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Outside Options in the Labor Market*

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Abstract

This paper develops a method to estimate workers' outside employment opportunities. We outline a matching model with two-sided heterogeneity, from which we derive a sufficient statistic, the "outside options index" (OOI), for the effect of outside options on wages, holding worker productivity constant. The OOI uses the cross-sectional concentration of similar workers across job types to quantify workers' outside options as a function of workers' commuting costs, preferences, and skills. Using German micro-data, we find that differences in options explain 20% of the gender wage gap, and that gender gaps in options are mostly due to differences in the implicit costs of commuting and moving.

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1 Introduction

In standard models of the labor market, wages depend on a worker's outside option. In a perfectly competitive market, an equally attractive outside option always exists, and competition between identical employers leads workers to earn their marginal product. However, in reality, a worker's next best option could require a different combination of their skills, could involve different working hours, or could be located in a different city. The number of outside options could be systematically lower for some workers because of the health or market structure of their local labor market, because they are unwilling or unable to commute, or because their skills are only valuable for a few employers or industries. Such differences could have significant implications for workers' incomes.

A key challenge for empirical research on this topic is that researchers do not typically observe a worker's option set. Researchers often assume that an individual's labor market consists of all jobs within the same commuting zone and industry or occupation (sometimes both). However, even workers in the same firm and occupation may face different option sets due to their specific set of skills, their preferences, or their constraints.

In this paper we develop an empirical procedure to uncover a key latent parameter in most wage-setting models: the value of an individual's option set. We show how this latent parameter can be derived from the cross-sectional concentration of similar workers across jobs. If similar workers are concentrated in a certain region, industry, occupation or other job characteristics, then the worker's options are more limited. We quantify this concentration in a single "outside options index" (OOI), which, in our model, is a sufficient statistic for the effect of outside options on compensation. Using administrative matched employer-employee data from Germany, we estimate the OOI for every worker in our dataset. We estimate the link between outside options and wages using a shift-share ("Bartik") instrument. Combining these two ingredients, we show that differences in outside options explain a substantial portion—one fifth—of the gender wage gap in Germany. This is driven entirely by differences in willingness to commute or move.

We start by outlining a static model of the labor market that illustrates how, with two-sided heterogeneity, differences in outside options lead to differences in compensation, even for equally productive workers (Dupuy and Galichon, 2014). Our model is based on the classic Shapley and Shubik (1971) assignment game—a two-sided matching model with transfers. Compensation in this setting is set to prevent workers from moving to their outside options; because of heterogeneity, workers' compensation is less than their full productivity in the first-best option. A direct implication is that workers' compensation is not only determined by what they produce, but also by their ability to produce in more places. Our focus on heterogeneity, rather than search frictions, is the standard approach in the industrial organization literature for analyzing market imperfections (see, for instance, Berry, Levinsohn, and Pakes, 1995); this approach has recently been adopted to analyze the labor market (Card et al., 2016; Azar, Berry, and I. E. Marinescu, 2019; Chan, Kroft, and Mourifié, 2019).[1]

We derive a sufficient statistic from this model, the "outside options index" (OOI), that summarizes the impact of options on compensation. Workers with more relevant jobs, as captured by the OOI, will, on average, have a better outside option, and be able to sort into better matches, conditional on their productivity. The OOI is estimated using information on the equilibrium distribution of workers into jobs.

The OOI is similar to a standard concentration index: workers with more options are those who, in equilibrium, are found in a greater variety of jobs. Under standard assumptions on the distribution of match quality (Choo and Siow, 2006; Dupuy and Galichon, 2014), the OOI is equal to the entropy index. This index, with a negative sign, is used in the industrial organization literature as a measure of market concentration (Tirole, 1988). It is similar to the Herfindhal-Hirschman Index (HHI), which has also been used to measure concentration in labor markets (Azar, I. Marinescu, and Steinbaum, 2020; Benmelech, Bergman, and Kim, 2020). Workers with more options (higher OOI) are those who are less concentrated across jobs, on all dimensions observed by the researcher.

However, the OOI is different from typical concentration indices in that, instead of generating a market-level measure of concentration, it quantifies the set of relevant jobs for an individual. This individual-level measure makes it possible to study differences in options between workers in the same occupation and establishment and to study gender gaps in options. The OOI also allows the researcher to incorporate multiple characteristics, including continuous characteristics, such as distance or task intensity.[2] The OOI is calculated without using any information on wages or wage offers.

We develop a method to estimate the OOI that is computationally feasible even in large datasets. In particular, we show that the problem of estimating the joint density of workers and jobs can be translated into a logistic regression framework. Using this density, it is straightforward to calculate the OOI for each worker.[3]

The OOI is the value of an individual worker's option set, constructed using a flexible definition for the worker's labor market. The OOI can account for differences in commuting preferences and

[1] An alternative approach would be to use a dynamic model with search frictions (Shimer and Smith, 2000; Lise and Postel-Vinay, 2020). In order to incorporate dynamic aspects of the labor market (that are beyond the scope of this paper), these models place simplifying assumptions on the types of options workers have. As a result, these models are less suitable for precisely estimating workers' option sets based on a large number of independent worker and job characteristics. That is the goal of this paper.

[2] We find, consistent with previous research documenting job applications fade out smoothly over space, that it is important to use continuous measures of distance (Skandalis, 2018; Manning and Petrongolo, 2017).

[3] We have developed an R package that implements this method. It is available to download through CRAN.

constraints, for differences in the quality of labor market opportunities, for skill transferability across industries/occupations, and for differences in workers' valuations of amenities. We show that the OOI predicts the relative rate of recovery for workers involved in the same mass-layoff, an event which forces workers to move to one of their outside options.

We demonstrate, using linked employer-employee data from Germany, that individuals' outside options are influential in determining between-group wage inequality. First, we document sizable between-group differences in OOI, by gender, education levels and citizenship status. We then estimate the (semi) elasticity between OOI and wages using a shift-share ("Bartik") instrument (Beaudry, Green, and Sand, 2012). We compare workers who work in the same industry, but have outside options in different industries because they reside in different parts of the country. We instrument for the growth in outside options in other industries with the national industry trends to exclude the impact of local productivity shocks. This approach yields a semi-elasticity between the OOI and wages of approximately 0.17, implying that access to 10% additional outside options increases wages by 1.7%.

Combining this elasticity with the estimated distribution of the OOI, we find that differences in outside options lower compensation for women by four percentage points, explaining roughly 20% of the overall gender wage gap in Germany. Differences in outside options also account for a two percentage points difference in compensation between immigrants and natives, which is 28% of the overall gap. We also find large effects on the return to education. The results are consistent with prior theoretical work, which has argued that some groups of workers—including women and minorities—may receive lower pay, in part because they face worse opportunities at other firms (see, e.g., Black, 1995).

In the last part of the paper we examine the reasons workers face different options. We use the underlying model to create a counterfactual distribution of the OOI, based on assuming workers have the same implicit costs of commuting. This exercise shows that the heterogeneity in the ability to commute or move is a key factor in explaining variation in outside options. This factor can account for the full gender gap in outside options. These results are consistent with recent work by Le Barbanchon, Rathelot, and Roulet (2021), who show that female workers have shorter maximum acceptable commutes than male workers. We also find that, without their higher willingness to work at more distant jobs, more educated workers would have fewer options than less educated workers. Our analysis suggests that this occurs because their skills tend to be more industry-specific (Amior, 2019).

The results in this paper contribute to a small, but growing, literature on the relationship between individuals' outside options and their wages. This literature has shown that there is a link between individuals' wages and their outside employment options (Beaudry, Green, and Sand, 2012; Caldwell and Harmon, 2018), and that changes in the outside option of non-employment do not impact wages (Jäger et al., 2020). The OOI provides a way to compare the employment opportunities available to different groups of workers, including those with similar skill sets. We show that heterogeneity in outside options explains a non-trivial portion of between-group wage inequality.

The results in this paper also contribute to a growing empirical literature on how to define a labor market (Manning and Petrongolo, 2017; Nimczik, 2017) and on concentration in the labor market (Azar, I. Marinescu, and Steinbaum, 2020; Benmelech, Bergman, and Kim, 2020; Schubert, Stansbury, and Taska, 2020; Rinz, 2020). A growing literature has noted that traditional labor market definitions, based only on occupation or commuting zone, may not capture a worker's option set.[4] The OOI provides a data-driven way to measure workers' labor markets.

From a policy perspective, our results on gender differences in outside options suggest that efforts to reduce women's commuting constraints are likely to help close the gender wage gap. These policies may include making childcare more widely available (especially in the hours before and after typical school hours) (Baker, Gruber, and Milligan, 2008). Our results also indicate that policies that improve workers' option sets may have sizable and heterogeneous general equilibrium effects. These effects can be studied through their effect on the OOI. In an appendix to the paper, we provide one such example, focusing on how the introduction of high speed rail in a small German town differentially affected workers' outside options, based on their education and gender. We find that higher educated workers gained access to more distant jobs due to the introduction of these trains. This is not surprising, given the high cost of taking the train. Women who are more likely to work close to home benefited from the new job openings in this town that followed the introduction of the train.

The paper proceeds as follows: Section 2 outlines a theoretical matching model, from which we derive the Outside Options Index (OOI). Section 3 explains how we estimate the OOI. Section 4 describes our empirical setting and data. Section 5 presents descriptive statistics on the OOI. Section 6 estimates the elasticity between the outside options index and wages, and analyzes the effect on wage inequality. Section 7 concludes.

2 A Model of Outside Options and Wages

This section outlines a model of a competitive labor market with two-sided heterogeneity, from which we derive an outside options index (OOI). The model is based on standard two-sided matching models with transferable utility (Shapley and Shubik, 1971; Becker, 1973).

[4] Manning and Petrongolo (2017) show that labor markets overlap across space and are more local than commuting zone measures might imply. Contemporaneous work by Schubert, Stansbury, and Taska (2020) shows that workers frequently change occupations when changing jobs.

2.1 Setup and Equilibrium

There is a continuum of workers $\mathcal{I}$ and a continuum of one-job firms $\mathcal{J}$. We treat $\mathcal{I}, \mathcal{J}$ as exogenous and of equal measure, which we normalize to 1. In Appendix A.5 we present an extended version of the model where participation decisions are endogenous and firms can have many jobs.[5]

A match between a worker $i \in \mathcal{I}$ and a job $j \in \mathcal{J}$, produces y_{ij} in output and a_{ij} in non-wage "amenities" that are valued by the worker. For instance, workers may derive utility from having flexible hours, or from working close to their home. Output is net of non-wage costs, including any cost of producing amenities. The total value of a match, τ_{ij}, is the sum of output and non-wage amenities. Employers and workers decide how to split this surplus into worker compensation (ω_{ij}) and employer profits (π_{ij}),

$$\tau_{ij} = \pi_{ij} + \omega_{ij} = y_{ij} + a_{ij}.$$

This division is accomplished via a set of transfers (wages) w_{ij}.

$$\begin{aligned} \overbrace{\pi_{ij}}^{\text{profits}} &= \overbrace{y_{ij}}^{\text{output}} - \overbrace{w_{ij}}^{\text{wages}} \\ \underbrace{\omega_{ij}}_{\text{compensation}} &= \underbrace{a_{ij}}_{\text{amenities}} + \underbrace{w_{ij}}_{\text{wages}} \end{aligned}$$

Employer profits are simply the total value of the output, minus transfers. Workers' compensation depends on the transfers they receive (wages) and on their valuation of the job's amenities. Both sides have perfect information.

We solve the model using an equilibrium notion based on cooperative game theory (Shapley and Shubik, 1971).[6] A stable equilibrium (core allocation) consists of an allocation, which is an invertible function $m : \mathcal{I} \to \mathcal{J}$, and a transfer w_{ij} for each matched pair (i, j) that satisfies

$$\forall i' \in \mathcal{I}, j' \in \mathcal{J} \quad : \quad \omega_{i',m(i')} + \pi_{m^{-1}(j'),j'} \geq \tau_{i'j'}. \tag{1}$$

This condition says that there is no single worker-employer combination that could deviate from their current allocation, produce together, and split the surplus in such a way that both the employer and the worker would be better off. This implies that surplus is split based on outside options: both sides must earn more in equilibrium than what they could earn in any off-equilibrium match.

[5] Including the option of non-participation yields similar results, except that compensation also depends on the value of non-participation. We do not include non-participation in the baseline model since prior work has shown that large changes to the value of non-employment do not impact compensation—even for workers likely to be unemployed (Jäger et al., 2020).

[6] While there are additional equilibrium concepts that lead to the same result, we focus on a cooperative framework, which does not require us to make any assumptions about how agents reach this equilibrium (e.g. who makes offers). Pérez-Castrillo and Sotomayor (2002) show one mechanism that leads to the same equilibrium under a sub-game perfect Nash equilibrium.

This model does not explicitly feature search frictions, but rather focuses on preference heterogeneity in driving the allocation of workers to jobs. Models that incorporate search frictions in heterogeneous labor markets (Shimer and Smith, 2000; Lise and Postel-Vinay, 2020) place simplifying assumptions on the type of worker and job heterogeneity that exists in the data, which excludes empirically important matching patterns (e.g. by distance). However, in practice, we are empirically able to account for some forms of frictions by attributing them to the value of $\tau_{i,j}$. In particular, $\tau_{i,j}$ is expected to be higher for matches between workers and jobs that are geographically closer. This could be because workers value taking jobs closer to where they currently live. However, it could also be because workers are more knowledgeable about job opportunities close to their homes (see, e.g., Skandalis, 2018).

2.2 Parametric Assumptions

Workers and jobs can be characterized by sets of characteristics $\mathcal{X} \subseteq \mathbb{R}^{d_x}$ and $\mathcal{Z} \subseteq \mathbb{R}^{d_z}$. We use X_i and Z_j to denote the observed worker and job characteristics, which have densities $d(X_i)$ and $g(Z_j)$ respectively.

We pin down the equilibrium using a standard separability assumption and by assuming a distribution for the portion of utility attributed to the unobservables (Choo and Siow, 2006). In particular, we follow Dupuy and Galichon (2014) and assume that the value of τ_{ij} conditional on the observables is the sum of two independent shocks drawn from continuous logit models. Both employers and workers have unobserved taste shocks: worker i receives additional idiosyncratic utility ε_{i,z_j} that does not depend on her observed characteristics, if she matches with an employer with characteristics z_j. Similarly, an employer j receives additional (unobserved) utility ε_{j,x_i} if it hires a worker with characteristics x_i. These taste shocks are drawn from two continuous logit models $CL(\alpha_x)$ and $CL(\alpha_z)$, which we describe in Appendix A.1. They are closely related to extremum value type-1 distributions but allow for continuous characteristics (Dagsvik, 1994).[7]

Assumption 1. *The match value τ_{ij} between a worker i with observable characteristics x_i, and a job j with observable characteristics z_j, can be written as*

$$\tau_{ij} = \tau(x_i, z_j) + \varepsilon_{i,z_j} + \varepsilon_{j,x_i}$$

[7] Every worker i draws ε_{i,z_j} shocks from a Poisson process on $\mathcal{Z} \times \mathbb{R}$. As a result, for every subset $S \subseteq \mathcal{Z}$, $\max_{z \in S}\{\varepsilon_{i,z}\} \sim EV_1(\alpha_z \log P(S), \alpha_z)$ where EV_1 is extremum-value type-1 distribution, and $P(S) = \int_S g(z)\,dz$. We do not make any assumption on whether ε_{i,z_j} or ε_{j,x_i} are shocks to the amenity a_{ij} or to output y_{ij}, since this does not affect our results. A similar process exists for ε_{j,x_i}. We depart from Dupuy and Galichon (2014) in how we parameterize the intensity of the Poisson Process in order to ensure that, as the number of options increases, workers' compensation (and employers' profits) do not become infinite. More details are provided in Appendix A.1.

where ε_{i,z_j} and ε_{j,x_i} are two independent draws from continuous logit models with scale parameters α_x and α_z

$$\begin{array}{rcl} \varepsilon_{i,z_j} & \perp & \varepsilon_{j,x_i} \\ \varepsilon_{i,z_j}, \varepsilon_{j,x_i} & \sim & CL\left(\alpha_z\right), CL(\alpha_x) \end{array}$$

This assumption allows us to derive a unique solution, which we can estimate in the data. However, it is strong for two reasons. First, it implies that workers (employers) have unobserved utility in or "taste" for jobs (workers) with specific observed characteristics, and that those unobserved preferences are uncorrelated, even between jobs (workers) with similar characteristics. Second, the assumption that $\varepsilon_{i,z_j} \perp \varepsilon_{j,x_i}$ implies that there are no interactions between the unobserved worker and job tastes. Because we place no restrictions on $\tau\left(x,z\right)$ and strong restrictions on ε_{i,z_j} and ε_{j,x_i}, our model is expected to perform better in settings where the researcher has more detailed information on workers and jobs. The parameters α_x and α_z scale the level of heterogeneity in the idiosyncratic taste shocks (of jobs for workers and of workers for employers).

Assumption 1 simplifies the matching procedure into two one-sided continuous logit choices (Dupuy and Galichon, 2014).

Theorem 1. *Under Assumptions 1, in equilibrium, worker i with characteristics x_i faces a continuous logit choice between employers who are offering*

$$\omega(x_i, z_j) + \varepsilon_{i,z_j}$$

and employers choose between workers who generate profits

$$\pi(x_i, z_j) + \varepsilon_{j,x_i},$$

where

$$\omega\left(x_i, z_j\right) + \pi\left(x_i, z_j\right) = \tau\left(x_i, z_j\right).$$

We provide proofs for all our results in Appendix A.2. Workers' expected compensation in equilibrium is therefore

$$E\left[\omega_{ij}|x_i\right] = E\left[\omega\left(x_i, z_j^*\right)|x_i\right] + E\left[\varepsilon_{i,z_j^*}|x_i\right], \tag{2}$$

where z_j^* denotes the characteristics of the job worker i takes in equilibrium.

2.3 The Outside Options Index

We define the OOI as the standardized expectation $\frac{1}{\alpha_z}E\left[\varepsilon_{i,z_j^*}|x_i\right]$.[8] This expression is a function of the equilibrium match probabilities. Using $f\left(x_i, z_j\right)$ to denote the joint distribution of matches based on workers' and employers' observed characteristics, and $f_{Z|X}\left(z_j|x_i\right) = \frac{f(x_i,z_j)}{d(x_i)}$ to denote the conditional distribution, we get the following expression for the OOI.

Lemma 1. *Under Assumption 1:*

$$OOI := \frac{1}{\alpha_z}E\left[\varepsilon_{i,z_j^*}|x_i\right] = -\int f_{Z|X}\left(z_j|x_i\right)\log\frac{f_{Z|X}\left(z_j|x_i\right)}{g\left(z_j\right)}. \tag{3}$$

The expression on the right of equation 3 is (minus) relative entropy, which takes a value in $(-\infty, 0]$. This expression is the continuous version of the Shannon entropy index, an index often used to measure industrial concentration. Workers have more options, as measured by our OOI, when their probability of being in any particular type of job is lower. As we explain in the next section, workers have more options when there are many jobs taken in equilibrium by workers with characteristics x_i, in which the worker receives roughly the same compensation, $\omega\left(x_i, z_j\right)$. This measure of concentration may depend on workers' occupation or industry; however it may also depend on other—possibly continuous—characteristics such as distance. The concentration is measured for each individual worker, accounting for the fact that workers could have different option sets, depending on their observable characteristics (e.g. their training, their gender, age, citizenship status).

The OOI depends both on a worker's ability to receive similar compensation in different types of jobs, and on the supply of available jobs (i.e. the distribution of z_j). Workers who have more transferable skills or who are more able to commute will, on average, have more opportunities. Jobs that a worker could, in theory, do (e.g. a trained surgeon could take a job as a plumber) but that workers of that type never take in equilibrium do not enter the OOI as the OOI is not affected by zero probability events. To give further intuition for the OOI—and for how, in this perfectly competitive model, equally productive workers with different OOI earn different wages—we present a simple parametric example in Appendix A.4.

The OOI can nest standard labor market definitions, including those that allow for transitions across occupations, industries, or locations. Given a vector of indicators for an individual's prior occupation x_i and a vector of occupation indicators for the set of jobs in the economy z_j, $f_{Z|X}(z_j|x_i = x)$ is simply the probability that an individual in occupation x moves to the occupation of job z_j (as in Schubert, Stansbury, and Taska (2020)). Because the OOI also allows for con-

[8] Note that $E[\varepsilon_{i,z_j^*}] > E[\varepsilon_{i,z_j}]$ because z_j^* is positively selected. Descaling by α_z guarantees that the value of the OOI is independent of the units we use to define $\tau\left(x, z\right)$.

tinuous covariates and for the inclusion of multiple covariates (including worker characteristics), it allows for richer substitution patterns across jobs than simple occupation- or industry-transition matrices. In our analysis below, we find that differential sensitivity to distance is key for explaining differences in sorting patterns between male and female workers with the same skill sets and between workers with different levels of education.[9] While our baseline model does not incorporate an explicit role for firms, in Appendix A.5 we show that the model can easily be modified to account for firm concentration; larger firms are able to impose larger mark-downs.

2.4 The Link Between Outside Options and Compensation

Workers with more options receive higher compensation in equilibrium for two reasons, both of which are captured in the OOI. First, a higher OOI implies a higher expected value of the unobserved portion of the utility ($E\left[\varepsilon_{i,z^*}|x_i\right]$ in Equation 2). Workers have a larger value of $E\left[\varepsilon_{i,z^*}|x_i\right]$ if they have more jobs with similar values of $\omega\left(x_i, z_j\right)$, perhaps because they live in a larger city, are willing to commute longer distances, or have more general skills. By contrast, workers that are concentrated (e.g. because they live in isolated areas, are unable to commute, or have specific skills) would have only few jobs with high $\omega\left(x_i, z_j\right)$ to choose from, and therefore would compromise for a lower value of ε_{i,z_j} in expectation.

Second, a higher OOI implies better outside options. Dupuy and Galichon (2015) show that the match surplus that is attributed to observable characteristics, $\tau\left(x, z\right)$, is divided between workers and employers based on their outside options.

Theorem 2. *In equilibrium, $\omega\left(x_i, z_j\right)$, the share of $\tau\left(x_i, z_j\right)$ that workers receive, satisfies*

$$\omega(x_i, z_j) = \frac{\alpha_x}{\alpha_x + \alpha_z} E[\omega_{ij}|x_i] + \frac{\alpha_z}{\alpha_x + \alpha_z} \left(\tau(x_i, z_j) - E[\pi_{ij}|z_j]\right). \qquad (4)$$

Equation 4 is reminiscent of standard equations for wage-determination in bargaining models where workers receive a weighted average of the value of their outside option (e.g. the value of working at their last best offer) and the value of their inside option (the value of working for this employer). The ratio $\frac{\alpha_z}{\alpha_x+\alpha_z}$ is comparable to the bargaining parameter β in standard search and matching models; it is higher when workers' idiosyncratic preferences are more dispersed relative to employers'. $E\left[\omega_{ij}|x_i\right]$ is comparable to the outside option. It is the expected value of an offer a worker with characteristics x_i would receive from other employers.[10]

Because they have better offers from other employers, workers with a higher OOI receive a

[9] We treat distance as a job characteristic, to allow for heterogeneity in sensitivity to distance across workers. See further details in Section 4.3.

[10] Because there is a continuum of workers with observable characteristics x_i, $E\left[\omega_{ij}|x_i\right]$ is not affected by the compensation worker i receives from her equilibrium employer.

larger portion of what they produce. A worker with a higher OOI will have higher compensation $E[\omega_{ij}|x_i]$ (Equation 2). They will then have a higher value of $\omega\left(x_i, z_j\right)$, which comes directly at the expense of the employer's profit $\pi\left(x_i, z_j\right)$. This equation generates a multiplier effect for having more options: having more options increases compensation directly by improving unobserved utility, and indirectly by increasing the portion each worker receives from what they produce. To get the overall impact we use the following decomposition by plugging in Equation 4 in Equation 2:

$$\underbrace{E\left[\omega_{ij}^{*}|x_i\right]}_{\text{Expected compensation}} = \underbrace{E\left[\tau\left(x_i, z_j^{*}\right)|x_i\right]}_{\text{Mean Production}} - \underbrace{E[\pi_{i,j^{*}}|x_i]}_{\text{Employer Rents}} + \underbrace{\left(\frac{\alpha_x+\alpha_z}{\alpha_z}\right) E\left[\varepsilon_{i,z_j^{*}}|x_i\right]}_{(\alpha_x+\alpha_z)\cdot OOI} \tag{5}$$

The OOI is a sufficient statistic for the effect of the size of the option set on a worker's compensation. To show this, we examine how compensation changes if we increase the access to options (λ_{x_i}) for workers with characteristics x_i, while keeping the quality of options constant.[11] Such an increase in options increases workers' compensation by its effect on the OOI times a constant.

Theorem 3. *Access to options λ_{x_i} has the following overall effect on expected worker compensation in equilibrium:*

$$\frac{dE\left[\omega_{i,j}\right]}{d\lambda_{x_i}} = \left(\alpha_x+\alpha_z\right)\frac{dOOI_i}{d\lambda_{x_i}}.$$

Theorem 3 shows that an increase in the size of a worker's option set, holding the quality of the options fixed, will affect compensation only through its effect on the third component in Equation 5. Because the quality of options remains fixed, mean production and employer profits stay the same. The overall effect summarizes the impact of having more options through the two channels described above. More options increase the unobserved portion of utility ($E\left[\varepsilon_{i,z_j^{*}}\right] = \alpha_z OOI_i$). They also increase the portion of the utility that is attributed to the observables, $\omega\left(x_i, z_j\right)$, by an amount of $\frac{\alpha_x}{\alpha_z}\left[\varepsilon_{i,z_j^{*}}\right] = \alpha_x OOI$.[12] Hence the impact of having more options on compensation depends only on the effect on the OOI, multiplied by the constant $\alpha_x+\alpha_z$.

In our empirical exercise, we both examine how the distribution of OOI varies across groups of workers, and examine how variation in the OOI contributes to wage inequality.

[11] Formally, λ_{x_i} is a parameter in the intensity of the Poisson process of the continuous logit model. A higher value of λ_{x_i} generates more options in every subset of jobs. Such shocks could include, e.g. the arrival of information shocks about other job options (which we do not model here), or drops in regulatory barriers such as non-compete agreements. Such shocks do not include shocks to productivity or preferences that are likely to change the quality of the jobs as well.

[12] $\frac{\alpha_x}{\alpha_z}$ is the result of the multiplier effect in Equation 4. $E\left[\varepsilon_{i,z_j}\right]$ directly affects $\omega\left(x_i, z_j\right)$ by another factor of the quasi-bargaining parameter $\frac{\alpha_x}{\alpha_x+\alpha_z}$. But increasing $\omega\left(x_i, z_j\right)$ also raises $E\left[\omega_{ij}|x_i\right]$ and so $\omega\left(x_i, z_j\right)$ further increases by a factor of $\frac{\alpha_x}{\alpha_x+\alpha_z}$ as well. The overall impact of $E\left[\varepsilon_{i,z_j}\right]$ is $\frac{\alpha_x}{\alpha_x+\alpha_z} + \left(\frac{\alpha_x}{\alpha_x+\alpha_z}\right)^2 + \left(\frac{\alpha_x}{\alpha_x+\alpha_z}\right)^3 + ... = \frac{\alpha_x}{\alpha_z}$.

3 Estimation

We use the cross-sectional allocation of observably similar workers to estimate the set of relevant options for each worker. Each individual's outside options index (OOI) depends on the ratio between her probability of working in jobs with characteristics z_j, $f_{Z|X}(z_j|x_i)$, and the distribution of such jobs in the economy, $g(z_j)$, for all jobs observed in the data (Equation 3).

3.1 Parameterization

In order to estimate these probabilities, we first parametrize this ratio as a function of the observables. We follow Dupuy and Galichon (2014) in assuming that the log density is linear in the interaction of worker and job characteristics. Note that x_i and z_j can include functions of worker and job characteristics (e.g. x_i could include a quadratic in age).

Assumption 2. *The log of the probability density is linear in the interaction of worker and job characteristics:*

$$\log \frac{f_{Z|X}(z_j|x_i)}{g(z_j)} = x_i A z_j + a(x_i) + b(z_j)$$

The matrix A includes all the coefficients on each of the interactions between worker and job characteristics. This assumption reduces the dimension of the problem significantly, without imposing restrictions on the relationship between pairs of covariates.[13]

3.2 Estimating Densities

Prior work has focused on estimating A by matching $E[XZ]$, the second moments of the joint distribution of X and Z (Dupuy and Galichon, 2014). We estimate A using a similar method that is computationally more efficient, especially in large datasets.

We first simulate data from a distribution $\widetilde{f}(x,z) = d(x) \cdot g(z)$, where x and z are independent.[14] We add these simulated data to our baseline dataset, and define a binary variable Y_k that equals one whenever match k is 'real' (taken from the data) and zero whenever it is simulated. As a result of Bayes rule

$$\frac{P(Y_k=1|x_k,z_k)}{P(Y_k=0|x_k,z_k)} = \frac{f(x_k,z_k)}{d(x_k)\,g(z_k)} \frac{P(Y_k=1)}{P(Y_k=0)} = \frac{f_{Z|X}(z_k|x_k)}{g(z_k)} \times const.$$

[13] Dupuy and Galichon (2014) show that A is proportional to the cross-derivative of $\tau: (\alpha_x+\alpha_z)A = \frac{\partial^2\tau}{\partial x \partial z}$, where α_x and α_z are the scale parameters of ε we defined in Assumption 1. Intuitively, if a worker characteristic and a job characteristic are complements, they will be observed more frequently in the data.

[14] We randomly sample an observed worker and an observed job independently. We simulate a total number of random matches equal to our original data size, so the share of real and simulated data is exactly one half.

Combining this result with Assumption 2 yields

$$\log \frac{P\left(Y_k=1|x_k,z_k\right)}{P\left(Y_k=0|x_k,z_k\right)} = x_k A z_k + a\left(x_k\right) + b\left(z_k\right). \tag{6}$$

We then use a logistic regression of Y_k on the matched worker and job characteristics (x_k, z_k), approximating $a(x)$ and $b(z)$with linear functions.[15] Under assumptions 1 and 2 this produces consistent estimates for $\widehat{A}, \widehat{a}(x_k)$, and $\widehat{b}(z_k)$.

We use the estimates from the logistic regression to estimate the log probability ratio for every worker-job combination:

$$\log \frac{\widehat{f_{Z|X}\left(z_j|x_i\right)}}{g\left(z_j\right)} = x_i \widehat{A} z_j + \widehat{a}\left(x_i\right) + \widehat{b}\left(z_j\right).$$

We estimate the densities as

$$f_{Z|X}\widehat{\left(z_j|\right.}x_i) = g\left(z_j\right)\exp\left[x_i \widehat{A} z_j + \widehat{a}\left(x_i\right) + \widehat{b}\left(z_j\right)\right], \tag{7}$$

where the sample weights are used for $g(z_j)$. We normalize the estimated densities so $\sum_{z_j} f_{Z|X}\widehat{(z_j|}x_i) = 1$ and calculate the OOI as:

$$\widehat{OOI}_i = -\sum_{z_j} f_{Z|X}\widehat{\left(z_j|\right.}x_i)\log\frac{\widehat{f_{Z|X}\left(z_j|x_i\right)}}{g\left(z_j\right)}. \tag{8}$$

Our method is similar to prior work by Dupuy and Galichon (2014) who matched moments of the form of $E[XZ]$. Appendix B.1 shows that if Assumption 2 is correctly specified, the first order conditions of the logistic regressions are equivalent to the moments matched by Dupuy and Galichon (2014). In particular,

$$E_{A,a,b}\left[XZ\right] = \int f_{A,a,b}\left(x,z\right) X_i Z_j = \widehat{\mu_{XZ}},$$

where $\widehat{\mu_{XZ}}$ are the observed values of $E[XZ]$. We also show that if we fully saturate the functions a and b our estimates converge to those of Dupuy and Galichon (2014) when the simulated data

[15] Dupuy and Galichon (2014) estimate a distribution $\widehat{f(x,z)}$ such that the estimated marginal distributions $\widehat{d}$ and $\widehat{g}$ are identical to their values in the data. To do so we would need to fully saturate the $a(x_i)$ and $b(z_j)$ functions, which we cannot given our data size. Therefore, we take linear functions of the X variables and Z variables. We also include indicators for district for both workers and jobs. These guarantees that our estimated distribution $\widehat{f(x,z)}$ generates the same first moment of all X and Z variables, and matches the shares of workers and jobs in each district to their value in the data. As we discuss in Appendix B.1 this specification fits the first moments of the marginal distributions when the simulated data are sufficiently large.

are sufficiently large.

4 Empirical Application

We use administrative data from Germany to generate measures of individual workers' outside options. The data includes detailed information on establishment and worker characteristics, including information on a variety of amenities provided by different establishments.

4.1 Empirical Setting: The German Labor Market

There are several distinctive features of the German labor market which are relevant for our analysis. First, there are different levels of secondary-school leaving certificates, which depend on the number of years and type of education. Our data allow us to distinguish between three categories: lower-secondary, which typically requires nine years of schooling, intermediate-secondary, which typically requires ten years of schooling, and high-secondary, which requires twelve to thirteen years of schooling, and allows the student to pursue a university degree. In our analysis we use indicators for the type of secondary education.

Second, in addition to (or sometimes instead of) formal education, many German workers receive on-the-job training through formal apprenticeships. Individuals in apprenticeship programs complete a prescribed curriculum and obtain occupation-specific certifications (e.g. piano maker). We include information on apprenticeships in our worker characteristics.

While wage setting in Germany was historically governed by strong collective bargaining agreements, employers today have considerable latitude in setting pay (Dustmann, Ludsteck, and Schönberg, 2009). While employers could always raise wages above the agreed-upon levels, "opening clauses", which allow employers to negotiate directly with workers to pay *below* the collectively bargained wage, emerged in the 1990s. Today these clauses are very common.

4.2 Data

Our analysis relies on the "LIAB Longitudinal" dataset, a matched employer-employee administrative dataset, based on a sample from the universe of German Social Security records from 1993-2014. The data come from the Integrated Employment Biographies (IEB) dataset, which is collected by the German Institute for Employment Research (IAB). Employers are required to report daily earnings (which is top-coded), education, occupation, and demographics for each of their employees at least once per year, and at the beginning of any new employment spell.[16] New spells

[16]Daily earnings are calculated as an average for the reported period. 11% of the sample is top-coded. Since we do not use wages to calculate the OOI, it is not affected by this censoring.

can arise due to changes in job status (e.g. part-time to full-time), establishment, or occupation. The data do not cover civil servants or the self-employed, who comprise 18% of the German workforce. One important limitation is that the data do not allow us to identify when jobs at different establishments are part of the same firm.

Establishment Surveys These data also include the answers from annual establishment surveys conducted by the German Institute for Employment Research (IAB). Each year a stratified random sample of establishments are asked a series of questions about their organizational structure (e.g. size, percentage of managers who are female), personnel policies (e.g. leave policies), and finances (e.g. annual sales, profits). This survey information is then merged with the complete (1993-2014) employment histories of all workers who ever worked at these establishments. Because LIAB samples at the establishment level, there are few establishments in each industry-district. This data sparsity motivates our decision to construct the OOI at the job level, rather than the employer level, in our baseline analysis.

BIBB Task Data We supplement these data with survey information on the characteristics of occupations and industries from the BIBB. The BIBB survey is conducted by the Federal Institute for Vocational Education and Training and includes information on respondents' occupation and industry, in addition to responses to questions related to organizational information, job tasks, job skill requirements, health and working conditions. These data are similar to the O*NET series, but allow us to account for possible differences in the task content of occupations between the United States and Germany, as well as differences in coding (Gathmann and Schönberg, 2010). These data allow us to account for the possibility that the "distance" in task space between different occupation codes may not be uniform. We provide more information on the data cleaning procedures in Appendix C.

4.3 Worker and Firm Characteristics

Estimating $f_{Z|X}\left(z_j|x_i\right)$ requires information on worker characteristics (x) and job characteristics (z).[17]

Worker Characteristics The worker characteristics x include gender, level of secondary education, an indicator for German citizenship, and a quadratic in age. The characteristics also include

[17] Appendix D investigates the performance of the OOI when computed with different sets of variables. We focus on sets of variables that are commonly available in survey data and in cross-sectional and panel linked employer-employee datasets. We find that it is important to include workers' training occupation and the worker and job locations; these data are available in many employer-employee datasets. Our baseline analysis uses both these and other variables (a described below) in order to improve the precision with which we estimate the OOI.

the occupation in which a worker undertook her apprenticeship (her "training occupation"). If we do not have information on a worker's apprenticeship (e.g. if it occurred before our data begin in 1993), or if a worker did not complete an apprenticeship, we use the first observed occupation, as long as it is at least ten years old.

Job Characteristics The z variables we use can be grouped into two categories: (1) characteristics of establishments, and (2) characteristics of jobs. First, we take several establishment-specific variables directly from the establishment survey: size and sales per worker. We also use the first two principal components of each of the five categories of the establishment survey: business performance, investments, working hours, vocational training, and a general category. Appendix Table A1 shows the most weighted questions in each category.

Second, we use industry and occupation characteristics. Because it would be infeasible to include interactions between all of our industry and occupation codes, we use data from the BIBB to project each industry or occupation into task space. The BIBB survey contains modules on working hours, task type, requirements, physical conditions and mental conditions. For each 3-digit occupation and 2-digit industry, we include the average value of the first two principal components for each module. We use these to code both the occupation and industry that describe the job, and the training occupation that describes the worker. Appendix Table A1 shows the most weighted questions in each module. We also include indicators for occupation complexity. These codes, available in the LIAB, group occupations into four categories based on the type of activity they require: (1) simple, (2) technical (3) specialist or (4) complex. The categories typically reflect the type of qualification needed to perform the job. For instance, these categories allow us to distinguish between a nursing assistant, nurse, specialist nurse and general practitioner. We also include an indicator for whether the job is part-time.

Geographical Distance Finally, we include one worker-firm specific variable: the geographic distance between a worker's last place of residence (before taking this job) and the location of the job.[18] This distance captures both the commuting and moving costs between places; empirically we cannot directly distinguish the two. Both locations are given at the district (kreis) level.[19]

To account for heterogeneity in willingness to commute or move, we interact a fourth degree polynomial in distance with all worker characteristics x (i.e. we treat distance as a job characteristic). This allows workers to be affected differently by distance, depending on their gender,

[18] A worker's current place of residence may depend on their choice of job. Her place of residence before taking the job better reflects the radius over which she searched for jobs.

[19] District size varies across the country and, importantly, highly populated areas have smaller districts. In many cases, the major city is its own district, allowing us to separately identify the city center and the suburbs. Though not perfect, this coding allows us to get a reasonable approximation of commuting and moving patterns by workers.

education, age, citizenship and training. As we discuss in Section 6.3, differential sensitivity to distance turns out to be a main driver of between-group differences in outside options.

4.4 Descriptive Statistics

Table 1 describes the characteristics of workers and matches in our sample. Because the model is static, we rely on repeated cross-sections of data. In much of our descriptive analysis we focus on the 2014 cross-section; in Section 6.1 we use data from 2004 and 2014 to examine how variation in the OOI affects wages. For each year, we define employment as of June 30th.

Column 1 of Table 1 shows that the mean age for a worker in our sample is forty-six years old and the vast majority (98%) are German citizens. About a third of workers have the highest level of education. More information on the education categories is in Appendix C. Men and women have similar age, education and citizenship status. However, women are much more likely to work in part-time jobs (53% compared to 12%) and work at smaller establishments. Women also work closer to their homes; Table 2 shows that their mean distance is 9.5 miles, compared with 15.8 miles for men.

The statistics in Table 2 are consistent with the findings in Manning and Petrongolo (2017), who show that labor markets are more local than standard commuting zone definitions would indicate. In our setting this is especially true for women, and for workers with less education. The statistics are also consistent with recent work by Amior (2019), documenting that high skilled workers have longer commutes.

5 Outside Options in the Labor Market

In this section we describe the distribution of the OOI, both between demographic groups and overall. We then validate the index by showing that the OOI is able to predict the ease with which a worker recovers from an involuntary job separation, which forces the worker to move to her outside option.[20] Finally, we explore the drivers of heterogeneity in the OOI.

5.1 The Distribution of the OOI

We start by plotting the distribution of OOI in the population. The dashed line in Figure 1 plots the distribution for all workers. The distribution of the OOI is skewed, with a long left tail, indicating that there are many workers who are extremely concentrated. The mean of the distribution is -4.82

[20] This result is consistent with standard dynamic models, in which workers with more outside options are able to recover more quickly from economic setbacks. Because our model is static, it does not provide any direct predictions on market adjustments after shocks.

and the standard deviation is 0.97. The red and blue lines plot the distribution for females and males separately.

We can interpret values of the OOI by considering a simple case where a worker only works in a share of p jobs in the overall economy, but is equally likely to work in any of these jobs. In this case, the probability density that the worker is found at any given job is either $\frac{1}{p}$ or 0; their OOI is $\log p$. In this scenario, a worker with an OOI of -4.82 (the mean in our sample) would be found in $p = 0.8\%$ of jobs. A worker with one standard deviation more options would be found in 2.1% of all jobs (160% more than a worker with the mean OOI), and a worker with one standard deviation fewer jobs would be found in 0.3% of all jobs (60% fewer than a worker with the mean OOI).

5.2 Validating the OOI Using Mass Layoffs

We next show that the OOI is a meaningful measure of workers' outside options. In particular, we show that it is able to predict the ease with which a worker recovers from an involuntary job separation, which forces the worker to move to one of her outside options.

A large literature has documented that workers involved in mass layoffs suffer long-run earnings losses (Jacobson, Lalonde, and Sullivan, 1993; Lachowska, Mas, and Woodbury, 2020). This is also true among our sample of workers. Following Jacobson, Lalonde, and Sullivan (1993), we identify mass layoffs by focusing on plants whose workforce has declined by at least thirty percent relative to the previous year. We only consider mass layoffs that occur in establishments with at least fifty workers, and restrict our analysis to workers under 55 who had been employed at the establishment for at least three years prior to the mass layoff. Our final sample consists of 13,404 workers from 581 distinct mass-layoffs. Table A2 presents descriptive statistics.

Appendix Figure A1 shows that, on average, and without adjusting for any covariates, workers lose 80% of their income in the month following the mass layoff. Workers' income only gradually returns to its previous level. This is consistent with previous work that has documented, using similar data, that German workers involved in a mass layoff suffer long-run earnings losses (J. Schmieder, Wachter, and Heining, 2018; Jarosch, 2021).

We compare the experiences of individuals involved in the same mass layoff who had different levels of OOI (as calculated using pre-layoff characteristics). In comparing workers within the same mass-layoff—and therefore not exploiting establishment-level variation—we deviate from previous mass layoffs literature (see, e.g., Lachowska, Mas, and Woodbury, 2020). We do this because comparing workers from the same mass layoff guarantees that the workers with different levels of OOI face similar market shocks.

The main outcome is relative income (current daily income/pre-mass layoff daily income, $\widetilde{w}_t = \frac{w_t}{w_0}$). Using relative income implies that permanent income differences between workers, such as those in a standard worker fixed effect model, would be canceled out. This is important since those permanent income differences could be directly correlated with the OOI. We use relative income without logs to better account for zero income, which is prevalent in this sample. See Appendix B.2 for further details. The second outcome is an indicator for employment ($e_{i,t}$). We regress these outcomes on the OOI, interacted with dummies for the number of months since the mass-layoff event, on original mass-layoff establishment-by-month fixed effects $\psi_{j_0(i),t}$, and, in some specifications, on individual controls X_{it} interacted with the number of months since the mass-layoff. We set wages to 0 during periods of unemployment or non-employment. Specifically, we estimate

$$\widetilde{w}_{i,t} = \sum_{\tau=0}^{36} \lambda_\tau OOI_i + \psi_{j_0(i),t} + \mu_t X_{it} + \nu_{i,t}, \quad (9)$$

$$e_{i,t} = \sum_{\tau=0}^{36} \lambda_\tau^{\text{emp}} OOI_i + \psi_{j_0(i),t}^{\text{emp}} + \mu_t^{emp} X_{it} + \nu_{i,t}^{\text{emp}}, \quad (10)$$

where λ_τ and λ_τ^{emp} are the coefficients on the interaction of the OOI with an indicator for τ months since the mass layoff.

Panels A and B of Figure 2 plot λ_τ and $\lambda_\tau^{\text{emp}}$. Panel A shows that, relative to her same-layoff peers, a worker with one unit higher OOI, or, slightly more than one standard deviation ($\sigma_{OOI} = .97$), has 10% higher earnings (as a share of her pre-layoff earnings) one year after the separation. That same worker is roughly 1.5% more likely to be employed, relative to her same-layoff peers. The employment effects cannot fully explain the wage effects. Panels A and B show that, the gaps are persistent over time. Within three years, there is still a statistically significant impact of having a higher OOI on either wages or employment.[21]

Table 3 presents estimates of equations 9 and 10, augmented with additional worker-level controls X_i, interacted with time since the layoff. In the interest of space, we report a subset of the coefficients in the table: $\lambda_3, \lambda_6, \lambda_{12}$, and λ_{24} are presented in Panel A and $\lambda_3^{\text{emp}}, \lambda_6^{\text{emp}} \lambda_{12}^{\text{emp}}$, and $\lambda_{24}^{\text{emp}}$ are presented in Panel B. The results in Column 1 are analogous to those presented in Figure 2.

The remaining specifications include additional controls for worker tenure (Column 2), demographics (Column 3), and prior occupation (Column 4) and their interaction with the number of months since the mass layoff. While these control variables capture a large portion of the variation in the OOI, there is still substantial variation—based on the interaction of these variables—as we

[21] Workers with higher OOI more easily recover than their low-OOI counterparts especially during non-recession years. This is potentially because the OOI measures employment opportunities, which are more relevant during non-recession periods.

show in the next section.[22] Across all specifications, we see that workers with greater OOI recover more quickly from a mass layoff event than their same-layoff peers.

5.3 Heterogeneity in Outside Options

We conclude this section by examining drivers of heterogeneity in the OOI across groups of workers. Figure 3 shows that workers in big cities with denser labor markets tend to have better options, as measured by the OOI.[23] This is not surprising as denser regions have more job opportunities within a given geographic radius.

Table 4 shows that there are also differences in average OOI across workers in different demographic groups. Columns 1 and 2 present the mean and standard deviation of the OOI; Columns 3–5 present the 25th, 50th and 75th quantiles. We find that men, citizens and more educated workers have a higher OOI, both on average, and throughout the distribution.

Looking at training occupations, we find that workers with more specific skills—as proxied by the occupational concentration of workers with the same training—also have a lower OOI (Gathmann and Schönberg, 2010). For each training occupation, we calculate the share of workers working in each occupation (Handwerker, Spletzer, et al., 2016). We measure skill specificity for each training occupation using the share of workers in the most common occupation; intuitively, workers whose training is more specific should be more concentrated in a certain occupation. Appendix Figure A2 presents a binned scatterplot of this skill specificity measure by the average OOI in each training occupation. We find lower levels of OOI for workers whose training is more specific.

However, workers whose training prepares them for a more specific (general) set of occupations do not necessarily have lower (higher) income. Equation 5 shows that individuals who develop skills specific to a narrow set of occupations experience two opposing effects. Wages could increase as productivity increases (higher $E\left[\tau\left(x_i, z_j^*\right)|x_i\right]$). However, wages could also decrease as these workers would cluster in a smaller set of occupations, narrowing their OOI. Appendix Figure A3 shows the correlation between the OOI and wages at the training occupation level. This figure shows that many high-skilled workers such as doctors or pilots have low values of the OOI (are very concentrated) despite having high wages, because their skills are very specific (Amior, 2019). This is in contrast to the correlation based on geography or demographic groups in which groups with higher OOI are typically those that are more highly paid. In the next section we use

[22]For instance, consider two fictional mass layoffs, one of which occurs in a geographically isolated town and the other of which occurs in a big city. Suppose there are two groups of workers—e.g. men and women—who differ in their sensitivity to distance. In the second layoff the gap in OOI is likely smaller, as there is a larger supply of nearby jobs. Hence, variation in the OOI would still exist even after controlling for location and workers type.

[23]This finding is robust to adding controls for worker demographics.

a standard shift-share approach to generate variation in workers' outside options, independent of productivity.

One of the key advantages of the OOI is that it identifies differences in options that emerge between workers with similar skills who work in similar labor markets. These differences could emerge due to preferences, constraints, or job availability. In order to examine these gaps more systematically, we next regress OOI on a vector of worker characteristics. Column 1 of Table 5 shows that, controlling for age (quadratic), education, and citizenship, the average OOI for women is still 0.295 units below the OOI of men. The average OOI for German citizen is 0.262 units higher OOI than that of the average non-citizen. Assuming similar distributions across jobs, this would imply that the average male (German citizen) has 34% (30%) more options than the average woman (non-citizen).[24] On average, workers with more education have higher values of OOI: lower-secondary (intermediate secondary) school workers' OOI are on average 0.60 (0.24) units lower than higher-secondary workers. This implies 82% (27%) more options assuming similar distribution across jobs. Together, these variables account for 13% of the variation in the OOI.

Columns 2-6 of Table 5 add controls for training occupation (Columns 2, 4, and 6), district of residence (Columns 3, 4, and 6), and establishment (Columns 5 and 6). An individual's prior occupation explains a significant fraction of the variation in OOI; the R-squared increases to over 0.25 once we control for an individual's training occupation (at the 3-digit level). The district of residence also explains a large fraction of the variation. However, across all columns of the table, we see a significant gap in the OOI between men and women: women have 0.237 units fewer OOI, controlling for establishment and training occupation. The gaps between education groups are only somewhat smaller when controlling for training occupation, district and establishment fixed effects. The OOI gaps between citizens and non-citizens are larger when we compare workers from the same district or establishment. This is because non-citizens are more concentrated in larger cities, which gives them access to more options. Column 7 of Table 5 shows that the results are robust to using a measure of the OOI calculated based on the distribution of vacant jobs, rather than the distribution of existing jobs. Information on how we calculate this distribution is provided in Appendix C.3.

6 Outside Options and Wage Inequality

Finally, we combine our estimates on the distribution of the OOI, with estimates of the OOI-wage semi-elasticity, to assess the overall effect of options on wage inequality. We first estimate the link between the OOI and wages using a shift-share instrument. We then decompose the between-group

[24] If two workers have the same distribution of working across jobs, but with different support, the differences in the OOI are differences in the measure of the support.

wage gap into the portion that can and cannot be explained by variation in the OOI. We conclude by examining which factors explain the OOI-induced wage gaps.

6.1 Linking Options and Wages

We use a shift-share ("Bartik") instrument to estimate the elasticity between wages and the OOI (Bartik, 1991; Blanchard and Katz, 1992). Identifying the relationship between options and wages is challenging for two reasons. First, the OOI estimator is a function of the observed characteristics X_i. These observables are likely to also capture differences in productivity, thus leading to omitted variable bias. Second, the OOI is estimated with noise, which may lead to attenuation bias. In order to cope with both issues, we use an instrumental variables strategy.

Our treatment follows prior work by Beaudry, Green, and Sand (2012), who used a shift-share instrument to show that there are spillovers in wages between industries. The instrument allows us to compare workers who work in the same industry, but who have different outside options, because of differences in the industry mix of their local labor markets. Because local growth of certain industries may be due to the impact of local productivity shocks, we use national industry trends to construct the instrument.

We estimate the national growth of different industries, controlling for region-wide shocks, by regressing the change in employment in industry j in region r between 2004 and 2014 on industry and region fixed effects[25]

$$\Delta_{04}^{14} \log \tilde{E}_{jr} = g_j + g_r + \varepsilon_{jr}.$$

Regions are defined by the administrative regions ("Regierungsbezirke") in Germany and industries are defined at the 3-digit level.[26] By construction, the estimator of $\widehat{g_j}$ is not driven by regional trends captured in $\widehat{g_r}$. This construction verifies that our instrument is not driven by local employment shocks in this region, or in nearby regions.

For each combination of region and industry, the instrument is the average predicted change in OOI we would observe if the distribution of jobs changed only as the result of (national) growth trends in each industry. In particular, we first construct $\widetilde{g}_{14}(z_j)$, the predicted distribution of jobs in 2014 based only on national trends in industry growth:

$$\log \widetilde{g}_{14}(z_j) = \log g_{04}(z_j) + \widehat{g}_j + c$$

[25]We use $\log \tilde{E_{jr}} = \log(E_{jr} + 1)$, which is a Bayesian correction of a uniform prior that adds one observation in each industry, region and year combination (Gelman et al., 2013).

[26]We take all 39 regions based on the NUTS2 level coding of the European Union. This includes historical administrative regions that have been disbanded. In results not reported, we find that the results are robust to the definition of a region and to using the NUTS3 level (district).

where $g_{04}(z_j)$ is the observed distribution of jobs in the base year (2004), $\widehat{g}_j$ is the national growth rate of industry j, and c is a normalization constant, which ensures that $\tilde{g}_{14}(z_j)$ integrates to 1.

We calculate the predicted OOI in 2014 using these probabilities and aggregate to the region-industry level. To do this, we use our existing estimates of $\widehat{f_{Z|X}(z_j|x_i)}$ (Equation 7) to calculate the predicted OOI based on Equation 8

$$\widetilde{OOI_{i,2014}} = -\sum_{z_j} \widehat{f_{Z|X}(z_j|x_i)} \left(\frac{\log \widehat{f_{Z|X}(z_j|x_i)}}{\log \widetilde{g}_{14}(z_j)} \right)$$

and a predicted OOI change using

$$\Delta_{04}^{14}\widetilde{OOI}_i = \widetilde{OOI_{i,2014}} - OOI_{i,2004}$$

This counterfactual OOI change captures the changes in options that are driven solely by national industry shocks. Since this counterfactual is still estimated with noise, we construct the instrument as the average counterfactual for each individual that is in industry j and region r in 2004. We denote the set of these individuals $\mathcal{S}(j,r)$. Formally, the instrument is:

$$Z_{j,r} = \frac{1}{|\mathcal{S}(j,r)|} \sum_{i \in \mathcal{S}(j,r)} \Delta_{04}^{14}\widetilde{OOI}_i$$

We then estimate the following system of equations:

$$\begin{array}{rcll} \Delta_{04}^{14}\log w_i & = & \alpha \Delta_{04}^{14} OOI_i & +\beta\Delta_{04}^{14}X_i + Ind_{j(i,2004)} + \upsilon_i \\ \Delta_{04}^{14} OOI_i & = & \gamma Z_{j(i,2004),r(i,2004)} & +\delta\Delta_{04}^{14}X_i + Ind_{j(i,2004)} + \epsilon_i, \end{array} \tag{11}$$

where we control for Ind_j^{04}, the worker's industry at the beginning of the period (2004), and for additional demographic controls, X_{ijr}. The parameter of interest is α, the semi-elasticity of wages with respect to options.[27] We use all observations of workers who are employed in both 2004 and 2014. We cluster standard errors at the region level.[28]

Table 6 presents the main results. Column 1 shows that a 0.1 increase in the instrument, translates to approximately .03 increase in the estimated OOI (about 3% more options), and .5% increase in wages. Combining both estimates yields a semi-elasticity of 0.17: a 10% increase in

[27] The coefficient α is analogous to $\alpha_x + \alpha_z$ in our model. Note that this is not the bargaining parameter $\frac{\alpha_x}{\alpha_x+\alpha_z}$. Rather, α captures the full impact of having more options on income, including both the ability to find a better match, and the ability to gain a larger share of the surplus, as discussed in Section 2.4.

[28] We do not use sampling weights in this exercise. Shift-share instruments do not yield the average treatment effect in the population (Goldsmith-Pinkham, Sorkin, and Swift, 2020). Therefore including sampling weights does not make the results representative of the population. Repeating the analysis using sampling weights yields similar point estimates, with lower statistical power.

relevant options leads to a 1.7% increase in wages. In column 2 we report results from a specification that adds additional demographic controls (gender, education level and quadratic in age), hence control for differential wage trends by demographics. The specification reported in Column 3 also includes regional controls for share of high-secondary graduates, population size, and share of non-citizen workers. Our results remain stable across the different specifications.

The identifying assumption is that growing industries are not systematically located in regions where wages are growing for other reasons (Borusyak, Hull, and Jaravel, Forthcoming). One way the assumption could be violated is if there are productivity spillovers. Workers that live near industries that are growing, may enjoy a local demand shock for their production due to the positive income effect on workers in that region. This could generate a wage increase, that is not driven by the improvement in their outside options. This is particularly a concern for workers who are producing non-tradable goods, whose productivity is set by local demand.

Following Beaudry, Green, and Sand (2012) we address this concern by showing that our results hold for workers in exporting industries, which are less likely to be affected by local demand shocks. We use information from the establishment survey to calculate the export share of each industry.[29] We divide our data into three groups based on the export share of the industry where the worker worked in 2004. If part of the effect is induced by demand-shock driven productivity increases, we would expect to find a smaller effect in exporting industries. Table 6 shows the results for each of the groups. We find a large and statistically significant elasticity between options and wages even among workers in industries with the highest exporting share. Column 4 indicates that, in response to a 0.1 increase in OOI, workers in these industries see their wages rise by 2.3%. If anything, this elasticity is somewhat higher than that in our baseline results (0.23 versus 0.17).

We examine heterogeneity by gender and education by re-estimating equations 11 within demographic groups. Columns 1–5 of Table 7 present the results of this exercise. While splitting the sample by gender or education increases the size of the confidence intervals, the point estimates are relatively stable. The estimated coefficients are slightly lower for women (.07) and higher secondary workers (.13), and higher for men (.22) and lower secondary workers (.25), but we cannot reject that they are all identical. This stability suggests that using the same semi-elasticity for all groups (as we do in our counterfactual exercise below) is a reasonable approximation.

We decompose the effect of access to more options into impacts for job stayers and movers. Because the choice of whether to move is endogenous, we view this as a descriptive exercise. We split our sample based on an indicator variable for whether a worker stayed at their establishments during this period. Table A3 shows that, as our model predicts, stayers saw smaller gains than

[29] This is a lower bound for demand from outside the region, as it does not include sales to other regions in Germany, which we cannot see in our data. We calculate the mean at the industry level because we do not have the share of sales from exports for all employers, only a representative sample.

movers. Our model provides one possible explanation. Stayers only benefit through an improvement in their outside options, while the larger effect on movers is consistent with the fact that both their outside options and match quality improve.[30]

6.2 Decomposing Wage Gaps

We next examine the contribution of the OOI to between-group wage inequality. As a baseline, we estimate a standard Mincer regression of log wages on demographic characteristics including indicators for each education group, a quadratic function of age, gender, and citizenship status. We also control for whether an individual's job is part-time. Because wages are top-coded, we use a Tobit model to estimate the coefficients, $\hat{\beta}_0$:

$$\log w_i = \beta_0 X_i + \epsilon_i. \tag{12}$$

These coefficients, presented in the red bars of Figure 4, show that the gender wage gap, accounting only for differences in age, citizenship, and education, is roughly 20% $(\exp(.19) - 1)$. Accounting for other demographic characteristics, German citizens earn 8% higher wages than non-citizens; workers with higher-secondary education earn 34% more than those with intermediate secondary education, who earn 13% more than workers with lower secondary education.

We then examine the extent to which these wage gaps are explained by the OOI. For each individual the non-OOI portion of wages is $\log w_i - \widehat{\alpha} OOI_i$. Because wages are top-coded, rather than simply subtracting $\hat{\alpha}$ times the OOI, we regress log wages on the same demographic characteristics as before, and OOI:

$$\log w_i = \underbrace{\widehat{\alpha}}_{.17} OOI_i + \beta_1 X_i + \nu_i. \tag{13}$$

We constrain the coefficient on OOI to be $\hat{\alpha} = .17$, as estimated in the previous section. While $\widehat{\beta_0}$ captures the overall gaps in wages between demographic groups, $\widehat{\beta_1}$ captures the gaps driven by factors other than the OOI.

The difference $\widehat{\beta_0} - \widehat{\beta_1}$ is the part that can be attributed to the differences in OOI. This difference is plotted in the light blue bars in Figure 4. This figure shows that differences in the OOI would imply a roughly 4% wage gap between men and women; this is about 20% of the overall gap (red bar). They also imply a 2% wage gap between citizens and non-citizens (28% of the total gap). The differences in OOI correspond to 3% (7%) higher wages for high-secondary graduates (intermediate secondary graduates), 9% (60%) of the overall return.

[30]The theoretical model suggests that the impact on movers depends on the level of unobserved heterogeneity of workers compared to jobs, and it is double that of stayers when the heterogeneity is the same ($\alpha_x = \alpha_z$); a 95% confidence interval for the ratio of the coefficients in Columns 2 of Table A3 includes 2.

6.3 The Role of Commuting Costs

Finally, we explore the role that commuting costs play in generating wage gaps. Our focus on commuting costs is informed by the analysis in section 5.3, which revealed that large OOI gaps between different demographic groups exist even between workers with the same training, between workers in the same district, and between workers in the same establishment (Table 5). It is also informed by the patterns in Table 2: we see large gaps in distance between male and female workers and between workers of different education groups.

We quantify the overall effect of differences in commuting and moving costs on wages through their effect on the OOI. We estimate the counterfactual wage gain for every worker, if they had the sensitivity to distance of a 40 year old male German citizen with the highest level of education. We generate a matrix $\widetilde{A}$ where the coefficients on the distance variables are set to this level for all workers. We then simulate the counterfactual probabilities $\widetilde{f(z_j|x_i)}$ using this matrix, and calculate the $\widetilde{OOI_i}$. This counterfactual should be thought of as changing only a zero measure number of workers each time, and keeping all other workers and employers unchanged, so that there are no general equilibrium effects. We then calculate workers' predicted log wages under this counterfactual.

A worker's counterfactual wage is constructed as the sum of the portion of wages that is not due to the OOI (the difference between log wages and $\hat{\alpha}$ times the actual OOI) and the portion that would come from the OOI under the counterfactual ($\hat{\alpha}\widetilde{OOI_i}$). This counterfactual wage is $\underbrace{\log w_i - \alpha OOI_i}_{\text{non}-\text{OOI portion}} + \underbrace{\hat{\alpha}\widetilde{OOI_i}}_{\text{counterfactual OOI}}$. Due to censoring in wages, rather than regress the counterfactual wage on X_i, we continue with the Tobit specification, running:

$$\log w_i = \underbrace{\hat{\alpha}}_{.17}\left(\mathrm{OOI}_i - \widetilde{OOI_i}\right) + \beta_2 X_i + \epsilon_i. \tag{14}$$

We constrain the coefficient on the OOI to be $\hat{\alpha} = .17$. The coefficients β_2 from this regression are the wage gaps that would emerge, if commuting and moving costs were equalized. The results of this exercise are presented in Figure 4. The figure plots the full gap ($\widehat{\beta}_0$, red bar), the portion that can be attributed to the OOI ($\widehat{\beta}_0 - \widehat{\beta}_1$, light blue bar), and the part that would be closed by equalizing commuting costs at the minimal level ($\widehat{\beta}_0 - \widehat{\beta}_2$, navy bar).

Differences in commuting costs seem to explain all of the gender gap that is driven by differences in the OOI. Equalizing commuting costs would increase wages for women by about 0.05 log units, relative to men. This is roughly 24% of the overall gender gap, and all of the gap that is explained by differences in the OOI.

The education gap in OOI reverses once we equalize commuting costs: workers with lower levels of education have more options than those with higher levels. Therefore, the higher (intermediate) secondary premium drops by 0.07 (0.09) log units, which is 24% (81%) of the overall premium. This is more than the full effect of the OOI difference between these groups. This implies that, in a given area, workers with lower levels of education have more relevant job options than workers with higher levels of education. It is only because more educated workers are willing to take jobs in more distant areas, that they end up with more options. This result can be explained by the fact that more educated workers tend to be more concentrated in occupations that have more industry specific skills.

These findings suggest that policies that affect workers' abilities to commute—such as transportation policies—are likely to have a heterogeneous impact on the options of workers in different demographic groups (Butikofer, Loken, and Willen, 2019). We test this prediction in Appendix E by using the OOI to study the effect on options of the introduction of a high speed rail in the small German town of Montabaur (Heuermann and J. F. Schmieder, 2018). We find that introducing these trains gave workers, especially those with more education, access to more distant jobs. It also generated new jobs in Montabaur that mostly benefited women, who are more likely to work in town.

7 Conclusion

In this paper we provided a distinctive and micro-founded approach to empirically estimate workers' outside options, and to measure the impact of outside options on the wage distribution. We used a two-sided matching model, to derive a sufficient statistic for the impact of outside options on wages, which we call the OOI. We then showed how this index can be estimated, even in large linked employer-employee datasets.

We documented, using linked employer-employee data from Germany, that the OOI can explain a meaningful portion of between-group wage inequality. We found that roughly 20% of the gender wage gap can be explained by differences in the OOI. Differences in the OOI between men and women are largely driven by differences in the implicit cost of commuting. Our findings are in line with other recent research documenting that individuals' labor markets are more local than commuting zone-based measures would suggest (Manning and Petrongolo, 2017; Le Barbanchon, Rathelot, and Roulet, 2021). The OOI provides a way to measure workers' option sets that does not depend on rigid occupation, industry, or geographic boundaries.

One direction for future research would be to use the OOI to examine the impact of particular policies on the labor market, such as restrictions on the use of non-compete clauses, re-training programs, or changes in transportation infrastructure. We provide an example of how such analysis could be done in Appendix E. It would also be interesting to use the OOI to study the outside options of employers and how changes in these outside options affect their wage setting decisions and profits.

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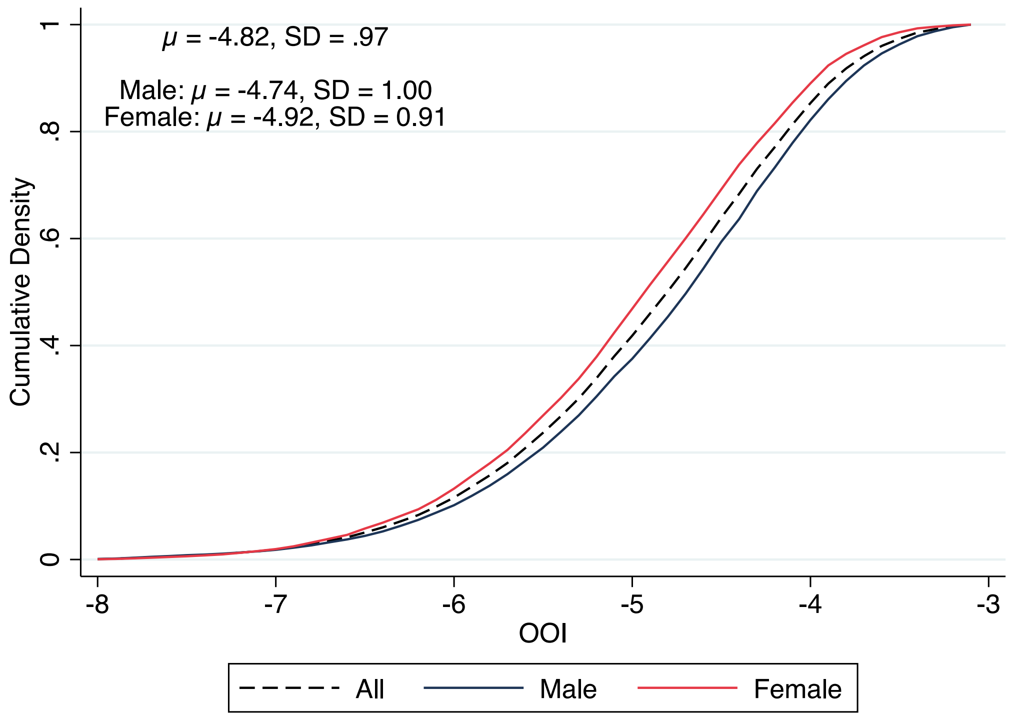
8 Figures and Tables

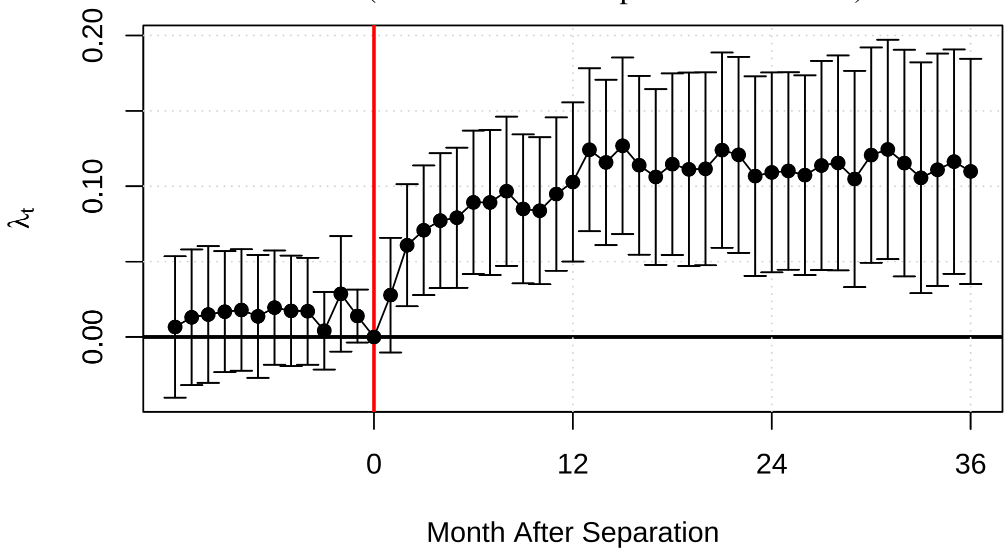
Figure 1: Distribution of the Outside Option Index



Note: This figure plots the cumulative distribution function of the outside options index overall and by gender, as calculated for the population of German workers as of June 30th, 2014. The OOI was calculated using the procedure described in Section 3. We construct the distributions using LIAB sample weights.

Figure 2: Mass-Layoffs Exercise

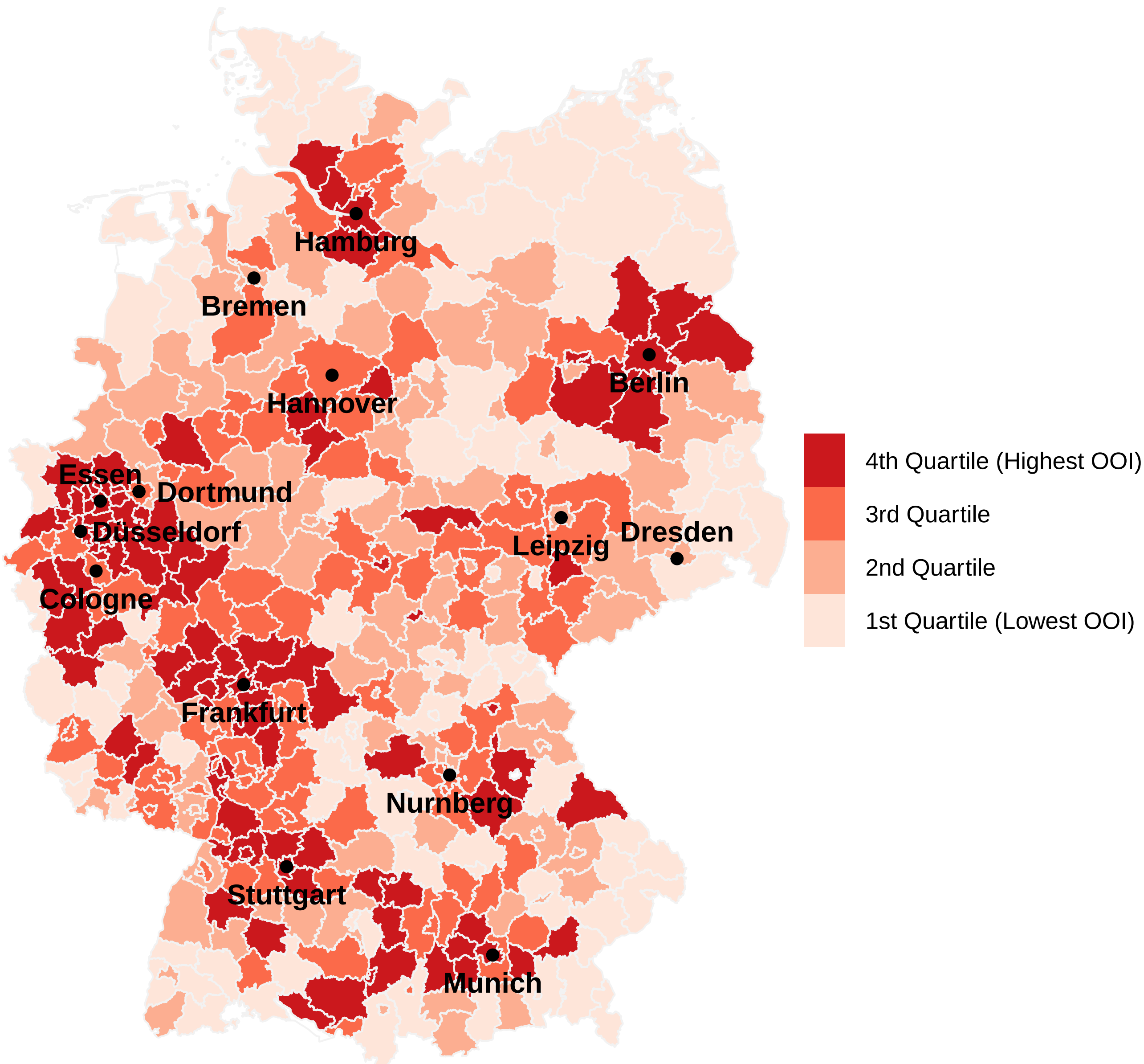
Panel A: Income (Relative to Pre-Displacement Income)



Panel B: Employment

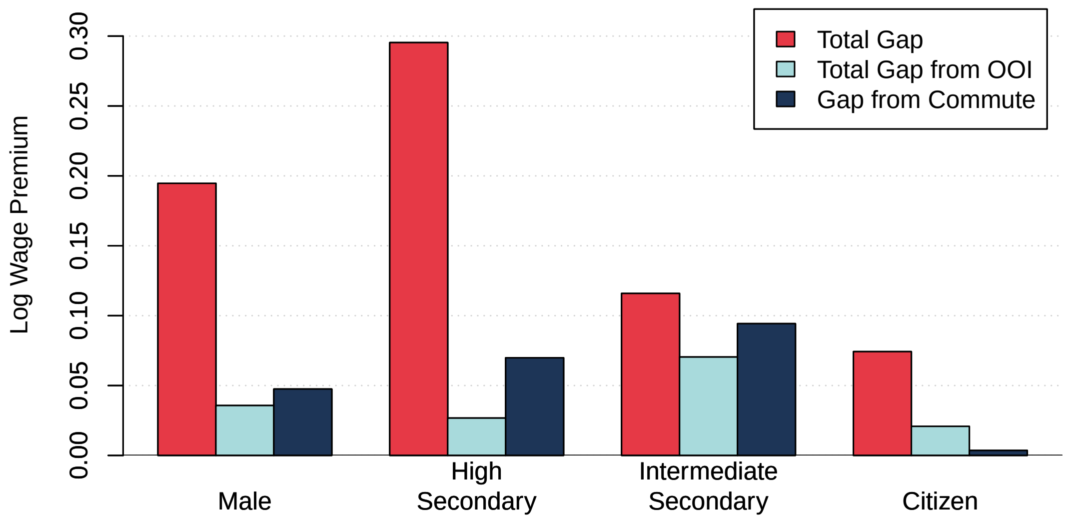
Note: This figure shows estimates of the coefficients of the OOI by month, λ_t and λ_t^{emp}, from Equations 9 (Panel A) and 10 (Panel B). A mass layoff occurs when an establishment with at least 50 workers reduces its workforce by at least 30% in a given year. The sample includes only workers below the age of 55 with at least three years of tenure before the layoff. The sample includes observations for workers up to 36 months following the mass layoff. The coefficients are taken from a regression of the outcome variable on the OOI, interacted with indicators for each month after separation (plotted), controlling for establishment-month fixed effects. Relative income (Panel A) is defined as the current daily income in that month divided by daily income before the layoff (month 0). Employment (Panel B) is an indicator for any positive income.

Figure 3: Distribution of OOI by District



Note: This figure plots the distribution of the outside options index by district (kreis) as calculated for the population of German workers as of June 30th, 2014. The OOI was calculated using the procedure described in Section 3. We use weights that adjust the population size of each district to its real value as measured by the Regional Database Germany ("Regionaldatenbank Deutschland") for the year of 2011 (Table 12111-01-01-4).

Figure 4: Effect of Commuting/Moving Costs



Note: This figure shows the extent to which the between-group wage gap can be explained by differences in the OOI. The red bars (Total gap) are the coefficients on the corresponding characteristic ($\hat{\beta}_0$) from a regression of log wages on Male, Citizen, an indicator for secondary-education category, a quadratic in age, and an indicator for part-time job (Equation 12). The light blue bars (Total gap from OOI) are the difference between the raw gaps ($\widehat{\beta_0}$) and remaining gaps without the OOI ($\widehat{\beta_1}$ in Equation 13). The navy bars capture the portion of the gap that is driven by differences in commuting and moving preferences/constraints. We simulate a counterfactual OOI for all workers, if they had the same sensitivity to distance as a 40 year old male citizen with the highest level of education. We then calculate counterfactual wages for all workers if they had this OOI by multiplying the OOI differences with the elasticity α. The navy bars capture the portion of the gap that is explained by commuting costs ($\hat{\beta}_0 - \hat{\beta}_2$ from Equation 14). See Section 6.3 for more details.

Table 1: Descriptive Statistics

	All		Male		Female	
	Mean	SD	Mean	SD	Mean	SD
	(1)	(2)	(3)	(4)	(5)	(6)
Workers						
Age	46.32	(11.64)	45.89	11.87	46.82	11.34
Female	46%	(0.50)	0%	---	1.00	---
German Citizen	98%	(0.14)	98%	0.16	0.99	(0.12)
Higher Secondary Degree	28%	(0.20)	27%	(0.20)	29%	(0.20)
Intermediate Secondary Degree	31%	(0.21)	27%	(0.20)	34%	(0.23)
Lower Secondary Degree	19%	(0.16)	19%	(0.15)	21%	(0.16)
Intermediate/Lower Education	22%	(0.17)	27%	(0.20)	16%	(0.14)
Daily Earnings	87.30	(51.23)	104.27	(50.87)	67.3	(43.90)
Distance	12.90	(39.15)	15.80	(43.71)	9.49	(32.64)
Jobs						
Establishment size	1547.75	(7665.13)	2183.74	(9368.63)	797.77	(4847.42)
Sales per worker in 2013 (€)	165341	(187464.80)	193785	(199633.30)	131798	(165859.70)
Part-time contract	31%	(0.46)	12%	(0.33)	53%	(0.50)
Observations	411,408		262,995		148,413	

Note: This table describes the workers and jobs in our sample, which consists of employment relationships as of June 30, 2014. Estimates are computed using LIAB sampling weights. We describe the data and sample restrictions in Section 4.2.

Table 2: Heterogeneity in Distance

	Distance (Miles) (1)	<5 Miles (2)	5-20 Miles (3)	20-50 Miles (4)	50+ Miles (5)
All	12.9	73.45%	15.51%	6.34%	4.71%
Male	15.8	69.28%	17.23%	7.37%	6.11%
Female	9.5	78.36%	13.48%	5.13%	3.02%
Higher Secondary Degree	22.1	62.50%	19.42%	9.10%	8.98%
Intermediate Secondary Degree	9.9	77.05%	13.97%	5.76%	3.20%
Lower Secondary Degree	9.4	77.78%	13.46%	5.58%	3.18%
Intermediate/Lower Education	8.0	79.04%	14.42%	4.08%	2.48%

Note: This table presents descriptive statistics on the distance between a person's current job and her place of residence before taking this job. Each number is the sample average of the indicated characteristic (column) for those in that subgroup (row). Estimates are computed using LIAB sampling weights. Section 4.2 provides information on the data and sample restrictions.

Table 3: Outside Options for Workers in a Mass Layoff

	(1)		(2)		(3)		(4)	
	A. Relative Wages							
3 Months (λ_3)	0.0708	***	0.0706	***	0.0674	***	0.0678	***
	(0.02)		(0.02)		(0.02)		(0.02)	
6 Months (λ_6)	0.0893	***	0.089	***	0.0832	***	0.0834	***
	(0.02)		(0.02)		(0.03)		(0.03)	
12 Months (λ_{12})	0.103	***	0.102	***	0.0888	***	0.0881	***
	(0.03)		(0.03)		(0.03)		(0.03)	
24 Months (λ_{24})	0.109	***	0.109	***	0.0785	**	0.0751	**
	(0.03)		(0.03)		(0.04)		(0.04)	
Observations	547,353		547,353		547,353		547,353	
	B. Employment							
3 Months (λ_3)	0.016	***	0.016	***	0.013	**	0.012	**
	(0.01)		(0.01)		(0.01)		(0.01)	
6 Months (λ_6)	0.008		0.008		0.004		0.002	
	(0.01)		(0.01)		(0.01)		(0.01)	
12 Months (λ_{12})	0.016	**	0.016	**	0.009		0.007	
	(0.01)		(0.01)		(0.01)		(0.01)	
24 Months (λ_{24})	0.017	***	0.017	***	0.011		0.007	
	(0.01)		(0.01)		(0.01)		(0.01)	
Observations	547,353		547,353		547,353		547,353	
Establishment-Month FE	✓		✓		✓		✓	
Tenure			✓		✓		✓	
Age					✓		✓	
Education					✓		✓	
Gender					✓		✓	
Training Occupation Characteristics							✓	
Workers	13,404		13,404		13,404		13,404	

Note: This table shows the results of regressing relative income and employment on OOI for workers that lost their jobs in a mass-layoff (λ_t and λ_t^{emp} in Equations (9) and (10)). Time is defined relative to the mass layoff. A mass layoff occurs when an establishment with at least 50 workers reduces its workforce by at least 30% in a given year. The sample includes only workers below the age of 55 with at least three years of tenure before the layoff. Relative wages (Panel A) are defined as current daily income divided by the last daily income before the layoff (w_{it}/w_{i0}). Employment (Panel B) is an indicator for any positive income. The tenure controls include a quadratic polynomial of days at the previous (mass layoff) establishment; the age controls include a quadratic polynomial in age; the education controls include dummies for the type of secondary education. All controls are interacted with months since mass layoff. See section 4.1 for details. Levels of significance: *10%, ** 5%, and *** 1%.

Table 4: Distribution of the OOI

	Mean	SD	Quantiles		
			25th	50th	75th
	(1)	(2)	(3)	(4)	(5)
All	-4.82	0.97	-5.37	-4.70	-4.14
Male	-4.74	1.00	-5.28	-4.59	-4.05
Female	-4.92	0.91	-5.47	-4.83	-4.27
Citizen	-4.82	0.95	-5.36	-4.70	-4.14
Non-Citizen	-5.10	1.37	-5.52	-4.86	-4.34
Higher Secondary Degree	-4.58	0.92	-5.01	-4.45	-3.99
Intermediate Secondary Degree	-4.76	0.87	-5.32	-4.67	-4.11
Lower Secondary Degree	-4.91	0.95	-5.47	-4.80	-4.22
Intermediate/Lower Education	-5.14	0.93	-5.69	-5.08	-4.46

Note: This table describes the distribution of the OOI for different groups of workers. We compute the OOI using data from our baseline cross-section (June 30, 2014). We list the X and Z variables used to construct the OOI in Section 4.3. Estimates are computed using LIAB sampling weights.

Table 5: Heterogeneity in the OOI

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Female	-0.295 ***	-0.268 ***	-0.283 ***	-0.255 ***	-0.201 ***	-0.237 ***	-0.344 ***
	(0.009)	(0.011)	(0.007)	(0.008)	(0.008)	(0.008)	(0.009)
Non-Citizen	-0.262 ***	-0.226 ***	-0.553 ***	-0.498 ***	-0.539 ***	-0.494 ***	-0.675 ***
	(0.036)	(0.032)	(0.030)	(0.026)	(0.022)	(0.020)	(0.025)
Lower-Secondary Certificate	-0.601 ***	-0.535 ***	-0.526 ***	-0.474 ***	-0.504 ***	-0.464 ***	-0.374 ***
	(0.014)	(0.014)	(0.011)	(0.010)	(0.011)	(0.010)	(0.010)
Intermediate	-0.236 ***	-0.211 ***	-0.110 ***	-0.110 ***	-0.129 ***	-0.129 ***	-0.098 ***
	(0.011)	(0.011)	(0.008)	(0.008)	(0.009)	(0.008)	(0.009)
Age Controls	Quadratic	Quadratic	Quadratic	Quadratic	Quadratic	Quadratic	Quadratic
Training Occupation FE		✓		✓		✓	✓
District FE			✓	✓			✓
Establishment FE					✓	✓	
OOI Based on Vacancies							✓
Adjusted R-Squared	0.133	0.253	0.530	0.629	0.573	0.627	0.562
Observations	375,765	375,765	375,765	375,765	375,765	375,765	375,765

Note: Each column in this table presents results from a regression of OOI_i on the covariates listed in that column. In columns 1-6, OOI_i is calculated using the full distribution of jobs (Equation 8). In column 7, the dependent variable is OOI_i^v , which is calculated based on the distribution of vacant jobs (Equation 22). The sample includes all workers with complete information that were employed on June 30th, 2014. We use LIAB sampling weights. Training occupation fixed effects are at the 3-digit level. Levels of significance: *10%, ** 5%, and *** 1%.

Table 6: Linking Outside Options and Wages

	Full Sample			By Exporting Share of Sales: More than 33%	By Exporting Share of Sales: Between 1 and 33%	By Exporting Share of Sales: Less than 1%
	(1)	(2)	(3)	(4)	(5)	(6)
First Stage	0.299 ***	0.276 ***	0.242 ***	0.353 ***	0.204 ***	0.272 ***
	(0.064)	(0.048)	(0.064)	(0.104)	(0.059)	(0.080)
Reduced Form	0.0517 **	0.0504 **	0.038	0.080 ***	0.009	0.031
	(0.021)	(0.021)	(0.024)	(0.026)	(0.026)	(0.023)
2SLS	0.173 ***	0.183 ***	0.156 *	0.227 ***	0.046	0.114
	(0.063)	(0.068)	(0.092)	(0.071)	(0.123)	(0.096)
Industry FE	✓	✓	✓	✓	✓	✓
Age Controls	✓	✓	✓	✓	✓	✓
Demographic Controls		✓	✓			
Regional Controls			✓			
F (First Stage)	21.95	32.82	14.5	11.52	12.04	11.56
Number of industry-regions	5510	5510	5510	2195	2525	790
Observations	435,586	435,586	435,586	144,039	147,529	144,018

Note: This table shows estimates from model 11. We use a two-stage least squares set-up where the outcome variable is the change in log daily wages 2004–2014 and the endogenous variable is the change in the OOI. The instrument is the average predicted change in the OOI in each 3-digit industry-region based on the national growth rate of each industry (see Section 6.1). All columns control for industry (in 2004) and age. The demographic variables (Columns 2 and 3) include gender, citizenship, education category and age squared. The regional controls (Column 3) include the share of high secondary, the population size and the share of non-citizens in each region. Columns 4–6 present the same results within subgroups defined by the exporting share of the worker's 2004 industry. The share of exports is calculated for each 3-digit industry based on the establishment panel survey in 2014. Standard errors, presented in parentheses below the coefficients, are clustered at the region level. Levels of significance: *10%, ** 5%, and *** 1%.

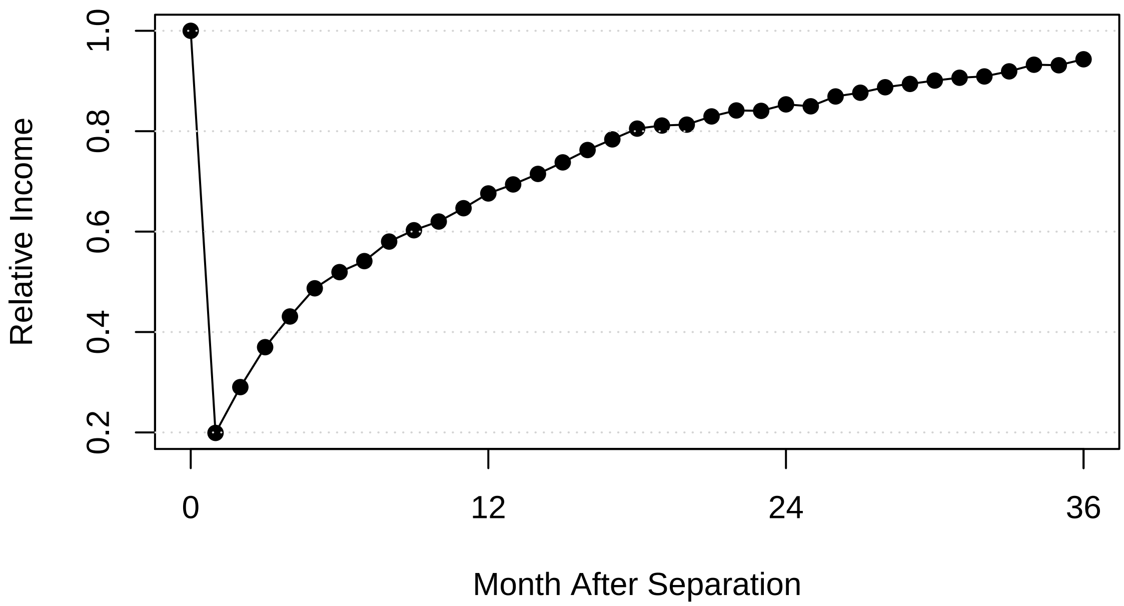
Table 7: Heterogeneity in Shift-Share Results

	By Gender			By Education						
	Male		Female		Higher Secondary		Intermediate Secondary		Lower Secondary	
	(1)		(2)		(3)		(4)		(5)	
First Stage	0.309	***	0.266	***	0.232	***	0.203	***	0.321	***
	(0.080)		(0.050)		(0.079)		(0.053)		(0.049)	
Reduced Form	0.0673	***	0.019		0.031		0.046	**	0.080	***
	(0.021)		(0.022)		(0.022)		(0.022)		(0.026)	
2SLS	0.218	***	0.071		0.134		0.228	**	0.247	***
	(0.059)		(0.086)		(0.099)		(0.103)		(0.078)	
Industry FE	✓		✓		✓		✓		✓	
Age Controls	✓		✓		✓		✓		✓	
F (first stage)	14.77		27.97		8.56		14.89		43.45	
Observations	283,550		152,036		96,148		148,136		91,793	

Note: This table shows estimates from model 11 estimated in different demographic groups. We use a two-stage least squares set-up where the outcome variable is the change in log daily wages 2004–2014 and the endogenous variable is the change in OOI. The instrument is the predicted change in the OOI in each 3-digit industry-region based on the national growth rate of each industry (see Section 6.1). All columns control for industry (in 2004), and age. The standard errors, presented in parentheses below the coefficients, are clustered at the region level. Levels of significance: *10%, ** 5%, and *** 1%.

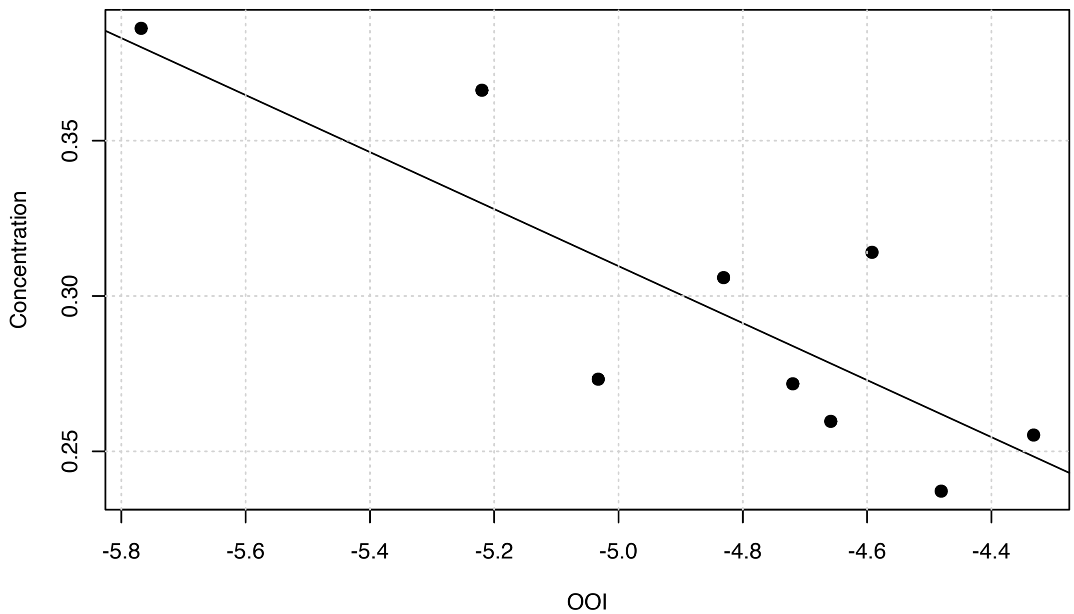
Appendix Figures and Tables

Figure A1: Impact of a Mass Layoff on Wages



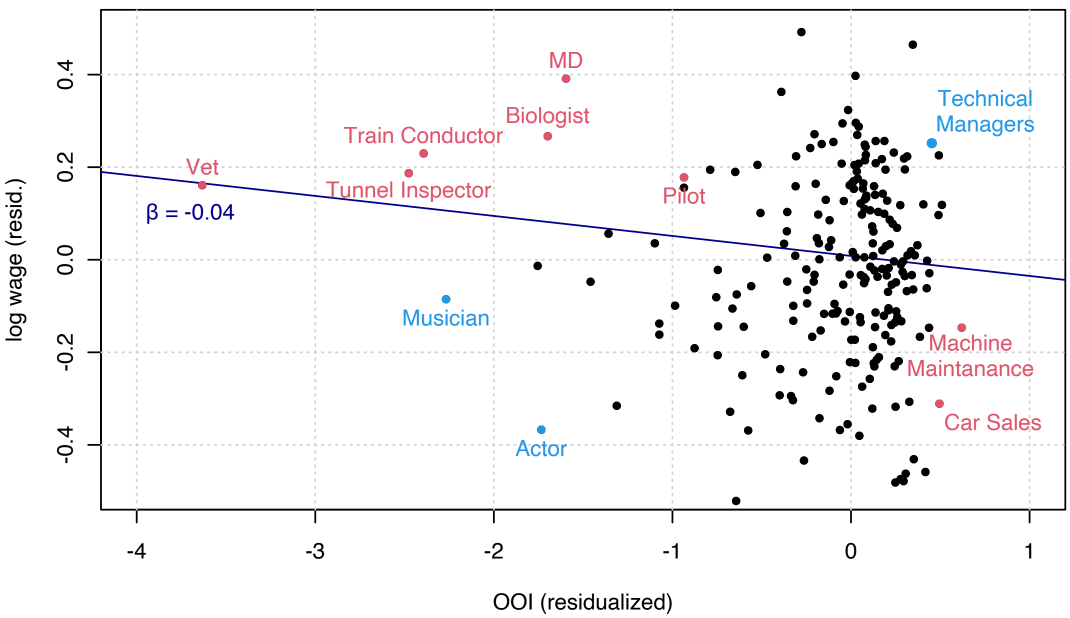
Note: This figure shows the impact of a mass layoff on workers' wages. Each dot is an estimate of β_τ from a regression of daily income on months since the layoff (relative to pre-layoff income) on a vector of dummies for months since the mass-layoff and for mass-layoff fixed effects. Time is defined relative to the mass layoff. A mass layoff occurs when an establishment with at least 50 workers reduces its workforce by at least 30% in a given year. The sample includes only workers below the age of 55 with at least three years of tenure before the layoff. The sample includes observations for workers up to 36 months following the mass layoff.

Figure A2: Skill Specificity and the OOI



Note: This figure shows a bin scatterplot of training occupations' concentration based on their average OOI as calculated for the population of German workers as of June 30th, 2014. Concentration is the share of workers who work at the most common occupation for workers with this training. The OOI was calculated using the procedure described in Section 3. Each dot shows the average OOI and concentration for training occupations that represent approximately 10% of the German population, sorted by the average training occupation OOI. Averages are calculated using the LIAB sample weights to make the distribution representative of the population in the occupation.

Figure A3: OOI by Training Occupation



Note: This figure plots the mean residualized outside options index and log wages by training occupation as calculated for the population of German workers as of June 30th, 2014. The OOI was calculated using the procedure described in Section 3. Residuals for the OOI and log wages were taken from a regression on gender, a quadratic in age, education category, citizenship status and district of residence. Means are calculated using the LIAB sample weights to make the distribution representative of the population in the occupation. See Section 4.2 for variable definitions.

Table A1: Most Weighted Questions in PCA

Panel A. Establishment Survey

	N	First Component	Second Component
Business Performance	8824	Member of chamber of industry	Profit
Investment & Innovation	8824	IT investment	Total investment
Hours	8824	Vacation credit policy	Flexible hours
Vocational Training	8824	Offer apprenticeship	Ability to fill training
General	8824	Family managed	Staff representation

Panel B. BIBB Survey

	N	First Component	Second Component
Hours	11021	Work on Sundays and public holidays	Hours per week like to work
Type of Task	15035	Have responsibility for other people	Cleaning, waste, recycling
Requirements	10904	Face acute pressure and deadlines	Highly specific regulations
Physical	20036	Oil, dirt, grease, grime	Pathogens, bacteria
Mental	17790	Support from colleagues	Often missing information about work

Note: This table shows the survey question that received the most weight in the first and second principal components in each survey category. We include the first two principal components from each survey category in our calculation of the OOI.

Table A2: Descriptive Statistics for Mass-Layoff Workers

	Main Sample		Mass Layoff Sample	
	Mean	SD	Mean	SD
	(1)	(2)	(3)	(4)
Workers				
Age	46.32	(11.64)	38.64	(10.62)
Female	0.46	(0.50)	0.40	(0.49)
German Citizen	0.98	(0.14)	0.98	(0.14)
Higher Secondary Degree	28%	(0.20)	18%	(0.15)
Intermediate Secondary Degree	31%	(0.21)	23%	(0.18)
Lower Secondary Degree	19%	(0.16)	20%	(0.16)
Intermediate/Lower Education	22%	(0.17)	39%	(0.24)
Daily Earnings	87.30	(51.23)	66.35	(85.93)
Workers	411,408		13,404	

Note: This table shows descriptive statistics for workers who lost their jobs in a mass-layoff (columns 3 and 4). The descriptive statistics of the main sample are presented for reference in columns 1 and 2. The OOI is computed using pre-layoff characteristics. Following Jacobson, Lalonde, and Sullivan (1993), we identify mass layoffs by focusing on establishments with more than 50 workers whose workforce declines by at least 30% in a given year. The sample includes only workers who have worked for at least three years before the layoff and are below the age of 55.

Table A3: Shift-Share Results by Job Mobility

	Stayers		Movers	
	(1)		(2)	
First Stage	0.243	***	0.324	***
	(0.040)		(0.092)	
Reduced Form	0.0703	**	0.00881	
	(0.034)		(0.021)	
2SLS	0.288	**	0.027	
	(0.137)		(0.057)	
Industry FE	✓		✓	
Age Controls	✓		✓	
Observations	190,545		245,041	

Note: This table shows the impact of OOI on wages interacted with whether a worker stayed at the same establishment. The outcome variable is the change in log wages 2004-2014. We instrument for the change in OOI over this time period using the average of a 3-digit industry national employment growth weighted by the initial share of every industry in a region (see Section 6.1). Stayers are defined as workers employed at the same establishment on June 30, 2004 and June 30, 2014. The standard errors, presented in parentheses below the coefficients, are clustered at the region level. Levels of significance: *10%, ** 5%, and *** 1%.

Table A4: Out of Sample Prediction for Different OOI Models

	Mean	SD	Quantile		
			25%%	50%	75%
	(1)	(2)	(3)	(4)	(5)
Baseline	325.9	1742.1	10.8	47.1	220.2
Panel Employer-Employee	318.2	1614.3	11.0	47.6	219.6
Cross-Sectional Employer-Employee	153.2	468.2	9.9	31.9	151.0
Population Survey	1.3	1.2	0.5	1.1	1.8
Observations			47,902		

Note: This table describes the distribution of the ratio between the predicted probability implied by the OOI model in each row and the probability implied by random matching for workers who switched firms after June 30, 2014. The predicted probabilities are estimated as described in Section 3.2 using the sample of workers employed on June 30, 2014. We compute the probability implied by random matching using the marginal distribution of job characteristics, estimated using LIAB sample weights. The estimates in each row come from models using different subsets of variables to estimate the predicted probabilities. The "population survey" model includes worker education (five categories), gender, age, citizenship, occupation characteristics and industry characteristics (both projected to top PCA components), and an indicator for part-time job. The "cross-sectional employer-employee" model adds two variables: the distance between each worker and job and establishment size. The "panel employer-employee" adds workers' training occupation characteristics (projected to top PCA components). The baseline model adds the top PCA components from the establishment survey and establishment size. More details are in Appendix D.

A Theoretical Appendix

A.1 Continuous Logit Distribution

We follow Dagsvik (1994) in defining the continuous logit models that produces ε_{i,z_j} and ε_{j,z_i}. In this section we define the distribution of ε_{i,z_j}. The distribution of ε_{j,x_i} is defined similarly.

Using continuous logit (rather than, for instance a multinomial logit) allows both workers and jobs to have characteristics that are continuous. For instance, it allows us to account for the possibility that the distance between a worker and a job affects the quality of a match in a continuous manner (e.g. a worker's preference for jobs may not vary discontinuously at boundaries of labor markets).

Each worker $i \in \mathcal{I}$ knows about a random subset of the total set of available jobs $\mathcal{J}$. For each of the jobs the worker is informed of, she draws ε_{i,z_j} shocks from a Poisson process on $\mathcal{Z} \times \mathbb{R}$ with intensity

$$g(z)\, dz \times e^{-\epsilon} d\epsilon$$

where $g(z)$ is the probability density function of job characteristics Z.[31] Denoting by P_i the infinite but countable points chosen in the process, every worker has a set of options for ε_{i,z_j}

$$\left\{\varepsilon_{iz_j} = \alpha_z \epsilon \middle| (z, \epsilon) \in P_i\right\}$$

This process yields a distribution of ε_{i,z_j} that has several similarities to finite extremum value type-1 distribution. These similarities are all derived from one basic property of this point process.

Proposition 1. *Let $h : \mathcal{Z} \to \mathbb{R}$ be a function that satisfies*

$$\int_{\mathcal{Z}} e^{h(z)} g(z)\, dz < \infty$$

and let $S \subseteq \mathcal{Z}$ be some Borel measurable subset. Define

$$\psi_S^g = \max_{z \in S \cap P_i} \left\{h(z) + \varepsilon_{i,z_j}\right\}$$

Then

$$\psi_S^g \sim EV_1\left(\alpha_z \log \int_S \exp \frac{h(z)}{\alpha_z} g(z)\, dz, \alpha_z\right)$$

and

$$S_1 \cap S_2 = \phi \iff \psi_{S_1}^{g_1} \perp \psi_{S_2}^{g_2}$$

[31] We deviate from the Poisson process used in Dupuy and Galichon 2014 as the density $g(z)$ also affects the intensity. This allows the joint distribution $f(x, z)$ to be properly defined over a larger class of functions for $\tau(x, z)$, including a constant, or simple polynomials.

Proof. This proposition stems from the fact that in a Poisson process, the amount of points chosen in two disjoint Borel measurable sets B_1, B_2 has an independent distribution $N(B_i) \sim Poisson(\Lambda(B_I))$ with

$$\Lambda(B_i) = \int_{B_i} \lambda(x)\, dx$$

Therefore, in our context the cumulative distribution function of ψ_S^g is

$$P(\psi_S^g \le x) = P(N((S \times \mathbb{R}) \cap \{h(z) + \alpha_z \epsilon > x\}) = 0)$$

From the Poisson distribution this is

$$\begin{aligned}
\log P(\psi_S^g < x) &= -\Lambda\left(S \times \left\{\epsilon > \tfrac{x-h(z)}{\alpha_z}\right\}\right) \\
&= -\int_S \int_{\frac{x-h(z)}{\alpha_z}}^{\infty} g(z)\, e^{-\epsilon} dz d\epsilon \\
&= -\int_S e^{-\frac{x-h(z)}{\alpha_z}} g(z)\, dz \\
&= -\exp\left[-\frac{x - \alpha_z \log \int_S \exp \frac{h(z)}{\alpha_z} g(z) dz}{\alpha_z}\right]
\end{aligned}$$

which is exactly a cumulative distribution function of $EV_1\left(\alpha_z \log \int_S \exp \frac{h(z)}{\alpha_z} g(z)\, dz, \alpha_z\right)$.

Since every draw of points in a Poisson process is independent, $S_1 \cap S_2 = \phi \iff \psi_{S_1}^{g_1} \perp \psi_{S_2}^{g_2}$. $\square$

This Proposition implies that even though ε_{i,z_j} is not defined for every $z \in \mathcal{Z}$, it is defined infinitely often for every Borel measurable subset that includes z, and the maximum for that set ψ_S^1 has an extreme-value type-1 distribution with variance α_z.

Since workers in equilibrium get a sum of a continuous function (which we denote by $\omega(x, z)$) and ε_{i,z_j} (Theorem 1), the maximum value they receive also has an EV_1 distribution for every Borel measurable set of jobs. Moreover, the probability density that a worker ends up in a job with observables z_j is similar to the finite case. Its exact value is

$$f_{Z|X}(z_j|x_i) = P\left(z_j = \arg\max_z \left\{\omega(x_i, z) + \varepsilon_{i,z_j}\right\}\right) = \frac{\exp\left[\frac{1}{\alpha_z}\omega(x_i, z_j)\right] g(z_j)}{\int_{\mathcal{Z}} \exp\left[\frac{1}{\alpha_z}\omega(x_i, z)\right] g(z)\, dz} \tag{15}$$

A direct link to the finite multinomial logit can be drawn if we divide $\mathcal{Z}$ into a finite number of disjoint sets $\mathcal{Z} = \bigcup_{i=1}^n S_i, S_i \cap S_j = \phi$. In this case, the value of the best job for worker i in each subset ($\psi_{S_i}^\omega$) is distributed EV_1. The maximization problem over each of the n job characteristics $z_{m(i)}$ is a finite multinomial logit. As n increases, the sets become smaller, and the choice becomes closer to an infinite options choice.

Like a standard multinomial logit, the continuous logit has the independence property: a worker's unobserved taste for jobs with characteristics in an neighborhood of z are uncorrelated with her unobserved taste for jobs with characteristics in a neighborhood of $z' \neq z$.

However, unlike in a standard multinomial logit, increasing the number of options to infinity does not yield infinite compensation. This is because when the number of options n grow, the mean measure of S_i decreases at a rate of $\frac{1}{n}$. As a result, the location parameter of each of the choices, decreases at a rate of $\log \frac{1}{n}$ (by the above proposition).

A.2 Theorems and Proofs

Theorem. *1 Under Assumptions 1, in equilibrium, worker i with characteristics x_i faces a continuous logit choice between employers who are offering*

$$\omega(x_i, z_j) + \varepsilon_{i,z_j}$$

and employers choose between workers who generate profits

$$\pi(x_i, z_j) + \varepsilon_{j,x_i}$$

where

$$\omega\left(x_i, z_j\right) + \pi\left(x_i, z_j\right) = \tau\left(x_i, z_j\right)$$

Proof. Consider two workers $i, i' \in \mathcal{I}$ and two jobs $j, j' \in \mathcal{J}$ that have the same observed characteristics: $X_i = X_{i'} = x_0$ and $Z_j = Z_{j'} = z_0$, where $m\left(i\right) = j$ and $m\left(i'\right) = j'$ (worker i is matched to job j and worker i is matched to job j').

By definition, the sum of worker and employer compensation equals total surplus

$$\omega_{ij} + \pi_{ij} = \tau\left(x_0, z_0\right) + \varepsilon_{i,z_0} + \varepsilon_{j,x_0} \tag{16}$$

$$\omega_{i'j'} + \pi_{i'j'} = \tau\left(x_0, z_0\right) + \varepsilon_{i',z_0} + \varepsilon_{j',x_0} \tag{17}$$

In equilibrium it must be that workers i, i' and jobs j, j' do not have a profitable deviation:

$$\omega_{ij} + \pi_{i'j'} \geq \tau\left(x_0, z_0\right) + \varepsilon_{i,z_0} + \varepsilon_{j',x_0} \tag{18}$$

$$\omega_{i'j'} + \pi_{ij} \geq \tau\left(x_0, z_0\right) + \varepsilon_{i',z_0} + \varepsilon_{j,x_0} \tag{19}$$

Inequality 18 says that the amount that worker i gets from being matched with employer j plus the amount that employer j' gets from being matched with worker i' must be weakly greater than the amount that the worker i and j' could get from matching with each other. Because we have assumed that the workers and jobs have the same observed characteristics this is $\tau(x_0, z_0)$ plus the sum of the idiosyncratic values of worker i (ε_{i,z_0}) and employer j (ε_{j,x_0}). Inequality 19 shows that employer j and worker i' also are better off in their equilibrium matches than they are from matching to each other.

Because the sum of the two weak-inequalities (18 and 19) is equal to the sum of the two equalities (16 and 17), the two weak inequalities must hold with equality. In particular, we can take the difference between 16 and 19 and the difference between 17 and 18 to write:

$$\omega_{ij} - \omega_{i'j'} = \varepsilon_{i,z_0} - \varepsilon_{i',z_0}$$

$$\pi_{ij} - \pi_{i'j'} = \varepsilon_{j,x_0} - \varepsilon_{j',x_0}$$

In other words, compensation for workers and employers in matches with the same characteristics is constant up to ε. As a result we can write

$$\omega_{ij} = \omega\left(x_0, z_0\right) + \varepsilon_{i,z_0}$$

$$\pi_{ij} = \pi\left(x_0, z_0\right) + \varepsilon_{j,x_0}$$

where ω_{ij} is the compensation worker i (with characteristics x_0) gets if she is matched with employer j (with characteristics z_0). Taking the sum of these equations and using Equation 16 yields

$$\omega\left(x_0, z_0\right) + \pi\left(x_0, z_0\right) = \tau\left(x_0, z_0\right)$$

□

Lemma. *1 Under Assumption 1:*

$$OOI := \frac{1}{\alpha_z} E\left[\varepsilon^*_{i,z_j} | x_i\right] = -\int f_{Z|X}\left(z_j|x_i\right) \log \frac{f_{Z|X}\left(z_j|x_i\right)}{g\left(z_j\right)}$$

Proof. Taking logs over Equation 15, using the fact that $\log \int_{\mathcal{Z}} \exp\left[\frac{1}{\alpha_z}\omega\left(x_i, z\right)\right] g\left(z\right) dz = \frac{1}{\alpha_z} E\left[\omega_{ij}|x_i\right]$ we get

$$\log f_{Z|X}\left(z_j|x_i\right) = \frac{1}{\alpha_z}\omega\left(x_i, z_j\right) + \log g\left(z\right) - \frac{1}{\alpha_z} E\left[\omega_{ij}|x_i\right]$$

Since $E\left[\omega_{ij}|x_i\right] = E\left[\omega\left(x_i, z^*_j\right)|x_i\right] + E\left[\varepsilon^*_{i,z_j}|x_i\right]$ and by taking expectations over both sides we get

$$E\left[\log f_{Z|X}\left(z_j|x_i\right)\right] = E\left[\log g\left(z\right)\right] - \frac{1}{\alpha_z} E\left[\varepsilon^*_{i,z_j}|x_i\right]$$

and so

$$\frac{1}{\alpha_z} E\left[\varepsilon^*_{i,z_j}|x_i\right] = -\int f_{Z|X}\left(z_j|x_i\right) \log \frac{f_{Z|X}\left(z_j|x_i\right)}{g\left(z_j\right)}$$

□

Theorem. 2 *In equilibrium, $\omega\left(x_i, z_j\right)$, the share of $\tau\left(x_i, z_j\right)$ that workers receive, satisfies:*

$$\omega(x_i, z_j) = \frac{\alpha_x}{\alpha_x + \alpha_z} E[\omega_{ij}|x_i] + \frac{\alpha_z}{\alpha_x + \alpha_z} \left(\tau(x_i, z_j) - E[\pi_{ij}|z_j]\right)$$

Proof. Define $Q_{z_j|x_i}$ to be the measure of jobs z_j that are chosen by workers with characteristics x_i and, analogously, $Q_{x_i|z_j}$, the measure of workers x_i that are chosen by jobs (firms) with characteristics z_j.

$$\begin{aligned} Q_{z_j|x_i} &= d(x_i) f_{Z|X}(z_j|x_i) \\ Q_{x_i|z_j} &= g(z_i) f_{X|Z}(x_i|z_j) \end{aligned}$$

In equilibrium, the measures of workers and firms choosing each other must be equal. Equalizing $Q_{z_j|x_i}$ and $Q_{x_i|z_j}$ we obtain:

$$\begin{aligned} Q_{z_j|x_i} &= Q_{x_i|z_j} \\ \underbrace{\frac{\exp \frac{1}{\alpha_z}\omega(x_i, z_j)}{\exp \frac{1}{\alpha_z} E[\omega_{ij}|x_i]} \times g(z_j)}_{f(z_j|x_i)} \times d(x_i) &= \underbrace{\frac{\exp \frac{1}{\alpha_x}\pi(x_i, z_j)}{\exp \frac{1}{\alpha_x} E[\pi_{ij}|z_j]} \times d(x_i)}_{f(x_i|z_j)} \times g(z_j) \end{aligned}$$

and, taking logs,

$$\frac{1}{\alpha_z}\omega(x_i, z_j) - \frac{1}{\alpha_x}\pi(x_i, z_j) = \frac{1}{\alpha_z} E[\omega_{ij}|x_i] - \frac{1}{\alpha_x} E[\pi_{ij}|z_j]$$

By construction,

$$\omega(x_i, z_j) + \pi(x_i, z_j) = \tau(x_i, z_j)$$

Adding $1/\alpha_x$ of this to the expression before yields:

$$\omega(x_i, z_j)\left(\frac{1}{\alpha_z} + \frac{1}{\alpha_x}\right) = \frac{1}{\alpha_z} E[\omega_{ij}|x_i] + \frac{1}{\alpha_x}\left(\tau(x_i, z_j) - E[\pi_{ij}|z_j]\right)$$

We end up with

$$\omega(x_i, z_j) = \frac{\alpha_x}{\alpha_x + \alpha_z} E[\omega_{ij}|x_i] + \frac{\alpha_z}{\alpha_x + \alpha_z} \left(\tau(x_i, z_j) - E[\pi_{ij}|z_j]\right)$$

and

$$\omega_{ij} = \omega(x_i, z_j) + \varepsilon_{i,z_j}$$

When $\alpha_x = \alpha_z = \alpha$ the expression for $\omega(x_i, z_j)$ further simplifies to:

$$\omega(x_i, z_j) = \frac{1}{2} E[\omega_{ij}|x_i] + \frac{1}{2} \left(\tau(x_i, z_j) - E[\pi_{ij}|z_j]\right)$$

□

A.3 Linking Outside Options and Wages

We next show how the OOI is a sufficient statistic for the impact of outside options on workers' wages. We examine how compensation changes if we increase the size of the option set for workers with observed characteristics x_i, while keeping the quality of options constant. To do so, we assume that the intensity of the Poisson process is multiplied by some constant λ_{x_i} (previously we assumed $\lambda_{x_i} = 1$)

$$\lambda_{x_i} \times g(z)\, dz \times e^{-\epsilon} d\epsilon$$

As we discussed in Appendix A.1, in the continuous logit model, the intensity of the Poisson process governs the number of options workers have in every subset of jobs. A higher value of λ_{x_i} generates more options in every subset of jobs. Such shocks could include, e.g. the arrival of information shocks about other job options (which we do not model here), or drops in regulatory barriers such as non-compete agreements. They do not however include shocks to productivity or preferences that are likely changing the quality of the jobs as well. Note that we keep λ fixed for all other values of x.

Theorem. *3 Access to options λ_{x_i} has the following overall effect on expected worker compensation in equilibrium*

$$\frac{dE[\omega_{i,j}]}{d\lambda_{x_i}} = (\alpha_x + \alpha_z) \frac{dOOI_i}{d\lambda_{x_i}}$$

Proof. The value of $\tau(x_i, z_j)$ does not depend on λ_{x_i}. Employers' expected profits are also not affected by any change in λ_x because the set of workers with characteristics x_i has zero measure. Theorem 2 then implies that any change to $\omega(x, z)$ is independent of z:

$$\forall z_j : \frac{d\omega(x_i, z_j)}{d\lambda_{x_i}} = \frac{\alpha_x}{\alpha_x + \alpha_z} \frac{dE[\omega_{ij}|x_i]}{d\lambda_{x_i}} = \gamma(x_i)$$

Because $\omega(x_i, z_j)$ changes by a constant, Equation 15 implies that $f(z|x_i)$ is unchanged. Therefore, the first two components of the decomposition in Equation 5 are unchanged. Therefore any change in λ_{x_i} will only affect compensation through the third component, $(\alpha_x + \alpha_z) OOI_i$, Since α_x, α_z are fixed

$$\frac{dE[\omega_{i,j}]}{d\lambda_{x_i}} = (\alpha_x + \alpha_z) \frac{dOOI_i}{d\lambda_{x_i}}$$

$\square$

In the generalized version of our model, the OOI depends on λ. The best offer in every subset S the best offer is distributed

$$EV_1\left(\alpha_z \log \int_S \exp \frac{\omega\left(x_i, z_j\right)}{\alpha_z} g\left(z\right) \lambda_{x_i} dz, \alpha_z\right)$$

Using similar calculation as in the proof of Lemma 1 we find that

$$OOI_i = \frac{1}{\alpha_z} E\left[\varepsilon^*_{i,z_j} | x_i\right] = -\int f_{Z|X}\left(z_j|x_i\right) \log \frac{f_{Z|X}\left(z_j|x_i\right)}{g\left(z_j\right)\lambda_{x_i}}$$

hence

$$\frac{dE\left[\omega_{i,j}\right]}{d\lambda_{x_i}} = \left(\alpha_x + \alpha_z\right) \frac{dOOI_i}{d\lambda_{x_i}} = \frac{\alpha_x + \alpha_z}{\lambda_{x_i}}$$

This follows the intuition that the OOI is similar to the log of the size of the option set.

A.4 Simple Parametric Example: Competition on a Line

In order to illustrate the intuition of the model, we provide a simple parametric example where workers are characterized by their productivity and their amount of options. For simplicity we assume $\alpha_z = \alpha_x = 1$.

Assume workers and jobs are equally dispersed across the $[0, 1]$ continuum. Each worker can be described as a 3-dimensional tuple $x_i = (l_i, y_i, d_i)$ which consists of her location on the real line, her productivity and the maximal distance she is able to commute. For simplicity, suppose d_i is either 0.1 or 0.01 with equal probability regardless of y_i, l_i:

$$\forall y_i, l_i : P\left(d_i = 0.1|y_i, l_i\right) = P\left(d_i = 0.01|y_i, l_i\right) = \frac{1}{2}$$

We can think of d_i as a worker's commuting radius: some workers can take jobs within $d = 0.1$ and others are limited to jobs within $d = 0.01$. There is free entry of firms such that equilibrium profits are 0.

Jobs are identical other than their location $z_j = l_j$. The value of a match is

$$\tau_{ij} = \begin{cases} y_i + \varepsilon_{i,z_j} + \varepsilon_{j,x_i} & |l_i - l_j| < d_i \\ -\infty & \text{otherwise} \end{cases}$$

where $\varepsilon_{i,z_j} + \varepsilon_{j,x_i}$ are the sum of two continuous logit distribution as before.

Equilibrium It is easy to show that in equilibrium, all workers will be matched with firms such that $|l_i - l_j| < d_i$. Because ε_{i,z_j} and ε_{j,x_i} are not correlated with $|l_i - l_j|$ within this interval, workers are evenly dispersed across jobs within this interval. For a worker who is not close to either edge (i.e. for whom $l_i \in [d_i, 1-d_i]$): $f_{Z|X}(z_j|x_i) = 1/2d_i$ if $|l_i - l_j| < d_i$ and 0 otherwise.

OOI Given this distribution of workers across firms, we can compute the OOI. Focusing on workers with $l_i \in [d_i, 1-d_i]$:

$$\begin{aligned}
\text{OOI} &= -\int_0^1 f_{Z|X}(l_j|x_i) \log \frac{f_{Z|X}(l_j|x_i)}{g(l_j)} \\
&= -\int_{-d_i}^{d_i} \frac{1}{2d_i} \log(1/2d_i) \\
&= -\log \frac{1}{2d_i} \\
&= \log 2d_i
\end{aligned}$$

This is the log measure of jobs that are available to the worker. Differences in OOI between workers are simply due to differences in the (log) measure of relevant options.

Wages We can go one step further and derive workers' equilibrium compensation

$$\begin{aligned}
\underbrace{E\left[\omega^*_{ij}|x_i\right]}_{\text{Expected compensation}} &= \underbrace{E\left[\tau\left(x_i, z^*_j\right)|x_i\right]}_{\text{Mean Production}} - \underbrace{E[\pi_{i,j^*}|x_i]}_{\text{Employer Rents}} + \underbrace{\left(\frac{\alpha_x + \alpha_z}{\alpha_z}\right) E\left[\varepsilon_{i,z^*_j}|x_i\right]}_{(\alpha_x+\alpha_z)\cdot OOI} \\
&= y_i - 0 + \log 2d_i
\end{aligned}$$

This example shows clearly how two workers who are, on average, equally productive (have identical y_i), could still earn different wages due to differences in outside options. Assume $l_1 = l_2$, $y_1 = y_2$, and $d_1 < d_2$. Worker 2 will earn a a higher wage because her OOI is greater—despite the fact that workers 1 and 2 are equally productive at every job in $[l - d_1, l + d_1]$. Because worker 2 has a higher price, most jobs in this range would prefer to hire worker 1. Still, as a result of idiosyncratic tastes, some employers are willing to pay a higher price.

A.5 Allowing for Entry and Size-Based Monopsony Power

While the baseline model considers a world where workers match with "jobs", jobs are typically organized into firms. We can augment the model by allowing jobs to be concentrated in a countable number of firms. In this case market power may arise both due to heterogeneity (as above) and due to size. We also incorporate entry decisions.

Define a firm as a positive measure of jobs and define $k : \mathcal{J} \to \mathcal{K}$ as the function that assigns jobs to firms. Jobs within a firm may differ according to their observed (z) and unobserved (ε) characteristics. We assume that all jobs with exactly the same observed characteristics are at the same firm.[32] We assume a linear production function between jobs within a firm; the total value produced by a firm k_0 is then $\int_{j\in\{k(j)=k_0\}} \tau_{m^{-1}(j),j} dj$.

The equilibrium is defined as before: an allocation and a set of transfers such that no worker and firm would be better off by pairing with each other instead of their existing match. However, in this case, i cannot deviate to a different job at the same firm. We also add entry conditions for both sides, which require them to receive larger compensation than their non-participation outside options.

$$\begin{aligned} \forall i' \in \mathcal{I}, j' \in \mathcal{J} / \{k(j') = k(m(i'))\} &\ : \quad \omega_{i'} + \pi_{j'} \geq \tau_{i'j'} \\ \forall i' \in \mathcal{I} &\ : \quad \omega_{i'} \geq u_{i'} \\ \forall j' \in \mathcal{J} &\ : \quad \pi_{j'} \geq v_{j'} \end{aligned} \tag{20}$$

In this setting, a worker's compensation depends only on her outside options from outside the firm; her income does not depend on her particular job within the firm. We next assume that employers are profit maximizing:

Assumption 3. *Firms allocate workers to maximize profits. Specifically, for every pair of workers in the same firm i_1, i_2 that are matched to jobs j_1, j_2*

$$\pi_{i_1 j_1} + \pi_{i_2, j_2} \geq \pi_{i_1 j_2} + \pi_{i_2, j_1}$$

Since compensation does not depend on the particular job a worker takes, by adding $\omega_{i_1} + \omega_{i_2}$ on both sides we get that

$$\tau_{i_1 j_1} + \tau_{i_2, j_2} \geq \tau_{i_1 j_2} + \tau_{i_2, j_1}$$

The allocation is the same as in the one-job firm case. This is because both models generate the optimal allocation of workers into jobs. Any other allocation would either violate profit maximization or Equation 20.

However, the transfers are not necessarily the same. One can prove that the compensation from the base case is still an equilibrium (as it satisfies a stronger set of conditions). However, it is no longer the unique equilibrium. Instead, it is an upper bound on workers' compensation. Any equilibrium that involved a worker earning more would either violate Equation 20 (if the deviation involves a job at a different firm) or Assumption 3 (if the deviation involves a job at the same firm).

[32] If jobs with the same characteristics exist in more than one firm, the equilibrium solution would be identical to the one-job employer case.

A worker's equilibrium compensation is bounded from below by the maximum a competing *firm* would be willing to pay her. This differs from the baseline case if the worker's first and second most productive jobs are housed in the same firm.

$$\omega_{ij} \geq \max_{j|k(j)\neq k(m^{-1}(i))} \{\tau_{ij'} - \pi_{j'}\}$$

This is a tight lower bound as setting $\omega_i = \max_{j|k(j)\neq k(m^{-1}(i))} \{\tau_{ij'} - \pi_{j'}\}$ satisfies the equilibrium conditions.

We assume the distribution of match quality τ_{ij} satisfies Assumption 2. We also assume the value of non-participation for workers has a type-1 extremum value distribution:

Assumption 4. *A worker i that chooses not to participate in the labor market receives a compensation of*

$$u(x_i) + \varepsilon_{i,u}$$

This assumption incorporates the decision of participation in the same multinomial logit structure of choosing where to work.

Theorem 4. *Under the above assumptions*

$$\underline{\omega_{ij}} = \max_{j|k(j)\neq k(m^{-1}(i))} \left\{\omega(x_i, z_j) + \varepsilon_{i,z_j}\right\},$$

where $\omega(x, z)$ is the same as the one-job firm case

Proof. We prove that this is a tight lower bound for ω by showing that any equilibrium that involves lower compensation for workers would have to violate one of the equilibrium conditions and that such equilibrium exists.

Suppose $\omega_{ij}^{eq} < \underline{\omega_{ij}}$ for some i, j, and let z_0 be the characteristics of the job that solve

$$\max_{j|k(j)\neq k(m^{-1}(i))} \left\{\omega(x_i, z_j) + \varepsilon_{i,z_j}\right\}$$

These are the characteristics of the job worker i found most attractive under the one-job firms case, considering only the set of jobs housed in *outside* firms. Because there is a continuum of workers, there is at least one worker i' with the same characteristics x_i, who is matched to a job with characteristics z_0 and is indifferent in the one-job firms case between job z_0 and non-participation. Because the allocation is the same, and because non-participation compensation is the same, this worker must earn at least her compensation in the single-worker case $\omega(x_i, z_0) + \varepsilon_{i',z_0}$.

Use $k(z_0)$ to denote the competing firm. Because the profits of this firm from hiring i' are $\tau(x_i, z_0) - \omega(x_i, z_0) + \varepsilon_{x_i,j}$ for all jobs with characteristics z_0 (regardless of the exact identity), firm $k(z_0)$ would earn higher profits by firing i' and hiring worker i at $.5 \times (\omega_{ij}^{eq}) + .5 \times$

$\left(\omega\left(x_i, z_j\right) + \varepsilon_{i,z_j}\right)$. As this is higher than worker i's equilibrium compensation $\underline{\omega_{ij}}$, she would also be better off than in her equilibrium match, which contradicts the model assumption (Equation 20).

This is a tight bound since setting $\omega_{ij} = \underline{\omega_{ij}}$ satisfies all of the equilibrium conditions. □

Using this theorem we can find an expression for the expected *maximal* markdown: the maximum difference between workers' compensation in a fully competitive market (the one-job firms case) and their worst-case wages.

Theorem 5. *Under the above assumptions the difference between workers' maximal and minimal compensation is*

$$E\left[\overline{\omega_{ij}} - \underline{\omega_{ij}}\right] = -\sum_k \log\left(1 - p_{k,i}\right)$$

where $p_{k,i}$ is the probability that a worker with the same characteristics as worker i works in firm k.

Proof. The difference between the upper and lower bounds on a worker's compensation is:

$$\max_{j|k(j)=k(m^{-1}(i))} \left\{\omega\left(x_i, z_j\right) + \varepsilon_{i,z_j}\right\} - \max_{j|k(j)\neq k(m^{-1}(i))} \left\{\omega\left(x_i, z_j\right) + \varepsilon_{i,z_j}\right\}$$

according to the previous theorem. The first term is simply her compensation in the one-job firms case. The second term is the lower bound from the previous theorem; this is the maximum compensation worker i would receive from another firm, where $\omega(x_i, z_j)$ is the same as the one-job firms case.

In a slight abuse of notation we use $\omega_k\left(x_i\right)$ to denote the best compensation offer in the one-job firms case from jobs in some firm k and $\omega_{-k}\left(x_i\right)$ to denote the best compensation offer she receives from jobs in a firm other than firm k.

$$\omega_k\left(x_i\right) = \max_{j\in\{j|k(j)=k\}} \left\{\omega\left(x_i, z_j\right) + \varepsilon_{i,j}\right\}$$

$$\omega_{-k}\left(x_i\right) = \max_{j\in\{j|k(j)\neq k\}} \left\{\omega\left(x_i, z_j\right) + \varepsilon_{i,j}\right\}$$

Based on Proposition 1 (Appendix A.1) both $\omega_k\left(x_i\right)$ and $\omega_{-k}\left(x_i\right)$ have a type-1 extremum value distribution and are independent. Therefore, their difference has a logistic distribution with mean $E\left[\omega_k\left(x_i\right) - \omega_{-k}\left(x_i\right)\right] = \log\frac{p_{k,i}}{1-p_{k,i}}$ where $p_{k,i} = P\left(\omega_k\left(x_i\right) - \omega_{-k}\left(x_i\right) > 0\right)$. The expected compensation difference at firm k given that firm k is the optimal choice for worker i is

$$E\left[\omega_k\left(x_i\right) - \omega_{-k}\left(x_i\right)|\omega_k\left(x_i\right) - \omega_{-k}\left(x_i\right) > 0\right]$$

This is the expectation of a truncated logistic distribution and so

$$E\left[\omega_k\left(x_i\right)-\omega_{-k}\left(x_i\right)\left|\omega_k\left(x_i\right)-\omega_{-k}\left(x_i\right)>0\right]=-\frac{1}{p_{k,i}}\log\left(1-p_{k,i}\right)\right.$$

The overall expectation is then

$$E\left[\overline{\omega_i}-\underline{\omega_i}\right]=\sum_k p_{k,i}E\left[\omega_k\left(x_i\right)-\omega_{-k}\left(x_i\right)\left|\omega_k\left(x_i\right)-\omega_{-k}\left(x_i\right)>0\right]=-\sum_k\log\left(1-p_{k,i}\right)\right.$$

This expression is similar to the HHI.[33] It is also similar to the expression derived in Jarosch, Nimczik, and Sorkin (2019), which focuses on the role that employer concentration plays in wage determination. □

[33]Taking a second order Tayler series of this function gives $\sum_k -p_{k,i}-\frac{p_{k,i}^2}{2}=-1-\frac{HHI}{2}$.

B Methodological Appendix

B.1 $f(x, z)$ Estimation

To estimate a logistic regression following Equation 6, we maximize

$$\max_{\theta} \sum_{k} \log P\left(y_k | x_k, z_k; \theta\right)$$

where θ are the parameters defined in this equation, including matrix A. We rewrite Equation 6 in a more general form. Note $p_k(\theta) = P\left(Y_k = 1 | X = x_k, Z = z_k\right)$:

$$\log \frac{p_k(\theta)}{1 - p_k(\theta)} = \sum_{j=1}^{K} \beta_j h_j\left(x_k, z_k\right)$$

where K is the number of moments h_j we control for in this regression.

Then the K first order conditions converge asymptotically to

$$E\left[p_k(\theta)\, h_j\left(x_k, z_k\right)\right] = E\left[h\left(x_k, z_k\right) | y_k = 1\right] s$$

where $s = P(Y = 1)$ is the share of real data (in our case $\frac{1}{2}$). Using $\frac{p_k(\theta)}{1-p_k(\theta)} = \frac{f(x,z)}{d(x)g(z)} \frac{s}{1-s}$ we can write

$$E\left[\frac{f(x, z)}{sf(x, z) + (1 - s)\, d(x)\, g(z)} h_j(x, z)\right] = E\left[h_j(x, z) | real\right]$$

The right side of this expression is the moment of $h_j(x, z)$ in the real data. The left side is the moment of $h_j(x, z)$ in the full data (real and simulated), weighted by the probability it is real.

If the model is correctly specified and the functional form assumption on $\frac{f(x,z)}{d(x)g(z)}$ is true, θ will be estimated consistently. This is because

$$E\left[\frac{f(x, z)}{sf(x, z) + (1 - s)\, d(x)\, g(z)} h_j(x, z)\right] =$$

$$\int \frac{f(x, z)}{sf(x, z) + (1 - s)\, d(x)\, g(z)} h_j(x, z)\left(sf(x, z) + (1 - s)\, d(x)\, g(z)\right) dxdz =$$

$$= \int h(x, z)\, f(x, z)\, dxdz = E\left[h(x, z) | real\right]$$

If the model is misspecified, our estimate of $\frac{f(x,z)}{sf(x,z)+(1-s)d(x)g(z)}$ will not converge to the real density ratios. Instead we will equalize moments of some other weighted average of h_j

$$E\left[w(x, z, \theta)\, h_j(x, z)\right] = E\left[h_j(x, z) | real\right] \tag{21}$$

where

$$w(x_k, z_k, \theta) = s^{-1} \frac{\exp \sum_{j=1}^{K} \beta_j h_j(x_k, z_k)}{1 + \exp \sum_{j=1}^{K} \beta_j h_j(x_k, z_k)}$$

We next analyze these weights as the simulated data grows asymptotically such that $s \to 0$. In order to calculate the OOI, we simulate values from $d(x)$ and $g(z)$, and re-weight them based on the following weights (Equation 7).

$$\frac{\widehat{f(x, z)}}{d(x)\, g(z)} = \frac{1-s}{s} \exp \sum_{j=1}^{K} \beta_j h_j(x_k, z_k)$$

Because $s \neq \frac{1}{2}$ we also need to multiply by $\frac{1-s}{s}$. These weights converge to $w(x_k, z_k, \theta)$

$$\lim_{s \to 0} \frac{w(x, z, \theta)}{\frac{\widehat{f(x,z)}}{d(x)g(z)}} = \lim_{s \to 0} \frac{\frac{1}{1+\exp \sum_{j=1}^{K} \beta_j h_j(x_k, z_k)}}{1-s} = 1$$

where the last equality is because $\lim_{s \to 0} \exp \sum_{j=1}^{K} \beta_j h_j(x_k, z_k) = 0$.

The density of the full data approaches the density of the simulated data. Hence the logit FOC (Equation 21) becomes:

$$E[w(x, z, \theta)\, h_j(x, z) \,|sim] = E[h_j(x, z) \,|real]$$

The above equation guarantees that after re-weighting we sample from a distribution with same moment value for every $h_j(x, z)$, even if the model is misspecified.

Dupuy and Galichon 2014 produce a distribution with the same second moments as the data, and same marginal distributions. Therefore, when $s \to 0$, and h includes all X, Z interactions, as well as indicators for every x_k and z_k value (that is, $h(x, z) = 1_{x=x_k}$ or $h(x, z) = 1_{z=z_k}$ for every k), we have a distribution that satisfies exactly the same moment conditions. Moreover, since both methods require that $f(x, z) = \widetilde{a}(x)\, \widetilde{b}(z)\, K(x, z)$, K has the same functional form, and there is a unique solution that satisfies these conditions, the two methods yield exactly the same distribution.

B.2 Specification for Mass Layoff Analysis

In Section 5.2, we examine the extent to which OOI predicts (within - establishment) recovery from a mass layoff. When examining the impact on wages, we use a specification where the dependent variable is earnings, relative to pre-displacement earnings: $w_{i,t}/w_{i,0}$.

This specification is motivated by the idea that individuals vary in (time-invariant) productivity θ_i, which may be correlated with OOI_i. We follow the standard assumption that θ_i has a linear effect on log wages, such that $\log w_{i,\tau} = \theta_i + \varepsilon_{i,\tau}$, where $\varepsilon_{i,\tau}$ represents independent, time-varying shocks. Assume that t months after a separation, if the worker is reemployed, her income is lower by some proportion $\eta_{i,t}$ such that

$$\log w_{i,t} = \theta_i + \eta_{i,t} + \varepsilon_{i,t}$$

$\eta_{i,t}$ is most often negative, and represents loss of specific human capital, or employment at a worse match etc. We expect $\eta_{i,t}$ to positively depend on the OOI as workers with more options can find higher quality matches more quickly.

In addition, assume that workers find a new job at some probability p_i. This probability is also likely to positively depend on the OOI as workers with more options can find a new job faster. Taken together, wages t months after the separation are

$$w_{i,t} = 1_{e(i,t)} \exp\left(\theta_i + \eta_{i,t} + \varepsilon_{i,t}\right)$$

where $1_{e(i,t)}$ is an indicator for employment.

We do not use a log specification, as in the rest of this paper, because a large portion of our sample is not employed after the separation. Log specification does not support zero income. Furthermore, functions that are close to log, such as $\log\left(w_{i,t} + c\right)$ or $\sin h^{-1}$ will also be biased because of the correlation with θ_i. Marking such a function with L we get that the difference between two periods is

$$L\left(w_{i,t}\right) - L\left(w_{i,0}\right) \approx 1_{e(i,t)}\left(\theta_i + \eta_{i,t} + \varepsilon_{i,t}\right) - \theta_i - \varepsilon_{i,0} = \theta_i\left(1_{e(i,t)} - 1\right) + \eta_{i,t} + \varepsilon_{i,t} - \varepsilon_{i,0}$$

In this case, θ_i does not cancel out since many people are not working and so $1_{e(i,t)} \neq 1$. Therefore, correlations between the OOI and persistent income differences would prevail using such an outcome.

Instead, we use the wage ratio which equals

$$\frac{w_{i,t}}{w_{i,0}} = 1_{e(i,t)} \exp\left(\eta_{i,t} + \varepsilon_{i,t} - \varepsilon_{i,0}\right)$$

In this case, the outcome is zero for all people who are not working, and therefore θ_i cancels out. Our specification in Equation 9 captures the linear approximation of the conditional expectation of this expression and is therefore not affected by any correlation between θ_i and the OOI.

C Data Appendix

C.1 Data Cleaning

This section provides more information on the data cleaning procedure.

Demographics We set German citizenship to one, if a worker was ever reported as a German citizen by her employer.

Education Within a year, we take the highest level of education that is reported up to that year. All upper secondary school certificates are coded as upper-secondary. In some years intermediate and lower secondary education are coded with the same value. In these cases, if we observe the worker in other years and can infer their schooling level we use that. Otherwise, we code these workers in a separate category for either lower or intermediate secondary education.

Wages The OOI is computed without information on workers' wages. This makes it ideally suited to settings where wage information may not be available, or where the available wage information is heavily censored. In our analysis of the link between workers' OOI and wages, the main outcome variable is log daily earnings. When studying between-group inequality we use a Tobit model to account for top-coding of earnings.

Past Location We define an individual's past location as the district they were living in before taking their current job. If this location is not available, we use the district of the last firm they were working at. If this is also not available, we take the first district that is available in the data for them.

Distance We calculate distance at the district level. For each district, we calculate the district center, by taking the weighted average of the latitude and longitude coordination of each city in this district, weighted by its population. We then calculate the distance between the districts, taking into account the concavity of the earth.

Occupation The LIAB contains occupations that are coded using KldB2010 classification. Occupations are coded with 5-digits where the last digit is used for occupation complexity (see main text). Occupation characteristics are calculated at the 3-digit level of the KldB 2010 coding using the BIBB data. We treat the last digit (occupation complexity) as a separate variable.

Training Occupation We define an individual's training occupation as the occupation in which workers spent the longest time in training. We use the employment status indicators (erwstat) to identify workers who are in training. For the large majority of workers, there is only one occupation in which they have completed vocational training. In rare cases where workers have conducted training in more than one occupation, we use the occupation in which the training was longer.

Some workers do not have any completed vocational training that we can observe. In this case, our we define their training occupation as the occupation in which they completed an internship (again, measured using the employment status variable). If an individual has not completed either vocational training or an internship, we set their past occupation to be the first occupation they were observed working in, provided that at least ten years have elapsed since the first point at which they enter the data.

C.2 BIBB Survey

We use data from the 2011-2012 wave of the German Qualification and Career Survey conducted by the Federal Institute of Vocational Training (BIBB).[34] The data cover 20,000 employed individuals between the ages of 16 and 65. Our data – the Scientific Use File – contain 3-digit occupations coded using the 2010 Kldb scheme and 2-digit industries. We run PCA on this survey by questions category and aggregate the results by 2-digit industry and 3-digit occupations. We then link the results to our main data at this level. The top questions in each category are reported in Table A1.

C.3 Vacancy-Based OOI

We use the establishment survey to construct the distribution of vacant jobs. The survey asks each establishment to report the number of vacancies for both "menial" and "skilled" jobs. Because the establishment does not report the characteristics of these jobs, we assume that each vacancy has—with equal probability—the same characteristics as each of the existing "menial" and "skilled jobs" in the establishment. We assume that skilled jobs are those taken by workers with high-secondary education; menial jobs are those that hire workers with lower levels of education.

We derive the distribution of vacant job characteristics by re-weighting the overall distribution of jobs. We weight each job j in our sample by the LIAB sample weight for that establishment ($m_{e(j)}$). In order to move between the distribution of filled jobs and the distribution of vacant jobs, we then multiply the weights by the number of vacant jobs at that establishment-skill group ($N^{vac}_{e(j),s(j)}$), divided by the number of existing jobs at that establishment-skill group ($N_{e(j),s(j)}$).

[34] This survey was conducted in cooperation with the Institute for Labor Market Research (IAB) until 1992.

This produces weights that can be applied to the overall distribution of jobs.

$$m_j^{vac} = m_{e(j)} \frac{N_{e(j),s(j)}^{vac}}{N_{e(j),s(j)}}$$

This exercise assigns a weight of zero to jobs in establishments that do not report vacancies for that skill category. The weights characterize the marginal distribution of vacancies $g^v\left(z_j\right)$. In order to calculate OOI^v , we plug this marginal distribution into equation 8:

$$OOI_i^v = -\sum_{z_j} \widehat{f_{Z|X}\left(z_j|x_i\right)} \log \frac{\widehat{f_{Z|X}\left(z_j|x_i\right)}}{g^v\left(z_j\right)} \tag{22}$$

D Applicability of the OOI in Different Environments

In order to examine the broader applicability of our approach, we re-estimate the OOI using different subsets of variables. We focus on three subsets of variables that correspond to three types of commonly used labor market datasets:

1. "Population Survey": Our most narrow subset includes variables typically available in a cross-sectional population survey such as the Current Population Survey in the United States. The set of worker characteristics is: their age, education (categories), their gender, and their citizenship status. The set of job characteristics is: the occupation (projected onto PCA components) and industry (projected onto the same PCA components), as well as an indicator for whether the job is part-time.

2. "Cross-Sectional Employer-Employee": Our second subset includes variables that are available in most employer-employee datasets. The set of worker characteristics is identical to those in the "population survey" subset. The set of job characteristics includes those in the "population survey" subset, as well as establishment size. The distance between the worker's location and the job's location is also included as an additional variable.

3. "Panel Employer-Employee": Our third subset includes variables that are available in most European employer-employee panels. The set of worker characteristics includes those in the "cross-sectional employer-employee" model, as well as the worker's training occupation (projected onto a set of 10 PCA components). The set of job characteristics includes those in the "cross-sectional employer-employee" model. Distance is included, as before.

Our baseline estimates in the paper come from a model that adds establishment survey variables (projected onto two PCA components for each of five modules, as well as establishment sales) to the "panel employer-employee" model. Table A1 shows the survey questions that receive the most weight in each module.

We measure the performance of each subset of variables by comparing their out-of-sample predictions. For each subset of variables, we estimate the conditional distribution of jobs given worker characteristics using logistic regression, as described in section 3.2. The estimation is performed using all workers in our main sample: workers employed as of June 30, 2014. In order to avoid over-fitting, we then focus on the 47,902 workers who, by the end of the year, have switched firms. For each of these workers, we calculate the probability implied by each of the four models (the three subsets + our baseline specification) that the worker takes a job with the characteristics of the job they have at the end of the year (on December 31). We assess the quality of each model's predictions by examining the ratio between this probability, and the probability that a worker randomly assigned to jobs would end up in a job with those characteristics (the marginal distribution of jobs):

$$\frac{\widehat{f\left(z_j|x_i\right)}}{g\left(z_j\right)}$$

The marginal distribution of jobs $g(z_j)$ is computed using LIAB sampling weights, as before.

Table A4 presents a summary of our results. For each subset of variables, we calculate the mean, standard deviation and the 25th, 50th, and 75th percentile of the above ratio. Using the full set of variables, the average ratio is 326. This implies that we are able to predict workers' next job 326 times better than a random guess.

The "panel employer-employee" prediction performs almost as well: on average it is able to predict the worker's next job 318 times better than a simple random guess. This is not very different from the performance of the baseline model, which includes information from the establishment survey. This implies that the OOI can be calculated on more commonly available employer-employee data sets, without much loss of precision.

By contrast, the "cross-sectional employer-employee" specification does not perform as well, and the "population survey" specification does not improve much on a random guess. Using our cross-sectional employer-employee specification we are able to predict workers' new jobs 153 times better than random, about half as well as in wide employer employee specification. This implies that the training occupation, which was omitted in this specification, includes important information about workers' option sets.[35] The "population survey" model only predicts 1.3 times better than a random draw, and in 25% of cases the model predicts a probability that is less than half of the probability of a random draw. As the key difference between this model and the "cross-sectional employer employee" model is the inclusion of distance, this indicates that information on worker and job location is critical for any reasonable assessment of the workers' outside options.

[35] This information is not available in most linked employer-employee datasets in the United States.

E Application: High-Speed Commuter Rail

In this section we show how the OOI can be used to understand the channels through which workers may benefit from infrastructure improvements. We focus on the case of the introduction of high speed commuter rail stations in Germany. As discussed in previous work by Heuermann and J. F. Schmieder, high speed rail was first introduced in Germany between 1991 and 1998 (2018).[36] During that period, stations were placed in major cities such as Berlin and Munich. In this paper we focus on the second wave, which expanded access to high speed rail by installing new stations in cities along pre-existing routes. As discussed by Heuermann and J. F. Schmieder (2018), the selection of cities in this second phase was based primarily on political factors, not factors related to anticipated labor market trends, or even anticipated train usage. This increase in infrastructure led to an increase in commuting probability. Figure A4 shows the map of districts that got stations in each wave.

Our treatment group consists of workers who, in 1999, lived in Montabaur, a small town (~12,000 residents) between Cologne and Frankfurt that received a station in the second wave.[37] We construct a matched control group using the set of workers from the same state (Rhineland-Palatinate) who, in 1999, lived in districts that never received stations. We use nearest-neighbor matching to match workers from the treatment and the control group based on their gender, age, citizenship, education level, training occupation characteristics, and lagged income using nearest-neighbor matching with replacement. We require the match to be exact on gender, education, and 2-digit occupation. Table A5 shows that this procedure produces a set of workers who are statistically indistinguishable. Because the workers reside in different districts, there is a difference in their baseline OOI in 1999 and their average distance to work.

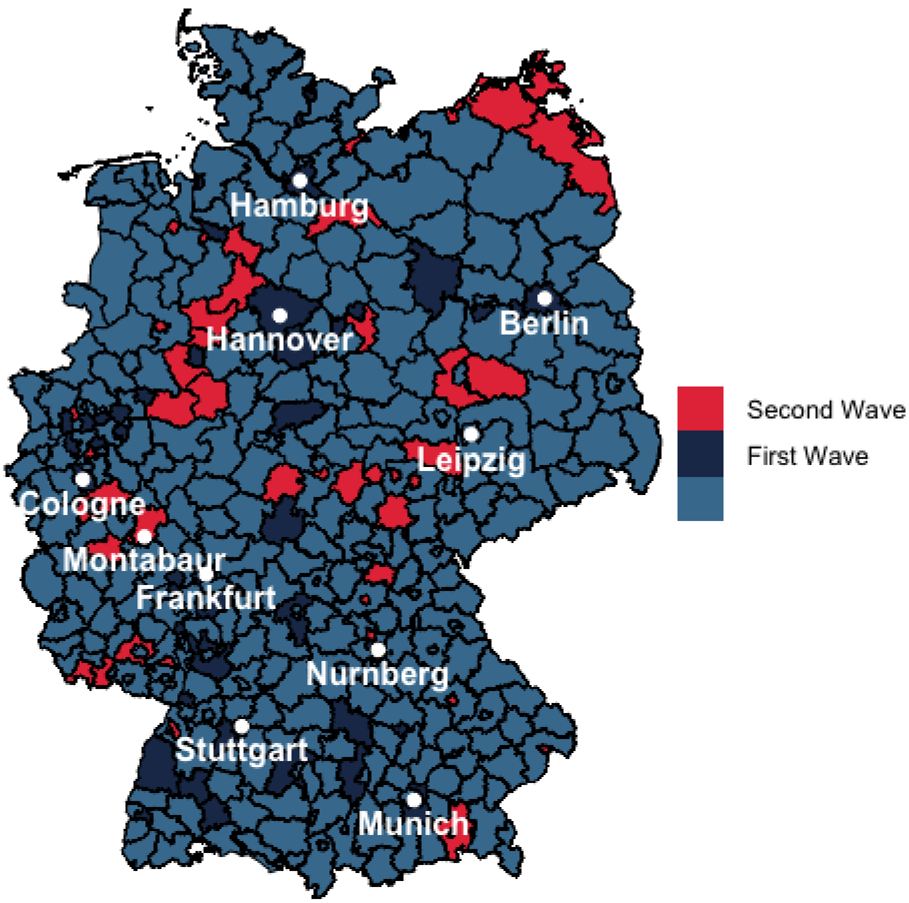
We first show that the introduction of the high speed rail stations impacted workers' OOI. We estimate the OOI separately in 1999 and 2012, using the full cross-sectional distribution of the German population. In this exercise, we use a fourth degree polynomial of both physical distance and trip duration by train, to estimate match probabilities. The left bars in Figure A5 show the results for the full Montabaur population and by gender (Panel A), as well as by education level (Panel B). Relative to the control group, the treatment group saw a 0.37 unit increase in OOI. Given a semi-elasticity of 0.17 (estimated in Section 6.1), this would suggest that wages in Montabaur should grow by around 7=6.3%.[38] The OOI has also grown similarly for both genders and all three education groups.

[36] Daniel Heuermann and Johannes Schmieder generously provided the train data for our use in this project.

[37] We do this for comparability with the previous literature. Heuermann and J. F. Schmieder (2018) focus on Montabaur as their key case study, as it is the prime example of the randomness in the selection process, and is an ideal natural experiment.

[38] We do not analyze the actual impact on wages due to our small sample size.

Figure A4: ICE Stations



Note: This figure shows the locations of ICE train stations by districts. The first wave includes all stations that were opened pre-1999. The second wave includes all stations that were opened post-1999.

Table A5: Treatment-Control Comparison

	Treated (1)	Control (2)
	A. Variables Used for Matching	
Female	31.3%	31.3%
Age (1999)	47.98	47.70
Higher Secondary Degree	16.0%	16.0%
Intermediate Secondary Degree	31.5%	31.5%
Lower Secondary Degree	52.5%	52.5%
Log Daily Income 1999	4.32	4.34
	B. Variables Not Used for Matching	
OOI 1999	-5.52	-5.31
Distance 1999	10.33	13.25
Part-Time 1999	0.15	0.18
Observations - Treated	613	
Observations - Total	5,299	

Note: This table compares the characteristics of individuals in the treatment and control groups for the trains exercise. The treatment group consists of all workers who, in 1999, lived in Montabaur. We construct a matched control group using nearest neighbor matching with replacement, drawing from the set of individuals who, in 1999, lived in a district in Rhineland-Paltitinate (the same state as Montabaur) that never received a high speed rail station. Matching is done exactly on gender, education group, citizenship status, state and 2-digit training occupation and continuously on age, and PCA components for training occupation (the third digit).

Infrastructure investments may increase the OOI and affect wages both (1) by changing the productivity of workers' matches and (2) by changing the value of workers' outside options. After the introduction of high speed rail, workers may find it easier to commute to high-quality matches that were previously too far away. It is also possible that the introduction of high speed rail encouraged new job openings in Montabaur itself. Even workers who did not end up commuting and did not end up matching with new job opportunities in Montabaur may have benefited by having these jobs in their option set.

Using our framework, we can assess the drivers of the increase in workers' outside options. We compute two counterfactual OOI: First, $\widetilde{OOI}_{X,A}$ where the values of X and A are taken from their 2012 values, while the distribution of jobs (Z) and the train schedules are kept at their 1999 level. Second, $\widetilde{OOI}_{X,A,Z}$ where X, A, Z are at their 2012 levels while only train schedules remains at its 1999 level, before the introduction of the fast commuter rails. We decompose the overall increase in OOI into three components (1) changes in worker characteristics due to, e.g. aging, and the matching function (difference between $\widetilde{OOI}_{X,A} - OOI^{1999}$), (2) changes in the distribution of available jobs (difference between $\widetilde{OOI}_{X,A,Z} - \widetilde{OOI}_{X,A}$) and (3) changes in commuting times ($OOI^{2012} - \widetilde{OOI}_{X,A,Z}$). For each component we compare the treatment and the control groups.

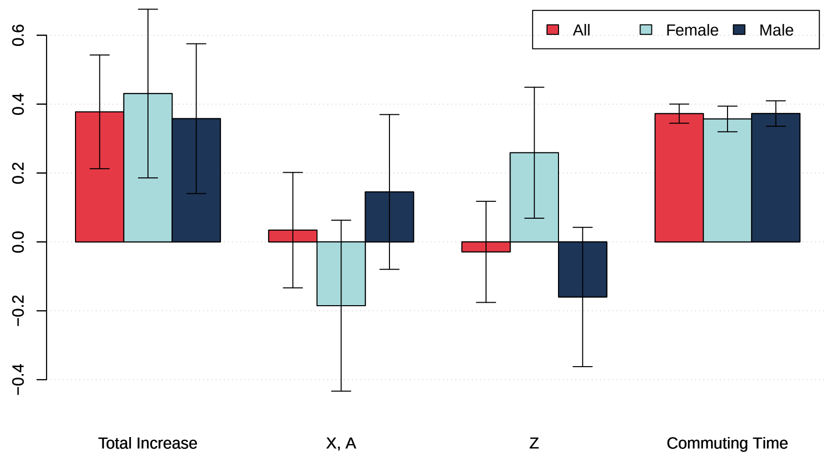
We find the trains affected workers differently, depending on their gender and education. The second bar in Figure A5 shows what the change in OOI would have been if only X_i and A had changed. This explains a relatively small fraction of the increase in outside options (9%). The third bar shows what the change in OOI would have been if the distribution of jobs Z_j had also changed, but commuting times remained the same. Overall, this reduced the OOI by about -.03 (-7% of the overall change). Yet the impact on female workers is positive and significant—0.25 (vs. -.16 for men) —consistent with the idea that the new jobs in Montabaur were primarily taken by women, who work closer to home. The final bar in each panel shows the change in OOI due to the change in commuting time. This component explains most of the increase in OOI (99%). The effect is significantly larger for higher educated workers. While we do not directly observe ridership, we do observe that more educated workers are more likely to work at districts to which they can commute using the fast trains, and so these workers were more likely to use the train.[39] The high cost of the high speed train made them impractical for lower-wage workers, who have lower levels of education on average.

To conclude we find that the various impacts of the new trains affected workers differently. Workers that are more likely to use the fast train to commute, such as higher educated workers, benefited from the access to more distant jobs. Workers that tend to work closer to home, such as female workers, benefited from the increase in supply of local jobs.

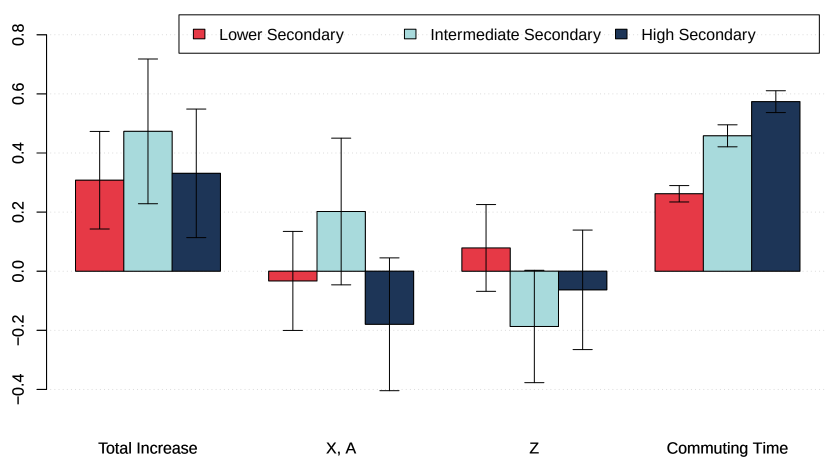
[39]Similar patterns were found by Butikofer, Loken, and Willen (2019) who analyze the impact of the Oresund bridge on workers.

Figure A5: Impact of Express Trains on Options and Wages

A. By Gender



B. By Education Group



Note: This figure shows the results of the impact of express trains on outside options. We calculate the OOI before the trains—OOI^{1999}—and after the trains—OOI^{2012}. The first set of bars shows the difference in the overall changes in OOI between the treatment and control groups: $\mathrm{OOI}^{2012} - \mathrm{OOI}^{1999}$. The second set of bars shows what the change in OOI would have been if only X_i and A had changed ($\widetilde{OOI}_{X,A} - OOI^{1999}$). The third set of bars shows the effect of the change in Z, holding X, A and train schedule at their 2012 value, and train schedules at their 1999 value ($\widetilde{OOI}_{X,A,Z} - \widetilde{OOI}_{X,A}$). The final set of bars shows the impact of the new train schedule holding X, A and Z at their 2012 values ($OOI^{2012} - \widetilde{OOI}_{X,A,Z}$). The whiskers depict 95% confidence intervals calculated using the methodology in Abadie and Imbens (2006).