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Search, Dating, and Segregation in Marriage

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Search, Dating, and Segregation in Marriage*

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Abstract

We study statistical discrimination in a marriage market where agents, characterized by attractiveness (e.g., wealth, education) and background (e.g., race, ethnicity), engage in time-consuming search. Upon meeting, couples date to learn about their match's quality. Following Phelps (1972), different backgrounds impede such learning. We show that even absent any bias, equilibrium features segregation. Moreover, welfare improvements enhance segregation. In particular, radical improvements in search technologies induce complete segregation and a "dating apocalypse" where agents replace partners frequently. We show that, in line with empirical findings, segregation is decreasing in couples' attractiveness, and provide conditions for (probabilistic) positive sorting by attractiveness.

Keywords: Statistical Discrimination, Segregation, Dating, Matching with search frictions, Learning.

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1 Introduction

In 1967, *Loving v. Virginia* 388 U.S. 1, the Supreme Court overturned remaining anti-miscegenation laws, legalizing interracial marriage throughout the U.S. Since then, the approval rate of intermarriage – marriage among partners from different racial or ethnic backgrounds – in the U.S. have soared (according to a recent survey by Gallup Polls, the interracial approval rate stands at 94%; see McCarthy, 2021). Moreover, since the beginning of the 21st century, dating apps have gradually displaced the roles that family and friends once played in bringing couples together (Rosenfeld, Thomas and Hausen, 2019), making it easier to meet partners from different backgrounds (see, e.g., Lewis, 2013). Yet, the actual rate of intermarriage among newlyweds remains surprisingly low – around 19% according to the American Community Survey, 2020.[1] Moreover, the increase in the observed rate of intermarriage is consistent with demographic changes alone: Figure 1 shows the dynamics of the actual rate of intermarriage, and the rate that would be observed in a counterfactual world in which all newlyweds in a given year are matched randomly.

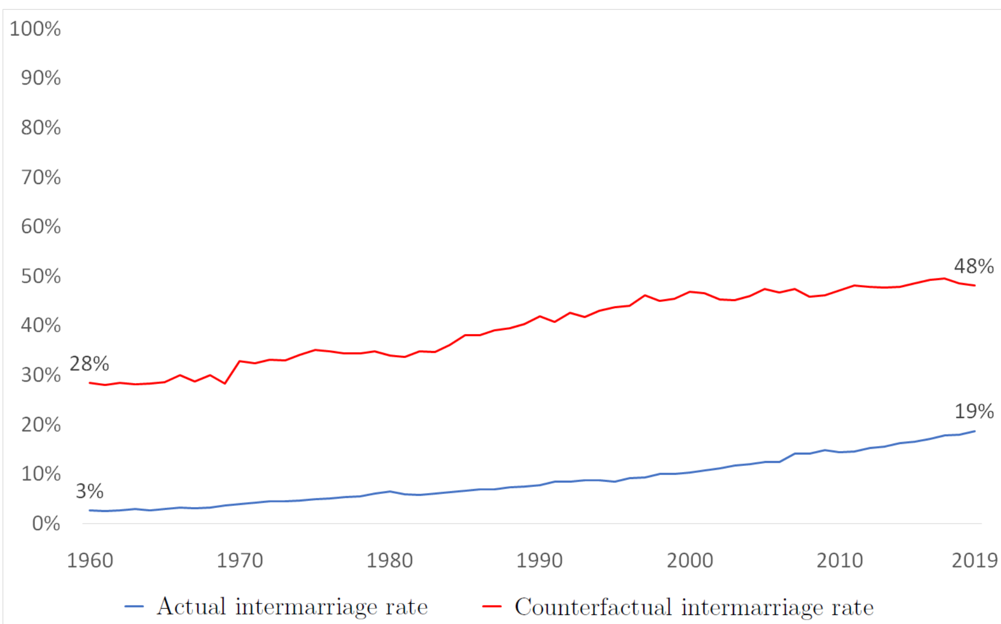

Figure 1: Actual vs. counterfactual intermarriage rates in the U.S.

Despite widespread approval of intermarriage, people remain far more likely to marry partners who share the same racial or ethnic background. This begs the question: with the

[1]This rate includes both interracial and interethnic marriages. Similarly, 18% of cohabitants had a partner of different race or ethnicity; see Livingston and Brown (2017). See Fryer (2007) for an analysis of interracial marriage using census data from 1880–2000.

approval rate of intermarriage so high, and with online dating technologies easily connecting people across backgrounds, why does the intermarriage rate remain so low?

One possible explanation for such segregation in marriage is the presence of bias (implicit or explicit). We show that *even in the absence of any form of bias*, segregation arises when agents search, date, and marry optimally; in particular, segregation arises when agents optimally choose how much time to invest in evaluating the prospects of each potential spouse that they meet. Moreover, the segregation patterns that emerge from such behavior are consistent with the empirical ones.

The key assumption underlying our analysis is that the evaluation of the quality of a match with a potential partner from a different background (race, ethnicity) is noisier than the evaluation of a partner with whom one shares the same background. This assumption is based on the central feature in the literature on statistical discrimination in the tradition of Phelps (1972).[2] Various explanations for such differences in evaluation have been proposed in the literature. Lang (1986) suggests that differences in "language" (verbal and nonverbal) impede communication between people from different backgrounds. Similarly, Arrow (1972) and Cornell and Welch (1996) propose that differences in background may hinder the assessment of intangible but relevant qualities. Aigner and Cain (1977) suggest that, in the eyes of each individual, members of their own "group" face a more homogeneous set of environmental determinants of quality, resulting in lower variation and hence less noisy evaluation.

This paper studies steady-state equilibria of a marriage market with nontransferable utility where agents – who are characterized by their attractiveness (e.g., wealth, education) and background (e.g., race, ethnicity) – spend time not only searching for partners, but also evaluating the quality of the match with each partner that they meet. In particular, we develop a model of matching with both search and learning frictions in which, contrary to standard assumptions in the existing literature, potential partners do not immediately learn the value of their match upon meeting, and need not immediately (and irreversibly) decide whether to accept or reject a prospective match. Rather, partners who meet may decide to first spend time learning more about their match's prospects. Introducing such learning into the classic matching-with-search-frictions literature (see Chade, Eeckhout and Smith, 2017, for a comprehensive review of this literature) allows us to incorporate Phelps' approach into this framework.

Our main results concern segregation in marriage. We show that segregation arises in

[2]Statistical discrimination theories explain group inequality without assuming preference bias or various forms of prejudice. This is in contrast to "taste-based" theories of discrimination (e.g., Becker, 2010).

any steady-state equilibrium, and that there is a tension between Pareto efficiency and segregation: if an equilibrium E Pareto dominates another equilibrium E', it must also exhibit higher segregation. Thus, any policy aimed at improving the welfare of all market participants inevitably results in further segregation. Furthermore, we find that advances in search technologies that allow people to meet many potential partners in a short span of time can backfire when it comes to segregation in marriage. In particular, as search frictions vanish, the market becomes fully segregated: there is no marriage between agents from different backgrounds.

Our model yields predictions about equilibrium sorting patterns along the dimensions of background and attractiveness. First, even though there is segregation (i.e., sorting along the dimension of background) at all attractiveness levels, segregation rates are lower for more attractive agents than for less attractive ones; i.e., less attractive agents are more likely to marry agents from their own background. Interpreting attractiveness as years of schooling, this prediction is in line with the American Community Survey – see Figure 3 below – which shows that segregation in marriage is less common among the more educated (see also Fryer, 2007). Second, complementarities in partners' attractiveness result in a probabilistic form of positive assortative matching: highly attractive agents are more likely to marry other highly attractive agents, but on occasion may marry less attractive ones as well. This prediction is also in line with data from the American Community Survey; see Figure 5 below. Finally, we show that even in the presence of such complementarities, positive assortative matching (by attractiveness) need not be efficient from a social perspective: a social planner may find it optimal to induce *negative* assortative matching along the dimension of agents' attractiveness, while maintaining segregation.

Beyond sorting and segregation patterns, our analysis provides a possible explanation for the phenomenon often referred to as the "dating apocalypse" (see, e.g., Sales, 2020), where the growing ease with which people are able to find dating partners through modern dating apps makes it more difficult to establish long-term relationships. In our model, as search frictions vanish, the average number of partners that agents date before marrying goes to infinity, and the amount of time they dedicate to each potential partner goes to zero.

The contribution of this paper is twofold. At the applied level, we study the connection between search, dating, and segregation patterns in the marriage market. In addition to contributing to the statistical discrimination literature, at the methodological level, this paper develops a novel tractable framework that incorporates pre-match information acquisition into the matching-with-search-frictions paradigm.

Related literature

This paper contributes to the matching-with-search-frictions literature by introducing pre-match information acquisition. This extensive literature explores the properties of equilibrium matching under various assumptions on the search technology, match payoffs, search costs, the ability to transfer utility, and agents' rationality.[3]

While at the methodological level our paper is most closely related to Smith (2006), at the applied level it is closely related to Eeckhout (2006). Eeckhout (2006) considers a marriage market where agents repeatedly search to form a partnership in which they choose to cooperate or defect, and then decide whether to continue or separate. Agents are homogeneous except for a payoff-irrelevant characteristic, color. Eeckhout shows that while color-blind equilibria exist, they are inferior to equilibria in which there is segregation between colors, and that minority bias can increase welfare.[4] The driving force behind his results is that color can be used as a public randomization device that creates better incentives for long-term cooperation. By contrast, in our model, segregation is driven by differences in the effectiveness of learning about partners (Phelps, 1972).[5] As a result, in our model equilibria always exhibit segregation. Moreover, since in our model agents are heterogeneous, we can also explore questions such as (i) how the level of segregation changes with agents' attractiveness (e.g., wealth, education), and (ii) which sorting patterns arise in equilibrium.

Mailath, Samuelson and Shaked (2000) study statistical discrimination in a model of the labor market with two-sided directed search. In their model, workers are either "red" or "green" and color is payoff irrelevant. They show that besides the color-blind equilibrium, there exists an asymmetric equilibrium in which firms search only for green workers, green workers invest more in acquiring skills, skilled green workers receive a higher wage, and skilled red workers suffer from a higher unemployment rate. More recently, Gu and Norman

[3]See, e.g., McNamara and Collins (1990), Smith (1992), Morgan (1996), Burdett and Coles (1997), Eeckhout (1999), Bloch and Ryder (2000), Shimer and Smith (2000), Chade (2001, 2006), Adachi (2003), Atakan (2006), Smith (2006), Lauermann and Nöldeke (2014), Bonneton and Sandmann (2021,2022*b*), Coles and Francesconi (2019), Lauermann, Nöldeke and Tröger (2020), and Antler and Bachi (2022).

[4]These findings bear a resemblance to those of Norman (2003), who shows – in the context of a labor market without search frictions – that statistical discrimination may be efficient.

[5]Following Phelps, statistical discrimination due to noise in evaluation has been studied in many settings. See, e.g., Aigner and Cain (1977), Borjas and Goldberg (1978), Lundberg and Startz (1983), Cornell and Welch (1996), and more recently Bohren, Imas and Rosenberg (2019), Bardhi, Guo and Strulovici (2020), Chambers and Echenique (2021), and Fershtman and Pavan (2021). By contrast, Arrow's (1973) theory of statistical discrimination does not assume differences in evaluation across groups, but instead relies on coordination failures; see, e.g., Coate and Loury (1993) and Moro and Norman (2004). See Fang and Moro (2011) for a comprehensive review of the theoretical statistical discrimination literature.

(2020) show that statistical discrimination can arise in labor markets due to a combination of occupational choice, search externalities, and a signal extraction problem. Moreover, they show that nondiscriminatory equilibria may become unstable due to the introduction of group characteristics, which may rationalize the use of affirmative action policies.

Our paper is not the first to incorporate incomplete information into the matching-with-search-frictions literature. Chade (2006) assumes that agents receive only a noisy signal about the payoff from marrying a potential partner before making an irreversible decision whether or not to marry that partner. This leads to an *acceptance curse*: the prospects of a marriage with a partner, conditional on the latter agreeing to the marriage, are lower than the unconditional prospects of such a marriage. Information frictions have also been incorporated into search-and-matching models in the related context of the labor market.[6] Building on Jovanovic (1984), Moscarini (2005) develops a theory of job turnover and wage dynamics in which employers and workers make inferences about the productivity of their match from the produced output. Besides the different context and questions studied, the main difference between Moscarini's (2005) model and ours is that, in the former, new matches are always accepted (workers and firms are symmetric ex ante) and learning occurs entirely ex post, whereas we focus on pre-match learning as it is central to the marriage market context.

2 A Model

We consider a marriage market with nontransferable utility. There is a set of men and a set of women, each containing a unit mass of agents. Each agent is characterized by two observable characteristics: her/his attractiveness $\omega \in [0, 1]$ (which represents attributes such as wealth or education) and background $\theta \in \{A, B\}$ (which represents attributes such as race, ethnicity, and religion). We denote an agent's type by $x = (\omega, \theta) \in X \equiv [0, 1] \times \{A, B\}$, and often identify an agent by her/his type. Following standard vector notation we denote agent x's attractiveness and background by x_ω and x_θ, respectively. To focus on the role of learning frictions across (racial/ethnic) groups, we assume that the distribution of types on both sides of the market is the same, and that groups are symmetric, i.e., both the size

[6] Information frictions have been incorporated into decentralized matching models in other contexts as well. For example, Lauermann and Wolinsky (2016), Lauermann, Merzyn and Virág (2018), and Mauring (2017) study two-sided search models in which buyers and/or sellers make inferences about an *aggregate state* from the terms of trade they encounter. Anderson and Smith (2010) show that information frictions can upset equilibrium sorting in a model without search frictions. In their model, agents choose a partner not only to maximize their current production, but also to signal their productivity to future partners.

and distribution of types are identical in both groups. Furthermore, we assume that the distribution of ω in the population admits a continuous density $g(\omega)$ that is bounded in $[\underline{g}, \overline{g}]$ for some with $0 < \underline{g} < \overline{g} < \infty$.

Any pair of potential partners are either compatible or not. We assume that the prior probability that agents x and y are compatible depends only on their attractiveness, and denote this probability by $q_0(x_\omega, y_\omega)$. Furthermore, $q_0(\cdot,\cdot)$ is strictly increasing in each of its arguments, symmetric, differentiable, and has derivatives that are bounded in $[\underline{q}, \overline{q}]$ for some $0 < \underline{q} < \overline{q} < \infty$. The compatibility (or lack thereof) of a couple determines whether their marriage would be *happy* or *unhappy*. The flow payoff each agent receives while in a happy marriage is normalized to 1, whereas the flow payoff from an unhappy marriage is $-z < 0$. We assume that $q_0(1,1) - (1 - q_0(1,1))z < 0$, which implies that (i) every couple are incompatible with positive probability, and that (ii) no couple marry without first receiving (positive) information about their compatibility.

The market operates in continuous time and agents discount the future at rate $r > 0$. Agents transition between three states: singlehood, dating, and marriage. While an agent is single, s/he meets singles on the other side of the market according to a quadratic search technology with parameter $\mu > 0$. That is, for any subset $Y \subseteq X$, if the measure of agents in the singles pool with types in Y is ν, then agent x meets such agents at a rate of $\mu\nu$.

When a pair of potential partners meet, they immediately observe each other's types. However, they do not observe whether they are compatible or not. Upon meeting, they can either begin dating (to learn about their compatibility) or reject the match and return to the singles market. At any point in time while a couple are dating, each agent can unilaterally break up with her/his partner. Following a breakup, both agents immediately return to the singles market to search for new potential partners. Dating is exclusive; that is, while a couple are dating they do not meet other potential partners.[7] Agents incur a small flow cost $c > 0$ while dating.

While dating, couples learn about their compatibility as follows. If x and y are compatible, they observe a positive signal that arrives at rate $\lambda_{xy} > 0$ according to an exponential distribution, whereas if they are incompatible, such a signal never arrives.[8] Let $\lambda_{xy} = \lambda$ if x and y share the same background, and $\lambda_{xy} = \beta\lambda$ otherwise. As discussed in the Introduction, we follow Phelps (1972) and make the following assumption.

[7]Relaxing the assumption that dating is mutually exclusive (e.g., assuming that while dating agents meet potential partners at a lower rate) does not affect our qualitative results.

[8]This learning technology is borrowed from the strategic experimentation literature (e.g., Keller, Rady and Cripps, 2005).

Assumption 1 (Inter-background noise) *The rate of learning λ_{xy} is slower when x and y have different backgrounds; that is, $\beta < 1$.*

Upon the arrival of a signal, a couple infer that they are compatible and marry immediately. In the absence of a signal, the couple gradually become more pessimistic about their compatibility until one of the partners chooses to separate. We denote by $q_t(x, y)$ the couple's joint belief about their compatibility after having dated for $t > 0$ units of time without observing a signal.[9]

We assume that each marriage is dissolved at a flow rate of $\delta > 0$. Such a dissolution can be interpreted as the arrival of bad news about attributes that cannot be revealed prior to marriage (e.g., a spouse's parenting style) that can turn a happy marriage into an unhappy one. When dissolution occurs, both agents return to the singles market. Finally, we assume that no agent is entirely excluded from the market, i.e., that $c(r + \delta) < \beta\lambda q_0(0, 0)$.

We analyze the steady state of the model. In a steady-state equilibrium, (i) the agents' decisions – whether and for how long to date each potential partner – are optimal given the endogenous composition of the singles pool, and (ii) the flows of individuals into and out of each of the three states – singlehood, dating, and marriage – are balanced. That is, the distributions of singles, dating couples, and married couples are stationary. In Section 5, we prove that such an equilibrium exists.

The (stationary) strategy of each agent is a function that specifies the maximal amount of time for which s/he is willing to date each potential partner. Thus, agent x's strategy is a mapping $T_x : X \to \mathbb{R}_+$, where $T_x(y)$ is the maximal amount of time that agent x is willing to date an agent of type y. Note that by setting $T_x(y) = 0$, agent x effectively rejects agent y immediately. As dating requires mutual consent, after agents x and y meet, they date for at most $\min\{T_x(y), T_y(x)\}$ units of time: if they learn that they are compatible beforehand, they marry, and otherwise they separate after dating for $\min\{T_x(y), T_y(x)\}$ units of time. We focus on symmetric strategy profiles, i.e., profiles in which strategies are symmetric with respect to gender and background.

Agents' dating strategies determine the probability that a couple $\langle x, y\rangle$ marry conditional on meeting one another. This probability, which we refer to as a couple's *conversion rate*, is denoted by $\alpha(x, y)$ and will be useful throughout the analysis.

Balanced Flow. We now derive the *balanced-flow condition*, which guarantees that the distributions of singles, dating couples, and married couples are stationary.

[9]The interim belief $q_t(\cdot, \cdot)$ is a function of both attractiveness and background, as the former affects the prior and the latter affects the learning rate. By contrast, the prior $q_0(\cdot, \cdot)$ is only a function of attractiveness.

At each point in time, the outflow of agents of type x from the singles pool is given by the measure of such agents who meet a partner y such that both x and y are willing to date one another for a positive amount of time. Note that agents date for a positive amount of time if and only if they marry with positive probability. Thus, the outflow of type-x agents from the singles pool is given by

$$\mu u(x) \int_{\{y:\alpha(x,y)>0\}} u(y)dy,$$

where $u(y)$ denotes the (steady-state) measure of agents of type y in the singles pool.

There are two circumstances in which a type-x agent returns to the singles pool. First, s/he may break up with an agent whom s/he has been dating (whether due to her/his own decision or due to that of her/his partner). The couples $\langle x, y\rangle$ that break up at a given point in time are those that met exactly $\min\{T_x(y), T_y(x)\}$ units of time ago and did not receive a signal while dating. The probability that such a couple did not receive a signal is $1-\alpha(x, y)$, and hence in a steady state the flow of agents of type x into the singles pool due to failed dating is given by

$$\mu u(x) \int_{\{y:\alpha(x,y)>0\}} u(y)(1-\alpha(x,y))dy.$$

Second, agent x may return to the singles pool due to the dissolution of her/his marriage. Denote by $d(x, y)$ the measure of type-x agents who are dating a type-y agent. The measure of type-x agents who are dating is then

$$d(x) = \int_{\{y:\alpha(x,y)>0\}} d(x,y)dy.$$

Thus, the measure of type-x agents who are married is $g(x) - u(x) - d(x)$, and the flow of such agents who return to the singles pool due to the dissolution of marriages is

$$\delta(g(x) - d(x) - u(x)).$$

By equating the inflow and outflow derived above and rearranging, we obtain that the flow of type-x agents into and out of singlehood is balanced if

$$\mu u(x) \int_{\{y:\alpha(x,y)>0\}} \alpha(x,y)u(y)dy = \delta(g(x) - d(x) - u(x)). \tag{1}$$

Note that the LHS of (1) is the inflow of type-x agents into marriage, and the RHS of (1) is the outflow of such agents from marriage. Hence, (1) guarantees that the flows into and out of marriage are also balanced. Thus, when (1) holds for every $x \in X$, the distributions of singles, dating couples, and married couples are all stationary.

Continuation values and capital gains. In the subsequent analysis we often make use of the continuation value of agent x when s/he is single. We denote this continuation utility by $W_s(x)$. In the literature (e.g., Smith, 2006), the flow of the continuation value of single agents represents their expected capital gain from meeting (and marrying) partners in their "acceptance set." In our model, it represents the capital gain from meeting (and beginning to date) potential partners. Formally,

$$rW_s(x) = \mu \int_X V_d(x; y) u(y) dy, \tag{2}$$

where $V_d(x; y)$ denotes agent x's capital gain from dating agent y.[10]

3 Dating and Segregation in Equilibrium

In our model, agents must choose not only which potential partners to date, but also how much time to spend dating each partner. To understand these choices, consider agent x's marginal value of dating agent y (i.e., the value of dating y for an additional dt units of time) after the couple have dated for $t \geq 0$ units of time. With probability $q_t(x, y)\lambda_{xy} dt$ the couple receive a signal and marry, in which case agent x enjoys the capital gain from a happy marriage. The flow value of a happy marriage is one, and thus the capital gain from a happy marriage is $(1 - rW_s(x))/(r + \delta)$. The marginal cost of dating consists of the flow cost of dating, c, and the flow value of singlehood, $rW_s(x)$, which must be forgone since dating is exclusive. Hence, agent x's marginal value of dating agent y is

$$\lambda_{xy} q_t(x, y) \frac{1 - rW_s(x)}{r + \delta} - (rW_s(x) + c). \tag{3}$$

Standard arguments show that $\dot{q}_t(x, y) = -\lambda_{xy} q_t(x, y)(1 - q_t(x, y))$, which implies that agent x's marginal value of dating agent y decreases over time.

Agent x's choice of whether or not to continue dating agent y is relevant only if the latter chooses to continue dating agent x. As in many other two-sided matching models, the

[10] Agent x's capital gain from dating agent y is given by evaluating Equation (A.1) (in the Appendix) at the equilibrium dating time of the two agents. We discuss optimal dating choices in detail in Section 3.

mutual consent requirement can sustain an equilibrium in which any two agents reject one another. To preclude this type of equilibrium, the matching-with-search-frictions literature typically assumes that an agent accepts any match whose benefit is strictly greater than the agent's reservation value. In this paper, we make the analogous assumption that agent x chooses to continue dating agent y as long as agent x's marginal value of dating agent y is positive.

The next result uses the above assumption and the fact that the marginal value of dating decreases over time to establish that agent x's optimal dating choices are characterized by a single threshold belief over her/his compatibility with the current partner, which is independent of the attractiveness of that partner.

Proposition 1 (Optimal dating) *Agent x's equilibrium behavior is characterized by a threshold belief*

$$q^\star(x) \equiv \frac{rW_s(x) + c}{1 - rW_s(x)} \times \frac{r + \delta}{\lambda}, \tag{4}$$

such that s/he chooses to continue dating agent y from the same (resp., from a different) background as long as $q_t(x, y) > q^\star(x)$ (resp., as long as $q_t(x, y) > q^\star(x)/\beta$).

Proposition 1 implies that agents are more selective about dating potential partners from a different background – both in terms of whom they are willing to begin dating in the first place and in terms of when they will break up with a partner they are dating. When an agent x meets a potential partner with attractiveness y_ω such that $q_0(x_\omega, y_\omega) \in (q^\star(x), q^\star(x)/\beta)$, agent x is willing to date her/him only if they share the same background. Furthermore, if an agent x meets an agent whom s/he would date regardless of the latter's background (i.e., $q_0(x_\omega, y_\omega) > q^\star(x)/\beta$), then there exist circumstances (i.e., $q_t(x, y) \in (q^\star(x), q^\star(x)/\beta)$) under which agent x will wish to continue dating the latter only if they share the same background.

Since dating requires mutual consent, it follows that within each couple, the agent with the higher breakup threshold is the one that ultimately chooses when to break up. Therefore, the *effective breakup threshold* of agents x and y is given by

$$Q^\star(x, y) = \frac{\max\{q^\star(x), q^\star(y)\}}{\lambda_{xy}/\lambda},$$

where $\lambda_{xy}/\lambda \in \{1, \beta\}$ is equal to 1 if the agents share the same background, and β otherwise.

3.1 Equilibrium Segregation and Social Welfare

Having characterized the agents' behavior at the individual level, we can now study the equilibrium of our model. In particular, we establish that segregation occurs in every equilibrium, and show that there is an inherent tension between social integration and social welfare.

We start by defining our notion of segregation. Roughly speaking, we say that there is segregation if the conversion rate of agents who share the same background is higher than the conversion rate of agents who have different backgrounds. Owing to the symmetry between groups, if this is true for agents in group A it is also true for agents in group B. For convenience, in our definitions of both segregation (Definition 1) and the level thereof (Definition 2) we use group A as the benchmark group.

Definition 1 (Segregation) *We say that there is segregation if for every two levels of attractiveness $\omega, \omega' \in [0, 1]$ it holds that*

$$\alpha\left((\omega, A), (\omega', A)\right) \geq \alpha\left((\omega, A), (\omega', B)\right) \tag{5}$$

with strict inequality if the LHS is strictly positive.

To examine whether segregation occurs in equilibrium, we first calculate the equilibrium conversion rates.

Lemma 1 (Conversion rates) *The equilibrium conversion rate of a couple $\langle x, y\rangle$ is*

$$\alpha^{\star}(x, y) = \frac{q_0(x_\omega, y_\omega) - Q^{\star}(x, y)}{1 - Q^{\star}(x, y)} \tag{6}$$

if $Q^{\star}(x, y) < q_0(x_\omega, y_\omega)$, and zero otherwise.

An immediate corollary of Proposition 1 and Lemma 1 is that every equilibrium exhibits segregation. Segregation arises due to the fact that an agent x who is willing to date agents with attractiveness ω from both social groups is always willing to invest more in dating the agent who shares her/his background (Proposition 1).

Corollary 1 (Equilibrium segregation) *Every equilibrium exhibits segregation.*

The next result establishes that there is a tension between integration and social welfare. In order to compare the levels of segregation of different equilibria, we define a *measure* of segregation, which we dub the *segregation ratio*. For any ω, ω' the segregation ratio $\rho(\omega, \omega')$ is the ratio between the conversion rate of a couple with attractiveness levels ω and ω' who share the same background and the conversion rate of a couple with attractiveness levels ω and ω' who do not share the same background.

Note that $\rho(\omega, \omega')$ is ill defined in situations where $\alpha((\omega, A), (\omega', B)) = 0$. When it also holds that $\alpha((\omega, A), (\omega', A)) > 0$, segregation is complete, and so we write $\rho(\omega, \omega') = \infty$. By contrast, in the case where $\alpha((\omega, A), (\omega', A)) = 0$ there is no meaningful way to say whether or not there is segregation at these levels of attractiveness, and so we do not define $\rho(\cdot, \cdot)$ in such cases.

Definition 2 (Level of segregation) *For any $\omega, \omega' \in [0, 1]$ such that $\alpha((\omega, A), (\omega', A)) > 0$, define the segregation ratio as*

$$\rho(\omega, \omega') = \begin{cases} \frac{\alpha((\omega,A),(\omega',A))}{\alpha((\omega,A),(\omega',B))} & , \textit{if } \alpha((\omega, A), (\omega', B)) > 0 \\ \infty & , \textit{otherwise} \end{cases} . \tag{7}$$

The next result shows that an equilibrium that Pareto dominates another equilibrium exhibits a higher level of segregation. This comparison can be interpreted either as a comparative statics exercise or as a comparison between multiple equilibria in the same setting.

Proposition 2 (Segregation vs. efficiency) *Fix λ and β. If an equilibrium E Pareto dominates an equilibrium E', then the segregation ratio in E is higher (pointwise) than in E'.*

Figure 2 provides a graphical explanation for Proposition 2. Consider two agents who come from different backgrounds, say, $x = (\omega, A)$ and $y = (\omega', B)$. The effective breakup threshold for them is $Q^\star(x, y)$ (i.e., they date until the joint belief drops from $q_0(\omega, \omega')$ to $Q^\star(x, y)$ or until observing a positive signal). Were x to date a potential partner from the same background $z = (\omega', A)$, their effective breakup threshold would be $Q^\star(x, z) = \beta Q^\star(x, y)$. We interpret the interval of beliefs $L_j = [Q^\star(x, y), q_0(\omega, \omega')]$ as the joint dating region since x dates agents from both backgrounds in this interval, and interpret the interval of beliefs $L_s = [\beta Q^\star(x, y), Q^\star(x, y)]$ as the segregative dating region since within this region x dates only agents from her/his own background. The segregation ratio is represented by $(L_j + L_s)/L_j$. Note that a Pareto improvement would raise the effective threshold $Q^\star(x, y)$ as $Q^\star(\cdot, \cdot)$ is increasing in the agents' continuation value $W_s(\cdot)$ (Equation (4)). The increase in $Q^\star(x, y)$ increases the size of the segregative dating region $(1 - \beta)Q^\star(x, y)$, reduces the size of the joint region $q_0(\omega, \omega') - Q^\star(x, y)$, and, as a result, increases the ratio $(L_j + L_s)/L_j$.

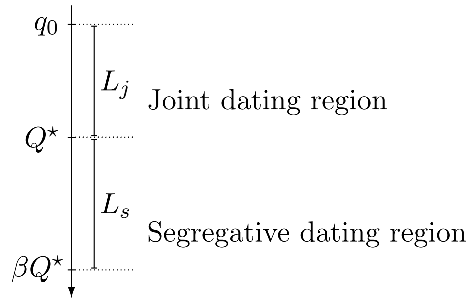

Figure 2: Segregation and efficiency.

3.2 Improvements in Online Dating Technologies

In recent years, the dating market has undergone radical changes due to the introduction of new dating apps that have replaced more traditional matching channels.[11] Such advances in search technologies enable people to meet many potential partners in a short span of time. In our model, such changes can be represented as an increase in the meeting rate μ. In this section we investigate the implications of such changes on segregation, dating, and marriage patterns.

Recall that Proposition 2 allows for changes in the model's parameters (except for λ and β) in the transition from E' to E. Thus, if an increase in μ improves the welfare of all market participants, then it must result in more segregation.[12]

Corollary 2 (Search frictions and segregation) *If an increase in μ leads to a Pareto improvement, then it also increases segregation.*

The introduction of dating apps allowing individuals to meet partners virtually instantaneously can be interpreted as search frictions becoming arbitrarily small. Various papers have studied the implications of vanishing search frictions on marriage-market outcomes in different contexts. Eeckhout (1999) and Adachi (2003) show that when search frictions vanish, the equilibrium matching converges to a matching that is pairwise stable in the Gale and Shapley (1962) sense and hence is efficient. Under vertical heterogeneity, pairwise stability also implies positive assortative matching in the classic frictionless nontransferable utility

[11] According to a recent survey by Pew Research Center (2020), "Roughly half or more of 18- to 29-year-olds ... say they have ever used a dating site or app."

[12] While deriving comparative statics of a steady-state equilibrium in search-and-matching models is notoriously difficult, for the special case of the model in which all agents have the same level of attractiveness, we can show that a reduction in search frictions leads to higher welfare and hence more segregation. We formally derive comparative statics results for that case in an earlier working paper version of this paper (Antler, Bird and Fershtman, 2022).

marriage model of Becker (1973). Lauermann and Nöldeke (2014) show that equilibrium matching can converge to a matching that is not pairwise stable if (i) mixed strategies are allowed and (ii) there are at least two pairwise stable matchings.[13] Antler and Bachi (2022) establish that vanishing search frictions lead to radically different results when agents' reasoning is coarse: in the frictionless limit agents search indefinitely and never marry.[14]

In this paper, we examine how a drastic reduction in search frictions affects both segregation in marriage and dating patterns. We start by showing that when search frictions vanish, the market becomes fully segregated.

Proposition 3 (Full segregation as search frictions vanish) *In the limit as search frictions vanish ($\mu \to \infty$), there is full segregation: agents date only partners with whom they share the same background.*

To understand the mechanism behind this result, let us assume for a moment that the singles market is fixed (in equilibrium, the size and composition of the singles market are endogenous). The key tradeoff agents face in our model is between learning more about a potential partner and searching for a more promising one. As the process of learning about a partner from a different background is noisier than that of learning about a partner with whom one shares the same background, the marginal benefit from learning about the former is lower. The opportunity cost of learning – searching for a different partner – is independent of the current partner's background. When the speed of search increases, it becomes easier to meet other singles, which raises the opportunity cost of learning more about a specific partner. Agents therefore invest less in dating each potential partner. Because learning across backgrounds is noisier, it follows that when search frictions are sufficiently small, agents prefer to avoid dating agents from a different background altogether, and instead wait for partners with whom they share the same background. The challenge in the proof is in arguing that the above intuition holds given the *endogeneity* of the (size and composition of the) equilibrium singles market.

3.2.1 Vanishing Search Frictions and the "Dating Apocalypse"

Although recent changes in dating technologies have drastically reduced search frictions, many individuals have reported a growing difficulty in establishing long-term relationships.

[13] In Adachi (2003) (i) is violated and in Eeckhout (1999) (ii) is violated.

[14] A parallel strand of the literature studies the implications of declining search frictions on product design, vertical differentiation, and growth in product and labor markets (e.g., Albrecht, Menzio and Vroman, 2021; Martellini and Menzio, 2021; Menzio, 2021).

This joint phenomenon is often referred to as the "dating apocalypse" (see, e.g., Sales, 2020). Our model can rationalize such findings by showing that when search frictions vanish, agents invest a negligible amount of time in dating each partner that they meet, and hence date an arbitrarily large number of partners before marrying.

In Proposition 3 we established that as $\mu \to \infty$, agent x meets singles instantaneously. Hence, in this case, agent x has no reason to date a potential partner for more than an infinitesimal amount of time: s/he can break up with the partner if a signal does not arrive immediately and can instantaneously begin dating a new one of the same type. Thus, in the limit $\mu \to \infty$, agent x chooses to date only the most attractive agents who are willing to date her/him, and hence the breakup threshold $q^\star(x)$ converges to $q_0(x_\omega, x_\omega)$. Moreover, as search frictions vanish, the investment in dating each potential partner converges to zero.

Corollary 3 (Dating apocalypse) *As $\mu \to \infty$, agents are willing to date only agents with their own level of attractiveness and above: $q^\star(x) \underset{\mu\to\infty}{\longrightarrow} q_0(x_\omega, x_\omega)$ for every $x \in X$. Hence, as search frictions vanish, (i) the maximal dating time between any two agents converges to zero, (ii) the average number of partners each agent dates before marrying goes to infinity, and (iii) dating becomes fully assortative in the attractiveness dimension.*

4 Segregation Patterns

In this section we study how one's set of partners changes with one's level of attractiveness. In Section 4.1 we establish that the segregation ratios decrease with attractiveness and show that this pattern is consistent with empirical findings. In Section 4.2, motivated by empirical patterns, we define a new probabilistic notion of assortative matching that captures the idea that individuals who are more attractive are more likely to marry highly attractive individuals (regardless of background) but, on occasion, may marry less attractive individuals. We then establish that complementarities between partners' attractiveness are sufficient for such probabilistic sorting to arise in equilibrium. Finally, in Section 4.3 we compare socially efficient sorting patterns with equilibrium sorting patterns.

Before we proceed to the analysis, it will be useful to derive a number of properties that the continuation value of single agents $W_s(\cdot)$ must satisfy. The first property is its strict monotonicity in attractiveness. To gain intuition, note that since agents' breakup thresholds are independent of their potential partner's attractiveness, an agent can mimic the *dating times* of a less attractive agent (from the same background) with *every* potential partner. Moreover, the more attractive agent is strictly more likely to be compatible with any given partner compared to a less attractive agent. Hence, were the more attractive agent to use this mimicking strategy, s/he would have a higher probability of (happily) marrying every partner that s/he meets compared to the less attractive agent. It follows that $W_s(\cdot)$ is strictly increasing in ω, which, by (4), implies that $q^\star(\cdot)$ is also strictly increasing in ω. The second property is the continuity of $W_s(\cdot)$ in attractiveness, which follows from an analogous mimicking argument: an agent who is slightly less attractive than x can obtain a similar continuation value as x by mimicking the latter's strategy.[15]

Lemma 2 *$W_s(\omega, \theta)$ and $q^\star(\omega, \theta)$ are strictly increasing and Lipschitz continuous in ω.*

4.1 Interracial Segregation by Education

There is considerable evidence that segregation is more common among the less educated. For instance, according to Fryer (2007), "[t]here seems to be some conventional wisdom that interracial marriages are concentrated among those with lower levels of education. But while this claim used to be true several decades ago, the pattern has reversed itself. Interracial marriages are now more concentrated among those with higher levels of education." This claim is also visible in Figure 3, which shows the rate of interracial marriage for high-education couples versus that of low-education couples. A high (resp., low) education couple is a couple in which both partners have (resp., do not have) a college degree.[16]

Fryer (2007) argues that "the data are most consistent with a Becker-style marriage market model in which objective criteria of a potential spouse, their race, and the social price of intermarriage are central." But, at the same time, Fryer also states that "[t]he evidence in favor of the classic Becker model is far from overwhelming." We suggest that premarital dating can also explain this evidence, even without any social price of intermarriage.

The next result establishes that, in the equilibrium of our model, agents' dating choices lead to more segregation at the bottom of the attractiveness distribution as long as μ is not too large (extreme values of μ lead to complete segregation as established in the previous section). Recall that one interpretation of an agent's attractiveness is her/his level of education. Under this interpretation, our model's prediction is consistent with the empirical evidence described in Figure 3.

[15] In the proof, we show that $W_s(\cdot)$ and $q^\star(\cdot)$ are Lipschitz continuous in ω, which will be useful in the sequel.

[16] The data shown in Figure 3 was obtained from the American Community Survey, 2008–2020. The trends shown in the figure are the same as those that can be seen from looking at the rate of interracial marriages of high/low-education individuals.

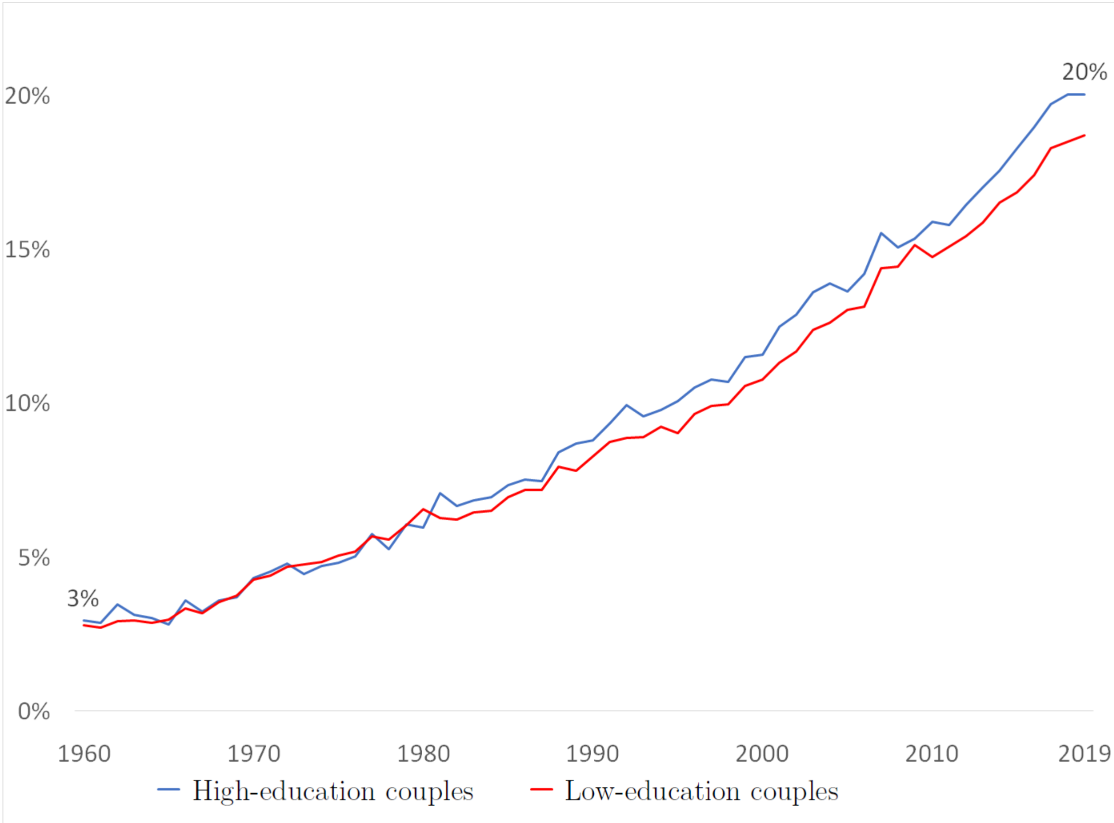

Figure 3: Intermarriage rates as a function of education in the U.S.

Proposition 4 (Intermarriage by level of attractiveness) *There exists $\mu^* > 0$ such that if $\mu < \mu^*$, then $\rho(\omega, \omega)$ is decreasing in ω; that is, there is less segregation among more attractive agents.*

In the proof of Proposition 4 we provide a condition that characterizes $\mu^\star$. It is worth pointing out that μ^* is linked to the variation in the marginal effect of an agent's attractiveness on the prior probability that a couple are compatible. In particular, when the variation in the derivative of $q_0(\cdot,\cdot)$ decreases (i.e., when $q_0(\cdot,\cdot)$ is "more linear"), $\mu^\star$ is greater. That is, segregation is less common among the most attractive agents for a wider region of parameters.

4.2 Sorting by Attractiveness

We now examine under what conditions assortative matching along the dimension of attractiveness is obtained, a question that has been central to the two-sided matching literature since the seminal work of Becker (1973). In particular, various conditions on the degree of complementarity of the match production function that yield assortative matching have been derived in the literature on matching with search frictions (see Chade, Eeckhout and Smith, 2017). Assortative matching by attractiveness can be interpreted as another measure of segregation, e.g., segregation by wealth/education rather than by background.

The assortativity of a matching is generally studied by examining how changes in an agent's type alter her/his acceptance set (agents who are mutually acceptable belong in each other's acceptance set). The probabilistic manner in which meetings are converted to marriages in our setting requires a new notion of assortative matching that takes into account the fact that, in our setting, it matters not only who marries whom, but also with what probability. We refer to the new notion as *single crossing of conversion rates*. This notion is satisfied if, roughly speaking, for any two agents who have levels of attractiveness x'_ω and $x''_\omega > x'_\omega$ and share the same background, there exists a critical attractiveness level $y^\star_\omega$ such that agent x'' is more likely than agent x' to marry a partner more attractive than $y^\star_\omega$, whereas agent x' is more likely than agent x'' to marry a partner less attractive than $y^\star_\omega$. Under this notion, the difference $\alpha(x'', y) - \alpha(x', y)$ satisfies a single-crossing property in y_ω (illustrated in Figure 4).

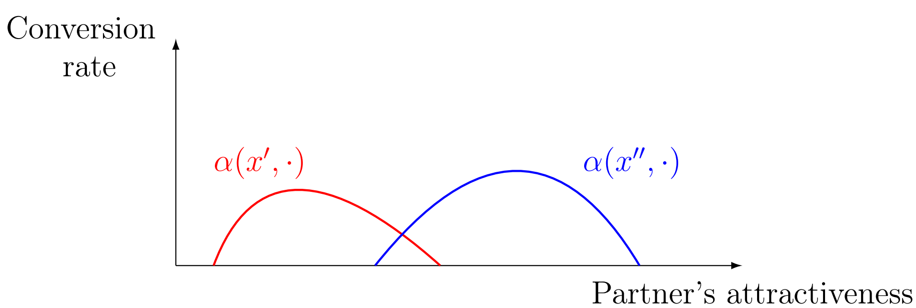

Figure 4: Illustration of the single crossing of conversion rates for $x'_\omega < x''_\omega$.

Formally, for any conversion-rate function $\alpha(\cdot, \cdot)$ and agents $x', x'' \in X$, we denote the set of agents who marry x' or x'' with positive probability by

$$A_\alpha(x', x'') = \{y : \alpha(x', y) > 0 \text{ or } \alpha(x'', y) > 0\}.$$

Our objective is to derive conditions for positive assortative matching along the attractiveness dimension. Hence, we focus on situations in which agents can be compared by their level of attractiveness alone. In particular, we fix two agents x', x'' (on the same side of the market) who share the same background and a potential partner y (possibly of a different background), and analyze how agent y's attractiveness affects the conversion rate of the couples $\langle x', y\rangle$ and $\langle x'', y\rangle$.

Definition 3 (Single crossing of conversion rates) *Fix two arbitrary backgrounds $x_\theta, y_\theta \in \{A, B\}$. A conversion-rate function $\alpha(\cdot,\cdot)$ satisfies the single-crossing property if for every two agents $x' = (x'_\omega, x_\theta)$ and $x'' = (x''_\omega, x_\theta)$ such that $x'_\omega < x''_\omega$, there exists $y^\star_\omega(x', x'')$ such that for any $(y_\omega, y_\theta) \in int(A_\alpha(x', x''))$, $\alpha(x'', (y_\omega, y_\theta)) > \alpha(x', (y_\omega, y_\theta))$ if and only if $y_\omega > y^\star_\omega(x', x'')$.*

The next result establishes that complementarity in agents' attractiveness leads to single-crossing of conversion rates.

Proposition 5 (Equilibrium sorting by attractiveness) *If $q_0(\cdot,\cdot)$ is supermodular, then every steady-state equilibrium conversion-rate function satisfies the single-crossing property.*

An implication of the above result is that highly attractive individuals are more likely to marry highly attractive partners, but on occasion may marry less attractive partners instead. Figure 5 shows that this prediction is consistent with empirical patterns (where attractiveness is measured by years of schooling). In particular, we split the population into four groups according to their level of education: high school and below, 1–3 years of college, 4 years of college, 5+ years of college. For each group, Figure 5 depicts the normalized empirical distribution of the educational level of partners. The markers in Figure 5 represent partners with exactly these four levels of education.[17] The single-crossing property (for every pair of education levels) is visible in the figure.

It is tempting to interpret the prior $q_0(\cdot,\cdot)$ as the analog of the match-production function $f(\cdot,\cdot)$, and to compare our condition for assortative matching with those derived in the literature. However, were we to interpret the production function $f(\cdot,\cdot)$ as the expected match value, we would have $f(\cdot,\cdot) = q_0(\cdot,\cdot)\cdot 1 + (1 - q_0(\cdot,\cdot))\cdot(-z)$. Hence, the supermodularity assumptions on $f(\cdot,\cdot)$ and $q_0(\cdot,\cdot)$ have different meanings and implications, which precludes a meaningful comparison.

4.3 Socially Efficient Sorting

In this section we argue that positive assortative matching by attractiveness may be inefficient from a social perspective even if $q_0(\cdot,\cdot)$ is supermodular. That is, it is inefficient from the

[17] This figure is based on marriages in 2019 that were recorded in the American Community Survey 2019. The distribution is normalized by the size of each group.

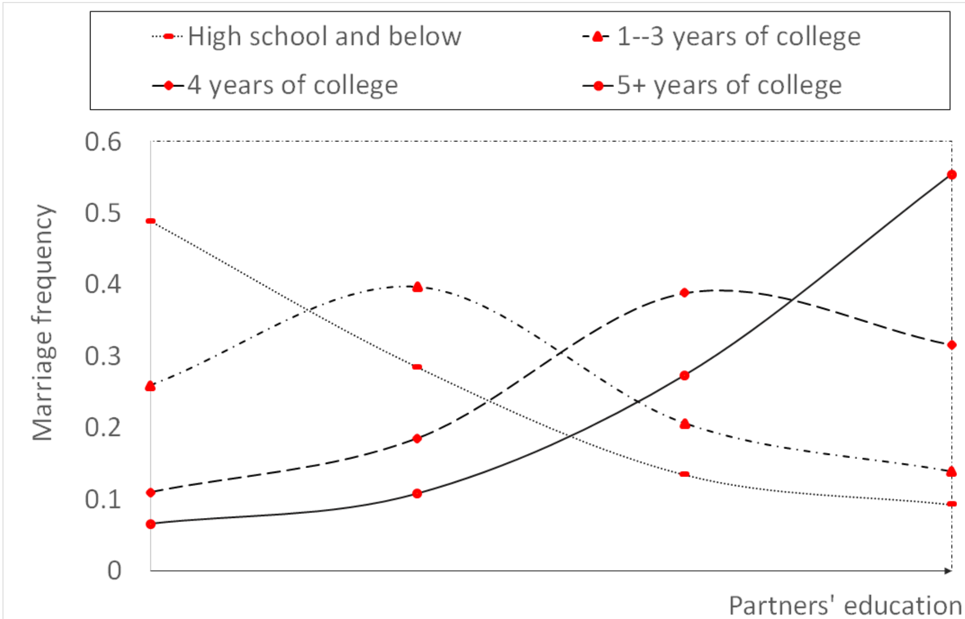

Figure 5: Single crossing in U.S. marriage probabilities.

perspective of a social planner who chooses the agents' stationary dating times without knowing their compatibility in order to maximize the weighted average of the continuation value of single agents.[18] In particular, we show that such a planner may induce negative assortative matching by attractiveness and positive assortative matching by background (i.e., segregation).

To gain intuition as to why efficient allocations may exhibit negative assortative matching, consider the expected length of agents' singlehood episodes. From a social perspective, the length of these episodes should be minimized as only married agents can obtain positive payoffs. Agents with low levels of attractiveness are unlikely to be compatible with one another, and so if they only date other agents with low levels of attractiveness, then, on average, they will date many partners before eventually finding the one that they will marry. This results in long singlehood episodes for such agents. By contrast, if highly attractive agents are allowed to date one another, they need to date relatively few potential partners before marrying and have short singlehood episodes. The average length of a singlehood episode for agent x is determined by $\mathbb{E}(1/\alpha(x,\cdot))$. Since this function is concave in $\alpha(x,\cdot)$ the social planner has an incentive to smooth out variation in the conversion rate, which may lead her to induce negative assortative matching that leads to lower variation in α.

[18]The stationarity assumption makes the social optimum analysis consistent with the equilibrium analysis. Bonneton and Sandmann (2022*a*) show that nonstationary equilibria may also exist in a two-sided search setup (without dating).

Example 1 (Negative Assortative Matching) *Suppose that there are only two levels of attractiveness,* $x^h_\omega = 1/2$ *and* $x^l_\omega = 2/5$*, where* $Pr(x_\omega = x^l) = 3/4$*, and that* $q_0(x_\omega, y_\omega) = x_\omega y_\omega$*. Moreover, suppose that* $c = 1/20, \delta = 1/100, r = 1/10, \mu = 2, \lambda = 1$*, and* $\beta = 3/4$*. Table 1 presents the socially optimal allocation:*

$\alpha(\cdot,\cdot)$	x^l_ω, x^l_ω	x^l_ω, x^h_ω	x^h_ω, x^h_ω
Same Background	12%	16%	0%
Different Background	10%	14%	0%

Table 1: Socially optimal conversion rates by background and attractiveness.

Note that even though the social planner induces negative assortative matching by the agents' level of attractiveness, she maintains the segregation across groups. This is the case as noisier learning is costly both at an individual level and at a social one.

Example 1 shows that the equilibrium sorting patterns may indeed be inefficient: while there is positive assortative matching (by attractiveness) in equilibrium, there is negative assortative matching in the socially efficient outcome. This phenomenon is intrinsically related to dating, which enriches the set of possible conversion rates that the social planner can induce. In the absence of dating, the conversion rate in the acceptance set is 1 regardless of the agents' type, and so the (expected) length of an agent's singlehood episode is determined entirely by the measure of agents in her/his acceptance group who are single. By contrast, when agents engage in premarital dating, the length of singlehood episodes is inversely related to the conversion rate, which is strictly less than one for any couple.

5 Equilibrium Existence

So far we have analyzed agents' equilibrium behavior in a steady-state equilibrium under the assumption that such an equilibrium exists. We now prove that a steady-state equilibrium indeed exsits. Recall that in a steady-state equilibrium, (i) agents set their breakup thresholds $q^\star(\cdot)$ optimally given the (endogenous) size and composition of the singles pool, (ii) agents' continuation utilities are consistent with equilibrium dating strategies, and (iii) the flows between singlehood, dating, and marriage are balanced. Formally:

Definition 4 (Equilibrium) *A* steady-state equilibrium *is a tuple* $\langle W_s(\cdot), q^\star(\cdot), u(\cdot)\rangle$ *that consists of the value functions of single agents, the agents' breakup thresholds, and the measure of single agents, such that Equations* (1)*,* (2)*, and* (4) *hold.*

Following the approach pioneered by Shimer and Smith (2000), we establish the existence of a steady-state equilibrium in the value-function space. While existence proofs in the literature typically rely on matching and acceptance sets, which reflect binary accept/reject choices, in our model the agents make richer decisions, choosing for how long, if at all, to date each potential partner. Moreover, in our model, agents transition between three states – singlehood, dating, and marriage – rather than just two states.

Proposition 6 (Equilibrium existence) *A steady-state equilibrium exists.*

We prove the existence of a steady-state equilibrium by invoking a fixed point argument. This requires establishing that the mappings (i) from value functions to conversion rates, and (ii) from convergence rates to the distribution of singles, are continuous and well defined. The latter is the analog of Shimer and Smith's (2000) fundamental matching lemma. Equilibria are then fixed points of an appropriately defined mapping. Using Schauder's fixed point theorem, we establish that such a fixed point indeed exists.

6 Concluding Remarks

We develop a model of two-sided search in which potential partners can spend time learning about their compatibility before agreeing to their match. To our knowledge, this paper is the first to incorporate premarital information acquisition into the classic search-and-matching framework, and to study explicitly the tradeoff between learning about a potential partner and searching for other, perhaps more promising, potential partners.

The new framework enables us to capture the idea that evaluating the propspects of a potential match depends on the partners' observable characteristics. In particular, for every potential couple, the partners' attractiveness determines the probability that they are compatible, whereas their background determines the effectiveness of their mutual learning process. The latter assumption, inspired by Phelps (1972), plays a central role in our analysis of segregation in marriage.

We find that, in equilibrium, individuals sort along two dimensions: attractiveness and background. However, this may not be beneficial from a social planner's point of view. A key finding of this paper is that there is a tradeoff between social integration and welfare. In particular, if advances in search technologies (e.g., the emergence of dating apps) improve all individuals' welfare, they inevitably result in more segregation. In fact, we show that radical improvements in the speed of search will lead to complete segregation.

While our model assumes a simple and commonly used learning technology, the driving force behind our results is that learning across backgrounds is more noisy. Thus, we expect that our qualitative results will extend to other commonly used learning technologies that exhibit such a difference in the effectiveness of learning across backgrounds (e.g., a Brownian motion in which learning across groups has a higher variance).

Throughout the paper, we use the marriage market and dating terminology and focus on segregation in marriage. However, pre-match learning is also prominent in other markets where agents trade bilaterally and engage in time-consuming search. Outside the marriage market context, learning may take different forms. For example, in the context of the labor market it can take the form of job interviews or probationary periods during the hiring process. In markets where developers and founders match to develop joint ventures, learning may take the form of a due diligence process. Our framework may help shed light on central aspects of these additional contexts, such as how hiring processes impact the division of surplus in the labor market, and how pre-match investment choices are resolved in markets for entrepreneurship.

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A Appendix

Proof of Proposition 1. Agent x's capital gain from meeting a potential partner y and then dating her/him for (at most) T units of time is

$$q_0(x_\omega, y_\omega) \int_0^T \lambda_{xy} e^{-\lambda_{xy} t} \left(e^{-rt} \frac{1 - rW_s(x)}{r + \delta} - \frac{1 - e^{-rt}}{r} (c + rW_s(x)) \right) dt \\ - \left(1 - q_0(x_\omega, y_\omega)(1 - e^{-\lambda_{xy} T})\right) \frac{1 - e^{-rT}}{r} (c + rW_s(x)). \quad \text{(A.1)}$$

The first term in this expression is agent x's expected gain in case a signal arrives while dating agent y, and the second term represents the cost agent x incurs when a signal does not arrive and the couple eventually separate without marrying. Antler, Bird and Oliveros (2023) show formally that the value of learning in such problems is concave. Hence, agent x's preferred dating time is either zero, or is given by equating the derivative of (A.1) with respect to T with zero. This derivative is given by (3), and equating it with zero yields (4).[19] ■

Proof of Lemma 1. Integrating $\dot{q}_t = -\lambda_{xy} q_t (1 - q_t)$ implies that

$$\frac{q_t(x, y)}{1 - q_t(x, y)} = e^{-\lambda_{xy} t} \frac{q_0(x_\omega, y_\omega)}{1 - q_0(x_\omega, y_\omega)}. \quad \text{(A.2)}$$

Hence, if a dating couple $\langle x, y \rangle$ break up when they believe that they are compatible with probability q, the maximal length of time for which they will date, $T(x, y, q)$, is given implicitly by

$$e^{-\lambda_{xy} T(x,y,q)} \frac{q_0(x_\omega, y_\omega)}{1 - q_0(x_\omega, y_\omega)} = \frac{q}{1 - q}.$$

The probability that they will eventually marry is therefore

$$\alpha(x, y, q) = \begin{cases} q_0(x_\omega, y_\omega)(1 - e^{-\lambda_{xy} T(x,y,q)}) = \frac{q_0(x_\omega, y_\omega) - q}{1 - q} & \text{, if } q < q_0(x_\omega, y_\omega) \\ 0 & \text{, otherwise} \end{cases}. \quad \text{(A.3)}$$

This establishes that the conversion rate is given by (6). ■

Proof of Proposition 2. From Lemma 1 and Equation (7) it follows that in the interme-

[19] It is straightforward to show that this preferred dating time is given by $T_x^\star(y; q^\star(x)) = \max\left\{0, \frac{1}{\lambda_{xy}} \log\left(\frac{q_0(x_\omega, y_\omega)(\frac{\lambda_{xy}}{\lambda} - q^\star(x))}{(1 - q_0(x_\omega, y_\omega)) q^\star(x)}\right)\right\}$.

diate region in which $\rho(\omega,\omega')$ is not constant it is given by

$$\rho(\omega,\omega') = \frac{(\beta - Q^\star((\omega,A),(\omega',A))(q_0(\omega,\omega') - Q^\star((\omega,A),(\omega',A))}{(1-Q^\star((\omega,A),(\omega',A)))(\beta q_0(\omega,\omega') - Q^\star((\omega,A),(\omega',A)))}. \tag{A.4}$$

A Pareto improvement will increase $Q^\star((\omega,A),(\omega',A))$ (see Equation (4)). Differentiating $\rho(\omega,\omega')$ with respect to $Q^\star((\omega,A),(\omega',A)$ yields

$$\frac{(1-\beta)(1-q_0(\omega,\omega'))}{(1-Q^\star((\omega,A),(\omega',A))^2(Q^\star((\omega,A),(\omega',A) - \beta q_0(\omega,\omega'))^2}\left(\beta q_0(\omega,\omega') - (Q^\star((\omega,A),(\omega',A))^2\right).$$

The ratio in this derivative is clearly positive. The second term of this derivative is also positive as $Q^\star((\omega,A),(\omega',A)) < 1$, and the segregation ratio is defined only for couples for whom $\beta q_0(\omega,\omega') > Q^\star((\omega,A),(\omega',A)$. ■

Proof of Proposition 3. We begin by deriving a connection between the measure of singles and the conversion rate. Fix $\alpha(\cdot,\cdot)$ and consider a couple $\langle x,y\rangle$ for which $\alpha(x,y) > 0$. From Equation (A.3) it follows that they date for at most

$$T^\star_\alpha(x,y) = -\frac{1}{\lambda_{xy}}\log\left(1 - \frac{\alpha(x,y)}{q_0(x_\omega,y_\omega)}\right)$$

units of time. Hence, the measure of couples of type $\langle x,y\rangle$ who are dating (under the chosen α) is

$$d_\alpha(x,y) = \mu u_\alpha(x)u_\alpha(y)\int_0^{T^\star_\alpha(x,y)}(1 - q_0(x_\omega,y_\omega)(1-e^{-\lambda_{xy}t}))dt.$$

Integrating over t yields

$$d_\alpha(x,y) = \mu u_\alpha(x)u_\alpha(y)\frac{\alpha(x,y) - (1-q_0(x_\omega,y_\omega))\log(1-\frac{\alpha(x,y)}{q_0(x_\omega,y_\omega)})}{\lambda_{xy}}. \tag{A.5}$$

Integrating (A.5) yields

$$\begin{aligned} d_\alpha(x) &= \int_X \mu u_\alpha(x)d(x,y)u_\alpha(y)dy \\ &= u_\alpha(x)\int_X \frac{\mu}{\lambda_{xy}}\left(\alpha(x,y) + (1-q_0(x_\omega,y_\omega))\log\left(\frac{q_0(x_\omega,y_\omega)}{q_0(x_\omega,y_\omega)-\alpha(x,y)}\right)\right)u_\alpha(y)dy. \end{aligned}$$

Rearranging the balanced flow condition (1) yields

$$u(x)\left(1+\frac{\mu}{\delta}u(x)\int_X \alpha(x,y)u(y)dy\right)=g(x)-d(x).$$

Plugging in the expression for the measure of dating agents derived above and rearranging yields

$$u(x)=\frac{g(x)}{1+\mu\int_X\left(\frac{\lambda_{xy}+\delta}{\lambda_{xy}\delta}\alpha(x,y)+\frac{1-q_0(x_\omega,y_\omega)}{\lambda_{xy}}\log\left(\frac{q_0(x_\omega,y_\omega)}{q_0(x_\omega,y_\omega)-\alpha(x,y)}\right)\right)u(y)dy}. \tag{A.6}$$

Next, we derive the limit result. Assume by way of contradiction that $\liminf_{\mu\to\infty} u(x)=0$ for some $x\in X$. From (A.6), it follows that there exists $Y\subset X$ with strictly positive measure such that for every $y\in Y$, (i) $\alpha(x,y)>0$, and (ii) $\liminf_{\mu\to\infty}\mu u(y)=\infty$. By construction, every agent of type $y\in Y$ is willing to date agent x for a strictly positive amount of time. Since agent x can start dating any $y\in Y$ after searching for an arbitrarily small amount of time, it follows that, in the limit, for almost all $y\in Y$ it must be the case that $\alpha(x,y)=0$. Hence, any $Y\subset X$ that satisfies properties (i) and (ii) must be of measure zero, contradicting the assumption that Y has a positive measure. Therefore, $\lim_{\mu\to\infty}\mu u(x)=\infty$ for all $x\in X$.

Since each agent must wait an arbitrarily small amount of time before meeting a potential partner of every possible type, each agent will date only the partner for whom the marginal value of learning is highest among those partners who are willing to date her/him. Hence, $\lim_{\mu\to\infty} q^\star(x)=q_0(x_\omega,x_\omega)$. ■

Proof of Corollary 3. The main part of this result was established in the proof of Proposition 3. Parts i) and ii) then follow immediately from the calculations in Lemma 1. Part iii) follows immediately from the fact that dating requires mutual consent. ■

Proof of Lemma 2. We start by establishing the strict monotonicity of $W_s(\omega,\theta)$. Assume by way of contradiction that $W_s(\cdot,\cdot)$ is not strictly increasing in ω. Then there exist agents x' and x'' for whom $x'_\omega<x''_\omega$ and $W_s(x')\geq W_s(x'')$. Under this assumption, for every $y\in X$ it holds that $V_d(x'';y)>V_d(x';y)$; that is, the capital gain of dating agent y is greater for agent x'' than for agent x'. This follows from three observations: (i) for every $y\in X$, the couple $\langle x'',y\rangle$ are more likely to be compatible than the couple $\langle x',y\rangle$, (ii) every $y\in X$ is willing to date x'' for at least as much time as s/he is willing to date x', and (iii) the opportunity cost for dating is lower for agent x'' than for agent x'. However, by (2), $W_s(x)$ is a convex combination of $V_d(x;\cdot)$, a contradiction. The strict monotonicity of $q^\star(\omega,\theta)$ in ω follows immediately from condition (4) and the fact that $W_s(\cdot) < \frac{1}{r}$ (the latter is the continuation value of being married to a compatible partner indefinitely).

We now establish the Lipschitz continuity of both functions. By (2), we have that

$$W_s(x_\omega + \epsilon, \theta) - W_s(x_\omega, \theta) = \frac{\mu}{r} \int_X \left(V_d((x_\omega + \epsilon, \theta); y) - V_d((x_\omega, \theta); y)\right) u(y) dy. \quad \text{(A.7)}$$

Fix $x_\omega \in [0, 1], y \in X$ and $\epsilon > 0$, such that $x_\omega + \epsilon \leq 1$. We begin by showing that there exists $K > 0$ independent of x_ω, y, and ϵ, such that $V_d((x_\omega + \epsilon, \theta); y) - V_d((x_\omega, \theta); y) < K\epsilon$.

The probability that any couple is compatible is at most $q_0(1, 1) < 1/(1 + z)$. Moreover, in optimum, every pair of agents who start dating either marry or break up before their belief about their compatibility reaches q_{min}, where $q_{min}\lambda\frac{1}{r} = c$. By the assumption that dating costs are not prohibitive, $q_{min} < 1/(1 + z)$. In combination with the fact that $\dot{q}_t = -\lambda_{xy} q_t(1 - q_t)$, it follows that $\epsilon\tilde{K}$, where

$$\tilde{K} = \frac{1}{\min_{q \in [\frac{cr}{\lambda}, \frac{1}{1+z}]} \{\beta\lambda q(1 - q)\}},$$

is a uniform upper bound on the amount of time it takes a couple's belief about their compatibility to drift down by ϵ.

Denote by τ the amount of time a couple $\langle(x_\omega + \epsilon, \theta), y\rangle$ must spend dating so that, in the absence of a signal arriving, they believe that they are compatible with probability $q_0(x_\omega, y_\omega)$ (i.e., τ is defined implicitly by $q_\tau((x_\omega + \epsilon, \theta), y) = q_0(x_\omega, y_\omega)$). Note that $\tau < \tilde{K}\overline{q}\epsilon$. The marginal gain from dating is bounded from above by λ/r for every agent. Since the marginal cost of dating is nonnegative, it follows that the capital gain that agent $(x_\omega + \epsilon, \theta)$ derives from the first τ units of time for which s/he dates y is bounded from above by $\tilde{K}\lambda\overline{q}\epsilon/r$.

For any $t > 0$, couple $\langle(x_\omega + \epsilon, \theta), y\rangle$ receive a signal after having dated for $t + \tau$ units of time with the same probability that couple $\langle(x_\omega, \theta), y\rangle$ receive a signal after having dated for t units of time, conditional on neither couple receiving a signal earlier. Moreover, since $W_s(\omega, \theta)$ is increasing in ω, the marginal cost of dating is higher for agent $(x_\omega + \epsilon, \theta)$ than for agent (x_ω, θ). Furthermore, the monotonicity of $q^\star(\omega, \theta)$ in ω implies that the dating time of couple $\langle(x_\omega, \theta), y\rangle$ is greater than the amount of time for which the couple $\langle(x_\omega + \epsilon, \theta), y\rangle$ continues to date after they have already dated for τ units of time. It follows that the additional capital gain that agent $(x_\omega + \epsilon, \theta)$ derives from dating y after they have already dated for τ units of time is less than agent (x_ω, θ)'s capital gain from dating y. We have therefore shown that (i) the capital gain agent $(x_\omega + \epsilon, \theta)$ derives from dating y for τ units of time is less than $\tilde{K}\lambda\overline{q}\epsilon/r$, and (ii) the capital gain agent (x,θ) derives from dating agent y is greater than the additional capital gain that agent $(x_\omega+\epsilon,\theta)$ derives from dating agent y after τ units of time passed without a signal arriving. Hence, $V_d((x_\omega+\epsilon,\theta);y) - V_d((x_\omega,\theta);y) < \tilde{K}\overline{q}\lambda\epsilon/r$.

Since $\int_X u(y)dy \leq 1$, from (A.7) it follows that $W_s(x_\omega+\epsilon,\theta) - W(x_\omega,\theta) \leq \overline{K}\epsilon$, where $\overline{K} = (\mu/r)(\lambda\overline{q}/r)\tilde{K}$. Since $W_s(\omega,\theta)$ is increasing in ω, this implies that $W_s(\omega,\theta)$ is Lipschitz continuous in ω with modulus $\overline{K}$. The Lipschitz continuity of $q^\star(\omega,\theta)$ in ω then follows immediately from condition (4) and the observation that $rW_s \leq r(\mu/(r+\mu))/r < 1$. The first inequality follows from the fact that rW_s is bounded from above by r times the expected discounted time at which the next partner is met, $\mu/(r+\mu)$, and the payoff from a marriage that lasts forever, $1/r$. ■

Proof of Proposition 4. From (A.4) it follows that

$$\rho(\omega,\omega) = \frac{(\beta - q^\star(\omega)(q_0(\omega,\omega) - q^\star(\omega))}{(1-q^\star(\omega))(\beta q_0(\omega,\omega) - q^\star(\omega))},$$

where $q^\star(\omega)$ is the breakup threshold of an agent who has an attractiveness level of ω. Since $q^\star(\omega,\theta)$ is Lipschitz continuous in ω (Lemma 2) and $q_0(\cdot,\cdot)$ is differentiable by assumption, $\rho(\omega,\omega)$ is differentiable almost everywhere and continuous. This derivative (where it exists) is

$$(1-\beta)\frac{(1-q_0(\omega,\omega))\frac{\partial q^\star(\omega)}{\partial\omega}\,(\beta q_0(\omega,\omega) - q^\star(\omega)^2) + (1-q^\star(\omega))q^\star(\omega)(q^\star(\omega)-\beta)\frac{\partial q_0(\omega,\omega))}{\partial\omega}}{(1-q^\star(\omega))^2(\beta q_0(\omega,\omega) - q^\star(\omega))^2}$$

The sign of this derivative is the same as the sign of

$$\Psi = \underbrace{(1-q_0(\omega,\omega))\left(\beta q_0(\omega,\omega) - q^\star(\omega)^2\right)\frac{\partial q^\star(\omega)}{\partial\omega}}_{\Psi_1} + \underbrace{(1-q^\star(\omega))q^\star(\omega)(q^\star(\omega)-\beta)\frac{\partial q_0(\omega,\omega))}{\partial\omega}}_{\Psi_2}.$$

To show that $\Psi < 0$ (i.e., the segregation ratio is decreasing in attractiveness), we replace both Ψ_1 and Ψ_2 with their upper bounds, and derive conditions under which that upper bound on Ψ is negative.

Bounding Ψ_1*:* Since $q^\star(\omega)$ is increasing (Lemma 2) and $\rho(\cdot,\cdot)$ is defined only for ω such that $q_0(\omega,\omega) > q^\star(\omega)/\beta$, it follows that Ψ_1 is the product of three positive terms.

From (4) it follows that

$$\frac{\partial q^\star(\omega)}{\partial \omega} = \frac{(c+1)r(\delta+r)}{\lambda(rW_s(\omega)-1)^2}\frac{\partial W_s(\omega)}{\partial \omega}.$$

The continuation value of any single agent is bounded from above by her/his expected value from meeting compatible partners at rate μ while single, marrying them immediately, and returning to the market once the marriage is hit by a dissolution shock. This upper bound is given by $\overline{W} \equiv \frac{\mu}{(r+\mu+\delta)r}$. Since $W_s(\cdot,\cdot)$ is Lipschitz continuous in ω with modulus $\tilde{K}\frac{\mu\lambda\overline{q}}{r^2}$ (Lemma 2), it follows that

$$\frac{\partial q^\star(\omega)}{\partial \omega} \leq \frac{(c+1)\overline{q}\tilde{K}\mu(\delta+\mu+r)^2}{r(\delta+r)}.$$

Hence,

$$\Psi_1 \leq \frac{(c+1)\overline{q}\tilde{K}\mu(\delta+\mu+r)^2}{r(\delta+r)}\left(\beta q_0(\omega,\omega)-(q^\star(\omega))^2\right)(1-q_0(\omega,\omega)).$$

Note that this bound is decreasing in $q^\star(\omega)$, and so we can replace $q^\star(\omega)$ with its lower bound of $q_{min} = \frac{cr}{\lambda}$ (see proof of Lemma 2) and get that

$$\Psi_1 \leq \frac{(c+1)\overline{q}\tilde{K}\mu(\delta+\mu+r)^2}{r(\delta+r)}\left(\beta q_0(\omega,\omega)-\frac{c^2r^2}{\lambda^2}\right)(1-q_0(\omega,\omega)).$$

This bound is concave in $q_0(\cdot,\cdot)$, and so it has a unique maximum over all possible values of $q_0(\cdot,\cdot)$. Evaluating the bound at that maximizer gives

$$\Psi_1 \leq \frac{(c+1)\tilde{K}\mu(\delta+\mu+r)^2\left(c^2r^2-\beta\lambda^2\right)^2}{4\beta\lambda^4 r(\delta+r)}\overline{q}.$$

Bounding Ψ_2*:* The expression $(1-q^\star(\omega))q^\star(\omega)(q^\star(\omega)-\beta)$ as a function of $q^\star(\omega)$ is convex in the region $[0,\beta]$. Moreover, it is strictly negative in the interior of this region. Since $q^\star(\omega) \in [q_{min}, \frac{1}{1+z}\beta]$, this expression is bound from above by $-M$ for some finite and strictly positive M. Hence,

$$\psi_2 \leq -M\frac{\partial q_0(\omega,\omega))}{\partial \omega}.$$

Thus, the segregation ratio is decreasing in attractiveness ($\Psi < 0$) if

$$\frac{(c+1)\overline{q}\tilde{K}\mu(\delta+\mu+r)^2\left(\beta\lambda^2-c^2r^2\right)^2}{4\beta\lambda^4 r(\delta+r)} < M\frac{\partial q_0(\omega,\omega))}{\partial \omega}.$$

Note that $\frac{\partial q_0(\omega,\omega)}{\partial \omega} \geq 2\underline{q}$, and so the above condition holds if

$$\frac{(c+1)\tilde{K}\mu(\delta+\mu+r)^2\left(\beta\lambda^2-c^2r^2\right)^2}{8\beta\lambda^4 Mr(\delta+r)} < \underline{q}/\overline{q}.$$

Finally, since M and $\tilde{K}$ are independent of μ, the LHS of the above condition converges continuously to zero as μ decreases. ■

Proof of Proposition 5. We provide a proof for the case where $x_\theta = y_\theta$. The proof for the other case is identical up to dividing $q^\star(\cdot)$ by β every time it appears below.

Fix $x''_\omega > x'_\omega$ and let $x'' = (x''_\omega, x_\theta)$, $x' = (x'_\omega, x_\theta)$. First, consider potential partners $y = (y_\omega, y_\theta)$ for whom $y_\omega \geq x''_\omega$. Since $q^\star(\cdot)$ is increasing in attractiveness, in both the couple $\langle x'', y\rangle$ and the couple $\langle x', y\rangle$, agent y is the one that breaks up with her/his partner. Moreover, s/he does so when the belief about the couple's compatibility drops to $q^\star(y)$. Thus, from (A.3) and the fact that $q_0(\cdot,\cdot)$ is increasing, it follows that

$$\alpha(x'',y) = \max\{\frac{q_0(x''_\omega, y_\omega) - q^\star(y)}{1-q^\star(y)}, 0\} > \max\{\frac{q_0(x'_\omega, y_\omega) - q^\star(y)}{1-q^\star(y)}, 0\} = \alpha(x', y),$$

for all such y for which either $\alpha(x'',y) > 0$ or $\alpha(x',y) > 0$. Hence, for any $y \in A_\alpha(x', x'')$ such that $y_\omega \geq x''_\omega$ it holds that $\alpha(x'',y) > \alpha(x',y)$

Next, consider potential partners who have level of attractiveness $y_\omega < x''_\omega$. For such potential partners, x'' is the one who breaks up with y. Moreover, x'' marries such partners with positive probability if and only if $q_0(x''_\omega, y_\omega) > q^\star(x'')$. Since $q_0(\cdot,\cdot)$ is increasing, the levels of attractiveness of the agents whom x'' marries with positive probability are an interval. Thus, if either (i) the sets of potential partners who have level of attractiveness $y_\omega \leq x''_\omega$ whom x'' and x' marry with positive probability do not intersect, or (ii) $\alpha(x'',y) > \alpha(x',y)$ for all $y \in A_\alpha(x',x'')$ such that $y_\omega \leq x''_\omega$, then the proposition is established.

In the remainder of the proof we therefore assume that there exists $y_\omega < x''_\omega$ such that $\alpha(x', (y_\omega, y_\theta)) > \alpha(x'', (y_\omega, y_\theta)) > 0$. Since $q^\star(\omega, \theta)$ is continuous in ω, it follows that the probability that a couple marry is also continuous in the attractiveness of both agents. Since $\alpha(x', x'') < \alpha(x'', x'')$, the intermediate value theorem implies that there exists $y^\star_\omega < x''_\omega$ for which $\alpha(x', (y^\star_\omega, y_\theta)) = \alpha(x'', (y^\star_\omega, y_\theta))$.

To conclude the proof, we show that $\alpha(x', y) > \alpha(x'', y)$ for every potential partner $y \in A_\alpha(x', x'')$ of attractiveness $y_\omega < y^\star_\omega$ and background y_θ. For such y, the couple $\langle x'', y\rangle$ break up when $q_t(x, y) = q^\star(x'')$. Thus, by (A.3), if a couple $\langle x'', y\rangle$ date, they marry with probability

$$\alpha(x'', y) = \frac{q_0(x''_\omega, y_\omega) - q^\star(x'')}{1 - q^\star(x'')}.$$

Hence, for any such y for which $\alpha(x'', y) > 0$, it holds that

$$\frac{d\alpha(x'', (y_\omega, y_\theta))}{dy_\omega} = \frac{\frac{dq_0}{dy_\omega}(x''_\omega, y_\omega)}{1 - q^\star(x'')}.$$

Similarly, the probability that a dating couple $\langle x', y \rangle$ marry is

$$\alpha(x', (y_\omega, y_\theta)) = \frac{q_0(x'_\omega, y_\omega) - q^\star((\max\{x'_\omega, y_\omega\}, x_\theta))}{1 - q^\star(\max\{x'_\omega, y_\omega\}, x_\theta))}.$$

Since $q^\star(\omega, \theta)$ is monotone in ω, it is differentiable in ω almost everywhere. Hence, for almost all y with $y_\omega < y^\star_\omega$,

$$\frac{d\alpha(x', (y_\omega, y_\theta))}{dy_\omega} = \frac{\frac{dq_0}{dy_\omega}(x'_\omega, y_\omega)}{1 - q^\star(\max\{x'_\omega, y_\omega\}, x_\theta))} - \frac{1 - q_0(x'_\omega, y_\omega)}{(1 - q^\star(\max\{x'_\omega, y_\omega\}, x_\theta)))^2} \frac{dq^\star(\max\{x'_\omega, y_\omega\}, x_\theta))}{dy_\omega}.$$

Since $x''_\omega > \max\{x'_\omega, y_\omega\}$, $q^\star(\cdot)$ is increasing in attractiveness, and $q_0(\cdot, \cdot)$ is supermodular, it follows that

$$\frac{d\alpha(x'', (y_\omega, y_\theta))}{dy_\omega} > \frac{d\alpha(x', (y_\omega, y_\theta))}{dy_\omega}$$

for all $y_\omega < y^\star_\omega$ at which $q^\star(\cdot)$ is differentiable. By Lemma 2, $q^\star(\cdot)$ is Lipschitz continuous in ω and hence absolutely continuous, which in turn implies that $\alpha(\cdot, \cdot)$ is also absolutely continuous. Since, by definition, $\alpha(y^\star, x'') = \alpha(y^\star, x')$, the proposition follows from the fundamental theorem of calculus. ■

Proof of Proposition 6. To establish the existence of a steady-state equilibrium, we show that (1) value functions have a continuous impact on the conversion rate of any two agents, (2) conversion rates have a continuous impact on the distribution of agents in the singles pool, and (3) the value functions are given by a fixed point of a continuous operator. We then invoke Schauder's fixed point theorem to establish that a fixed point exists.

Recall that $\overline{W} \equiv \frac{\mu}{(r+\mu+\delta)r}$ is an upper bound on the continuation value of single agents (see proof of Proposition 4). Define the family $\mathcal{F}$ of functions from X to $[0, \overline{W}]$ that are weakly increasing and Lipschitz continuous (in x_ω) with modulus $K^\star > 0$. $\mathcal{F}$ is a subset of $C[0, \overline{W}]$ that is nonempty, bounded, closed, and convex. We endow this family of functions with the sup norm $||W_s|| = sup_{x \in X} |W_s(x)|$.

As explained in the proof of Lemma 2, every agent either marries or breaks up with her/his partner by the time the belief about the couple's compatibility reaches $q_{\min} = rc/\lambda$. Therefore, for every $x, y \in X$, it holds that $\alpha(x,y) \in [0, \frac{q_0(x_\omega,y_\omega)-q_{min}}{1-q_{min}}]$.

We denote by $q^{W_s}(x)$ the minimum between (i) agent x's breakup threshold when her/his continuation value while single is $W_s(x)$ (as given by the optimality condition (4)), and (ii) the maximal probability that a couple is compatible. That is,[20]

$$q^{W_s}(x) \equiv \min\{\frac{rW_s(x)+c}{1-rW_s(x)}\frac{r+\delta}{\lambda}, q_0(1,1)\}.$$

We denote by $\alpha^{W_s}: X^2 \to [0, \frac{q_0(x_\omega,y_\omega)-q_{min}}{1-q_{min}}]$ a mapping that specifies the conversion rate for any pair of agents, when they behave according to the breakup thresholds given by q^{W_s}. We endow this family with the sup norm $||\alpha^{W_s}|| = sup_{(x,y)\in X^2}|\alpha^{W_s}(x,y)|$.

Lemma A.1 *$\alpha^{W_s}(\cdot,\cdot)$ is continuous in W_s.*

Proof of Lemma A.1. From (A.3) it follows that

$$\alpha^{W_s}(x,y) = \begin{cases} \frac{q_0(x_\omega,y_\omega)-\frac{\lambda}{\lambda_{xy}}\max\{q^{W_s}(x),q^{W_s}(y)\}}{1-\frac{\lambda}{\lambda_{xy}}\max\{q^{W_s}(x),q^{W_s}(y)\}} & \text{, if } \frac{\lambda}{\lambda_{xy}}\max\{q^{W_s}(x),q^{W_s}(y)\} \le q_0(x_\omega,y_\omega) \\ 0 & \text{, otherwise} \end{cases}.$$

Since $W_s \in [0, \overline{W}]$, it holds that rW_s is bounded away from 1. Thus, from (4) it follows that $\frac{dq^{W_s}(x)}{dW_s(x)}$ is uniformly bounded from above. The derivative of the conversion rate of the couple $\langle x,y\rangle$ with respect to $\max\{q^{W_s}(x),q^{W_s}(y)\}$ is bounded by $\frac{q_0(x_\omega,y_\omega)-1}{(1-\max\{q^{W_s}(x),q^{W_s}(y)\})^2}$. Since q^{W_s} is bounded from above by $q_0(1,1) < \frac{1}{1+z}$, the absolute value of this derivative is also uniformly bounded. It follows that $\alpha^{W_s}(\cdot,\cdot)$ is continuous in W_s in the sup norm. ■

Let u_α and d_α denote, respectively, the steady-state measure of agents in the singles pool and the measure of agents who are dating, as functions of the conversion rate $\alpha(\cdot,\cdot)$. We endow both measures with the sup norm. The next lemma establishes that for any viable $\alpha(\cdot,\cdot)$ there is a unique u_α for which the balanced-flow condition (1) holds, and that this mapping is continuous. The proof of this lemma is analogous to the proof of step 1 of Lemma 4 in Shimer and Smith (2000).

Lemma A.2 *u_α and d_α are well defined and continuous.*

[20]In equilibrium, all agents must date someone, and so for any $W_s(x)$ that is part of an equilibrium, $q^{W_s}(x)$ is equal to the breakup threshold given by (4). The need to define q^{W_s} in this way is due to the fact that $\overline{W}$ is a loose bound on the value function.

Proof of Lemma A.2. First, we show that for any viable $\alpha(\cdot,\cdot)$ there is a unique u_α for which the balanced-flow condition (1) holds.

In the proof of Proposition 3 we derived Equation (A.5) that connects the measure of dating couples and the conversion rate. Plugging this equation into the balanced-flow condition (1) and rearranging yields

$$u_\alpha(x) = \frac{g(x)}{1+\int_X \frac{\mu}{\lambda_{xy}}\left\{\frac{\lambda_{xy}}{\delta}\alpha(x,y)-(1-q_0(x_\omega,y_\omega))\log\left(\frac{q_0(x_\omega,y_\omega)-\alpha(x,y)}{q_0(x_\omega,y_\omega)}\right)\right\}u_\alpha(y)dy}. \tag{A.8}$$

Define Ω to be the space of measurable functions from X to $[\log(\underline{l})-\log(1+\frac{\mu}{\beta\lambda}\bar{l}),\log(\bar{l})]$, where $\underline{l}=\underline{g}$ and

$$\bar{l}=\bar{g}\cdot\max\{1,\int_X\left\{\frac{\lambda}{\delta}\frac{q_0(x_\omega,y_\omega)-q_{min}}{1-q_{\min}}-(1-q_0(x_\omega,y_\omega))\log\left(\frac{q_{\min}(1-q_0(x_\omega,y_\omega))}{q_0(x_\omega,y_\omega)(1-q_{\min})}\right)\right\}dy\}.$$

For all $x\in X$ and $\nu\in\Omega$, define

$$\Psi_\alpha\nu(x)=\log\left(\frac{g(x)}{1+\int_X \frac{\mu}{\lambda_{xy}}\left\{\frac{\lambda_{xy}}{\delta}\alpha(x,y)-(1-q_0(x_\omega,y_\omega))\log\left(\frac{q_0(x_\omega,y_\omega)-\alpha(x,y)}{q_0(x_\omega,y_\omega)}\right)\right\}e^{\nu(y)}dy}\right),$$

where $u\equiv e^\nu$. Note that u_α solves the balanced-flow condition (1) if and only if $\nu=\Psi\nu$. Next, we show that Ψ is a contraction mapping, and so it has a unique fixed point.

It is straightforward to verify that Ψ_α is a map from Ω to Ω. For any $x\in X$ and $\nu^1,\nu^2\in\Omega$, it holds that

$$\begin{aligned}
&\Psi_\alpha\nu^2(x)-\Psi_\alpha\nu^2(x)\\
&=\log\left\{\frac{1+\int_X \frac{\mu}{\lambda_{xy}}\left\{\frac{\lambda_{xy}}{\delta}\alpha(x,y)-(1-q_0(x_\omega,y_\omega))\log\left(\frac{q_0(x_\omega,y_\omega)-\alpha(x,y)}{q_0(x_\omega,y_\omega)}\right)\right\}e^{\nu^1(y)}dy}{1+\int_X \frac{\mu}{\lambda_{xy}}\left\{\frac{\lambda_{xy}}{\delta}\alpha(x,y)-(1-q_0(x_\omega,y_\omega))\log\left(\frac{q_0(x_\omega,y_\omega)-\alpha(x,y)}{q_0(x_\omega,y_\omega)}\right)\right\}e^{\nu^2(y)}dy}\right\}\\
&\le\log\left[\frac{1+\int_X \frac{\mu}{\lambda_{xy}}e^{||\nu^1-\nu^2||}\left\{\frac{\lambda_{xy}}{\delta}\alpha(x,y)-(1-q_0(x_\omega,y_\omega))\log\left(\frac{q_0(x_\omega,y_\omega)-\alpha(x,y)}{q_0(x_\omega,y_\omega)}\right)\right\}e^{\nu^2(y)}dy}{1+\int_X \frac{\mu}{\lambda_{xy}}\left\{\frac{\lambda_{xy}}{\delta}\alpha(x,y)-(1-q_0(x_\omega,y_\omega))\log\left(\frac{q_0(x_\omega,y_\omega)-\alpha(x,y)}{q_0(x_\omega,y_\omega)}\right)\right\}e^{\nu^2(y)}dy}\right].
\end{aligned} \tag{A.9}$$

The first inequality uses the fact that the integrand is positive, and that $e^{\nu^1(y)}\le e^{\nu^2(y)}e^{||\nu^1-\nu^2||}$ for all $y\in X$ under the sup norm. The term in square brackets is increasing in $\alpha(\cdot,\cdot)$ and $u_\alpha\le\bar{g}$. Since $\alpha\le\frac{q_0(x_\omega,y_\omega)-q_{min}}{1-q_{\min}}$, it follows that the integral is bounded from above by $\bar{l}$.

Since $e^{||\nu^2-\nu^1||} > 1$, the fraction in (A.10) is increasing in the integral, and so

$$\Psi_\alpha \nu^2(x) - \Psi_\alpha \nu^1(x) \leq \log\left\{\frac{1+\frac{\mu}{\beta\lambda}e^{||\nu^1-\nu^2||}\bar{l}}{1+\frac{\mu}{\lambda}\bar{l}}\right\}.$$

Finally, observe that

$$\frac{\log\{1+e^{||\nu^1-\nu^2||}\frac{\mu}{\beta\lambda}\bar{l}\} - \log\{1+\frac{\mu}{\lambda}\bar{l}\}}{||\nu^1-\nu^2||} \leq \frac{\log\{\underline{l}+\frac{\mu}{\beta\lambda}\bar{l}^2(1+\frac{\mu}{\beta\lambda}\bar{l})\} - \log\{\underline{l}(1+\frac{\mu}{\beta\lambda}\bar{l})\}}{\log\{\bar{l}\} - \log\{\underline{l}\} + \log\{1+\frac{\mu}{\beta\lambda}\bar{l}\}} \equiv \chi \in (0,1).$$

It follows that $|\Psi_\alpha \nu^2(x) - \Psi_\alpha \nu^1(x)| \leq \chi||\nu^1-\nu^2||$. Thus, Ψ_α is a contraction mapping, and there is a unique steady-state mass of singles that is consistent with any viable $\alpha(\cdot,\cdot)$. This, in turn, implies that $d_\alpha(x,y)$ is well defined for any $x, y \in X$ (Equation (A.8)), and hence d_α is well defined.

We now establish the continuity of u_α. Rearranging (A.8) yields

$$\int_X u_\alpha(y)\left\{\alpha(x,y)(\frac{\mu}{\delta}+\frac{\mu}{\lambda_{xy}}) - \frac{\mu}{\lambda_{xy}}(1-q_0(x_\omega,y_\omega))\log\left(\frac{q_0(x_\omega,y_\omega)-\alpha(x,y)}{q_0(x_\omega,y_\omega)}\right)\right\}dy = \frac{g(x)}{u_\alpha(x)} - 1. \tag{A.10}$$

Consider how changing the value of $\alpha(\cdot,\cdot)$ by at most ϵ (for any element in its domain) impacts the term in curly brackets in (A.10). The change in the first term inside the curly brackets is at most

$$\epsilon(\frac{\mu}{\delta}+\frac{\mu}{\beta\lambda}).$$

Since $\alpha(x,y) \leq \frac{q_0(x_\omega,y_\omega)-q_{min}}{1-q_{min}}$, it follows that $q_0(x_\omega,y_\omega)-\alpha(x,y) \geq q_{min}\frac{1-q_0(x_\omega,y_\omega)}{1-q_{min}}$, and so the absolute value of the change in the second term inside the curly brackets is at most

$$\frac{\mu(1-q_{min})}{\beta\lambda q_{min}}\epsilon.$$

Moreover, note that since $u_\alpha(x) \in [0,\overline{g}]$ for any α, the absolute value of the change in $u_\alpha(x)$ due to any change in α is at most $\overline{g}$. It follows that such a change in α can change the absolute value of the LHS of (A.10) by at most

$$\overline{g}\left(\frac{\mu}{\delta}+\frac{\mu}{\beta\lambda}+\frac{\mu(1-q_{min})}{\beta\lambda q_{min}}\right)\epsilon.$$

Therefore, u_α is continuous in α in the sup norm. ■

Next, we construct the operator whose fixed points represent the set of equilibria in our model. By (2), an equilibrium value function must satisfy

$$rW_s(x) = \mu \int_X (W_d^{W_s}(x;y) - W_s(x))u^{W_s}(y)dy,$$

where $W_d^{W_s}(x;y)$ is agent x's continuation utility upon meeting agent y given W_s (note that $W_d^{W_S}(x;y)$ is not the capital gain from dating), and u^{W_s} are the densities in the singles pool that are consistent with value functions W_s. Adding the expectation of $W_s(x)$ to both sides and rearranging, we define the operator Γ by

$$\Gamma W_s(x) = \frac{\mu}{r + \mu \overline{u}^{W_s}} \int_X W_d^{W_s}(x;y)u^{W_s}(y)dy, \tag{A.11}$$

where $\overline{u}^{W_s} = \int_X u^{W_s}(y)dy$.

Lemma A.3 *If $K^\star$ is sufficiently large, then $W_s \in \mathcal{F}$ implies that $\Gamma(W_s) \in \mathcal{F}$.*

Poof of Lemma A.3. First, we show that $\Gamma W_s(x)$ lies in $[0, \overline{W}]$. Since $W_d^{W_s}$ is nonnegative, it is immediate that $\Gamma W_s(x) \geq 0$. An upper bound on agent x's value upon meeting agent y is given by her/his value from marrying a compatible partner immediately, separating when the dissolution shock occurs, and then being matched again to agent y according to a Poisson process with arrival rate μ. Thus,

$$W_d^{W_s}(x;y) \leq \frac{1}{r+\delta} + \frac{\delta}{r+\delta}\frac{\mu}{r+\mu}W_d^{W_s}(x;y),$$

which implies that

$$W_d^{W_s}(x;y) \leq \frac{\mu + r}{r(\delta + \mu + r)}.$$

It follows that

$$\Gamma W_s(x) \leq \frac{\mu \overline{u}^{W_s}}{r + \mu \overline{u}^{W_s}} \frac{\mu + r}{r(\delta + \mu + r)} \leq \overline{W}.$$

Next, note that $\Gamma W_s(\cdot, \theta)$ is weakly increasing. This follows from $W_s(\cdot, \theta)$ being increasing, as it is an element of $\mathcal{F}$, which implies that an agent who has level of attractiveness x_ω can perfectly duplicate the dating time of an agent who has level of attractiveness $x'_\omega < x_\omega$ with any potential partner.

Finally, we show that for sufficiently large $K^\star$, $\Gamma W_s(\cdot, \theta)$ is Lipschitz continuous with modulus $K^\star$. Since $\Gamma W_s(\cdot, \theta)$ is increasing, it suffices to show that $\Gamma W_s(x_\omega + \epsilon, \theta) - \Gamma W_s(x_\omega, \theta) \leq \epsilon K^\star$. Note that

$$W_d^{W_s}(x;y) = V_d^{W_s}(x;y) + W_s(x),$$

where $V_d^{W_s}(x;y)$ is agent x's capital gain from meeting agent y, given value functions W_s. Using this representation, it follows that

$$\Gamma W_s(x_\omega+\epsilon,\theta)-\Gamma W_s(x,\theta)=\frac{\mu}{r+\mu\overline{u}^{W_s}}\int_X(V_d^{W_s}((x_\omega+\epsilon,\theta);y)-V_d^{W_s}((x_\omega,\theta);y))u^{W_s}(y)dy$$
$$+\frac{\mu}{r+\mu\overline{u}^{W_s}}\int_X(W_s(x_\omega+\epsilon,\theta)-W_s(x_\omega,\theta))u^{W_s}(y)dy.$$

Since $W_s(\cdot,\theta)$ is increasing and continuous (as it is an element of $\mathcal{F}$), the same arguments used in the proof of Lemma 2 show that $\int_X V_d^{W_s}(x;y)u^{W_s}(y)dy$ is Lipschitz continuous in x_ω with modulus $\frac{r}{\mu}\overline{K}$. Thus,

$$\Gamma W_s(x_\omega+\epsilon,\theta)-\Gamma W_s(x,\theta)\leq\frac{\mu}{r+\mu\overline{u}^{W_s}}\int_X(\frac{r}{\mu}\overline{K}\epsilon+K^\star\epsilon)u^{W_s}(y)dy\leq(\frac{r}{r+\mu}\overline{K}+\frac{\mu}{r+\mu}K^\star)\epsilon.$$

Hence, if $K^\star$ is sufficiently large, then $\Gamma W_s(\cdot,\theta)$ is Lipschitz continuous with modulus $K^\star$ for $\theta\in\{A,B\}$, and thus $\Gamma:\mathcal{F}\to\mathcal{F}$. ■

Lemma A.4 *The operator* Γ *is continuous.*

Proof of Lemma A.4. If W_s^1 and W_s^2 are close under the sup norm, then the dating times of any couple are close under these two value functions. Since small changes in W_s also induce small changes in u^{W_s} (by Lemmata A.1 and A.2), it follows that small changes in W_s have a small impact on ΓW_s. ■

We have therefore shown that $\mathcal{F}$ is closed, bounded, convex, and nonempty. Moreover, since $\mathcal{F}$ is a family of Lipschitz continuous functions with the same modulus, it is equicontinuous. We have also shown that Γ is a continuous mapping from $\mathcal{F}$ to $\mathcal{F}$. Thus Schauder's fixed point theorem (Theorem 17.4 in Stokey and Lucas, 1989) establishes that Γ has a fixed point, which proves Theorem 6. ■