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Optimal Defined Contribution Pension Plans: One-Size Does Not Fit All*

Kathrin Schlafmann Ofer Setty Roine Vestman

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Abstract

We study the role of defined contribution pension plans for individuals' welfare and ability to accommodate shocks. Using a rich life-cycle model, we find that common designs, with a fixed contribution rate out of income for all individuals at all times, are unnecessarily rigid. We propose a design where the contribution rate is a function of the individuals' age and account balance-to-income ratio. Compared to the typical rigid contribution rate, our design leads to the same average replacement rate, 25.6 percent, but reduces the cross-sectional standard deviation from 10.8 to 4.0 percent. Furthermore, our proposed rule provides both liquidity and consumption benefits for the first 17 years. Consumption increases by as much as 4.9 percent. The design implies a welfare gain of 3.3 percent in consumption equivalent relative to the current fixed contribution rate.

JEL classification: D91, E21, G11, H55.

Keywords: Age-based investing, life-cycle model, pension plan design.

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1 Introduction

Developed countries are gradually undertaking reforms that separate pension systems from the fiscal budget. A consequence of this move is that more of the economic risks are borne by workers rather than employers or the government. A typical feature of such reforms is to rely more on defined contribution (DC) pension plans. OECD (2017) reports that 32 out of 34 member countries have mandatory or quasi-mandatory second-tier pension provision for workers. Fifteen member countries have DC pension plans. Notably, there is a great deal of heterogeneity in these plans. The mandatory DC contribution rate varies between two and fifteen percent, suggesting arbitrariness in the systems' design. Furthermore, two key design features are at odds with optimal savings behavior according to the life-cycle consumption-savings model. Those features are the constant contribution rate out of income and the inability to halt contributions to the account in the presence of transitory shocks. Thus, existing DC pension plans appear to be unnecessarily rigid.

The purpose of this paper is to investigate the consequences of offering flexibility in DC pension plans beyond existing designs. We set up a life-cycle consumption-savings model where replacement rates are a function of the design of a first and second tier pension system. The first tier is a safe government-mandated savings account that guarantees a replacement rate of around fifty percent of labor income. The second tier pension system consists of a DC pension plan, which could be government-mandated or quasi-mandatory. Our calibration is based on Swedish micro data and the Swedish institutional setting, which is often considered a model for other countries. We report two main findings.

First, we demonstrate two disadvantages of the typical existing DC pension plan design. The first one concerns illiquidity. The motive to save for retirement is very limited for individuals younger than 50 years. That is, before that stage in the life-cycle, the value of contributing to DC pension plans is low if accumulated balances are illiquid and hence inaccessible to smooth against income fluctuations. The second concerns the cross-sectional dispersion in replacement rates. If contribution rates remain constant throughout, then the replacement rate out of the DC plan varies wildly, depending on the history of shocks to income and returns that individuals have faced.The cross-sectional standard deviation in our baseline calibration is 10.8 percent, which is large relative to an average of 25.6. In this sense, a fixed contribution rate is an inadequate policy instrument for allowing the policy maker to ensure that individuals obtain a prespecified replacement rate.

Second, we propose a rule for the optimal contribution rate. Our proposal is a contribution rate that depends on the individual's age and account balance-to-income ratio relative to the average in her birth cohort. Every year, the contribution rate should unconditionally increase by 0.3 percentage points. Furthermore, an investor who deviates by -0.1 from the cohort's asset-to-income ratio should deviate by $+17.5$ percent in her contribution rate (with a floor at 0 and a ceiling at 50 percent). This implies that relatively small shifts in the account balance or income shifts contribution rates considerably. This rule reduces the cross-sectional standard deviation in replacement rates from 10.8 to 4.0 percent while preserving the average at 25.6 percent by construction. Furthermore, our proposed rule provides both liquidity and consumption benefits for the first 17 years. Consumption increases by as much as 4.9 percent for the youngest. Consequently, the rule implies a substantial welfare gain. In terms of consumption equivalent variation, the gain is on average 3.3 percent. Importantly, ex ante the proposed rule is Pareto improving – that is, there are no losers from implementing the rule. The average gain is sizable. It can be contrasted to a hypothetical world in which there is no DC pension plan. Abolishing it would render an average gain of 4.9 percent but it would also lead to lower replacement rates in terms of pay-outs out of total assets and hence manifest itself in terms of lower consumption among retired individuals. Thus, our proposed rules attains two thirds (3.3/4.9) of a hypothetical gain while maintaining the average replacement rate of the baseline.

Our analysis relates to three strands of the literature on pension plan design and savings rates of individuals and households. First, there is a ongoing debate about auto-enrollment into pension plans and auto-escalation of contribution rates, in particular for the U.S. where designs of DC pension plans vary more (see Beshears, Choi, Laibson, and Madrian (2018) for a discussion of defaults). Our results suggest that designing defaults that involve auto enrollment and automatic adjustments of the contributions depending on individual circumstances, as our proposed rule, would cause little harm while simultaneously provide a strong nudge to save, as is typical for default options. Second, there is strong concern in the literature that many consumers lack the financial literacy to make informed retirement planning decisions (see Lusardi and Mitchell (2014) for an overview). As a mandatory DC pension plan, the design that we propose relieves individuals from the majority of these complex questions. Moreover, in our proposed design the mandatory contribution rates flexibly adjust to the financial situation of the household. This has the potential of allowing financially illiterate households to get closer to the optimal retirement savings behavior. Third, our emphasis on flexibility is in line with for instance Beshears, Choi, Iwry, John, Laibson, and Madrian (2019) who discuss different designs of savings accounts that would enable individuals to build emergency savings and self-insurance to the kind of expense shocks or transitory income shocks that we consider.

Our analysis also relates to an ongoing policy debate fueled by the Covid-19 pandemic which concerns whether individuals should be able to withdraw balances from retirement accounts, such as 401(k). There are good arguments in favor of either side; on one hand if individuals are living hand-to-mouth the welfare gain from allowing early withdrawals in emergency situations are large. On the other hand, individuals may miss out on high returns as financial markets reverses. Our proposal does not involve early withdrawals and thus avoids the associated perils. Put differently, the analysis centers attention to the cash-flow relief from automatic adjustments of contributions. We wish to highlight that the cash-flow reliefs associated with our proposed rule for the contribution rate attains two thirds out of the maximum in our model. This suggests that allowing for prewithdrawals (possibly against a penalty fee) at most implies an additional average gain of one third. In other words, our analysis implies that a flexible design of contribution rates substantially diminishes the value of pre-retirement withdrawals.

Relative to the existing literature, Sandris Larsen and Munk (2019) is perhaps most similar to our study. They study how to design optimal contribution rates given that pension plans are mandatory. They investigate contribution rates that depend on age whereas we also consider the DC account balance-to-income ratio. Pries (2007) uses a quantitative life-cycle model to study labor supply responses and welfare effects associated with reform of U.S. Social Security to a system with individual accounts with age-dependent contribution rates.

The paper proceeds as follows. Section 2 provides an overview of the Swedish pension system. Section 3 presents our life-cycle model and its calibration. Section 4 examines the economic forces that determine individuals' savings motive. Section 5 presents the implications of a flexible design of the contribution rate into the DC pension plan. Finally, Section 6 concludes.

2 The Swedish pension system

The Swedish pension system rests on three pillars: public pensions, occupational pensions, and private savings. Below, we describe the public and occupational pensions.

The public pension system was reformed in 2000.[1] It has two major components referred to as the income-based pension and the premium pension. A means-tested benefit provides a minimum guaranteed pension.

The contribution to the income-based pension is 16% of an individual's income, though the income is capped (in 2014 the cap was SEK 426,750, or approximately USD 62,200).[2] The return

[1] Individuals born between 1938 and 1954 are enrolled in a mix of the old and new pension systems, while individuals born after 1954 are enrolled entirely in the new system.

[2] In 2014 the SEK/USD exchange was around seven. During our sample period, the exchange rate has fluctuated between six and ten SEK per USD. We henceforth report numbers in SEK.

on the contribution equals the growth rate of aggregate labor income measured by an official "income index." Effectively, the return on the income-based pension is similar to that of a real bond. The income-based pension is notional in that it is not reserved for the individual but is instead used to fund current pension payments as in a traditional pay-as-you-go system. It is worth mentioning that the notional income-based pension is also DC, but to avoid confusion we simply refer to it as the notional pension.

The contribution to the premium pension is 2.5% of an individual's income (capped as above). Unlike the income-based pension, the premium pension is a fully funded DC account used to finance the individual's future pension. Individuals can choose to allocate their contributions to up to five mutual funds from a menu of several hundred. The premium pension makes it possible for individuals to gain equity exposure. Indeed, most of the investments in the system have been in equity funds (see, e.g., Dahlquist et al., 2015). A government agency manages a default fund for individuals who do not make an investment choice. Up to 2010, the default fund invested mainly in stocks but also in bonds and alternatives. In 2010, the default fund became a life-cycle fund. At the time of retirement, the savings in the income-based pension and the premium pension are transformed into actuarially fair life-long annuities.

In addition to public pensions, approximately 90% of the Swedish workforce is entitled to occupational pensions. Agreements between labor unions and employer organizations are broad and inclusive and have gradually been harmonized across educational and occupational groups. For individuals born after 1980, the rules are fairly homogeneous, regardless of education and occupation. The contribution is 4.5% of an individual's income (capped as above) and it goes into a designated individual DC account. For the part of the income that exceeds the cap, the contribution rate is greater in order to achieve a high replacement rate even for high-income individuals. While the occupational pension is somewhat more complex and tailored to specific needs, it shares many features with the premium pension. Specifically, it is an individual DC account.

3 The model

We set up a life-cycle model for the economic situation of a Swedish individual. The model is an extension of Dahlquist, Setty, and Vestman (2018) which in turn builds on Viceira (2001), Cocco et al. (2005), and Gomes and Michaelides (2005). It includes risky labor income, a consumption–savings choice, and a portfolio choice. We augment the model with a pension system in which individuals save in two pension accounts, from which their pension is received as annuities. One of the accounts belongs to the first pillar of the pension system and is pay-as-you-go but with an individual notional balance. The other account is a standard defined contribution pension account where contributions are either rigid or flexible. It represents the second pillar of the pension system.[3]

Next we describe the model's building blocks.

3.1 Demographics

We follow individuals from age 25 until the end of their lives.[4] The end of life occurs at the latest at age 100, but could occur before as individuals face an age-specific survival rate, ϕ_t. The life cycle is split into a working, or accumulation, phase and a retirement phase. From the ages of 25 to 64 years, individuals work and receive labor income exogenously. They retire at age 65.

[3]Our model relates to Gomes et al. (2009), who consider portfolio choice in the presence of tax-deferred retirement accounts, and to Campanale et al. (2014), who consider a model in which stocks are subject to transaction costs, making them less liquid.

[4]We choose age 25 as the start of the working phase, as Swedish workers do not fully qualify for occupational pension plans before that age.

3.2 Preferences

The individuals have Epstein and Zin (1989) preferences over a single consumption good. At age t, each individual maximizes the following:

$$U_t = \left(c_t^{1-\rho} + \beta \phi_t E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \tag{1}$$

$$U_T = c_T, \tag{2}$$

where β is the discount factor, $\psi = 1/\rho$ is the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion, and $t = 25, 26, ..., T$ with $T = 100$. For notational convenience, we define the operator $\mathcal{R}_t(U_{t+1}) \equiv E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$.

3.3 Labor income

Let Y_{it} denote the labor income of employed individual i at age t. During the working phase (up to age 64), the individual faces a labor income process with a life-cycle trend and persistent income shocks:

$$y_{it} = g_t + z_{it}, \tag{3}$$

$$z_{it} = z_{it-1} + \eta_{it} + \theta \varepsilon_t, \tag{4}$$

where $y_{it} = \ln(Y_{it})$. The first component, g_t, is a hump-shaped life-cycle trend. The second component, z_{it}, is the permanent labor income component. It has an idiosyncratic shock, η_{it}, which is distributed $\mathrm{N}\left(-\sigma_\eta^2/2, \sigma_\eta^2\right)$, and an aggregate shock, ε_t, which is distributed $\mathrm{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$. The aggregate shock also affects the stock return, and θ determines the contemporaneous correlation between the labor income and the stock return. We allow for heterogeneity in income as early as age 25 by letting the initial persistent shock, z_{i25}, be distributed $\mathrm{N}(-\sigma_z^2/2, \sigma_z^2)$.

During the retirement phase (from age 65 and onwards), the individual has no labor income.[5] Pension is often modeled as a deterministic replacement rate relative to the labor income just before retirement.[6] However, in our model, the replacement rate is endogenously determined. The individual relies entirely on annuity payments from savings accounts. Later we discuss these accounts in detail.

3.4 Asset returns

The gross return on the stock market, R_{t+1}, develops according to the following log-normal process:

$$\ln(R_{t+1}) = \ln(R_f) + \mu + \varepsilon_{t+1}, \tag{5}$$

where R_f is the gross return on a risk-free bond and μ is the equity premium. Recall that the shock, ε_t, is distributed $\mathrm{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$, so $E_t(R_{t+1} - R_f) = \mu$. Also recall that ε_t affects labor income in (4), and that the correlation between stock returns and labor income is governed by the weight θ.

3.5 Three accounts for financial wealth

An individual has three financial savings accounts: (i) a liquid account outside the pension system (which we simply refer to as financial wealth), (ii) a fully-funded DC account in the pension system, and (iii) a notional account belonging to the pension system. The notional account, which provides the basis for the pension, is income based and evolves at the rate of the risk-free bond. The DC account is also income based but the investor can choose how to allocate between bonds and stocks; it features mandatory contribution rates and those are the contribution rates we wish to design.

[5] Hence, the retirement decision is not endogenous as in French and Jones (2011). More generally, we do not consider endogenous labor supply decisions as in Bodie et al. (1992) and Gomes et al. (2008).

[6] One exception is that of Cocco and Lopes (2011), who model the preferred DB or DC pension plan for different investors.

The account outside the pension system is accessible at any time. Each individual chooses freely how much to save and withdraw from it. In contrast, the contributions to the pension accounts during the working phase are determined by the pension policy (rather than by the individual) and are accessible only in the form of annuities during the retirement phase. Importantly, the two pension accounts include insurance against longevity risk.

Financial wealth

The individual starts the first year of the working phase with financial wealth, A_{i25}, outside the pension system. The log of initial financial wealth is distributed $\mathrm{N}(\mu_A - \sigma_A^2/2, \sigma_A^2)$. In each subsequent year, the individual can freely access the financial wealth, make deposits, and choose the fraction to be invested in risk-free bonds and in the stock market. However, the individual cannot borrow:

$$A_{it} \geqslant 0, \tag{6}$$

and the equity share is restricted to be between zero and one:

$$\alpha_{it} \in [0, 1]. \tag{7}$$

Taken together, (6) and (7) imply that individuals cannot borrow at the risk-free rate and that they cannot short the stock market or take leveraged positions in it.

3.6 Stock market participation costs and investor types

To enter the stock market outside the pension system, the individual must pay a one-time participation cost, κ_i. (The financial wealth and the decision to invest in the stock market are described later.) A one-time entry cost is common in portfolio-choice models (see, e.g., Alan, 2006; Gomes and Michaelides, 2005, 2008).

The state variable, I_{it}, tracks whether stock market entry has occurred between age 25 and age t; its initial value is zero (i.e., $I_{i25} = 0$). The law of motion for I_{it} is given by:

$$I_{it} = \begin{cases} 1 \text{ if } I_{it-1} = 1 \text{ or } \alpha_{it} > 0 \\ 0 \text{ otherwise} \end{cases} \tag{8}$$

where α_{it} is the fraction of financial wealth invested in the stock market. The cost associated with stock market entry then becomes $\kappa_i(I_{it} - I_{it-1})$.

We allow for different costs for different investors. We assume a uniform distribution of the cost:

$$\kappa_i \quad \sim \quad U(\underline{\kappa}, \bar{\kappa}), \tag{9}$$

where $\underline{\kappa}$ and $\bar{\kappa}$ denote the lowest and highest costs among all investors, respectively. We justify the dispersion in cost with reference to the documented heterogeneity in financial literacy and financial sophistication (see Lusardi and Mitchell, 2014, for an overview). By introducing a cost distribution, we can replicate the fairly flat life-cycle participation profile in the data.[7] On average, low-cost investors will enter early in life whereas high-cost investors will enter later or never at all. With a sufficiently low value of $\underline{\kappa}$, some low-cost investors will enter immediately. At the end of life, more high-cost than low-cost investors will remain non-participants. For simplicity, we assume that the cost is independent of other characteristics.

[7]Fagereng et al. (2015) present an alternative set-up to account for the empirical life-cycle profiles on portfolio choice. Their set-up involves a per-period cost and a loss probability on equity investments.

The first pillar of the pension system

The first pillar of the pension system is a notional account. Its balance evolves as follows during the working phase:

$$N_{it+1} = (N_{it} + \lambda^{\mathrm{N}} \min\{Y_{it}, \overline{Y}\})R_f, \tag{10}$$

where λ^{N} is the contribution rate for the notional account. Note that the ceiling on the contributions to the DC and notional accounts captures the progressive feature of the pension system.

The second pillar of the pension system

The second pillar of the pension system is a defined contribution pension account with a balance B_{it}. During the working phase, the contribution rate equals $\lambda_{it} \geqslant 0$. This is the central parameter of our analysis. Going forward, we will label a pension system for which λ_{it} is a positive constant independent of age and and individual characteristics (i.e., $\lambda_{it} = \lambda > 0$) as a rigid pension system. Before retirement ($t \leqslant 64$), the law of motion for the DC account balance is:

$$B_{it+1} = (B_{it} + \lambda_{it} Y_{it}) R^B_{t+1}. \tag{11}$$

If λ_{it} is zero then individuals need to rely entirely on the first pillar and their liquid financial wealth, A_{it}, to support themselves in retirement.

Annuitization of the pension accounts

Upon retirement at age 65, the DC account and the notional account are converted into two actuarially fair life-long annuities. They insure against longevity risk through within-cohort transfers from individuals who die to survivors. The notional account provides a fixed annuity with a guaranteed minimum. If the balance of the account is lower than is required to meet the guaranteed level at age 65, we let the individual receive the remainder at age 65 in the form of a one-time transfer from the government. The annuity from the DC account is variable and depends on the choice of the equity exposure as well as realized returns. In expectation, the individual will receive a constant payment each year. The conversion from account balances to annuity payments are functions denoted by $h^B(.)$ and $h^N(.)$. They take the respective balances as arguments.

3.6.1 Budget constraint and laws of motions

The budget constraint at all stages in life is the same one and independent of the design of λ_{it}:

$$C_{it} + A_{it} + \kappa_i(I_{it} - I_{it-1}) = X_{it}. \tag{12}$$

where X_{it} denotes (liquid) cash-in-hand. The law of motion for X_{it} is:

$$X_{it+1} = \hat{Y}_{it+1} + A_{it}R^a_{t+1} \tag{13}$$

$$\hat{Y}_{it+1} = \begin{cases} Y_{it+1}\text{exp}(\omega_{it+1}) - \lambda_{it+1}Y_{it+1} & \text{if } t < 64 \\ h^B(B_{it+1}) + h^N(N_{it+1}) & \text{otherwise} \end{cases} \tag{14}$$

where ω_{it+1} is an idiosyncratic expense shock distributed $\text{N}(-\sigma^2_\omega/2, \sigma^2_\omega)$.[8] The law of motion for B_{it} after retirement ($t > 64$) is given by:

$$B_{it+1} = (B_{it} - h^B(B_{it}))R^B_{t+1} \tag{15}$$

and similarly for N_{it}.

[8] Notice that in Equation (14) we do not subtract the notional contribution (fraction λ^N out of gross income) from income $Y_{i,t+1}$, since $Y_{i,t+1}$ is defined as net of the notional account contribution.

3.7 The individual's problem

Next we describe the individual's problem. To simplify the notation, we suppress the subscript i. Let $V_t\left(X_t, B_t, z_t, \kappa, I_t\right)$ be the value of an individual of age t with cash in hand X_t, DC account balance B_t, a persistent income component z_t, cost κ, and stock market participation experience I_t.

The following describes the individual's problem.

The participant's problem

An individual who has already entered the stock market solves the following problem:

$$\begin{aligned} V_t\left(X_t, B_t, z_t, \kappa, 1\right) &= \max_{A_t, \alpha_t} \left\{ \left(\left(X_t - A_t\right)^{1-\rho} + \beta \phi_t \mathcal{R}_t \left(V_{t+1}\left(X_{t+1}, B_{t+1}, z_{t+1}, \kappa, 1\right)\right)^{1-\rho} \right)^{\frac{1}{1-\rho}} \right\} \\ & \text{subject to equations (3)–(15).} \end{aligned}$$

The entrant's problem

Let $V_t^+\left(X_t, B_t, z_t, \kappa, 0\right)$ be the value for an individual with no previous stock market participation experience who decides to participate at t. This value can be formulated as:

$$\begin{aligned} V_t^+\left(X_t, B_t, z_t, \kappa, 0\right) &= \max_{A_t, \alpha_t} \left\{ \left(\left(X_t - A_t - \kappa\right)^{1-\rho} + \beta \phi_t \mathcal{R}_t \left(V_{t+1}\left(X_{t+1}, B_{t+1}, z_{t+1}, \kappa, 1\right)\right)^{1-\rho} \right)^{\frac{1}{1-\rho}} \right\} \\ & \text{subject to equations (3)–(15).} \end{aligned}$$

The non-participant's problem

Let $V_t^-(X_t, B_t, z_t, \kappa, 0)$ be the value for an individual with no previous stock market participation experience who decides not to participate at t. This value can be formulated as:

$$\begin{aligned} V_t^-(X_t, B_t, z_t, \kappa, 0) &= \max_{A_t} \left\{ \left((X_t - A_t)^{1-\rho} + \beta \phi_t \mathcal{R}_t \left(V_{t+1}(X_{t+1}, B_{t+1}, z_{t+1}, \kappa, 0) \right)^{1-\rho} \right)^{\frac{1}{1-\rho}} \right\} \\ & \quad \text{subject to equations (3)–(15).} \end{aligned}$$

Note that as $\alpha_t = 0$, the return on financial wealth is simply R_f.

Optimal stock market entry

Given the entrant's and non-participant's problems, the optimal stock market entry is given by:

$$V_t(X_t, B_t, z_t, \kappa, 0) = \max\left\{ V_t^+(X_t, B_t, z_t, \kappa, 0), V_t^-(X_t, B_t, z_t, \kappa, 0) \right\}.$$

3.8 Calibration

In this section we describe our calibration strategy. Table 1 reports the values of key parameters. Most parameters are set either according to the existing literature or to match Swedish institutional details; those parameters can be said to be set exogenously. Four parameters are set to match the data as well as possible; those parameters can be said to be determined endogenously.

Exogenous parameters

There are six sets of exogenous parameters.

First, we set the elasticity of intertemporal substitution to 0.5, which is a common value in life-cycle models of portfolio choice (see, e.g., Gomes and Michaelides, 2005).

Second, we set the equity premium to 4% and the standard deviation of the stock market return to 18%. These choices are in the range of commonly used parameter values in the literature. We set the simple risk-free rate to zero, which in other calibrations is often set to 1–2%. We argue that this is correct in our model as labor income does not include economic growth. Thus, we deflate the account returns by the expected growth to obtain coherent replacement rates. As replacement rates in our model are a function of returns, rather than a function of final labor income, this choice is more important to the present model than to previous models. Simulations of the labor income process and contributions to the pension accounts validate our strategy. These simulations indicate that replacement rates at age 65 relative to labor income at age 64 are coherent with Swedish Pensions Agency forecasts.

Third, we set labor income according to Swedish data. The level of the income profile (g_t) is first set to match gross labor income. Then the profile is adjusted further to account for the fact that gross labor income in the data is after deductions of DC plan contributions. Typical contribution rates are 7% – the sum the premium pension account with a contribution rate of 2.5% and the occupational pension account with a typical contribution rate of 4.5%.[9] We therefore scale up the income profile by a factor of 1.07. Following Carroll and Samwick (1997), we estimate the riskiness of labor income. To abstract from other transfers of the welfare state, progressive taxation, etc we estimate the risk on disposable income. We find that the standard deviation of permanent labor income (σ_η) equals 0.072 and that the standard deviation of transitory risk is 0.102. We use this value for our expense shock (σ_ω). We set the one-year correlation between permanent income growth and stock market returns to 10%. This corresponds to a θ of 0.040. We approximate the distribution of initial labor income and financial wealth using log-normal distributions. The mean financial wealth for 25-year-old default investors is set to SEK 111,300. The cross-sectional standard deviations are set to 0.391 (σ_z) and 1.365 (σ_A) to match the data for

[9] This corresponds to the ITP1 pension plan for birth cohorts 1979 and younger but abstracting from the increase in contributions above the cap of the notional account.

25-year-old individuals.

Fourth, we match the contribution rates to Sweden. As mentioned before, the total contribution rate to DC accounts are 7% out of observable gross labor income. This corresponds to a contribution rate in the model of 6.54% (0.07/1.07). The contribution rate for the notional account is set to 14.95% (0.16/1.07). The maximum contribution to this account is capped (corresponding to a labor income ceiling of SEK 344,250).

Fifth, we determine the annuity divisor for the notional account in retirement. We use the unisex mortality table of Statistics Sweden to determine ϕ_t. We assume that the notional account continues to be invested in the risk-free bond and allow for inheritances within a cohort from dying to surviving individuals, incorporating those into the returns of the survivors. We then use the standard annuity formula to reach an annuity factor of 5.6% out of the account balance at age 65. We use the same formulas for the DC account, though we adjust the expected return to the endogenous choice of the DC equity share in retirement.

Finally, we determine the DC equity share profile of the calibration. We use glide path 100-minus-age which is a very common allocation and similar to the default fund in the premium pension plan.

Endogenous parameters and model fit

Four parameters are treated as endogenous in the calibration. We consider data from the working phase.[10] The discount factor (β) is calibrated to match the average ratio of financial wealth to labor income during the working phase (0.922). A β of 0.942 provides an exact fit to the data. The top-left panel of Figure 1 shows the full life-cycle profile of financial wealth. The model fits the financial wealth well up to age 60 and undershoots after that.

[10]Note that we match the model to data from 2007. This does not allow us to extract cohort or time effects as in, e.g., Ameriks and Zeldes (2004). However, Vestman (2019) finds that cohort and time effects are not strongly present in the data.

The support of the cross-sectional distribution of participation costs is set so that we match the average stock market participation rate between ages 25 and 64. As can be seen in the top-right panel of Figure 1, participation is almost flat over the life-cycle. Intuitively, the parameter $\underline{\kappa}$ affects the participation rate among the young, who are poor in terms of financial wealth and reluctant to enter the stock market if the cost is high. The relatively high participation rate of young individuals therefore leads us to set $\underline{\kappa}$ equal to zero. The parameter $\bar{\kappa}$ is then determined to match the average participation rate from age 25 to 64, which is 0.452 in the model and 0.511 in the data. We obtain this participation rate by setting $\bar{\kappa} = 35,000$. As the distribution is uniform, this corresponds to an average participation cost of SEK 17,500. We find our modeling approach appealing as the uniform distribution of the cost enables the model to replicate the flat participation profile in the data.[11]

Finally, the relative risk aversion coefficient, γ, determines the conditional equity share. We weigh each age group's equity share equally. A relative risk aversion of 14 provides a reasonable fit. The conditional equity share is 0.522 in the model and 0.444 in the data. The lower-left panel of Figure 1 depicts the life-cycle profile. The model overshoots the data when financial wealth is low and undershoots when liquid financial wealth is high. We are reluctant to increase the relative risk aversion above 14, as this would lead to a worse discrepancy close to retirement age. In the model there is a noticeable increase in the equity share after age 80; however, if value-weighted, this increase is negligible as the financial wealth is small then.

Figure 2 shows that the distribution of entry costs produces an endogenous sorting of individuals into stock market participants and non-participants that matches the data well. The left panel shows that the average labor income of non-participants is similar in the model and the data. The average labor income of participants is somewhat lower than in the data. The right panel shows the financial wealth in the model and in the data. The sorting by financial wealth to participants

[11] Technically, we approximate the uniform distribution using five equally weighted discrete types (the five costs are equally spaced between zero and SEK 35,000).

and non-participants is consistent with the data but weaker.[12] Financial wealth in the model peaks just before retirement, earlier than in the data. In the years after retirement, financial wealth decumulates in the model and the data, but much less so in the latter. In particular, the gap widens for participants. There could be several reasons for this, one being the lack of a bequest motive in the model.

4 Individuals' savings motive

We now investigate the determinants of individuals' savings motive. This gives us intuition for individuals' savings motives, which enables us to design a DC pension plan that is less rigid and more tailored to individual circumstances. We do so in two ways.

First, we decompose the precautionary savings motive from the retirement savings motive similar to Gourinchas and Parker (2002). Figure 3 compares life-cycle profiles of the baseline calibration with life-cycle profiles based on solutions and simulations where different components of risk have been turned off. The variable of main interest is financial wealth outside the DC account, reported in the fifth panel. If income shocks and expense shocks are turned off, individuals save substantially less before 45 to 50 years when labor income peaks. In this case financial wealth is still zero at age 50, compared to SEK 400,000 in the baseline calibration. This suggests that a DC pension plan whose savings are illiquid and cannot be used to self-insure against shocks should not have a constant contribution rate over the life-cycle. To be precise, we can derive the savings rate in financial wealth out of permanent income, denoted by λ_{it}^{A}, by combining equations (12) and (13):

[12] It is well known that it is difficult to generate wealth inequality in life-cycle models with incomplete markets. This has been addressed by incorporating heterogeneity in discount factors (Krusell and Smith, 1998) or a right-skewed income process (Castaneda et al., 2003). In our model the progressive feature of the pension system helps us match the data.

$$\lambda_{it}^{\mathrm{A}} = \frac{A_{it} - A_{it-1}R_t^a}{Y_{it}} = 1 - \frac{C_{it}}{Y_{it}} - \frac{\kappa_i(I_{it} - I_{it-1})}{Y_{it}}, \tag{16}$$

where the transitory expense shock ω_{it} is set to zero. This savings rate is showed in the bottom left panel of figure 3. It shows that the savings rate is zero or negative before age 51, provided that there are no income or expense shocks. In contrast, savings rates are positive from age 30 in the presence of idiosyncratic shocks. This graph on savings rates reveals two things. First, it shows that the DC account, which is illiquid until age 65, is a poor substitute for financial wealth which is voluntarily accumulated because of precautionary savings motives. Second, even in the presence of idiosyncratic income and expense risks, most individuals younger than 30 years do not wish to save. This suggests that contributions to DC pension plans should not be initiated too early in the life-cycle.

Second, we compute the replacement rate out of the DC account in the baseline and its cross-sectional dispersion, reported in column (1) of Table 3. The mean across individuals is 0.256 with substantial cross-sectional dispersion. The standard deviation is 0.108 and percentiles 99 and 1 are 0.608 and 0.103, respectively. This cross-sectional dispersion translates into considerable dispersion in wealth at 65. Panel A of Table 3 considers the thought experiment that the sum of financial wealth and the DC account balance would be annuitized at 65. It would yield a mean replacement rate of 0.815 with a standard deviation of 0.259 and percentile 99 corresponding to 1.788. These replacement rates can be contrasted to the wealth dispersion if there had been no DC pension plan, reported in column (2). In this setting, wealth accumulation is lower, resulting in a mean replacement rate of 0.687. It is however noteworthy that the cross-sectional dispersion in replacement rates out of wealth is considerably smaller, with a substantially smaller standard deviation, of 0.173, and percentile 99 of 1.455.

These statistics provide additional evidence that a constant contribution rate is too rigid relative to our life-cycle model's implications for optimal wealth accumulation. In Panel C the welfare gains associated with a shift to the no-DC setting are reported. The average ex ante welfare gain is substantial, at 0.049.[13] Notably, in expectation nobody loses from abolishing the DC plan. This is because our baseline assumes that all individuals are rational and because the insurance value against longevity that the DC plan offers is insufficient to outweigh the rigidity during working life. We will use the gains of moving to a No-DC setting as a yardstick when we propose more flexible designs of the contribution rate in the next section.

5 Flexible DC contribution rates

We now design a DC pension plan with a flexible contribution rate. Our starting point is a hypothetical policy function for λ_{it} which would rely on the state variables, $(t, X_t, B_t, z_t, \kappa, I_t)$, where z_t just as well can be represented by Y_t. Which of these variables are most suitable as conditioning variables for a more flexible design of the contribution rate? Given that our analysis suggests that individuals do not wish to save for retirement prior to age 50, age (t) is a natural candidate. Nevertheless, it is unlikely that conditioning on age by itself can reduce the substantial cross-sectional dispersion in replacement rates out of the DC account. A good predictive variable for this replacement rate is the account balance-to-income ratio, B_t/Y_t. We therefore consider flexible DC contribution rates of the form:

$$\lambda_{it} = \beta_0 + \beta_1 \times t + \beta_2 \times \left(\frac{B_{it}}{Y_{it}} - \frac{\overline{B_t}}{Y_t} \right), \tag{17}$$

[13]With Epstein-Zin utility it is straightforward to compute the consumption equivalent. It is proportional to the value function. Our reference to ex ante means that we use the value functions of the 25-year olds.

where t indicates the individual's age (minus 24) and $\frac{B_{it}}{Y_{it}} - \overline{\frac{B_t}{Y_t}}$ indicates the individual's account balance-to-income ratio, de-trended relative to the cross-sectional mean among individuals of age t.[14] This set of contribution rates allows for a great deal of flexibility. Yet, the conditioning variables are exogenous.

We perform a grid search over the parameters $(\beta_0, \beta_1, \beta_2)$ and compute the maximum welfare gain relative to the baseline calibration. Notice that equation (17) nests the baseline calibration which is given by $\lambda_{it} = 0.0654$. We impose several restrictions in the search. A common restriction in all our searches is that we require the average replacement rate out of the DC account to be maintained at a certain level, for instance 0.256 if we wish to target the DC replacement rate in the baseline setting. To facilitate comparisons and illustrations of mechanisms, we consider four subsets of contribution rates:

1. Constant contribution rates. We impose $\beta_1 = \beta_2 = 0$ and vary β_0 between the contribution rate of the baseline and the case of the No-DC plan.

2. Age-dependent contribution rates. We impose $\beta_2 = 0$ and then adjust β_0 so that for each value of β_1 we achieve a specific replacement rate of the DC account.

3. Asset-to-income (B/Y) dependent contribution rates. We impose $\beta_1 = 0$ and for ease of illustration of mechanisms, we adjust β_0 so that the average contribution rate is constant over the life-cycle.

4. Age and asset-to-income dependent contribution rates. In this case, we impose that the average contribution rate for each age group is equal to $\beta_0 + \beta_1 \times t$ to facilitate the interpretation of the role of $\frac{B_{it}}{Y_{it}} - \overline{\frac{B_t}{Y_t}}$.

[14]Notice that since Epstein-Zin utility is a homothetic function and since we impose that income shocks are permanent, see equation (4), the state space could be represented approximately by a scaled version, $(t, X_t/Y_t, B_t/Y_t, \kappa/Y_t, I_t)$. To be precise, computing $\overline{\frac{B_t}{Y_t}}$ requires us to solve for the cross-sectional average for a given set of parameters $(\beta_0, \beta_1, \beta_2)$ and then subsequently adjust β_0 until convergence. See Appendix A for details.

Columns (3) to (5) of Table 3 report our findings from grid searches over sets 2, 3, and 4, imposing that the average replacement rate out of the DC account should be approximately equal to the baseline.

The age-dependent contribution rate that maximizes welfare implies that contribution rates start at zero until age 36 and then increase by 0.7 percentage points per year. At 64, the contribution rate peaks at 20.8 percent.[15] As reported in Panels A and B this age-dependent contribution rate is able to reduce the cross-sectional dispersion in replacement rates to 0.072 and limits percentile 99 to 0.488. This results in an average welfare gain of 0.021 relative to the baseline. Thus, it bridges 43 percent of the welfare gap between the baseline and the No-DC plan. Intuitively, this achieved by improving individuals' ability to self-insure early in life when their marginal utility is high.

The asset-to-income dependent contribution rate that maximizes welfare implies that an investor who deviates by -0.1 from the cohort's asset-to-income ratio should deviate by $+17.5$ percent in her contribution rate (with a floor at 0 and a ceiling at 50 percent). This implies that relatively small shifts in the account balance or income shifts contribution rates. A fall in income or an increase in the account balance would lead to substantial cash-flow benefits.[16] Column (4) of Table 3 reports the associated statistics. According to Panel B this policy is able to further reduce the cross-sectional dispersion in replacement rates. The standard deviation is a mere 0.058. Panel C reports welfare gains. Interestingly, at 2.7 percent this rule for the contribution rate is associated with a higher welfare gain than the best age-dependent contribution rate. And the gains appear somewhat more evenly distributed.

Our proposal involves a combination of both instruments. The best combination is that contribution rates that unconditionally increases with 0.3 percentage points every year and that the

[15] $\lambda_{it} = \max\{0, -0.0717 + 0.007 \times t\}$.

[16] $\lambda_{it} = \min\{0.5, \max\{0, 0.0645 - 1.75 \times \left(\frac{B_{it}}{Y_{it}} - \overline{\frac{B_t}{Y_t}}\right)\}\}$. Note that we have a floor 0 and a ceiling at 0.5. Our results are qualitatively similar if we reduce the ceiling to 0.2.

correction based on the asset-to-income ratio is as in the pure asset-to-income rule.[17] The results are reported in column (5). Panel B reports a further decline in the cross-sectional distribution, with a standard deviation of only 0.040. Panel C reports the welfare gains. Interestingly, this rule adds an additional welfare benefit. The average welfare gain is 3.3 percent. Thus, the best rule for the contribution rate covers 67 percent of the gap between the baseline setting and the No-DC setting.

Figure 4 illustrates the implications of our proposal by comparing outcomes from the baseline setting (panels to the left) with outcomes from proposed rule (panels to the right). Contribution rates are low early in life. The average contribution is lower than the one in the baseline until age 42. By that stage in life, ten percent of individuals contribute ten percent of their income. Just before retirement the average contribution rate is thirteen percent. The increase in contribution rates over life implies that the DC account balance displays more exponential growth under the proposed rule and on average account balances do not reach the level of the baseline until a few years before retirement. The evolution of financial wealth is similar early in life but flatter later in life, suggesting that our proposed rule serves well as a substitute for additional voluntary retirement savings in liquid financial wealth. Finally, our rule has positive implications for consumption. Up until age 45, average consumption under the proposed rule exceeds the baseline. Among the youngest the increase is almost five percent. The increases are similar in magnitude also for the second and ninth deciles.

Figure 5 reports the outcome of a broader grid search, targeting different DC replacement rates. For each set of rules, the dots form a frontier of how large the welfare gain can be from abolishing a constant contribution rate and shifting to an alternative. The No-DC case serves as a yardstick, corresponding to a replacement rate of zero and a welfare gain of 0.049. The main take-away is how disadvantageous constant contribution rates are relative to our proposals. Two thirds of the

[17] $\lambda_{it}^{\mathrm{DC}} = \max\{0, 0.0099 + 0.003 \times t - 1.75 \times \left(\frac{B_{it}}{Y_{it}} - \overline{\frac{B_t}{Y_t}}\right).$

gain associated with the No-DC plan setting can be attained simply from a redesign that maintains a replacement rate north of 25 percent.

6 Concluding remarks

We investigate the implications of introducing a defined contribution (DC) pension plan that offers more flexibility than typical plans do. We model the first and second tiers of a pension system, where the second tier is a DC pension plan, and embed it into a life-cycle portfolio choice model. We find that a fixed, "rigid", contribution rate is at odds with optimal consumption-savings behavior. It backloads individuals' consumption and does not offer any insurance against shocks. To address this, we propose a flexible design. The design is based on a rule of thumb for the contribution rate. The rule takes age and the ratio of the DC balance to income as inputs. The rule leads to the same average replacement rate as the fixed contribution rate but at a lower cross-sectional dispersion. There are substantial welfare gains associated with this design. The average gain is 3.3 percent of consumption and nobody loses.

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Table 1: **Calibration – model parameters**

	Notation	Value
Preferences and stock market entry cost		
Discount factor*	β	0.942
Elasticity of intertemporal substitution	$1/\rho$	0.500
Relative risk aversion*	γ	14
Ceiling for stock market entry cost*	$\overline{\kappa}$	35,000
Floor for stock market entry cost*	$\underline{\kappa}$	0
Returns		
Gross risk-free rate	R_f	1.00
Equity premium	μ	0.04
Standard deviation of stock market return	σ_ε	0.18
Labor income, expense shock, and financial wealth		
Standard deviation of idiosyncratic labor income shock	σ_η	0.072
Weight of stock market shock in labor income	θ	0.040
Standard deviation of idiosyncratic expense shock	σ_ω	0.102
Standard deviation of initial labor income	σ_z	0.391
Standard deviation of initial financial wealth	σ_A	1.365
Mean of initial financial wealth	**	111,300
Ceiling for contributions to DC and notional accounts	$\overline{Y}$	344,250
Floor for notional pension	$\underline{Y}$	85,829
Contribution rates in pension accounts		
DC account	λ	6.54%
Notional account	λ^{N}	14.95%
Life-cycle profiles		
Labor-income profile (scaled by 1.07)	g_t	—
Survival rates	ϕ_t	—

The table presents the parameter values of the model. * The parameter value has been determined endogenously by simulation of the model. ** The mean initial financial wealth for 25-year-old default investors, $\exp(\mu_A - \sigma_A^2/2)$, is set to SEK 111,300. The labor-income profiles are discussed in detail in the main text. The survival rates are computed from unisex statistics provided by Statistics Sweden.

Table 2: **Matched Moments in Data and Model**

	Data	Model
Financial wealth-to-labor income ratio	0.922	0.921
Stock market participation	0.511	0.452
Equity share (conditional)	0.444	0.522

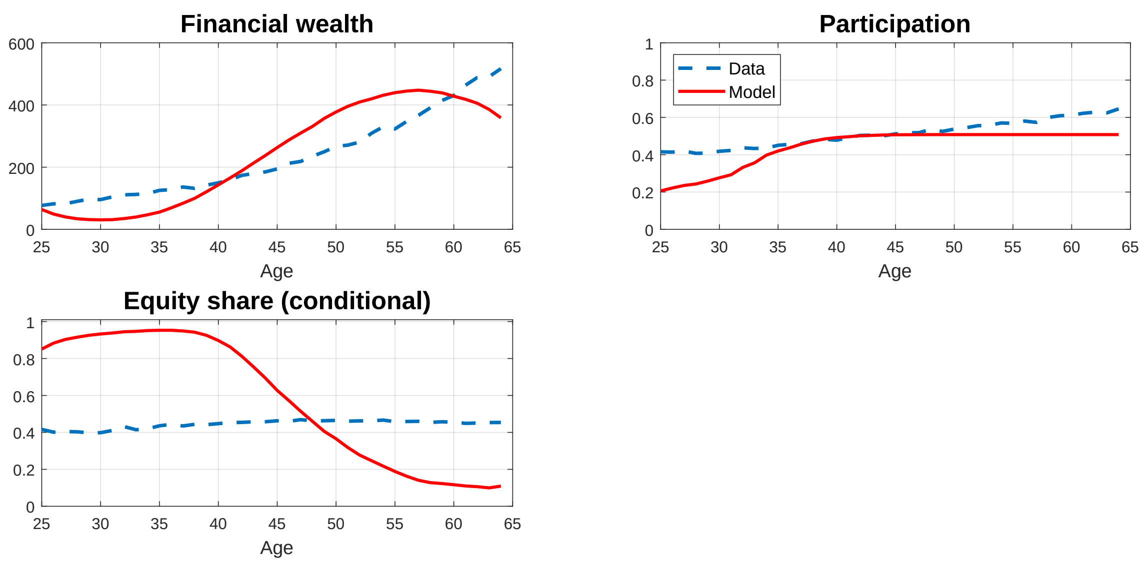
The table presents matched moments in the data and model.

Table 3: **Replacement rates and welfare gains**

	(1)	(2)	(3)	(4)	(5)
	Baseline	”No-DC setting”	Age dependency	B/Y dependency	Both age and B/Y dependency
Panel A: Replacement rate out of financial wealth and the DC account					
Mean	0.815	0.687	0.800	0.808	0.800
Standard deviation	0.259	0.173	0.242	0.233	0.228
Maximum	2.251	1.555	2.148	2.162	2.014
Percentile 99	1.788	1.455	1.747	1.742	1.726
Percentile 1	0.514	0.481	0.515	0.534	0.534
Minimum	0.433	0.412	0.435	0.452	0.451
Panel B: Replacement rate out of the DC account					
Mean	0.256	—	0.256	0.255	0.256
Standard deviation	0.108	—	0.072	0.058	0.040
Maximum	0.979	—	0.761	0.780	0.603
Percentile 99	0.608	—	0.488	0.474	0.402
Percentile 1	0.103	—	0.140	0.187	0.198
Minimum	0.069	—	0.099	0.160	0.163
Panel C: Welfare gain relative to baseline					
Mean	—	0.049	0.021	0.027	0.033
Standard deviation	—	0.006	0.003	0.003	0.004
Maximum	—	0.062	0.028	0.033	0.038
Minimum	—	0.027	0.006	0.015	0.015

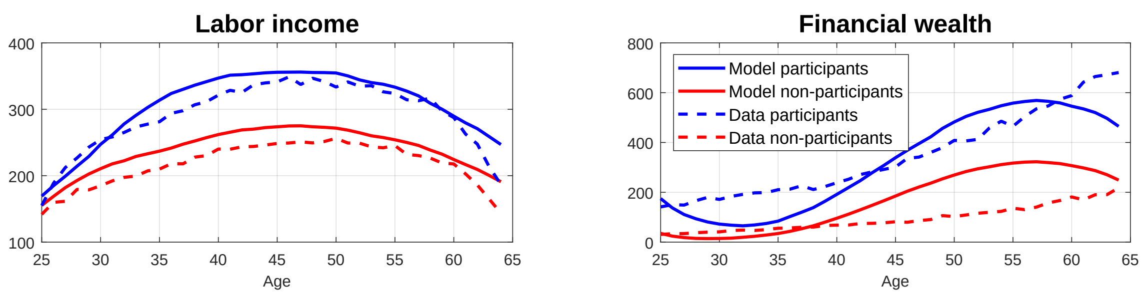
Panel A reports moments of replacement rates out of total wealth $((h^B(B_{65}+A_{65})+h^N(N_{65}))/Y_{64})$. Panel B reports moments of replacement rates out of the DC account $(h^B(B_{65}/Y_{64})$. Panel C reports moments of ex ante welfare gains associated with a shift from the baseline to each one of the other DC plan designs. The age-dependent policy in column (3) corresponds to $\lambda_{it} = \max\{0, -0.0717 + 0.007 \times t\}$. The B/Y-dependent policy in column (4) corresponds to $\lambda_{it} = \max\{0, 0.0645 - 1.75 \times \left(\frac{B_{it}}{Y_{it}} - \frac{\overline{B_t}}{Y_t}\right)\}$. The policy in column (5) corresponds to $\lambda_{it} = \max\{0, 0.0099 + 0.003 \times t - 1.75 \times \left(\frac{B_{it}}{Y_{it}} - \frac{\overline{B_t}}{Y_t}\right)\}$.

Figure 1: **Calibration**



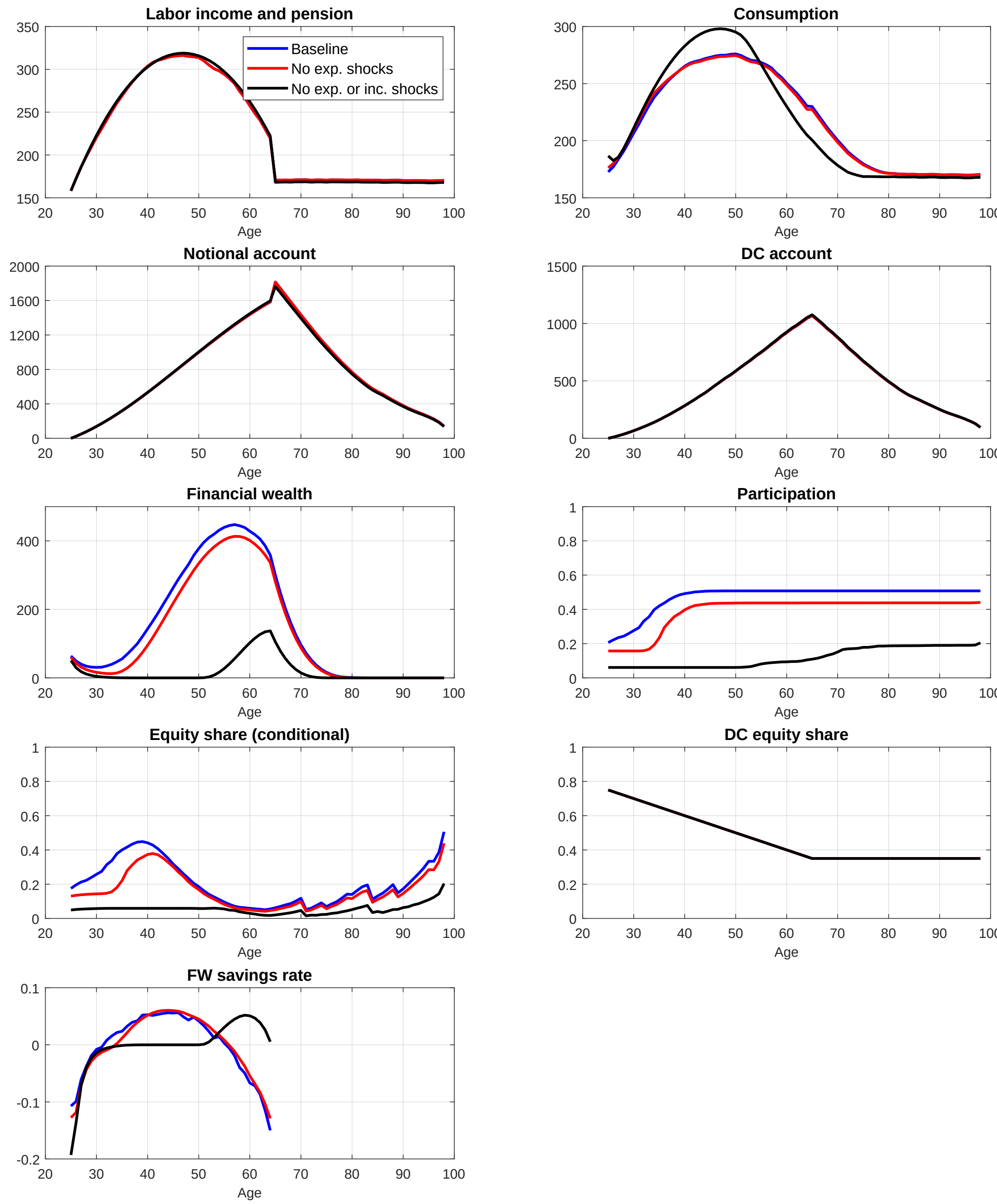
The figure shows the variables that the calibration targets. Financial wealth is expressed in SEK 1000s.

Figure 2: **Model fit**



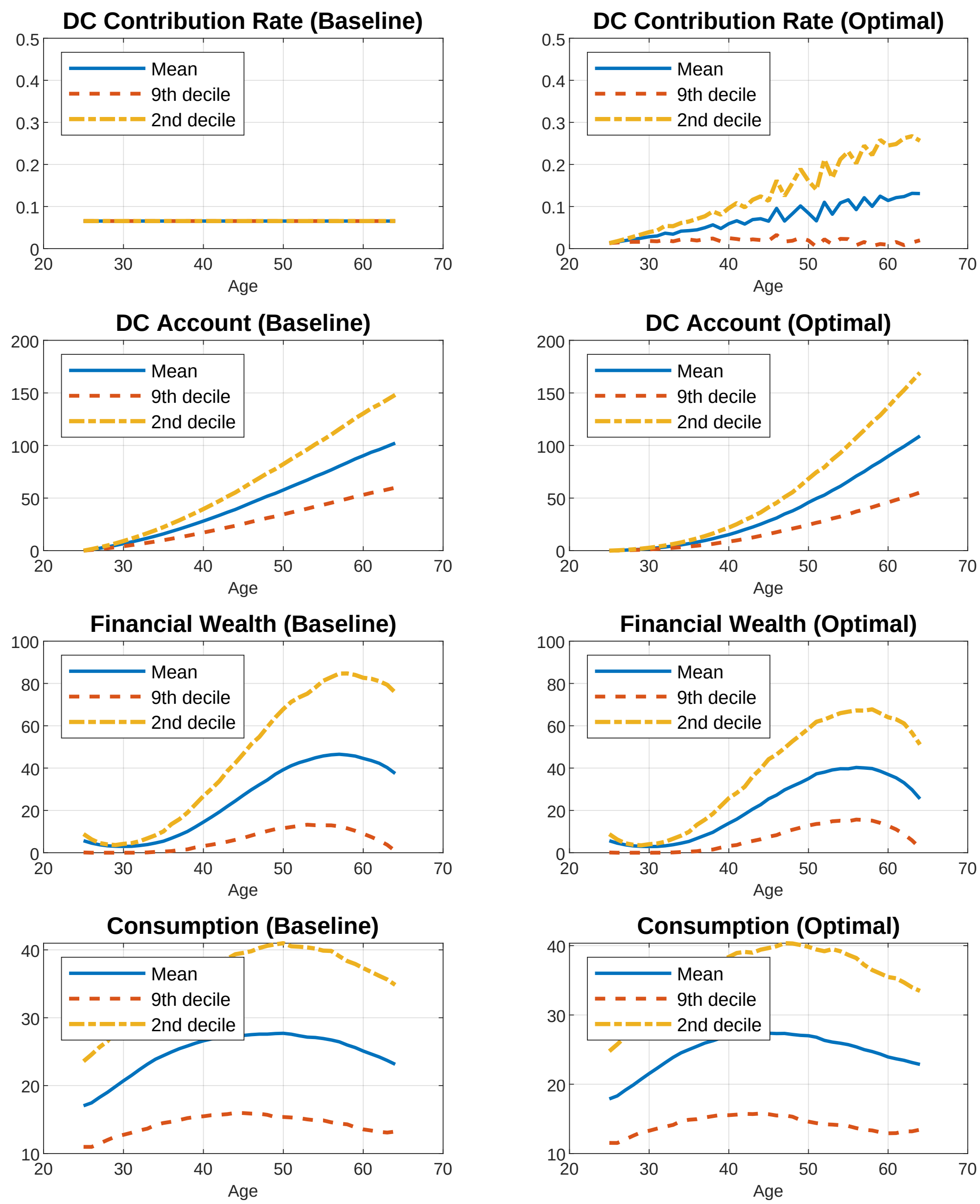
The figure shows labor income and financial wealth conditional on stock market participation. Financial wealth is expressed in SEK 1000s.

Figure 3: **Decomposition of savings motives**



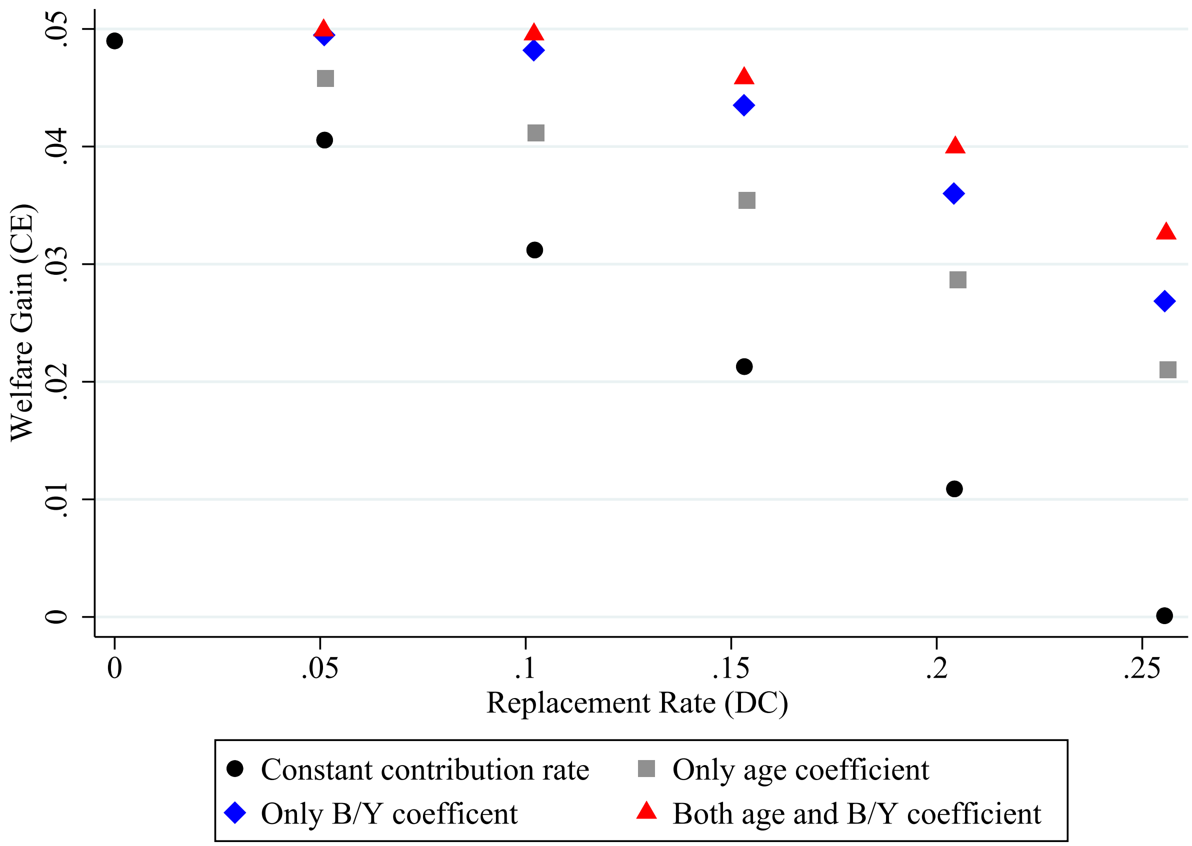
The figure shows averages of variables for three sets of parameter values: the baseline calibration, no expense shocks (σ_ω set to zero), and no expense shocks or income shocks (σ_ω and σ_η set to zero). The bottom left panel plots the savings rate in financial wealth ($\lambda^{\mathrm{A}}_{it}$). Values are expressed in SEK 1000s.

Figure 4: **Baseline vs. Flexible pension system**



The top panels show the average and the 2nd and 9th deciles of the contribution rate into the DC account (i.e., λ_{it}). The other panels show the corresponding values for other variables. Values are expressed in SEK 1000s.

Figure 5: **Replacement rates and welfare gains**



The figure shows replacement rates out of the DC account and welfare gains (consumption equivalents) for four types of contribution rate policies: (i) constant contribution rates ($\lambda_{it} = \beta_0$), (ii) only age coefficients ($\lambda_{it} = \beta_0 + \beta_1 \times t$), (iii) only B/Y coefficients ($\lambda_{it} = \beta_0 + \beta_2 \times \left(\frac{B_{it}}{Y_{it}} - \frac{\overline{B_t}}{Y_t}\right)$), and (iv) age and B/Y coefficients ($\lambda_{it} = \beta_0 + \beta_1 \times t + \beta_2 \times \left(\frac{B_{it}}{Y_{it}} - \frac{\overline{B_t}}{Y_t}\right)$).

Appendix for Online Publication

Optimal Defined Contribution Pension Plans: One-Size Does Not Fit All

Kathrin Schlafmann, Ofer Setty and Roine Vestman

A Algorithm to find optimal policies for the contribution rate

To select the optimal policy for contribution rates we aim to select the policy that delivers the highest welfare gain while achieving the same average replacement rate out of the DC account as the baseline constant contribution rate. We proceed in four steps:
For each target replacement rate we

1. solve for the constant contribution rate that delivers this average replacement rate

2. search for the optimal policy that allows for an age trend in the policy:

   (a) for each candidate trend in age solve for the required constant such that the policy achieves exactly the required average replacement rate

   (b) select the policy (i.e. candidate coefficient) with the highest welfare gain

3. search for the optimal policy that reduces the variance of replacement rates:

   (a) for each candidate for the coefficient of (B/Y - trend_BY(age)), solve for the required constant and vector trend_BY such that

      - the policy achieves exactly the required average replacement rate and
      - the average contribution rate is constant for all ages

   (b) select the policy (i.e. candidate coefficient) with the highest welfare gain

4. search for the optimal policy that reduces the variance of replacement rates while allowing for an age trend:

   (a) for each combination of candidates for the coefficient of (B/Y - trend_BY(age)) and the age trend, solve for the required constant and vector trend_BY such that

      - the policy achieves exactly the required average replacement rate and
      - the average contribution rate follows exactly the age trend

   (b) select the policy (i.e. combination of coefficients) with the highest welfare gain

A.1 Finding policies that reduce the variance of replacement rates (step 3)

To solve for the required constant and vector trend_BY, given a target replacement rate and a B/Y coefficient) we proceed according to the following algorithm:

I. find a good starting value for the search:

   (i) use an iterative algorithm to find the constant required to meet the replacement rate target *in the absence of variance reduction* (i.e. B/Y coefficient = 0 in the policy)

   (ii) simulate the model with this constant *and the B/Y coefficient* and record average B/Y by age

II. use the following algorithm to find the constant and trend_BY:

   (i) use the constant and trend_BY = mean(B/Y) from simulation in Iii as initial guess

   (ii) for the given constant, use an iterative algorithm to solve for trend_BY such that mean(contribution rate) = constant for all age groups

   (iii) simulate the average replacement rate under a policy with this constant and this trend_B/Y

   (iv) if the replacement rate target is met: stop; otherwise update guess for constant and return to step IIii (using trend_B/Y from step IIiii as initial guess)

A.2 Finding policies that reduce the variance of replacement rates and allows for age trend (step 4)

To solve for the required constant and vector trend_BY, given a target replacement rate, age trend and a B/Y coefficient) we proceed according to the identical algorithm as in section A.1, with two modifications:

- in step Ii we solve for the constant required for a policy that includes the given age trend
- in step IIii we require that mean(contribution rate) = constant + age trend