Appendix C. Proof of Existence of Equilibrium

There are costs $K \geq 3c$ to open a firm. A zero profit condition pins down the wage ($w = \frac{W}{3}$):

$$E^{|\delta_1 \delta_2} V(\delta_1, \delta_2; w; \widetilde{y}, c) = K \tag{29}$$

As we have seen, the hiring rule is independent of w (since it is independent of y). If y is sufficiently large relative to K, we know that $E^{|\delta_1 \delta_2} V(\delta_1, \delta_2; w; \widetilde{y}, c) > K$, and there exists a wage w^* that satisfies (29). We will now give a formal proof of existence, as well as sufficient conditions on the parameters that ensure existence and production in each period.

Define

$$\overline{V} \equiv E^{|\delta_1 \delta_2} V(\delta_1, \delta_2; 0; \widetilde{y}, c)$$

Given our assumption that the firm always produces until it is destroyed, it follows that

$$E^{|\delta_1 \delta_2} V(\delta_1, \delta_2; w; \widetilde{y}, c) = \overline{V} - \frac{W}{r'} \tag{30}$$

where $r' = r/(1+r)$ and where, as above, $W = 3w$. By assumption, $\overline{V} > 0$ (see below). It follows that there exists a unique W that solves the zero-profit condition given by

$$\overline{V} - \frac{W}{r'} = K \tag{31}$$

The solution is given by $W = r'(\overline{V} - K)$.

We will give conditions on parameters that ensure that $\overline{V} > 0$,and that firms, if entering, will produce even after the worst possible draws. The supremum of per-period output is $\widetilde{y}$ (obtained with $\delta_1 = \delta_2 = 0$). It follows that

$$\overline{V} < \frac{\widetilde{y}}{r'}$$

Suppose

$$K > \frac{4}{3} \frac{1}{r'} \tag{32}$$

From the zero profit condition it then follows that

$$W = r'(\overline{V} - K) < \widetilde{y} - 4/3 \tag{33}$$

The infimum of per period profit is $\pi^{\text{inf}} = \widetilde{y} - 4/3 - W$ (obtained when $\delta_1 = \delta_2 = 1/3$). From (33) it follows that

$$\pi^{\text{inf}} = \widetilde{y} - 4/3 - W > 0 \tag{34}$$

Hence a sufficient condition for firms to operate after the lowest possible draws is that (32) is satisfied.

We assume that the lower bound on wages is that $W \geq 0$. To ensure that $\overline{V} > K$, note that

$$\overline{V} > \frac{\widetilde{y} - 4/3}{r'}$$

since $\widetilde{y} - 4/3$ is the lowest per period output and a firm can always choose not to replace. Entry occurs in equilibrium if and only if it is profitable to enter when $W = 0$. Hence a sufficient condition for entry to occur i is that $\frac{\widetilde{y}-4/3}{r'} > K$ or that $\widetilde{y} \geq r'K + 4/3$ (tighter bounds can also be found).

13 Appendix D. Shapley values

The Shapley value of an agent is the agent's expected marginal contribution to output when the agents arrive to the coalition in a random order. The firm will only have a strictly positive marginal contribution if it arrives number 3 or number 4, which both happen with probability $\frac{1}{4}$. If the firm arrives as number 3, its marginal contribution is y_{ij}, where i and j are the identities of the workers already in place. Hence the expected contribution in this event is $\frac{y_{12}+y_{13}+y_{23}}{3}$. If the firm is the last to arrive, its marginal contribution is $y_{12}+y_{13}+y_{23}$. It follows that the Shapley value of the firm is $(\frac{1}{12}+\frac{1}{4})(y_{12}+y_{13}+y_{23}) = \frac{y_{12}+y_{13}+y_{23}}{3}$

For a worker, he will only have a strictly positive marginal contribution if he arrives as number 3 or 4, which both happens with probability 1/4. If the worker arrives as number 3, the firm will be in place with probability 2/3. Consider worker 1. If he arrives as number 3, his expected contribution will be $\frac{y_{12}+y_{13}}{3}$. If he arrives as number 4, his marginal contribution will be $y_{12}+y_{13}$. His Shapley value is thus $\frac{y_{12}+y_{13}}{3}$. More generally, the Shapley value of worker i is $\frac{\sum_{j=1, j\neq i} y_{ij}}{3}$.

As a consistency check, note that if we sum the pay-offs of the three workers and the firm we get that the total pay-off is equal to total pay-off $y_{12}+y_{13}+y_{23}$.

14 Appendix D. The Simulation Methodology

The entire simulation is run in Matlab with 100 iterations. In order to account for the variability of simulation output from iteration to iteration, we report the average and the standard deviation of the moments and the probability density functions, as obtained in 100 iterations.

Calculating the Value Function

We find the value function V numerically for the discretized space (δ_1, δ_2), using a fixed-point procedure. First we guess the initial value for V in each and every point of this two-dimensional space; we then mechanically go over all possible events (exit, in which case the value turns zero, forced or voluntary separation, with the subsequent draw of the third worker) to calculate the expected value in the next period, derive the optimal decision at each point (δ_1, δ_2), given the initial guess V, and thus compute the RHS of the value function equation below:

$$V(\delta_1.\delta_2) = \pi(\delta_1, \delta_2) + \beta \left[s \cdot 0 + (1-s) \cdot \begin{pmatrix} \lambda \cdot \left[E^{\Gamma^F} V(\delta_1', \delta_2') - c \right] \\ +(1-\lambda) \cdot E \max[V(\delta_1, \delta_2), E^{\Gamma} V(\delta_1', \delta_2') - c] \end{pmatrix} \right] \tag{35}$$

Next, we define the RHS found above as our new V and repeat the calculations above. We iterate on this procedure till the stage when the discrepancy between the V on the LHS and the RHS is less than the pre-set tolerance level.

The mechanical steps of the program are the following:

1. We assume that each of δ_1, δ_2 can take only a finite number of values between 0 and 1. We call this number of values BINS_NUMBER and it may be changed in the program.

2. However, not all the pairs (δ_1, δ_2) are possible, as by definition $\delta_2 \geq \delta_1$ and $\delta_2 \leq \frac{1}{2} - \frac{\delta_1}{2}$ (the latter ensures that the distances are measured "correctly" along the circle). We impose the above restriction on the pairs constructed earlier, and so obtain a smaller number of pairs, all of which are feasible. Note that all the distances in the pairs are proportionate to 1/BINS_NUMBER

3. In fact, the expected value of forced and voluntary replacement, $E^{q^F} V(\delta_1', \delta_2')$ and $EV^q(\delta_1', \delta_2')$, differ in only one respect: when the replacement is voluntary, two remaining workers are those with δ_1 between them, whereas when the replacement is forced, it might be any of the three: δ_1, δ_2

or $\min((\ \delta_1+\delta_2),1-(\ \delta_1+\delta_2))$, with equal probabilities. In the general case, if there are two workers at a distance δ, and the third worker is drawn randomly, possible pairs in the following period may be of the following three types: (i) δ turns out to be the smaller distance (the third worker falls relatively far outside the arch), (ii) δ turns out to be the bigger distance (the third worker falls outside the arch, but relatively close) (iii) the third worker falls inside the arch, in which case the sum of the distances in the next period is δ. In the simulation we go over all possible pairs to identify the pairs that conform with (i)-(iii). Note that because all the distances are proportionate to 1/BINS_NUMBER, it is easy to identify the pairs of the type (iii) described above. This can be done for any δ, whether it is δ_1, δ_2 or $\min((\ \delta_1+\delta_2),1-(\ \delta_1+\delta_2))$

4. Having the guess V, and given that all possible pairs are equally probable, we are then able to calculate the expected values of the firm when currently there are two workers at a distance δ. Call this value $EV(\delta)$. Then, if there is a firm with three workers with distances $(\delta_1.\delta_2)$, the expected value of voluntary replacement is $EV(\delta_1)$, and expected value of forced replacement is $1/3{\cdot}EV(\delta_1){+}1/3{\cdot}EV(\delta_2){+}1/3{\cdot}EV(\min((\delta_1+\delta_2),1-(\delta_1+\delta_2)))$. Thus we are able to calculate the RHS of equation (35) above and compare it to the initial guess V.

We iterate the process till the biggest quadratic difference in the values of LHS and RHS, over the pairs (δ_1,δ_2), of equation (35) is less than the tolerance level, which was set at 0.0000001.

Dynamic Simulations

Once the value function is found for all possible points on the grid, the simulation is run as follows.

1. The number of firms (N) and the number of periods (T) is defined. We use $N=1000, T=30$.
2. For each firm, three numbers are drawn randomly from a uniform distribution $U[0,1]$ using the Matlab function *unifrnd.*
3. The distances between the numbers are calculated, the middle worker is defined, and as a result, for each firm a vector (δ_1,δ_2) is found.
4. For each firm, the actual vector (δ_1,δ_2) is replaced by the closest point on the grid found above $\left(\widetilde{\delta}_1,\widetilde{\delta}_2\right)$.

5. According to $\left(\widetilde{\delta}_1, \widetilde{\delta}_2\right)$, using the calculations from previous section, we assign to each firm the value and the optimal decision in the current period.

6. It is determined whether an exit shock hits. If it does, instead of the current distances of the firm, a new triple is drawn in the next period. If it does not, it is determined whether a forced separation shock λ hits. If λ hits, a corresponding worker is replaced by a new draw and distances are recalculated in the next period. If it does not, and it is optimal not to replace, the distances are preserved for the firm in the next period, as well as the value. If it is optimal to replace, the worst worker is replaced by a new one, distances are re-calculated in the next period, together with the value.

Steps 4-6 are repeated for each firm over all periods.

As a result, we have a T by N matrix of firm values. The whole process is iterated 100 times to eliminate run-specific effects. We also record the events history, in a T by N matrix which assigns a value of 0 if a particular firm was inactive in a particular period,1 if it replaced voluntarily, 2 if it was forced to replace, and 3 if it was hit by an exit shock and ceased to exist from the next period on. We use this matrix to differentiate firms by states and to calculate firms' ages.

15 Appendix E. Changes in Parameters

Table E1
The Effects of Changes in Parameters

	Parameters				**Moments of ln(V) in period 30**			
	c	r	λ	s	mean	coef. of var.	skewness	excess kurtosis
1	0.01	0.04	0.1	0.1	1.46	0.13	−0.47	−0.40
2	0.05	–[12]	–	–	1.45	0.14	−0.55	−0.28
3	0.10	–	–	–	1.44	0.16	−0.68	0.06
4	–	0.01	–	–	1.60	0.10	−0.39	−0.53
5	–	0.04	–	–	1.46	0.13	−0.47	−0.40
6	–	0.10	–	–	1.15	0.20	−0.72	0.02
7	–	–	0	–	1.73	0.11	−0.67	−0.04
8	–	–	0.05	–	1.58	0.12	−0.58	−0.27
9	–	–	0.15	–	1.46	0.13	−0.41	−0.48
10	–	–	–	0	2.82	0.02	−0.21	−0.52
11	–	–	–	0.05	1.86	0.07	−0.41	−0.40
12	–	–	–	0.15	1.09	0.22	−0.53	−0.32
13	–	–	0	0	3.11	0.02	−0.12	−0.49

The implications of these changes are discussed in sub-section 5.5.

[12] As in the benchmark, row 1.