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Anabolic Persuasion

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Anabolic Persuasion[*]

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Abstract

We present a model of optimal training of a rational, sluggish agent. A trainer commits to a discrete-time, finite-state Markov process that governs the evolution of training intensity. Subsequently, the agent monitors the state and adjusts his capacity at every period. Adjustments are incremental: the agent's capacity can only change by one unit at a time. The trainer's objective is to maximize the agent's capacity - evaluated according to its lowest value under the invariant distribution - subject to an upper bound on average training intensity. We characterize the trainer's optimal policy, and show how stochastic, time-varying training intensity can dramatically increase the long-run capacity of a rational, sluggish agent. We relate our theoretical findings to "periodization" training techniques in exercise physiology.

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1 Introduction

Economists have a long tradition of invading other academic disciplines. Lazear (2000) celebrates this so-called economic imperialism, demonstrating its value for such diverse fields as sociology, criminology and organizational behavior. Proponents of economic imperialism maintain that the ideas of individual rationality, forward-looking behavior and equilibrium help us understand empirical regularities and guide policy interventions. As Becker (1976, p. 8) wrote: "I have come to the position that the economic approach is a comprehensive one that is applicable to all human behavior". Recently, economists applied the imperialistic approach to the field of epidemiology, in the context of the Covid-19 pandemic (Acemoglu et al. (2020)).

This paper carries the imperialistic approach to a new territory. It applies tools from economic theory to the field of *exercise physiology*, which studies the body's response and adaptation to exercise to maximize human physical potential (for an introduction to this field, see Glass et al. (2014)). Specifically, we focus on the question of how the body's muscle mass responds to patterns of physical exercise. Muscle mass adapts to physical stimuli, and the economic approach seeks to describe this adaptation as the result of maximizing behavior. We demonstrate that by modeling the body as a forward-looking optimizing agent, we gain insights into the effectiveness of popular physical training strategies.

To describe the body as an optimizing agent, we need to specify its preferences. On the one hand, maintaining muscle mass is costly in terms of energy expenditure (Zurlo et al. (1990)). On the other hand, if muscle mass is too low relative to the demands of exercise, the body may incur the energy costs of repairing torn muscle tissue or inflammation (see Frankenfield (2006) and Faulkner et al. (1993)). Moreover, if the body lacks adequate muscle mass, it will not be able to complete the required physical task. It is plausible to assume that the body will internalize this performance gap as a cost. This cost can be interpreted in terms of psychological motivation, which itself

may originate from evolutionary survival pressures (see Sagar and Stoeber (2009) and Lieberman (2015)). A more motivated trainee will record the performance gap as a larger cost relative to the muscle maintenance cost.

In a dynamic environment where the intensity of exercise changes stochastically over time, a key ingredient in modeling the body as an optimizing agent is its expectation of future demands. Here, too, we follow the economist's standard recipe and assume that the body has rational expectations - i.e., it knows the stochastic process that governs the future evolution of physical exercise, possibly as a result of some previous adaptive-learning phase.

Using these ingredients, we construct the following stylized discrete-time model. A "trainer" commits ex-ante to a strategy, which is a stochastic process that governs the evolution of exercise intensity. We restrict ourselves to stochastic processes that follow a finite-state Markov chain. Average intensity (according to the chain's invariant distribution) cannot exceed some integer μ more than negligibly. The parameter μ represents a "budget constraint" that limits the amount of resources that can be devoted to physical training.

Following the trainer's choice of strategy, at every subsequent period, the body (referred to as an "agent") monitors the state of the trainer's Markov process and chooses its muscle mass. We assume that the body can only make *incremental adjustments* to its muscle mass: at every period it can only change the mass by -1, 0 or 1 units. The body is an expected discounted utility maximizer, with a periodic payoff function that trades off the maintenance cost of muscle mass and the excess intensity of current physical exercise relative to current mass. We measure muscle mass and exercise intensity with the same units, such that excess intensity (also referred to as the performance gap) is a simple difference between the two numbers. We impose the constraint that the agent has a best-reply to the trainer's strategy that induces a Markov process (over an extended state space that also

includes muscle mass in the definition of a state) with a *unique* invariant distribution.

The trainer's problem is to choose the Markov process to maximize the agent's long-run mass - evaluated according to its *minimal* realization under the invariant distribution. Our use of such a "max-min" criterion is justified by the interpretation of muscle mass as a *capability*: the higher the agent's minimal long-run mass, the higher the intensity he can reliably withstand in the long run.

The sluggish adjustment of muscle mass is a fundamental assumption in our model. Exercise intensity can fluctuate wildly between periods, but clearly, the body cannot change its muscle mass instantaneously to any level (see DeFreitas et al. (2011) and Counts et al. (2017)). Our model is set up such that if the body had perfect flexibility in adjusting its muscle mass, the trainer's problem would be trivial: at every period, muscle mass would be set such that excess intensity would be exactly zero, and the long-run average mass will be at most μ. This is also the minimal long-run mass that the trainer can attain with a constant-intensity policy. Under such a policy, the distinction between sluggish and flexible agents is irrelevant. The question is whether using some non-degenerate Markov process will enable the trainer to outperform this benchmark when muscle adjustment is sluggish.

The trainer's problem is similar in spirit to Bayesian persuasion (Kamenica and Gentzkow (2011)). In a persuasion problem, the sender wants to increase the receiver's action; in our model the trainer wants to increase the agent's muscle mass. In a persuasion problem, the sender commits to a signal function; in our problem, the trainer commits to a Markov process. In a persuasion problem, the receiver's response to a signal realization is dictated by Bayesian updating; in our model, the agent's response to a realized state is constrained by sluggish adjustment. Finally, in a persuasion problem, the sender's ability to attain his objective is constrained by Bayes plausibility, which requires the average posterior belief to equal the prior; in our model,

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the trainer is constrained by the average intensity limit μ.

Our analysis focuses on two extreme cases in terms of the agent's discount factor. We begin by analyzing a myopic agent, whose adjustment rule is mechanical and independent of the trainer's strategy: mass moves up (down) a notch when current intensity is above (below) current mass. In this case, the trainer cannot attain a minimal long-run mass above $2\mu - 1$. He can implement this upper bound using a two-state Markov process with intensity levels 0 and 2μ; the transition from 0 to 2μ is deterministic, while the transition from 2μ to 0 occurs with *near* certainty. This random element ensures that regardless of the initial muscle mass, the agent eventually oscillates between mass levels 2μ and $2\mu - 1$.

We next turn to an arbitrarily patient agent. In this case, the trainer cannot attain a minimal long-run muscle mass above $\mu/c - 1$, where c is the maintenance cost per unit of muscle mass (we assume that μ/c is an integer, for convenience). The trainer can implement this bound using a two-state Markov process with intensity levels 0 and μ/c; one of the transitions between the two states occurs with certainty (which one depends on the value of c). This policy ensures that regardless of the initial mass, the agent eventually oscillates between muscle mass levels μ/c and $\mu/c - 1$. The transition probabilities are calibrated to make the body nearly indifferent to lowering muscle mass below $\mu/c - 1$.

Thus, when the agent is sluggish, a properly designed stochastic training program can increase long-run mass substantially relative to the flexible-adjustment benchmark. Our results suggest a rationale for the popular training technique of *periodization*, which structures the training regimen as a cycle with phases of high intensity physical load and recovery phases of low intensity. Since it first began in the 1960s, this methodology has gained popularity and is currently the dominant technique used by professional athletes. Numerous studies have documented the success of periodization in terms of increased muscle mass, increased muscle strength, greater endurance and

athletic performance (Bompa and Buzzichelli (2018)).[1] While the physiological literature offers biological explanations for the superiority of a cyclical training design (e.g., see Issurin (2019)), our results provide a complementary perspective: the effectiveness of periodization techniques may stem from rational yet sluggish adaptation to fluctuations in physical stimuli.

Although our paper strictly adheres to the model's exercise-physiology interpretation, its abstraction enables other interpretations. For example, m and d may represent cognitive capacity and the intensity of cognitive activity, such that our results can be viewed in terms of programs for maintaining cognitive skills. Moving to more conventionally economic settings, we can view the agent as an *organization* like a military or an emergency-management agency. The mission of such organizations is to maintain a level of preparedness against unexpected challenges. This can be achieved with a suitably designed regimen of *drills*. Sluggish adjustment is a natural assumption in this setting: organizations cannot drastically improve their level of preparedness overnight; and likewise, deteriorating preparedness tends to be gradual. Our analysis sheds light on the optimal design of a drill program for such organizations. More generally, we find the optimal design of a stochastic process for a sluggish agent to be an interesting (and, to our knowledge, new) problem from an abstract economic-theory perspective.

2   The Model

We consider a principal-agent model, in which the principal is referred to as a "trainer". We interpret the agent as a physiological system that is trained to increase its capacity. The trainer commits ex-ante to a pair (P, f), where P is a discrete-time, Markov process over some finite set of states S, and $f : S \to \mathbb{N}_+$ is an output function that assigns a challenge level to every

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[1]For recent discussions of various periodization techniques, see Issurin (2010), Kiely (2012) and Kiely et al. (2019).

state $s \in S$. We denote by d_t the challenge level in period t. When there is no risk of confusion we will replace the notation $f(s)$ with $d(s)$.

We impose the following constraints on (P, f). First, P has a unique invariant distribution λ_P. Second,

$$\sum_{s \in S} \lambda_P(s) f(s) \leq \mu + \varepsilon$$

where $\mu \geq 1$ is an integer and $\varepsilon > 0$ is arbitrarily small. That is, the long-run average challenge level cannot exceed μ by more than a negligible amount (the approximate formulation of the constraint is due to μ getting integer values).

The agent knows the trainer's choice of (P, f). At every period t, he observes the realized state s_t and then chooses a non-negative capacity level $m_t \in \{m_{t-1} - 1, m_{t-1}, m_{t-1} + 1\}$. Henceforth, we refer to m_t as the agent's "mass" at time t. Let $m_0 \in \mathbb{N}_+$ be the agent's initial mass. The restricted choice set for m_t reflects the sluggish adaptation of the agent's mass.

The agent is an expected discounted utility maximizer with discount factor δ. His payoff at period t is

$$-[cm_t + \max(0, d_t - m_t)]$$

where $c \in (0, 1)$, $d_t = f(s_t)$ and s_t is the state of P at period t. The body's periodic cost incorporates two factors. First, cm_t is the caloric maintenance cost of muscle mass m_t. Second, the gap between m_t and d_t (when the latter is higher) represents a performance shortfall because the agent's capacity is lower than the challenge it faces.

Given (P, f), the agent faces a Markov decision problem over an extended state space, where the state at period t is the pair (s_t, m_{t-1}). We impose the following additional constraint on the trainer: the extended Markov process over (s_t, m_{t-1}) that is induced by the agent's best-reply to (P, f) has a unique invariant distribution $\lambda^*_{(P,f)}$. This ensures that the minimal and average long-

run masses are well-defined and independent of the initial condition m_0.

The trainer aims to maximize the agent's lowest muscle mass in the support of the invariant distribution $\lambda^*_{(P,f)}$. The larger this mass, the higher the challenge level that the agent is guaranteed to meet in the long run. Formally, the trainer's problem can be stated as follows:

$$\max_{(P,f)} \min\{m \mid \lambda^*_{(P,f)}(s,m) > 0 \text{ for some } s \in S\}$$

subject to the feasibility constraint

$$\sum_{(s,m)} \lambda^*_{(P,f)}(s,m)f(s) \lessapprox \mu$$

The max-min criterion means that the trainer looks for the highest capacity that the agent's body *reliably* maintains in the long run. The symbol $\lessapprox$ represents the requirement that average intensity cannot exceed μ by more than a negligible amount.

Discussion of the model's interpretation

The level of physical challenge d can be interpreted in terms of duration (e.g. the number of repetitions of a given exercise), load (e.g. lifting weight) or effort (e.g. running speed).[2] The stylized nature of our model abstracts from such fine distinctions. However, the interpretation of the resource constraint does depend on the meaning of d. If it represents exercise duration, then μ is the average amount of time per period that the trainee can devote to physical exercise. If, however, d represents load or effort, μ is perhaps better viewed as a parameter of the trainer's problem than an exogenous resource constraint.

Our model endows the human body with rational expectations: it has knowledge of (P, f) when making its periodic decisions. The justification for

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[2] See Steele (2014) and Steele et al. (2017) for discussions of these different notions of intensity.

this assumption is that the body forms adaptive expectations based on a long memory. We find it reasonable to assume that in the long run, the body will learn finite-state Markov processes, especially when they have few states.

The adaptive-expectations rationale also underlies our restriction that the trainer cannot condition d_t on past realizations of m. If he could, he would have recourse to off-equilibrium threats. For instance, he could incentivize the agent to increase muscle mass using a policy of zero on-path challenges, sustained by a "grim" threat to switch to persistently extreme challenges if m fails to go up. We find such policies absurd in the physiological context and attribute this absurdity to the implausibility of full-throttle rational expectations in this content. We effectively rule out off-path threats by assuming that the trainer does not condition on m. Under our alternative interpretation of the agent as an organization, it is questionable whether the trainer can monitor m, which represents the organization's level of preparedness.[3]

Benchmark: Completely flexible adjustment

Suppose the agent could choose any $m_t \in \mathbb{N}_+$ at every period. Then, since $c \in (0, 1)$, he would choose $m_t = d_t$ at every t. This means that the long-run average of m_t would coincide with the long-run average of d_t, which by assumption cannot exceed μ more than negligibly. Therefore, the best the trainer can do according to his max-min criterion is play a constant strategy $d_t = \mu$ at every period, such that the flexible agent's mass will be μ as well. The same deterministic process attains the same long-run mass of μ also when the agent is sluggish (because the agent will eventually reach this mass and stay there indefinitely). The question is whether the trainer can outperform this benchmark with a non-degenerate Markov process.

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[3]We conjecture that if the trainer can condition d_t on m_{t-1}, the results in our paper will not change.

3 A Myopic Agent

In this section we analyze the trainer's problem when $\delta = 0$ - i.e., the agent is myopic.

Proposition 1 *Let $\delta = 0$. Then:*

(i) For any trainer strategy, the minimal long-run mass induced by the agent's best-reply is at most $2\mu - 1$.

(ii) This upper bound can be implemented by the following (P, f). The Markov process P has two states, H and L, and a transition matrix given by

$$
\begin{array}{c|cc}
\Pr(s_t \to s_{t+1}) & L & H \\
\hline
L & 0 & 1 \\
H & \beta & 1 - \beta
\end{array}
$$

where β is arbitrarily close to 1. The output function is $f(H) = 2\mu$ and $f(L) = 0$. In the $\beta \to 1$ limit, the invariant mass distribution assigns probability $\frac{1}{2}$ to $m = 2\mu$ and $m = 2\mu - 1$.

Thus, a slightly perturbed cyclic training program can dramatically increase the minimal long-run mass of a myopic sluggish agent. When μ is large (corresponding to a very sluggish agent, given that we normalized his adjustment increment to 1), the increase is by a factor of nearly 2 relative to the flexible-agent benchmark.

The trainer's training regime approximately consists of alternating periods of high intensity $(d = 2\mu)$ and rest $(d = 0)$. After a period of high intensity training, there is a small chance $1 - \beta$ that the high-intensity episode will be repeated. This stochastic perturbation ensures that the set of mass values $\{2\mu, 2\mu - 1\}$ is absorbing: the agent will reach it in finite time with probability one, regardless of m_0.

Proof of part (i) of Proposition 1

The proof proceeds by a series of steps. Recall that we use the notation $d(s)$ as a substitute for $f(s)$.

Step 1: *The agent's strategy*

Consider the agent's move at period t, given the extended state (s_t, m_{t-1}). A myopic agent will choose m_t to minimize $cm_t + \max(0, d(s_t) - m_t)$. Therefore, we can immediately pin down the agent's behavior, independently of the trainer's strategy. Since $c \in (0, 1)$, we obtain the following: if $d(s_t) > m_{t-1}$, the agent will choose $m_t = m_{t-1} + 1$; if $d(s_t) < m_{t-1}$, the agent will choose $m_t = m_{t-1} - 1$; and if $d(s_t) = m_{t-1}$, the agent will choose $m_t = m_{t-1}$. That is, the agent will always adjust his mass in the direction of the current level of d. $\square$

Consider an arbitrary strategy for the trainer, which induces an extended Markov process with a unique invariant distribution. Let $(m_{t-1}, d_t)_{t=1,2,...}$ be a possible sample path that results from the extended process. By the unique-invariant-distribution requirement, the extended process is ergodic. Therefore, the long-run frequency of every (m, d) in the sample path coincides with the probability of this pair according to the invariant distribution. Let $\lambda(m, d)$ denote the probability of (m, d) according to the invariant distribution, as well as the frequency of (m, d) in the sample path. Let X be the set of recurrent pairs (m, d) in the sample path. Partition X into three classes:

$$
\begin{aligned}
X^+ &= \{(m, d) \in X \mid d > m\} \\
X^- &= \{(m, d) \in X \mid d < m\} \\
X^0 &= \{(m, d) \in X \mid d = m\}
\end{aligned}
$$

Step 2: λ *satisfies*

$$\sum_{(m,d)\in X^+} \lambda(m,d)(m+1) = \sum_{(m,d)\in X^-} \lambda(m,d)m \tag{1}$$

Consider some period t along the sample path such that $(m_t, d_{t+1}) \in X^+$. By definition, this pair is recurrent. Therefore, m_t must be visited again in some later period. Let $t'+1$ be the earliest such period. Since m moves only in one-unit increments, it must be the case that $(m_{t'}, d_{t'+1}) \in X^-$ and $m_{t'} = m_t + 1$. We have thus defined a one-to-one mapping from periods t for which $(m_t, d_{t+1}) \in X^+$ to periods t' for which $(m_{t'}, d_{t'+1}) \in X^-$, such that $m_{t'} = m_t + 1$. In a similar way, we can define a one-to-one mapping from periods t for which $(m_t, d_{t+1}) \in X^-$ to periods t' for which $(m_{t'}, d_{t'+1}) \in X^+$, such that $m_{t'} = m_t - 1$. It follows that

$$\lim_{T\to\infty} \frac{\sum_{t=1}^T \mathbf{1}[(m_t, d_{t+1}) \in X^+] \cdot (m_t+1)}{T} = \lim_{T\to\infty} \frac{\sum_{t=1}^T \mathbf{1}[(m_t, d_{t+1}) \in X^-] \cdot m_t}{T}$$

which can be rewritten as (1). $\square$

Step 3: *The average long-run m is at most 2μ (approximately)*

The long-run average of m induced by the trainer's strategy can be written as

$$\mathbb{E}(m) = \sum_{(m,d)\in X^+} \lambda(m,d)m + \sum_{(m,d)\in X^-} \lambda(m,d)m + \sum_{(m,d)\in X^0} \lambda(m,d)m \tag{2}$$

By the feasibility constraint,

$$\sum_{(m,d)\in X^+} \lambda(m,d)d + \sum_{(m,d)\in X^-} \lambda(m,d)d + \sum_{(m,d)\in X^0} \lambda(m,d)d \lessapprox \mu$$

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By definition, $d \geq m+1$ for every $(m,d) \in X^+$, $d \geq 0$ for every $(m,d) \in X^-$, and $d = m$ for every $(m,d) \in X^0$. Therefore,

$$\sum_{(m,d)\in X^+} \lambda(m,d)(m+1) + \sum_{(m,d)\in X^-} \lambda(m,d) \cdot 0 + \sum_{(m,d)\in X^0} \lambda(m,d)m \lessapprox \mu$$

This means that

$$\sum_{(m,d)\in X^+} \lambda(m,d)m \leq \sum_{(m,d)\in X^+} \lambda(m,d)(m+1) \lessapprox \mu - \sum_{(m,d)\in X^0} \lambda(m,d)m$$

By (1), it follows that

$$\sum_{(m,d)\in X^-} \lambda(m,d)m \lessapprox \mu - \sum_{(m,d)\in X^0} \lambda(m,d)m$$

as well. Plugging the last two inequalities in (2), we obtain

$$E(m) \lessapprox 2\mu - \sum_{(m,d)\in X^0} \lambda(m,d)m \leq 2\mu$$

□

Step 4: *The minimal long-run m is at most $2\mu - 1$*

Suppose the long-run distribution over d is degenerate at some d^*. Therefore, $d^* \lessapprox \mu$. The agent's myopic best-reply implies that eventually, his mass coincides with d^*. It follows that to reach a minimal long-run mass above μ, the long-run distribution over d must assign positive probability to at least two values. This means there are infinitely many periods t in which $d_t \neq m_{t-1}$. By myopic best-replying, this precludes the possibility that the long-run distribution over m is degenerate. Since the long-run average of m cannot exceed 2μ by more than an infinitesimal amount, there must be infinitely many periods t in which $m_t \leq 2\mu - 1$. This completes the proof of part (i). □

Proof of part (ii) **of Proposition 1**

Consider the trainer's strategy described in part (ii) of the statement of the result. As long as $\beta \in (0,1)$, the Markov process over m that is induced by the strategy and the agent's best-reply (given by Step 1) has a unique invariant distribution, with $m = 2\mu$ and $m = 2\mu - 1$ being the only recurrent mass values. The reason is that if $m_t > 2\mu$, $m_{t+1} = m_t - 1$ with certainty; if $m_t < 2\mu - 1$, there is a positive probability that there will be a streak of realizations $d = 2\mu$ such that m will keep adjusting upward until it reaches $m = 2\mu$; and finally, if $d_t = 0$ then $d_{t+1} = 2\mu$ for sure, which means that once m hits 2μ and later goes down to $2\mu - 1$, it will return to 2μ immediately in the next period. As the exogenous upper bound on average intensity gets arbitrarily close to μ, β can be made arbitrarily close to one. In the $\beta \to 1$ limit, the invariant distribution over m assigns probability $\frac{1}{2}$ to each of the values $m = 2\mu$ and $m = 2\mu - 1$. ∎

4   A Patient Agent

In this section we characterize the solution to the trainer's problem when the agent is forward-looking and arbitrarily patient. For expositional convenience, we assume μ/c is an integer.

Proposition 2 *Let δ be arbitrarily close to 1. Then:*

(i) The minimal long-run mass at the solution to the trainer's problem is at most $\mu/c - 1$.

(ii) This upper bound can be implemented by (P, f) with the following properties. The Markov process P has two states, H and L, and a transition matrix given by

$\Pr(s_t \to s_{t+1})$	L	H
L	$1 - \alpha$	α
H	β	$1 - \beta$

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where $\alpha = 1$ if $c \geq \frac{1}{2}$, $\beta = 1$ if $c < \frac{1}{2}$, and $\alpha/(\alpha + \beta)$ is arbitrarily close to c from above. The output function is $f(H) = \mu/c$ and $f(L) = 0$. In the $\alpha/(\alpha + \beta) \to c$ limit, the invariant mass distribution assigns probability c to $m = \mu/c$ and probability $1 - c$ to $m = \mu/c - 1$.

The upper bound on the agent's minimal long-run mass is higher than in the myopic benchmark whenever $c < \frac{1}{2}$. Moreover, it gets arbitrarily high when $c \to 0$. As c gets closer to one, the highest minimal long-run mass approaches the flexible-agent benchmark μ.[4]

The Markov process that attains the upper bound is similar to the one in Section 3. The main difference is that persistence of one of the two states occurs with non-vanishing probability. When $c < \frac{1}{2}$, a "rest period" (corresponding to the state L) is followed by another one with probability approximately equal to $(1 - 2c)/(1 - c)$. When $c > \frac{1}{2}$, a high-intensity period (corresponding to the state H) is followed by another one with probability $(2c - 1)/c$.

Compare this with Section 3. The myopic agent only responds to current realizations of d. In contrast, the patient agent reacts to the trainer's entire continuation strategy. When $c < \frac{1}{2}$, the trainer's program allows for a streak of $d = 0$ realizations. When this happens, the agent does not lower his mass below $\mu/c - 1$ because he takes into account the future loss $d - m$ in the event that d switches to $d = \mu/c$. The trainer designs the transition probabilities such that the patient agent's intertemporal trade-offs lead him to be nearly indifferent between lowering his mass and remaining at $m = \mu/c - 1$. In contrast, the myopic agent cannot be made indifferent when faced with a streak of $d = 0$ realizations: he repeatedly lowers his mass. This difference enables the trainer to achieve a higher minimal long-run mass when the agent is patient, as long as $c < \frac{1}{2}$.

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[4]Because μ/c is an integer, we rule out the possibility that c is arbitrarily close to one. In that case, the trainer cannot outperform the flexible-agent benchmark of μ.

We now turn to the proof of Proposition 2. In our proof of part (i), we actually prove a somewhat stronger result: in order to attain a strictly positive minimal long-run mass, the *average* long-run mass cannot exceed $\mu/c - 1 + c$. The Markov process we construct in part (ii) approximates this upper bound. This means that among all trainer strategies that attain the minimal long-run mass of $\mu/c - 1$, this process cannot be outperformed in terms of average mass.

Proof of part (i) of Proposition 2

Let p be the unique invariant distribution over (d_t, m_t) that results from the trainer's strategy and the agent's best-replying strategy. (Note the different time subscripts of d and m, compared with the proof of Proposition 1 in Section 3; our different notation highlights this difference.) We abuse notation and write $p(d)$, $p(m)$ and $p(d \mid m)$ to represent marginal and conditional distributions induced by p. As in the myopic-agent case, we first derive an upper bound on the expected mass according to p, which we use to derive the upper bound on the minimal long-run mass. Then, we show how to implement this upper bound.

In Section 2, we saw that the trainer can implement a minimal long-run mass of at least μ (by playing $d = \mu$ at every period). Therefore, we take it for granted that the minimal value of m in the support of p is at least $\mu \geq 1$.

Step 1: $p(d > 0) \geq c$

Consider the following deviation by the agent. Pick some period-t history for which $m_{t-1} \geq 1$ is at the lowest value according to p. Therefore, $m_t = m \in \{m_{t-1}, m_{t-1} + 1\}$. At this history, the agent deviates to $m_t' = m - 1$. Subsequently, the agent behaves according to his original strategy *as if the deviation did not occur.*

This deviating strategy induces an invariant distribution p' such that for every (d, m) in the support of p, $p'(d, m - 1) = p(d, m)$. Therefore, the deviation saves c at every period, but raises costs by one unit per period whenever $d \geq m$ under the original strategy. In order for this deviation

to be unprofitable for an arbitrarily patient agent, it must be the case that $p(d \geq m) \geq c$. Since $m > 0$ with probability one, $p(d > 0) \geq p(d \geq m)$, hence $p(d > 0) \geq c$. $\square$

Step 2: *The expectation of m according to p is at most $\mu/c - 1 + c$*

Assume the contrary. Then, the agent's average long-run cost exceeds

$$c \cdot [\frac{\mu}{c} - 1 + c] = \mu - c(1 - c)$$

Now consider a deviation to the following strategy. Descend from m_0 to $m = 0$, and then implement the following rule: $m_t = 0$ whenever $d_t = 0$, and $m_t = 1$ whenever $d_t > 0$. When the agent is arbitrarily patient, the average long-run cost from this strategy is approximately

$$\begin{aligned} p(d &= 0) \cdot 0 + p(d > 0) \cdot [c + \sum_{d>0} p(d \mid d > 0)d - 1] \\ &\lessapprox p(d > 0)(c - 1) + \mu \end{aligned}$$

Since $c < 1$, Step 1 implies that

$$p(d > 0)(c - 1) + \mu < \mu - c(1 - c)$$

such that the deviation is profitable, a contradiction. $\square$

Step 3: *The minimal long-run mass is at most $\mu/c - 1$*

Since μ/c is an integer, $\mu/c - 1 + c$ is not an integer. Hence, in order for the average long-run cost to be weakly below $\mu/c - 1 + c$, the minimal long-run mass cannot exceed $\mu/c - 1$.[5] $\square$

Proof of part (ii) of Proposition 2

Consider the strategy described in the statement of part (ii). Our objective is to show that given this strategy, there is a best-reply for the agent such that

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[5] The proof of this step utilizes the convenient assumption that μ/c is an integer. An alternative proof that does not rely on this assumption is analogous to Step 4 in the proof of Proposition 1.

for every sufficiently high t, $m_t = \mu/c$ whenever $s_t = H$ and $m_t = \mu/c - 1$ whenever $s_t = L$.

Since the agent faces a Markovian decision problem with an extended state space (s, m), there exists a best-reply that is Markovian with respect to this state space. To derive such a best reply, we proceed in four steps.

Step 1: *There is no best-reply in which the invariant distribution assigns probability one to a single m.*

Proof. Assume the contrary. If $m < \mu/c$, then it is profitable for the agent to deviate to a strategy that plays $m + 1$ whenever $s = H$ and m whenever $s = L$. Likewise, if $m > 0$, it is profitable for the agent to deviate to a strategy that plays m whenever $s = H$ and $m - 1$ whenever $s = L$. $\square$

Step 2: *The set of recurrent values of m (according to the unique invariant distribution induced by the two parties' strategies) is a set of consecutive numbers $\underline{m}, \underline{m} + 1, ..., \overline{m}$, where $\overline{m} \leq \mu/c$.*

Proof. The agent's sluggishness implies that if the agent visits two non-adjacent masses m and m', then he must also visit every m'' between them. Therefore, if m and m' are recurrent, so is m''. Suppose $\overline{m} > \mu/c$. Then, there is a profitable deviation for the agent that instructs him to remain at $\overline{m} - 1$ whenever the original strategy instructs him to switch to $\overline{m}$. $\square$

Step 3: *There is a best-reply that induces an invariant distribution that assigns positive probability to exactly two values of m.*

Proof. Consider the invariant distribution over (d, m) induced by the trainer's strategy and the agent's best-reply. By Step 1, $\overline{m} - \underline{m} \geq 1$. If $\overline{m} - \underline{m} = 1$, we are done. Therefore, assume $\overline{m} - \underline{m} > 1$. There are two cases to consider.

First, let $\alpha = 1$ (this fits the case of $c \geq 1/2$). This means that whenever $s = L$, the state switches immediately to $s = H$ in the next period. Consider the top two values of m in the invariant distribution, namely $\overline{m}$ and $\overline{m} - 1$. By Step 2, $\overline{m} \leq \mu/c$. Moreover, when $s = L$ (at which d attains its lowest value according to the trainer's strategy), the agent strictly prefers $\overline{m} - 1$ to $\overline{m}$. Consider some t for which $m_t = \overline{m}$ (there are infinitely such periods because

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$\overline{m}$ is recurrent). If $s_{t+1} = L$, the agent necessarily switches to $m_{t+1} = \overline{m} - 1$. If, on the other hand, $s_{t+1} = H$, we need to consider two possibilities.

- Suppose that when $s_{t+1} = H$, it is not optimal for the agent to play $m_{t+1} = \overline{m}$. That is, the agent switches from $m_t = \overline{m}$ to $m_{t+1} = \overline{m} - 1$ for *any* realization of s_{t+1}. But this also means that if $m_{t'} = \overline{m} - 1$ at some period t' and $s_{t'+1} = H$, it cannot be optimal for the agent to switch to $m_{t'+1} = \overline{m}$. The reason is that by revealed preference, the agent prefers being at $\overline{m} - 1$ to being at $\overline{m}$ when the state is H. And since we already saw that the agent prefers being at $\overline{m} - 1$ to being at $\overline{m}$ when the state is L, this means that the agent will *never* switch from $\overline{m} - 1$ to $\overline{m}$, contradicting the definition of $\overline{m}$ as a recurrent state.

- Suppose that when $s_{t+1} = H$, it is optimal for the agent to play $m_{t+1} = \overline{m}$. This reveals a weak preference for $\overline{m}$ over $\overline{m} - 1$ when the state is H. Therefore, there is a best-reply for the agent that prescribes $m_{t+1} = \overline{m}$ whenever the extended state (s_{t+1}, m_t) is $(H, \overline{m} - 1)$ or $(H, \overline{m})$. We already saw that when the extended state is $(L, \overline{m})$, the agent switches to $\overline{m} - 1$. Since $\alpha = 1$, this means that we have constructed a best-reply for the agent such that once he reaches $\overline{m}$, he will only visit $\overline{m}$ and $\overline{m} - 1$ from that period on, contradicting the assumption that there are additional recurrent values of m.

Thus, we have ruled out the possibility that $\overline{m} - \underline{m} > 1$ when $\alpha = 1$. Now suppose $\beta = 1$ (this fits the case of $c \leq 1/2$). An analogous argument establishes that there is a best-reply for the agent that induces an invariant distribution with only two recurrent mass values, $\underline{m}$ and $\underline{m} + 1$.

It follows that we can restrict attention to strategies of the agent that induce an invariant distribution which assigns positive probability to precisely two consecutive mass values, m and $m - 1$, where $0 < m \leq \mu/c$. $\square$

Step 4: *There is a best-reply for the agent that induces an invariant distribution on the mass values μ/c and $\mu/c - 1$.*

Proof. Given Step 3, it is clearly optimal for the agent to be at m when $s = H$ and at $m - 1$ when $s = L$. In addition, when $m > \mu/c$ ($m < \mu/c - 1$), the agent clearly wants to move downward (upward).

The invariant distribution of the trainer's two-state Markov process assigns probability $\alpha/(\alpha + \beta)$ to state H and $\beta/(\alpha + \beta)$ to state L. Therefore, since the agent is arbitrarily patient, his long-run expected payoff is approximately

$$-\frac{\alpha}{\alpha + \beta} \cdot (cm + \frac{\mu}{c} - m) - \frac{\beta}{\alpha + \beta} \cdot c(m - 1)$$

It is now easy to see that given that $\alpha/(\alpha + \beta) > c$, this expression increases with m, such that the optimal value of m is μ/c. The expected value of m according to this strategy is

$$\frac{\alpha}{\alpha + \beta} \cdot \frac{\mu}{c} + \frac{\beta}{\alpha + \beta} \cdot (\frac{\mu}{c} - 1)$$

which is arbitrarily close to the upper bound. ∎

5   Discussion

In this section we discuss two features of our model.

5.1   The Importance of Randomization

Randomization is a feature of the optimal training strategy in our model. In the myopic-agent case, it ensures a unique invariant mass distribution. Randomization plays a different role in the patient-agent case. In particular, when $c < \frac{1}{2}$, a rest period is followed by another rest period with positive probability. Is this randomization necessary? Or can the same long-run mass be sustained by a deterministic strategy with the same long-run distribution

over d? The following example illustrates that the answer is negative.

Suppose $\mu = 4$ while c is slightly below $\frac{4}{11}$. Then, the optimal training strategy we presented in Proposition 2 induces an invariant distribution that assigns probability $\frac{4}{11}$ to $d = 11$ and probability $\frac{7}{11}$ to $d = 0$. The strategy sustains a minimal long-run mass level of $m = 10$.

Now consider a deterministic strategy that induces the same long-run frequencies of d. The strategy follows an 11-period cycle consisting of four consecutive periods of $d = 11$ and seven consecutive periods of $d = 0$. If the agent plays $m = 11$ when $d = 11$ and $m = 10$ when $d = 0$ - as he does against the strategy presented in Proposition 2 - the minimal long-run mass is $m = 10$. However, given the predictable evolution of d under the cyclic deterministic strategy, an agent with $\delta \to 1$ can do better. Suppose that he plays the following sequence of m against the cyclic sequence of d:

$$
\begin{array}{ccccccccccc}
d & 11 & 11 & 11 & 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
m & 11 & 11 & 11 & 10 & 9 & 8 & 7 & 7 & 8 & 9 & 10
\end{array}
$$

Compared with the benchmark strategy of playing $m = 11$ (10) against $d = 11$ (0), the agent saves approximately

$$
c \cdot (1 + 1 + 2 + 3 + 3 + 2 + 1) - 1 \approx \frac{41}{11}
$$

per cycle. Even if this is not a best-reply to the cyclic deterministic strategy, it means that any best-reply will lead to a minimal long-run mass below $m = 10$.

This example highlights a key role of the stochasticity of the trainer's optimal strategy. The fact that there is always a chance that the agent will be required to exert high effort following a rest period incentivizes the agent not to lower his mass. In contrast, the predictable nature of the cyclic deterministic strategy allows the agent to gradually lower his mass and gain it back by the time he needs to exert effort. In particular, it is profitable for the

agent to lower his mass already in the final period of the high-intensity phase of the cycle, even though this involves a costly performance gap, because this is more than offset by the cumulative maintenance-cost saving over the cycle. This is reminiscent of the phenomenon known as "overtraining", where an individual's performance begins to deteriorate during the high-intensity phase of a periodization strategy (see Cadegiani and Kater (2019)). The optimal stochastic strategy avoids this effect.

5.2 The Trainer's max-min Criterion

In our model, the trainer's objective is to maximize the agent's minimal long-run mass. Alternatively, we could use the long-run *average* mass as a criterion. However, this criterion is less attractive in our context because it does not reflect the idea of "preparedness" - namely, that the body should be able to perform at a *consistently* high level. In particular, the average criterion allows zero to be a recurrent value for the agent's mass (and consequently, his level of preparedness).

A by-product of our analysis in Section 3 is that in the myopic-agent case, 2μ is an upper bound on the average long-run mass that the trainer can attain. It can be shown that this upper bound can be approximated arbitrarily well, but this must come at the price of arbitrarily long recurrent stretches of $m_t = 0$ realizations (which are compensated for by periods in which m_t reaches arbitrarily high values). Obviously, such paths imply that the agent cannot consistently meet positive challenge levels. By comparison, the process we constructed in Section 3 induces an average long-run mass of approximately $2\mu - \frac{1}{2}$ and a minimal long-run mass of $2\mu - 1$.

A similar diagnosis pertains to the patient-agent case (we treat μ as a precise upper bound on average intensity, for the sake of the argument). An upper bound on the average long-run mass is μ/c. The reason is that if average mass exceeds this value, it implies that the agent's average long-run cost is above μ. However, the agent can ensure an average cost of μ by always

playing $m = 0$, hence a long-run mass in excess of μ/c is inconsistent with the agent's best-replying. We believe that as in the myopic-agent case, this upper bound can be approximated arbitrarily well. However, as in the myopic-agent case, recurrent stretches of $m_t = 0$ realizations are necessary for this - which, once again, fails the max-min criterion miserably. By comparison, the process we constructed in Section 4 induces an average long-run mass of approximately $\mu/c - 1 + c$, and a minimal long-run mass of $\mu/c - 1$.

6   Conclusion

In this paper we presented a theoretical approach to the subject of exercise physiology, based on the view of the human body as a forward-looking optimizing agent which is nevertheless constrained by sluggish adjustment. We saw that this very sluggishness is actually a boon to physical trainers: using a stochastic training strategy that resembles popular "periodization" techniques, the trainer can achieve a significantly higher long-run muscle mass than if the body could instantaneously adjust its mass to physical stress.

Our analysis focused on the two polar cases of $\delta = 0$ and $\delta \to 1$. While the optimal strategy is similar in the two cases, the logic that sustains them is different. Therefore, finding the optimal strategy for arbitrary $\delta \in (0,1)$ remains an open problem.

We believe that thanks to its abstraction, our modeling approach can be extended to related problems, such as the optimal design of dynamic dieting regimes. A model that describes the body's metabolism as a consequence of dynamic *sluggish* optimization with rational expectations may shed light on prevalent dieting programs such as carb cycles. We hope to pursue this approach before the next pandemic.

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