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(Asset) Pricing the Business Cycle

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(Asset) Pricing the Business Cycle

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Abstract

Optimal hiring and investment over the business cycle is governed by asset values – the expected present value of workers (Q^N), of capital (Q^K), and of the firm (Q). Importantly, labor and capital *Qs* are inter-related. The paper formalizes the connections between aggregate shocks (TFP, investment-specific, and matching technology), the afore-cited asset values, investment and hiring decisions, and the production of aggregate output.

Using aggregate U.S. data and structural estimation, time series for the unobserved *Qs* are derived; a local projections methodology is then used to study the cyclical behavior of asset values and of the decision variables.

Key words: Firm value, labor value, capital value, optimal investment, optimal hiring, business cycles.

JEL codes: E32, G31, J63.

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(Asset) Pricing the Business Cycle[1]

1 Introduction

When firms hire workers and invest in capital, they take into account both current costs of these activities and their expected, discounted, future gains. Hence these activities are essentially investment activities. Firms are facing expected present discounted values of workers and of capital and comparing them to costs. These asset values are unobserved; in formal terms they are the Lagrange multipliers of the relevant optimization conditions. This paper seeks to estimate them using aggregate U.S. data and to characterize their behavior over the business cycle. The study of business cycle behavior includes the response of investment and hiring to the estimated asset values and the changing sensitivities to these values and to specific components over the cycle. The paper formalizes the connections between aggregate shocks, the afore-cited asset prices, investment and hiring decisions, and the production of aggregate output.

I rely on the existence of frictions in investment and in hiring to formulate the relevant dynamic optimization problem, with emphasis on the joint optimality of investment and hiring. These give rise to this asset-pricing type of analysis, with three relevant shadow values, to be denoted by Qs. I denote by Q total firm value, i.e., the expected discounted value of the firm; by Q^N – the value of the worker to the firm in the same terms; and by Q^K– the value of capital to the firm, also in expected present value terms. Using the CRS properties of the production and costs functions, firm value Q is the sum of the two latter values, each multiplied by the number of input units. The latter two Qs are inter-related. Shocks affect each of the Qs and create cyclical fluctuations. These include TFP, investment-specific, and matching technology shocks. While I use the Q terminology of q, the analysis makes no use of financial market data or other firm price data. The idea here is to use U.S. data to structurally estimate these shadow values, pertaining to firm profits from capital and from labor using data on variables, such as GDP, wages, relative prices, hiring and investment, all in real terms. The resulting time series allow for the study of the following five empirical questions:

First, does the model fit U.S. aggregate data and how does it fare relative to the performance of related models in this context – the Tobin's q and search and matching models?

Second, what do we learn about the magnitude of asset values relevant to investment and hiring?

Third, studying firm optimal behavior over the cycle – how do investment and vacancy creation react to shocks? How do they co-move with the afore-cited asset values? What are the directions of co-movement, the strength of reaction to shocks, and the changing sensitivities over the cycle?

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Fourth, looking at cross effects, how do capital values affect vacancy creation, and how do labor values and labor market conditions affect capital investment?

Fifth, what are the implications for policy, in particular in terms of the corporate tax rate?

Following structural estimation, I use a local projections methodology, proposed by Jorda (2005) and Daly et al (2018), to study the cyclical behavior of the (estimated) *Qs* as well as of investment, hiring, and output. I study the cyclical behavior of the *Qs* and the decision variables (investment and vacancy creation), both statically (unconditional on shocks), and dynamically (conditional on the afore-cited shocks). The analysis yields an understanding of optimal investment and hiring over the business cycle, with asset prices, in the form of the afore-cited *Qs*, playing a key role.

The key findings are as follows. The model fits the data well. Prevalent models omit important cross effects (capital values on labor and vice versa). The model works well with moderate asset values; i.e., it does not need big frictions to fit thee data. Procyclical asset values engender pro-cylical investment and vacancy creation. There are, however, interesting aysmmetries: ELABORATE

The paper proceeds as follows: Section 2 places the paper in the relevant contexts in the literature. Section 3 presents the model and the relations to be examined empirically, elaborating on the various Q concepts. Three sections present the empirical work: Section 4 presents structural estimation, using aggregate U.S. data, deriving the shadow asset values, and comparing the estimates to prevalent specifications. Section 5 discusses the implications of the estimates for firm's optimal behavior, including the linkages between the decision variables and asset values. Section 6 presents the the cyclical analysis. Section 7 uses the results of the preceding three sections to provide an integrated picture and concludes. Derivations and other technical matters are relegated to appendices.

2 Literature

In what follows I briefly place the analysis of the current paper in the relevant contexts in the macroeconomic literature.

2.1 A Brief History of q

The seminal papers explicitly introducing the concept of q were the papers by Tobin (1969) and Tobin and Brainard (1969). *Tobin (1981) defined the variable* q^K *as "the ratio of market valuation of capital goods to normal replacement cost at time period" (p.21) and posited that investment is a function of* q^K*. He noted that "the deviations of* q^K *from 1 represent real costs of adjustment, including positive or negative rents, incurred by investing firms in changing the size of their installed capital." (p.22)*

The formulation used here was initially suggested by an important early literature on adjustment costs of factor inputs, studying the accumulation of all factors of production. Key papers include Lucas (1967) and Mortensen (1973), who derived firm optimal behavior with convex adjustment costs for n factors of production. *Mortensen's summary of Lucas (footnote 4 on p. 659), states that "Adjustment costs arise in the view of Lucas either because installation and planning involves the use of internal resources or because the firm is a monopsonist in its factor markets. Since Lucas rules out the possibility of interaction with the production process, the costs are either the value of certain perfectly variable resources used exclusively in the planning and installation processes or the premium which the firm must pay in order to obtain the factors at more rapid rates."* Treadway (1971) considered (p.878) "the marginal internal cost of investment ($-f_{\dot{x}}$) arising from the current product "lost" due to the expansion activity of the firm." Lucas and Prescott (1971) embedded these convex adjustment costs in stochastic industry equilibrium. Nadiri and Rosen (1969) considered interrelated factor demand functions for labor and capital with adjustment costs. A formalization of the q concept within the latter set of models was offered by Hayashi (1982). Over the years much empirical work was done. Prominent examples include Nadiri and Rosen (1969), Abel (1980), Summers (1981), Shapiro (1986), and Hall (2004). Chirinko (1993) and Smith (2008) offer reviews.

2.2 Modelling Hiring and Investment Costs

The current paper places emphasis on hiring frictions of various kinds, investment frictions, and their interactions. Hiring costs include costs of advertising, screening and testing, matching frictions, training costs, and more. Thus they pertain to vacancy posting, actual hires from non-employment, and hires from employment (job to job movements). Investment involves capital installation costs, implementation costs, learning the use of new equipment, etc. All activities may involve, in addition to production disruption, the implementation of new organizational structures within the firm and new production techniques. On the latter see Alexopoulos and Tombe (2012).

2.2.1 Functional Form

I use a convex cost function. While non-convexities were found to be significant at the micro level (plant, establishment, or firm), a number of papers have given empirical support for the use of a convex function in the aggregate, showing that such a formulation is appropriate at the macroeconomic level. Thus, Thomas (2002) and Kahn and Thomas (2008, see in particular their discussion on pages 417-421) study a dynamic, stochastic, general equilibrium model with nonconvex capital adjustment costs. One key idea which emerges from their analysis is that there are smoothing effects that result from equilibrium price changes. House (2014) shows that even though neoclassical investment models are inconsistent with micro data, they capture the relevant aggregate investment dynamics embodied in models with fixed investment adjustment costs. On page 99 he states that "This finding is highly robust and explains why researchers working in the DSGE tradition have found little role for fixed costs in numerical trials." This is due to the "The near-infinite elasticity of intertemporal substitution (which) eliminates virtually any role for microeconomic heterogeneity in governing investment demand."

2.2.2 Frictions and Their Interactions

Recent micro studies have looked at hiring costs in their various forms in detail; see Blatter et al (2016) and Mühlemann and Leiser (2018). The resulting picture is that of convex costs placed mainly on training, with a more limited role for vacancy costs.

In the model I make a distinction between job to job movements and hiring from non-employment. Thus, a micro study of a large hospital system by Bartel, Beaulieu, Phibbs, and Stone (2014) shows such distinction is warranted. They find that the arrival of a new nurse is associated with lowered productivity, but that this effect is significant only if the nurse is hired externally.

A recent theoretical and empirical literature has given foundations to investment-hiring costs interaction terms, which I use below. This new literature looks at the connections between investment in capital, the hiring of workers, and organizational and management changes. A general discussion and overview of this line of research is offered by Ichniowsky and Shaw (2013) and by Lazear and Oyer (2013). One specific example is provided by Bartel, Ichniowski and Shaw (2007), who study the effects of new information technologies (IT) on productivity. They use data on plants in one narrowly defined industry, valve manufacturing. Their empirical analysis reveals, inter alia, that adoption of new IT-enhanced capital equipment coincides with increases in the skill requirements of machine operators, notably technical and problem-solving skills, and with the adoption of new human resource practices to support these skills. They show how investment in capital equipment has a variety of effects on hiring and on training.

2.3 Previous Work

In previous work I have used a similar framework but explored different issues.

In Merz and Yashiv (2007) the focus was on production-based asset pricing. We investigated the links between the financial and labor markets, using a production-based model for firms' market value following Cochrane (1991).[2] We inserted into the model labor and capital adjustment costs, with the adjustment costs for labor interacting with those for capital. We showed that this framework can account well for U.S. stock prices.

In Yashiv (2016) I decomposed the future determinants of capital and job values. I found complementarity between the hiring and investment processes; important cross effects of the value of capital on the mean and the volatility of the hiring rate, and vice versa; and that future returns are shown to play a dominant role in determining capital and job values. I also used this framework to analyze U.S.labor market developments in the Great Recession and its aftermath 2007–2013.

The one point of overlap of this paper with the two cited papers is structural estimation of the firms' optimality equations. In the current paper the sample period is updated and the specification estimtaed is wider, i.e., it nests the previous ones as special cases.

[2] See also the more recent contributions of Cochrane (2007, 2017).

3 The Model

The model formulates optimal hiring and investment decisions in the in the presence of frictions. Investment and hiring are treated symmetrically, and interactions of investment and hiring play a major role. In developing the model I define a Q for capital Q_t^K, an analog concept for labor Q_t^N, and a general Q_t which is a function of both. The model, to be taken to the data, is a partial equilibrium model, intended to avoid potential misspecifications in the households sector, in financial markets, and in the formulation of monetary and fiscal policy. I include a discussion of important special cases, which are prevalent in the capital and labor literatures.

3.1 Firm Optimization

There are identical workers and identical firms. All agents live forever and have rational expectations.

Firms make gross investment (i_t) and vacancy (v_t) decisions. Once a new worker is hired, the firm pays him or her a per-period wage w_t. Firms use physical capital (k_t) and labor (n_t) as inputs in order to produce output goods y_t according to a constant-returns-to-scale production function f with TFP denoted by z_t:

$$y_t = f(z_t, n_t, k_t), \tag{1}$$

TFP follows the process:

$$\ln z_t = \kappa_1 + \ln z_{t-1} + \varepsilon_t^f \tag{2}$$

where ε_t^f is a shock and κ_1 a parameter.

Vacancies, gross hiring, and gross investment are subject to frictions, spelled out below, and hence are costly activities. I represent these costs by a function $g[i_t, k_t, v_t, h_t, n_t]$ which is convex in the firm's decision variables (i_t, v_t) and exhibits constant returns-to-scale, allowing hiring costs and investment costs to interact.

In every period t, the capital stock depreciates at the rate δ_t and is augmented by new investment i_t. Similarly, workers separate at the rate ψ_t and the employment stock is augmented by new hires $q_t v_t = h_t$. The laws of motion are:

$$k_{t+1} = (1 - \delta_t)k_t + i_t, \quad 0 \leq \delta_t \leq 1. \tag{3}$$

$$n_{t+1} = (1 - \psi_t)n_t + q_t v_t, \quad 0 \leq \psi_t \leq 1 \tag{4}$$

The vacancy filling rate q_t embodies a matching shock μ_t following the AR1 process:

$$\ln \mu_t = \kappa_2 + \rho_\mu \ln \mu_{t-1} + \varepsilon_t^\mu \tag{5}$$

where ε_t^μ is a matching shock and κ_2, ρ_μ are parameters.

The representative firm chooses sequences of i_t and v_t in order to maximize its profits as follows:

$$\max_{\{i_{t+j},v_{t+j}\}} E_t \sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} \rho_{t+i} \right) (1-\tau_{t+j}) \left(\begin{array}{c} f(z_{t+j}, n_{t+j}, k_{t+j}) - g\left(i_{t+j}, k_{t+j}, v_{t+j}, n_{t+j}\right) \\ -w_{t+j}n_{t+j} - \left(1-\chi_{t+j} - \tau_{t+j}D_{t+j}\right) \widetilde{p}^I_{t+j}\ i_{t+j} \end{array} \right) \tag{6}$$

subject to the constraints (3) and (4), where τ_t is the corporate income tax rate, w_t is the wage, χ_t the investment tax credit, D_t the present discounted value of capital depreciation allowances, $\tilde{p}^I_t$ the real pre-tax price of investment goods, and ρ_{t+j} is a time-varying discount factor.In line with the investment technology literature, $\tilde{p}^I_t$ is driven by an unanticipated IST shock as follows, using an AR1 process for the log of the inverse of the investment price:

$$\begin{aligned} \widetilde{p}^I_t &\equiv \frac{1}{\Theta_t} \\ \ln \Theta_t &= \kappa_3 + \rho_\Theta \ln \Theta_{t-1} + \varepsilon^I_t \end{aligned} \tag{7}$$

where ε^I_t is the shock and κ_3, ρ_Θ are parameters.

The firm takes the paths of the variables $z_t, q_t, w_t, \psi_t, \widetilde{p}^I_t, \delta_t, \tau_t$ and ρ_t as given. This is consistent with the standard models in the search and matching and Tobin's q literatures. The Lagrange multipliers associated with these two constraints are denoted Q^K_{t+j} and Q^N_{t+j}, respectively. These Lagrange multipliers can be interpreted as marginal Q for physical capital, and marginal Q for employment, respectively. I shall use the term capital value for the former, and labor value for the latter.

The first-order conditions for dynamic optimality are:[3]

$$Q^K_t = E_t \left[\rho_{t+1} \left[(1-\tau_{t+1}) \left(f_{k_{t+1}} - g_{k_{t+1}}\right) + (1-\delta_{t+1})\, Q^K_{t+1}\right]\right] \tag{8}$$

$$Q^K_t = (1-\tau_t) \left(g_{i_t} + p^I_t\right) \tag{9}$$

$$Q^N_t = E_t \left[\rho_{t+1} \left[(1-\tau_{t+1}) \left(f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1}\right) + \left(1-\psi_{t+1}\right) Q^N_{t+1}\right]\right] \tag{10}$$

$$Q^N_t = (1-\tau_t) \frac{g_{v_t}}{q_t} \tag{11}$$

These equations determine the capital value Q^K_t and the job value Q^N_t. Basically the capital value is the present value of expected marginal productivities, adjusted for taxes and depreciation; the job value is the present value of the profit flows from the marginal worker adjusted for taxes and separation rates.

I can summarize the firm's first-order necessary conditions from equations (8)-(11) by the following two expressions:

[3] where I use the real after-tax price of investment goods, given by:

$$p^I_{t+j} = \frac{1-\chi_{t+j} - \tau_{t+j}D_{t+j}}{1-\tau_{t+j}} \widetilde{p}^I_{t+j}$$

$$(1-\tau_t)\left(g_{i_t}+p_t^I\right) = E_t\left[\rho_{t+1}(1-\tau_{t+1})\left[\begin{array}{c} f_{k_{t+1}} - g_{k_{t+1}} \\ +(1-\delta_{t+1})(g_{i_{t+1}} + p_{t+1}^I) \end{array}\right]\right] \tag{12}$$

$$(1-\tau_t)\frac{g_{v_t}}{q_t} = E_t\left[\rho_{t+1}(1-\tau_{t+1})\left[\begin{array}{c} f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \\ +(1-\psi_{t+1})\frac{g_{v_{t+1}}}{q_{t+1}} \end{array}\right]\right] \tag{13}$$

These equations are at the focal point of the analysis. Following the explicit formulation of the costs function g I shall consider alternative special cases.

3.2 Investment and Hiring Costs

The costs function g, capturing the different frictions in the hiring and investment processes, is at the focus of the estimation work. The literature review above has spelled out what these frictions are. To summarize what is modelled here, investment involves implementation costs, capital installation costs, learning the use of new equipment, etc. Hiring costs include costs of advertising, screening and testing, matching frictions, training costs and more. Both activities may involve, in addition to production disruption, the implementation of new organizational structures within the firm and new production techniques. In sum g is meant to capture all the frictions involved in getting workers to work and capital to operate in production, and not, say, just capital adjustment costs or vacancy costs. One should keep in mind that this is formulated as the costs function of the representative firm, and not one of a single firm in a heterogenous firms set-up. In what follows I look more closely at the arguments and functional form of this function.

3.2.1 Hiring and Separation Flows[4]

Before formulating the function one needs to define worker flows. The flow from non-employment – unemployment (U) and out of the labor force (O) – to employment, E, is to be denoted $OE + UE$ and the separation flow in the opposite direction, $EU + EO$. Worker flows within employment – i.e., job to job flows – are to be denoted EE.

I shall denote:

$$\begin{aligned} \frac{h}{n} &= \left(\frac{h^1}{n}\right) + \left(\frac{h^2}{n}\right) \\ \frac{h^1}{n} &= \frac{OE+UE}{E}; \quad \frac{h^2}{n} = \frac{EE}{E} \end{aligned} \tag{14}$$

Hence h^1 and h^2 denote flows from non-employment and from other employment, respectively, and n is employment.

Separation rates are given in an analogous way by:

[4] I am indebted to Giuseppe Moscarini for very useful suggestions to this sub-section.

$$\begin{aligned} \psi &= \psi^1 + \psi^2 \\ \psi^1 &= \frac{EO + EU}{E}; \quad \psi^2 = \frac{EE}{E} = \frac{h^2}{n} \end{aligned} \tag{15}$$

Employment dynamics are thus given by:

$$\begin{aligned} n_{t+1} &= (1 - \psi_t^1 - \psi_t^2) n_t + h_t^1 + h_t^2 \\ &= (1 - \psi_t) n_t + h_t, \quad 0 \leq \psi_t \leq 1 \\ h_t^2 &= \psi_t^2 n_t \end{aligned} \tag{16}$$

Firms hire from non-employment (h_t^1) and from other firms (h_t^2). Each period, the worker's effective units of labor (normally 1 per person) depreciate to 0, in the current firm, with some exogenous probability ψ_t. Thus, the match suffers an irreversible idiosyncratic shock that makes it no longer viable. The worker may be reallocated to a new firm where his/her productivity is (temporarily) restored to 1. This happens with a probability of ψ_t^2. Those who are not reallocated join unemployment, with probability $\psi_t^1 = \psi_t - \psi_t^2$. So the fraction ψ_t^2 that enters job to job flows depends on the endogenous hiring flow h_t^2. The firm decides how many vacancies v_t to open and, given job filling rates (q_t^1, q_t^2), will get to hire from the pre-existing non-employed and from the pool of matches just gone sour. The job-filling or matching rates satisfy:

$$q_t^1 = \frac{h_t^1}{v_t}; \quad q_t^2 = \frac{h_t^2}{v_t}; \quad q_t = q_t^1 + q_t^2$$

3.2.2 Functional Form of the Costs Function

Following the literature review in sub-section 2.2.2, the parametric form I use is the following, generalized convex function.

$$g(\cdot) = \begin{bmatrix} \frac{e_1}{\eta_1}\left(\frac{i_t}{k_t}\right)^{\eta_1} \\ +\frac{e_2}{\eta_2}\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2} \\ +\frac{e_{31}}{\eta_{31}}\left(\frac{i_t}{k_t}\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}}\left(\frac{i_t}{k_t}\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}} \end{bmatrix} f(z_t, n_t, k_t). \tag{17}$$

The basic idea is of a convex function of the rates of activity – investment ($\frac{i_t}{k_t}$) and recruiting ($\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 h_t^1+\lambda_2 h_t^2}{n_t}$). This function is linearly homogenous in its arguments i, k, v, n. The parameters e_l, $l = 1, 2, 31, 32$ express scale, and the parameters $\eta_1, \eta_2, \eta_{31}, \eta_{32}$ express the convexity of the costs function with respect to its different arguments. λ_1 is the weight in the cost function assigned to hiring from non-employment ($\frac{h_t^1}{n_t}$), λ_2 is the weight assigned to hiring from other firms ($\frac{h_t^2}{n_t}$), and $(1 - \lambda_1 - \lambda_2)$ is the weight assigned to vacancy ($\frac{v_t}{n_t}$) costs. The weights λ_1 and λ_2 are thus related to the training and production disruption aspects, while the complementary weight is related to the vacancy creation aspect. The last two terms in square brackets capture interactions between investment and hiring. For these it differentiates between interaction of hiring from employment and those of hiring from non-employment. When a parameter is estimated, there is no constraint placed on its sign or magnitude.

This specification captures the idea that frictions or costs increase with the extent of the activity in question – vacancy creation, hiring and investment. This needs to be modelled relative to the size of the firm. The intuition is that hiring 10 workers, for example, means different levels of hiring activity for firms with 100 workers or for firms with 10,000 workers. Hence firm size, as measured by its physical capital stock or its level of employment, is taken into account and the costs function is increasing in the vacancy, hiring and investment rates, $\frac{v}{n}$ and $\frac{i}{k}$. The function used postulates that costs are proportional to output, i.e., the results can be stated in terms of lost output.

More specifically, the terms in the function presented above may be justified as follows (drawing on Garibaldi and Moen (2009)): suppose each worker i makes a recruiting and training effort h_i; as this is to be modelled as a convex function, it is optimal to spread out the efforts equally across workers so $h_i = \frac{h}{n}$; formulating the costs as a function of these efforts and putting them in terms of output per worker one gets $c\left(\frac{h}{n}\right)\frac{f}{n}$; as n workers do it then the aggregate cost function is given by $c\left(\frac{h}{n}\right)f$.

The terms $\frac{e_{31}}{\eta_{31}}\left(\frac{i_t}{k_t}\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}}$ and $\frac{e_{32}}{\eta_{32}}\left(\frac{i_t}{k_t}\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}}$ express the interaction of investment and hiring costs. They allow for a different interaction for hires from non-employment (h_t^1) and from other firms (h_t^2). These terms, absent in many studies, have important implications for the complementarity of investment and hiring.

3.3 Important Special Cases

Beyond the general model spelled out above, I specifically examine important special cases, which have been widely used in the capital and labor literatures.

3.3.1 A Single Factor Tobin's q Approach

Obvious special cases of the above follow the literature on Tobin's q, relying on the seminal contributions of Tobin (1969) and Hayashi (1982). This approach ignores the other factor of production (i.e., assumes no adjustment costs for it). In the current case, this is either convex costs of investment in capital, with no hiring costs or convex costs of hiring, with no investment costs. Typically quadratic costs are posited. Hence in the former case this has $e_2 = e_{31} = e_{32} = 0$ and $\eta_1 = 2$ and in the latter case $e_1 = e_{31} = e_{32} = 0$ and $\eta_2 = 2$. The optimality equations become:

$$(1-\tau_t)\left(e_1(\frac{i_t}{k_t})^{\eta_1-1} + p_t^I\right) = E_t\left[\rho_{t+1}(1-\tau_{t+1})\left[\begin{array}{c} f_{k_{t+1}} - g_{k_{t+1}} \\ +(1-\delta_{t+1})(g_{i_{t+1}} + p_{t+1}^I) \end{array}\right]\right] \quad (18)$$

$$(1-\tau_t)\frac{e_2}{q_t}\begin{bmatrix}(1-\lambda_1-\lambda_2)\\+\lambda_1 q_t^1+\lambda_2 q_t^2\end{bmatrix}^2\frac{v_t}{n_t}=E_t\frac{1}{\frac{f_t}{n_t}}\left[\rho_{t+1}(1-\tau_{t+1})\begin{bmatrix}f_{n_{t+1}}-g_{n_{t+1}}-w_{t+1}\\+(1-\psi_{t+1})\frac{g_{v_{t+1}}}{q_{t+1}}\end{bmatrix}\right] \tag{19}$$

3.3.2 The Standard Search and Matching Model

The standard search and matching model – see Pissarides (2000) for an overview – does not consider investment when formulating costs and refers to linear vacancy costs. In terms of the model above it has $e_1 = e_{31} = e_{32} = 0, \lambda_1 = \lambda_2 = 0$ and $\eta_2 = 1$. It thus formulates the optimality equation for vacancy creation (v_t) as follows, i.e., this is equation (13) for this particular model.

$$(1-\tau_t)\frac{e_2}{q_t}\frac{f_t}{n_t}=E_t\left[\rho_{t+1}(1-\tau_{t+1})\left[f_{n_{t+1}}-w_{t+1}+(1-\psi_{t+1})\frac{e_2}{q_{t+1}}\frac{f_{t+1}}{n_{t+1}}\right]\right] \tag{20}$$

This specification is a prevalent formulation in the labor search and matching literature, that has total costs be a linear function of vacancies, i.e., $\frac{e_2}{q_t}\frac{f_t}{n_t}v_t$ whereby the cost is proportional to labor productivity $\frac{f_t}{n_t}$ and depends on the average duration of the vacancy $\frac{1}{q_t}$ (q_t is the job filling rate, $q_t = \frac{h_t}{v_t}$).

3.4 Business Cycle Qs

I formulate the relevant asset prices inherent in the analysis. Aggregate Q_t is a function of the afore-cited Q_t^K and Q_t^N. Appendix A shows the full derivation, yielding:

$$Q_t = k_{t+1}Q_t^K + n_{t+1}Q_t^N, \tag{21}$$

In stationary terms, divided by GDP, this aggregate Q values is given by:

$$\frac{Q_t}{f_t}=\frac{k_{t+1}}{k_t}\frac{Q_t^K}{\frac{f_t}{k_t}}+\frac{n_{t+1}}{n_t}\frac{Q_t^N}{\frac{f_t}{n_t}} \tag{22}$$

The terms on the RHS of (22) are given by the following expressions, in terms of output per unit of input (using equations (8) –(11)) :

$$\frac{Q_t^K}{\frac{f_t}{k_t}}=(1-\tau_t)\left(\frac{g_{i_t}}{\frac{f_t}{k_t}}+\frac{p_t^I}{\frac{f_t}{k_t}}\right) \tag{23}$$

$$\frac{Q_t^N}{\frac{f_t}{n_t}}=(1-\tau_t)\frac{\frac{g_{v_t}}{q_t}}{\frac{f_t}{n_t}} \tag{24}$$

One can further sub-divide the capital value $\frac{Q_t^K}{\frac{f_t}{k_t}}$ into a term based on the price of investment p_t^I and a term relating to investment costs, as follows:

$$\frac{\widetilde{Q}_t^K}{\frac{f_t}{k_t}} = (1-\tau_t)\,\frac{g_{i_t}}{\frac{f_t}{k_t}} \tag{25}$$

$$\frac{Q_t^P}{\frac{f_t}{k_t}} = (1-\tau_t)\,\frac{p_t^I}{\frac{f_t}{k_t}} \tag{26}$$

4 Estimating Optimal Behavior and Asset Values

In order to be evaluated empirically, the afore-going optimality equations of the firm will be estimated. This allows for the derivation of time series of the various costs and of the different Qs (as formulated in equations (22) – (26)), which are unobserved.

4.1 The Data

The data are quarterly and pertain to the aggegate private sector of the U.S. economy. For a large part of the empirical work reported below the sample period is 1994-2016. The start date of 1994 is due to the lack of availability of job to job worker flows (h_t^2) data prior to that. For another part of the empirical work, the sample covers 1976-2016. The 1976 start is due to the availability of credible monthly CPS data, from which the gross hiring flows (h_t^1) series (from non-employment) is derived. This longer sample period covers five NBER-dated recessions, and both sample periods include the Great Recession (2007-2009) and its aftermath. The data include NIPA data on the NFCB GDP and its deflator, capital, investment, the price of investment goods and depreciation, BLS CPS data on employment and on worker flows, and Fed data computations on tax and depreciation allowances. Appendix B elaborates on the sources and on data construction. These data have the following distinctive features: (i) they pertain to the U.S. private sector; (ii) both hiring h_t and investment i_t refer to gross flows; likewise, separation of workers ψ_t and depreciation of capital δ_t are gross flows; (iii) the estimating equations take into account taxes and depreciation allowances.

4.2 Estimation Methodology

I structurally estimate the firms' first-order conditions – equation (12) and equation (13) – jointly, using Hansen's (1982) generalized method of moments (GMM). In what follows I outline the methodology and the alternative specifications used. Details are provided in Appendix C.

For the production function I use a standard Cobb-Douglas formulation, with a productivity shock $\exp(z_t)$:

$$f(z_t, n_t, k_t) = \exp(z_t) n_t{}^{\alpha} k_t^{1-\alpha},\ 0 < \alpha < 1. \tag{27}$$

The costs function g was spelled out above (see equation (17)). Estimation pertains to the parameters $\alpha; e_1, e_2, e_{31}, e_{32}; \eta_1, \eta_2, \eta_{31}, \eta_{32}, \lambda_1, \lambda_2$, or to a sub-set of these parameters.

Replacing expected values in these equations by actual ones and expectational errors ($j_t^{k,n}$), the estimation equations are given by (estimation is undertaken after dividing the investment equation by $\frac{f_t}{k_t}$ and the vacancy/hiring equation by $\frac{f_t}{n_t}$ to induce stationarity):

$$(1-\tau_t)\left(g_{i_t} + p_t^I\right) = \rho_{t+1}(1-\tau_{t+1})\left[\begin{array}{c} f_{k_{t+1}} - g_{k_{t+1}} \\ +(1-\delta_{t+1})(g_{i_{t+1}} + p_{t+1}^I) \end{array}\right] + j_t^k \tag{28}$$

$$(1-\tau_t)\frac{g_{v_t}}{q_t} = \rho_{t+1}(1-\tau_{t+1})\left[\begin{array}{c} f_{n_{t+1}} - g_{n_{t+1}} - w_{t+1} \\ +(1-\psi_{t+1})\frac{g_{v_{t+1}}}{q_{t+1}} \end{array}\right] + j_t^n \tag{29}$$

The moment conditions estimated are those obtained under rational expectations i.e., $E(\mathbf{Z}_t \otimes j_t) = 0$ where $\mathbf{Z}_t$ is the vector of instruments. I report the J-statistic χ^2 test of the over-identifying restrictions. Appendix C spells out the first derivatives included in these equations. Importantly, I check whether the estimated g function fulfills the convexity requirement. The instrument set includes 8 lags of the key variables – the hiring rate ($\frac{h}{n}$) and the investment rate ($\frac{i}{k}$) for both equations; the rate of growth of output per unit of capital ($\frac{f}{k}$) and the depreciation rate (δ) for equation (28); and the labor share ($\frac{wn}{f}$) and rate of separation (ψ) for equation (29).

I start from unconstrained estimation of all the parameters listed above. I do so for the shorter sample 1994-2016, including job to job flows. As some of the parameter estimates have high standard errors, I also report a specification constraining the g function to be linear-quadratic (i.e., I set $\eta_1 = \eta_2 = 2, \eta_{31} = \eta_{32} = 1$). I repeat the latter specification also for the longer sample 1976-2016, omitting job to job flows (hence restricting the equations to the case of $\lambda_2 = e_{32} = 0$). Finally, I estimate the special cases discussed in sub-section 3.3 above.

4.3 Estimation Results

I present the GMM estimates of equations (28) and (29) under the alternative specifications described above. I use three criteria to evaluate the estimates:

a. The J-statistic test of the over-identifying restrictions.

b. Fulfillment of the convexity requirement for the costs function g.

c. The magnitude of implied total and marginal costs. As in many cases of investment equations estimated in the q-literature, some specifications imply very high costs. These are deemed to be unreasonable.

Table 1 reports the results of estimation. The table reports the point estimates and their standard errors, Hansen's (1982) J-statistic and its p-value.

Table 1

Panel a reports the specifications derived from the model above. Panel b shows the estimates of the standard specifications discussed in sub-section 3.3 above.

Consider panel a. Row (a) estimates all 11 parameters. Eight of these are not precisely estimated, but suggest a quadratic g function with linear interactions, provide for a very reasonable estimate of the production function, and place the most weight on hiring costs associated with the h_t^1 gross flows from non-employment. The J-statistic result does not reject the null hypothesis.

Row (b) restricts four of the parameters of the costs function to the point estimates presented in row (a), yielding a quadratic function ($\eta_1 = \eta_2 = 2$) with linear interactions ($\eta_{31} = \eta_{32} = 1$). Here the 7 free parameters are precisely estimated, the resulting g function fulfills all convexity requirements, the estimate of α is around the conventional estimate of 0.66, and the J-statistic again has a high p-value. The point estimates are close to those of the unrestricted row (a).

Row (c) takes up the same specification as row (b) but ignores job to job flows, i.e., sets $\lambda_2 = e_{32} = 0$ and $h_t^2 = \psi_t^2 = 0$. This allows for the use of a longer data sample – 1976:1-2016:4, with 168 quarterly observations. It, too, yields a J-statistic with a high p-value, is, for the most part, precisely estimated, and the resulting g fulfills all convexity requirements. Evidently the estimates are not the same as those of row (b), but there is considerable affinity (see also Table 3 below).

The three rows yield similar results in terms of the implied costs reported in Table 2 below. The main take-aways from these estimates are: quadratic costs with linear interactions; the latter feature negative coefficients ($e_{31}, e_{32} < 0$), implying complementarity between hiring and investment; and the bigger weight of recruitment costs is assigned to actual hires from non-employment, i.e. λ_1 at around 2/3. This is in line with the results in Yashiv (2000) with different data.

Now consider panel b relating to prevalent specifications in the literature.

Row (a) follows the standard Tobin's q for capital model and looks at a quadratic specification. It sets $\eta_1 = 2, e_2 = e_{31} = e_{32} = 0$, i.e., has quadratic investment costs, with no role for labor (see equation (18)). There is no rejection of the model, but this specification implies high, marginal investment costs, as seen in Table 2 and discussed below. This is reminiscent of the results in much of the literature on Tobin'q models for investment

Row (b) posits the same "standard Tobin's q model" but this time for labor, ignoring capital, and looks at a quadratic specification. It thus sets $\eta_2 = 2, e_1 = e_{31} = e_{32} = 0$. Most parameters are imprecisely estimated and the J statistic rejects the null.

Row (c) reports the results of the standard (Pissarides-type) search and matching model formulation with linear vacancy costs and no other arguments, as formulated in equation (20), such that $\eta_2 = 1, e_1 = e_{31} = e_{32} = \lambda_1 = \lambda_2 = 0$. The J statistic implies rejection and the estimates imply high total costs, as discussed below.

Hence standard specifications are rejected or deemed implausible. Thus, in what follows, I take row (b) in panel a as the preferred estimates.

The estimates of rows (a) and (b) of panel a imply the following equations, to be used below.

For investment:

$$(1-\tau_t)\left(g_{i_t}+p_t^I\right)=E_t\left[\rho_{t+1}\left(1-\tau_{t+1}\right)\left[\begin{array}{c} f_{k_{t+1}}-g_{k_{t+1}} \\ +(1-\delta_{t+1})(g_{i_{t+1}}+p_{t+1}^I) \end{array}\right]\right] \tag{30}$$

where:

$$\frac{g_{i_t}}{\frac{f_t}{k_t}}=\left[e_1(\frac{i_t}{k_t})+e_{31}\left(\frac{q_t^1 v_t}{n_t}\right)+e_{32}\left(\frac{q_t^2 v_t}{n_t}\right)\right]$$

For vacancy creation:

$$(1-\tau_t)\frac{g_{v_t}}{q_t}=E_t\left[\rho_{t+1}\left(1-\tau_{t+1}\right)\left[\begin{array}{c} f_{n_{t+1}}-g_{n_{t+1}}-w_{t+1} \\ +(1-\psi_{t+1})\frac{g_{v_{t+1}}}{q_{t+1}} \end{array}\right]\right] \tag{31}$$

where:

$$\frac{g_{v_t}}{\frac{f_t}{n_t}}=\left[\begin{array}{c} e_2\frac{v_t}{n_t}\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2 \\ +e_{31}q_t^1\left(\frac{i_t}{k_t}\right)+e_{32}q_t^2\left(\frac{i_t}{k_t}\right) \end{array}\right]$$

4.4 The Estimated Frictions

Table 2 shows the mean and volatility of the estimated costs series in two panels corresponding to the GMM estimates of Table 1.

Table 2

Total costs. Total costs out of GDP ($\frac{g_t}{f_t}$) are estimated to be 3.2-3.3% across rows (a)-(c) of panel a with little variation. In panel b, the two Tobin's q specifications with one factor only indicate 1.1% for the capital case and 2.5% for the labor case; jointly this is somewhat higher than the panel a estimates. The standard search and matching model estimates imply more than double the costs, 6.8% of GDP, with a big increase in their volatility.

*Marginal investment costs.*These are expressed in Table 2 in terms of the percentage out of the marginal capital unit price, $\frac{g_{i_t}}{p_t^I}$. The results of rows (a) – (c) in panel a point to 2.4%-3.4%, i.e. for every dollar spent on the marginal unit of capital, these costs add 2.4–3.4 cents. These results correspond to those papers in the investment q-literature which reported low costs. The one relevant result in panel b, row (a), Tobin's q for capital, yields a much larger estimate, 7.5%, reflecting a long-running problem with this widely used specification.

Marginal hiring costs. These are expressed in Table 2 in terms equivalent to quarterly wages, using $\frac{g_{v_t}}{q_t w_t}$. The results of rows (a) – (c) in panel a point to the equivalent of 50%-65% of quarterly wages, or the equivalent of 6.4 to 8.4 weeks of wages, for marginal costs. The relevant estimates of panel b (rows b and c) are in the same ball park.

The take away from this discussion is that hiring and investment costs in rows (a)-(c) are very moderate. Hence, the analysis below does not rely on excessive or implausible costs, an issue that plagued the relevant literatures for decades.

The estimates of the costs evidently allows for the quantification of the unobserved asset values (Qs), which are a function of them, as shown in equations (22) – (26). I use the derived time series in the cyclical analysis below.

4.5 What Do We Learn?

The afore-going analysis has shown that the model fits U.S. data well, in a period of over four decades, including the decade of the Great Recession and its aftermath. Prevalent models, in the Tobin's q and search and matching traditions, fit much less well, as they ignore cross effects between investment and hiring frictions. The interaction estimates imply complementarity in these activities. The fit is achieved using very moderate estimates of the relevant frictions. The time series for the shadow asset values are derived and used extensively below.

5 Optimal Firm Behavior

This section uses the estimation results above to explicitly formulate the firms' investment and vacancies decisions and to analyze them quantitatively.

5.1 Decision Rules

The decision rules implied by equations (30) and (31), are as follows, using the preferred parameter estimates (Appendix D shows the full derivation).

For investment, using the FOC:

$$\frac{Q_t^K}{\frac{f_t}{k_t}} = (1-\tau_t)\left(\left[e_1(\frac{i_t}{k_t}) + e_{31}\left(\frac{q_t^1 v_t}{n_t}\right) + e_{32}\left(\frac{q_t^2 v_t}{n_t}\right)\right] + \frac{p_t^I}{\frac{f_t}{k_t}}\right)$$

I get:

$$\frac{i_t}{k_t} = \frac{1}{e_1 e_2 \Lambda_t^2 - \Omega_t^2}\left[e_2\Lambda_t^2\left(\frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}\right) - q_t\Omega_t\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right] \tag{32}$$

where:

$$\begin{aligned}\Lambda_t &\equiv (1-\lambda_1-\lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2\\ \Omega_t &\equiv e_{31}q_t^1 + e_{32}q_t^2\end{aligned}$$

Equation (32) has the following implications for the sensitivity of investment decisions with respect to asset values, matching rates, and the corporate tax rate.[5] When unmabiguous, I indicate the sign, noting that the estimates of Table 1 indicate that $e_1 e_2 \Lambda_t^2 - \Omega_t^2 > 0$ and that $\Omega_t < 0$:

$$\frac{\partial \frac{i_t}{k_t}}{\partial \frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}} = \frac{e_2 \Lambda_t^2}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} > 0 \tag{33}$$

$$\frac{\partial \frac{i_t}{k_t}}{\partial \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}} = \frac{-q_t \Omega_t}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} > 0 \tag{34}$$

$$\begin{aligned}\frac{\partial \frac{i_t}{k_t}}{\partial q_t^1} &= \frac{2 e_2 \lambda_1 \Lambda_t P_t^K - P_t^N \left[2 e_{31} q_t^1 + q_t^2 (e_{31} + e_{32})\right]}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} \\ &- \frac{\left[e_2 \Lambda_t^2 P_t^K - \Omega_t (q_t^1 + q_t^2) P_t^N\right] \left[2(e_1 e_2 \lambda_1 \Lambda_t - e_{31} \Omega_t)\right]}{\left[e_1 e_2 \Lambda_t^2 - \Omega_t^2\right]^2}\end{aligned} \tag{35}$$

$$\begin{aligned}\frac{\partial \frac{i_t}{k_t}}{\partial q_t^2} &= \frac{2 e_2 \lambda_2 \Lambda_t P_t^K - P_t^N \left[2 e_{32} q_t^2 + q_t^1 (e_{31} + e_{32})\right]}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} \\ &- \frac{\left[e_2 \Lambda_t^2 P_t^K - \Omega_t q_t P_t^N\right] \left[2(e_1 e_2 \lambda_2 \Lambda_t - e_{32} \Omega_t)\right]}{\left[e_1 e_2 \Lambda_t^2 - \Omega_t^2\right]^2}\end{aligned} \tag{36}$$

$$\frac{\partial \frac{i_t}{k_t}}{\partial \tau_t} = \frac{\left[e_2 \Lambda_t^2 \left(\frac{\frac{Q_t^K}{\frac{f_t}{k_t}}}{(1-\tau_t)^2} + \frac{p_t^I}{\frac{f_t}{k_t}} \right) - \Omega_t \frac{q_t \frac{Q_t^N}{\frac{f_t}{n_t}}}{(1-\tau_t)^2} \right]}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} > 0 \tag{37}$$

where:

$$\begin{aligned} P_t^K &\equiv \frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}} > 0 \\ P_t^N &\equiv \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}} > 0 \end{aligned}$$

The investment rate $\frac{i_t}{k_t}$ is unmabiguously a positive function of the asset values $\frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}$ and $\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$ and of the corporate tax rate. The intuition for the former result, is that as the present value of the marginal investment unit or of the marginal hire rise,

[5] Noting that matching rates and corporate tax rates are components of asset values.

the firm invests more. The latter result is explained by the fact that corporate taxes are reduced by costs being expensed, so, ceteris paribus, without changes in future rates, the current tax rate rise gives an incentive to invest more now.[6]

For vacancy creation, using the FOC:

$$\frac{Q_t^N}{\frac{f_t}{n_t}} = \frac{(1-\tau_t)\left[\begin{array}{c} e_2\frac{v_t}{n_t}\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2 \\ +e_{31}q_t^1\left(\frac{i_t}{k_t}\right)+e_{32}q_t^2\left(\frac{i_t}{k_t}\right)\end{array}\right]}{q_t}$$

I get:

$$\frac{v_t}{n_t} = \frac{1}{e_1 e_2 \Lambda_t^2 - \Omega_t^2}\left[e_1 q_t \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}} - \Omega_t \frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}\right] \tag{38}$$

I proceed as above, with details presented in Appendix D, to get:

$$\frac{\partial \frac{v_t}{n_t}}{\partial \frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}} = \frac{-\Omega_t}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} > 0 \tag{39}$$

$$\frac{\partial \frac{v_t}{n_t}}{\partial \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}} = \frac{e_1 q_t}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} > 0 \tag{40}$$

$$\frac{\partial \frac{v_t}{n_t}}{\partial q_t^1} = \frac{e_1 P_t^N - e_{31} P_t^K}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} - \frac{\left[e_1 q_t P_t^N - \left(e_{31} q_t^1 + e_{32} q_t^2\right) P_t^K\right]\left[2 e_1 e_2 \lambda_1 \Lambda_t - 2 e_{31} \Omega_t\right]}{\left[e_1 e_2 \Lambda_t^2 - \Omega_t^2\right]^2} \tag{41}$$

$$\frac{\partial \frac{v_t}{n_t}}{\partial q_t^2} = \frac{e_1 P_t^N - e_{32} P_t^K}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} - \frac{\left[e_1 q_t P_t^N - \left(e_{31} q_t^1 + e_{32} q_t^2\right) P_t^K\right]\left[2 e_1 e_2 \lambda_2 \Lambda_t - 2 e_{32} \Omega_t\right]}{\left[e_1 e_2 \Lambda_t^2 - \Omega_t^2\right]^2} \tag{42}$$

$$\frac{\partial \frac{v_t}{n_t}}{\partial \tau_t} = \frac{\left[\frac{e_1 q_t \frac{Q_t^N}{\frac{f_t}{n_t}}}{(1-\tau_t)^2} - \Omega_t\left(\frac{\frac{Q_t^K}{\frac{f_t}{k_t}}}{(1-\tau_t)^2} + \frac{p_t^I}{\frac{f_t}{k_t}}\right)\right]}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} > 0 \tag{43}$$

The vacancy rate $\frac{v_t}{n_t}$ is unmabiguously a positive function of $\frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}$ and $\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$ and of the corporate tax rate, for reasons similar to the ones presented above.

5.2 Quantifying the Responses of the Decision Variables

The equations above allow for a quantitative anlysis of the responses of the decision variables to asset values, matching rates, and the corporate tax. Table 3 presents this

[6]Note, too, that τ_t includes χ_t the investment tax credit, and D_t the present discounted value of capital depreciation allowances,

quantification for the sample period showing the elasticities based on the afore-going derivatives, namely the sensitivity of the decision variables to asset values, matching rates, and the corporate tax rate.

Table 3

The picture which emerges from Table 3 is analyzed in the next two sub-sections.

5.2.1 The Magnitude and Sign of the Response

One notable point is that labor values have stronger effects than capital values for both decision variables – elasticities of 0.78 and 0.90 as opposed to 0.22 and 0.10 with respect to the investment rate and the vacancy rate, respectively.

A second notable point is that the job filling rates have differential effects: the rate from non-employment is weakly positive on average with respect to investment and negative with respect to vacancies. The rate from other employment has a positive effect on both.

The third implication is that the current corporate tax rate has a positive effect, ceteris paribus, which is much stronger than that of the asset value, of which it is a part. Investment elasticity is much higher than vacancy elasticity.

5.2.2 Cyclicality

While the next section provides cyclical analysis using shock specifications and HP-filtered variables, much can be learned from Table 3. The decision variables, investment and vacancy creation, are pro-cyclical, as is well known. Job filling rates are counter-cyclical, also a well-known result. This is so as in booms (recessions) labor market conditions are good (bad) for the worker, typically with higher (lower) vacancies and lower (higher) unemployment. This makes job filling harder (easier) for firms. Hiring rates are the product of the pro-cyclical vacancy rate and the counter-cyclical job filling rates. Hiring rates from non-employment appear to be counter-cyclical, while hiring rates from other employment are pro-cyclical. The price of investment in terms of average output per capital is counter-cyclical, as in booms (recessions) the investment price falls (rises) and output per unit of capital rises (falls).

In terms of the estimated asset values, capital asset values ($\frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}$,net of the price of capital) are pro-cyclical. They have a positive effect on both investment and vacancy rates, as seen in the equations of this section. For both decision variables the elasticity with respect to P_t^K, a function of the above capital value, is pro-cyclical.

Labor asset values ($\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$) are also pro-cyclical and have a positive effect on both investment and vacancy rates. However, for both decision variables the elasticity with respect to P_t^N, a function of this labor value, is counter-cyclical.

There are two surprising findings here: the labor value elasticity rises in recessions, i.e., worker values become more important in recessions; and, as noted, the mean elasticity is much higher than the one for capital asset values, for both hiring and investment. These results point to the importance of worker values even for capital investment decisions, all the more so in recessions. This feature has not been explored in the literature.

Asset values embody, inter alia, matching rates and the corporate tax rate. In what follows I look at the cyclical behavior of the elasticity of the decision variables with respect to these two components of asset values.

The elasticity with respect ot the rate of matching from non-employment (q_t^1) is pro-cyclical (note, though the differential effects of this rate discussed above). As to the elasticity with respect to the rate of matching from other employment (q_t^2), for the investment rate it is counter-cyclical, while for the vacancy rate it is weakly pro-cyclical.

As noted, increases in the current corporate tax rate (τ_t), without changes in future rates, raise investment and vacancy creation, for the reasons elaborated above. Their effect rises in recessions i.e., the elasticity is counter-cyclical.

Taken the above results have the following implications for recessions.

When labor market conditions reflect the recession and so the non-employment job filling rate q_t^1 is high, the firm, ceteris paribus, reduces the vacancy rate; at the same time it raises the capital investment rate slightly. At these times the job filling rate from other employment q_t^2 is also high and this effect pushes firms to raise vacancy rates (and the investment rate). However, more broadly, labor asset values ($\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$), as well as net capital values, are low, and firms lower vacancy rates.

For capital investment, high job filling rates in recessions operate to raise investment. But, again, more broadly, in recessions investment rates are low with lower capital asset values (net of the price of capital, $\frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}$) and lower labor asset values.

6 The Cyclical Behavior of Hiring, Investment, and Q Values

This section studies in more depth the business cycle behavior of the decision variables (investment and vacancies), of their sensitivities, of asset values (the Qs), and of labor market conditions (embodied in the job filling rates (q^1, q^2)). There are three innovations here relative to the preceding discussion: one is that I look at three widely-used shocks, which generate cyclical behavior; the second is that I use logged, HP-filtered variables; the third is that the empirical methodology is geared towards estimating cyclical relations. Two main questions are examined here: what are the cyclical patterns of firms behavior in terms of investing in inputs (capital and labor), of asset values, and of job filling rates, which embody market conditions? How does the sensitivity of the firms' decisions to asset values and to market conditions change over the cycle?

In what follows I define the shocks and present their data sources (6.1), delineate the methodology (6.2), and present the results in terms of co-movement (6.3) and dynamic correlation (6.4) analyses.

6.1 The Shock Series

I use series on three widely-used shocks in the aggregate U.S. economy. I take these shock series from the authors of the following papers, using the relevant sub-period in their data, given that the analysis here focuses on the period 1994-2016.

The TFP shock series is taken from an online data base[7] and is described in Fernald (2014). It is defined as utilization-adjusted TFP growth and is computed by taking output growth less the contribution of capital and labor and subtracting from it estimates of the utilization of capital and labor (see equation 2 in Fernald (2014)). In terms of equation (2) above, this is utilization-adjusted ε_t^f.

The unanticipated IST shock series are taken from Ben Zeev and Khan (2015). Ben Zeev (2018) shows in his equations 2 and 3 the relevant stochastic specification, akin to equation (7) above.

The matching shocks were generated by Furlnaetto and Groshenny (2016). They derive the series within a medium-scale DSGE model applied to the U.S. economy. Their model features a Cobb Douglas matching function (see their equation 5), with a matching technology shock process akin to equation (5) above.

6.2 Methodology

I study the cyclical behavior of the key variables in two ways: in one, I use local projections (LP) methods to analyze the IRFs of all variables in response to each shock. In the other, I study the linear relations between the various variables on the one hand and GDP on the other hand, conditional on the local projections analysis. For both types of analysis I use a formulation based on Jorda (2005) and Daly et al (2018).

The local projection is given by the equation:

$$s_{t+h} = c_{ih}^s + \zeta_{it}\lambda_{ih}^s + R_t\Gamma_{ih}^s + e_{t+h}^s \tag{44}$$

where:

$$\begin{aligned}
s_{t+h} &\in \left\{f_{t+h}, x_{1,t+h} ... x_{J,t+h}\right\} \\
x_{it} &\in \left\{\frac{i_t}{k_t}, \frac{v_t}{n_t}, q_t^1, q_t^2, \frac{h_t^1}{n_t}, \frac{h_t^2}{n_t}, \frac{Q_t^N}{\frac{f_t}{n_t}}, \frac{Q_t^P}{\frac{f_t}{k_t}}, \frac{\widetilde{Q}_t^K}{\frac{f_t}{k_t}}, \frac{Q_t}{f_t}, \boldsymbol{\eta}_t\right\} \\
\boldsymbol{\eta}_t &\in \left\{\begin{array}{c}
\frac{\partial \frac{i_t}{k_t}}{\partial P_t^K}\frac{P_t^K}{\frac{i_t}{k_t}}, \frac{\partial \frac{i_t}{k_t}}{\partial P_t^N}\frac{P_t^N}{\frac{i_t}{k_t}}, \frac{\partial \frac{i_t}{k_t}}{\partial q_t^1}\frac{q_t^1}{\frac{i_t}{k_t}}, \frac{\partial \frac{i_t}{k_t}}{\partial q_t^2}\frac{q_t^2}{\frac{i_t}{k_t}}, \\
\frac{\partial \frac{v_t}{n_t}}{\partial P_t^K}\frac{P_t^K}{\frac{v_t}{n_t}}, \frac{\partial \frac{v_t}{n_t}}{\partial P_t^N}\frac{P_t^N}{\frac{v_t}{n_t}}, \frac{\partial \frac{v_t}{n_t}}{\partial q_t^1}\frac{q_t^1}{\frac{v_t}{n_t}}, \frac{\partial \frac{v_t}{n_t}}{\partial q_t^2}\frac{q_t^2}{\frac{v_t}{n_t}}
\end{array}\right\} \\
\zeta_{it} &\in \left\{\varepsilon_t^f, \varepsilon_t^I, \varepsilon_t^\mu\right\}
\end{aligned}$$

[7] See https://www.frbsf.org/economic-research/economists/jfernald/quarterly_tfp.xlsx

Equation (44) can be understood as follows:

On the LHS s_{t+h} is a predicted variable at horizon h; I examine 19 such variables – GDP (f_t), the decision variables $(\frac{i_t}{k_t}, \frac{v_t}{n_t})$, the job filling rates (q_t^1, q_t^2), the hiring rates $(\frac{h_t^1}{n_t}, \frac{h_t^2}{n_t})$, four asset values $(\frac{Q_t^N}{\frac{f_t}{n_t}}, \frac{Q_t^P}{\frac{f_t}{k_t}}, \frac{\widetilde{Q}_t^K}{\frac{f_t}{k_t}}, \frac{Q_t}{f_t})$ and eight sensitivities of the decision variables, i.e., elasticities of the decisions variables $(\boldsymbol{\eta}_t)$. All variables are logged and HP filtered. The forecasting horizon is denoted h and set to 15 quarters.

On the RHS each regression has a constant (c_{ih}^s) and an error term (e_{t+h}^s). The afore-cited shocks are denoted ξ_{it}; each regression features one of the three shocks delineated above. R_t is a vector of control variables. The estimated coefficients are λ_{ih}^s and Γ_{ih}^s , respectively.

I tried alternative formulations for R_t which have yielded similar results. In Table 4 below are estimates of equation (44), using for controls two lags of the shock and one lag of the asset values, $\frac{Q_{t-1}^N}{\frac{f_{t-1}}{n_{t-1}}}, \frac{Q_{t-1}^P}{\frac{f_{t-1}}{k_{t-1}}}, \frac{\widetilde{Q}_{t-1}^K}{\frac{f_{t-1}}{k_{t-1}}}$. The latter essentially capture the information known to the firm at the time of its decision on hiring and investment.

I then use the results of estimation for the following cyclical analysis. Consider first an OLS regression of the form:

$$x_{jt} = a_j + b_j f_t + e_{jt} \tag{45}$$

The estimated coefficient b_j is an indicator of the cyclicality of variable x_{jt} with respect to GDP f_t. This is an estimate unconditioned by shocks, and does not take into account any dynamics. Daly et al (2018) suggest to use the afore-going LP estimates of λ_{ih}^s to estimate a shock-conditional b_j using Classical Minimum Distance (CMD) as follows:

$$\widehat{b}_{ij} = (\widehat{\boldsymbol{\lambda}}_i^{f'} M \widehat{\boldsymbol{\lambda}}_i^f)^{-1} (\widehat{\boldsymbol{\lambda}}_i^{f'} M \widehat{\boldsymbol{\lambda}}_i^j) \tag{46}$$

with variance of $\widehat{b}_{ij}$ given by:

$$v_{ij} = (\widehat{\boldsymbol{\lambda}}_i^{f'} M \widehat{\boldsymbol{\lambda}}_i^f)^{-1} \tag{47}$$

and where:

$$M = \left(\Omega_i^j\right)^{-1} \tag{48}$$

$\widehat{\boldsymbol{\lambda}}_i^f$ is the $h \times 1$ vector of LP coefficients of shock i on f, $\widehat{\boldsymbol{\lambda}}_i^j$ is the $h \times 1$ vector of LP coefficients of shock i on variable j, and Ω_i^j a diagonal matrix with variance estimates of $\widehat{\boldsymbol{\lambda}}_i^j$ from equation (44).

The intuition here is to find the $\widehat{b}_{ij}$ which makes the IRF of f (to a shock i) as close as possible to the IRF of variable j (to a shock i).

6.3 Cyclical Co-Movement

I turn to study the co-movement of the key variables, both unconditionally and conditionally on shocks. Table 4 reports the point estimates and standard errors of b_j from equation (45) and b_{ij} from equation (46). Recall that the former captures the unconditional co-movement of any particular variable with GDP. The latter captures the conditional co-movement; it is the parameter that makes the IRF to a particular shock of a variable j as close as possible to the IRF of GDP to the same shock.

Table 4

6.3.1 Unconditional Co-Movement

Without conditioning on any shock the following are the cyclical patterns. *The decision variables* – investment and vacancy rates – are pro-cyclical. *The job filling rates* are counter-cyclical. The hiring rates are the product of the vacancy rate and the job-filling rates and are thus subject to opposing cyclical forces. *The hiring rate* from non-employment is counter-cyclical while the hiring rate from other employment is pro-cyclical. *Asset values* which relate to costs ($\frac{Q_t^N}{\frac{f_t}{n_t}}$ and $\frac{\widetilde{Q}_t^K}{\frac{f_t}{k_t}}$) are pro-cyclical while the one that relates to the price of investment ($\frac{Q_t^P}{\frac{f_t}{k_t}}$) is counter-cyclcial. Total firm value ($\frac{Q_t}{f_t}$) is dominated by the latter and so is counter-cyclical. *The sensitivity of investment and vacancy rates* to the part of the value of capital related to adjustment costs is pro-cyclical, while the elasticity related to the value of labor is counter-cyclical. These elasticities ($\eta_{\frac{i_t}{k_t},P_t^K}$ and $\eta_{\frac{i_t}{k_t},P_t^N}$) are correlated -1, by the model's equations. Hence in recessions firms' decisions are more tightly related to the value of labor and less tightly to the adjustment cost part of the value of capital. The elasticity of the decision variables with respect to the job-filling rate from non-employment is pro-cyclical and with respect to job filling from other employment is counter- (for investment) or a- (for vacancies) cyclical. In recessions, when market conditions generate high job filling rates, the decision variables become more sensitive to the job filling rate from non-employment and less sensitive to the job-filling rate from other employment. The elasticity with respect to the corporate tax is counter-cyclical, i.e., the effects of taxes increase in recessions.

6.3.2 Conditional Co-Movement

Conditioning on the three shocks, the following results emerge. *The cyclicality of the decision variables* – investment and vacancy rates stay pro-cyclical, but the pro-cyclicality is weaker. *The job filling rates* are counter-cyclical, as in the unconditional case, but more weakly so, to the point there are some non-significant coefficients. *The hiring rates* have non-significant negative coefficients in the case of flows from non-employment, suggesting weakening of the counter-cyclicality found above in the unconditional case. In the hiring flows from other employment they remain pro-cyclical but weaker too, with one non-significant coefficient. The coefficient for total *firm value* in terms of output is not significant, as is the value of capital related to the price of capital . The components related to marginal costs of investment of labor are both pro-cyclical. The cyclicality of the *elasticities* is qualitatively as in the unconditional case but less strong, with some non-significant coefficients.

Overall the conditional moments are qualitatively similar to the unconditional moments, but indicate weaker cyclicality with some insignificant coefficients.

6.4 Dynamic Cross Correlations

Table 5 shows the dynamic correlations of asset values with GDP, all logged and HP-filtered.

Table 5

The table shows that asset values lead the cycle. At the fourth lag they are correlated -0.4 to -0.5 with GDP for total Q and the part of capital value due to the price of investment, and $+0.6$ for the labor value. At second and first lags they are correlated -0.5 to -0.7 for total Q and the part of capital value due to the price of investment, and $+0.5$ for the labor value.

7 Summary and Conclusions

The afore-going analysis produces a plethora of results. In this section I provide for an integrated picture of firm behavior over the cycle. The overall picture emerging from the preceding analysis is as follows:

The decision variables – investment and vacancies - react positively to both capital and labor asset values. Standard specifications which do not feature cross effects (between capital value and vacancy creation and between labor value and investment) are thus omitting important variables. The effects of labor values is stronger on both, as measured by the relevant elasticities. It is of interest to see that there is a further asymmetry here. Higher labor values operate to increase both investment and vacancy creation but their effect is weaker in booms, while the positive effect of capital values is stronger in booms.

With pro-cyclical labor and net capital values, the decision variables are pro-cyclical. However if we look at hiring flows (h_t^1, h_t^2) then there is a role for job-filling rates (q_t^1, q_t^2) as well as for vacancy creation rates (v_t). The job filling rates are counter-cyclical. In the case of hiring flows from non-employment these dominate and thus this hiring flow is counter-cyclical.

The effects of the job filling rates, reflecting labor market conditions, on the decision variables is subsumed in asset values. Their partial effect varies in sign, strength and cyclicality. Thus the job filling rate from non employment has weak and variable effects on investment rates and relatively strong negative effects on vacancy rates, which strengthen even more in recessions. The job filling rate from other employment has positive effects on both decision variables, which tend to strengthen in recessions.

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8 Tables

Table 1
GMM Estimation Results

a. Preferred Specifications

	specification	η_1	η_2	η_{31}	η_{32}	e_1	e_2	e_{31}	e_{32}	λ_1	λ_2
a	all free	1.99	**1.98**	1.02	1.00	75	7	-12	-8	**0.64**	0.20
		(1.69)	(**0.86**)	(1.06)	(1.45)	(397)	(10)	(60)	(63)	(**0.10**)	(0.14
b	η_s restricted	2	2	1	1	**81**	**8**	$\mathbf{-11}$	$\mathbf{-10}$	**0.65**	**0.21**
	1994-2016 sample	–	–	–	–	(**8**)	(**2**)	(**2**)	(**3**)	(**0.06**)	(**0.07**
c	η_s restricted;	2	2	1	–	**80**	**2.4**	$\mathbf{-12}$	–	0.34	–
	$e_{32} = \lambda_2 = 0$	–	–	–	–	(**15**)	(**1.2**)	(**4**)		(0.24)	
	1976-2016 sample										

b. Standard Specifications

	specification	η_1	η_2	η_{31}	η_{32}	e_1	e_2	e_{31}	e_{32}	λ_1	λ_2	α
a	Tobin's Q for K	2	–	–	–	**31.7**	0	0	0	–	–	**0.70**
		–				(**12.9**)	–	–	–			(**0.00**)
b	Tobin's Q for N	–	2	–	–	–	2.72	0	0	0.44	0.11	**0.67**
		–	–				(6.76)	–	–	(0.47)	(0.86)	(**0.03**)
c	Std Matching Model	–	1	–	–	–	**0.34**	0	0	0	0	**0.65**
							(**0.08**)	–	–	–	–	(**0.01**)

Notes:

1. The tables report GMM point estimates with standard errors in parentheses. Bolded estimates are significant. The J-statistic is reported with p value in parentheses.

2. The sample period is 1994:1 – 2016:4 in Table a rows a and b and in all rows of Table b; and it is 1976:1-2016:4 in row c of Table a.

3. The estimates refer to the moment conditions $E(\mathbf{Z}_t \otimes j_t) = 0$ where $\mathbf{Z}_t$ is the vector of instruments. This set consists of 8 lags of the following variables – the hiring rate ($\frac{h}{n}$) and the investment rate ($\frac{i}{k}$) for both equations; the rate of growth of output per unit of capital ($\frac{f}{k}$) and the depreciation rate (δ) for equation (12); and the labor share ($\frac{wn}{f}$) and rate of separation (ψ) for equation (13).

4. In all specifications I estimate scaling parameters, which multiply p^I and $\frac{v}{n}$, as these are indices. These parameter estimates are not reported.

Table 2

Estimated Frictions

a. Preferred Specifications

	specification		$\frac{g_t}{f_t}$	$\frac{\frac{g_{i,t}}{\frac{f_t}{k_t}}}{\frac{P_t^I}{\frac{f_t}{k_t}}}$	$\frac{g_{v,t}}{\frac{q_t}{w_t}}$
a	all free	mean	0.032	0.027	0.50
		std.	0.004	0.013	0.05
b	η_s restricted	mean	0.033	0.024	0.55
	1994-2016 sample	std.	0.004	0.014	0.06
c	η_s restricted; $e_{32} = \lambda_2 = 0$	mean	0.032	0.034	0.65
	1976-2016 sample	std	0.015	0.024	0.34

b. Standard Specifications

	specification		$\frac{g_t}{f_t}$	$\frac{\frac{g_{i,t}}{\frac{f_t}{k_t}}}{\frac{P_t^I}{\frac{f_t}{k_t}}}$	$\frac{g_{v,t}}{\frac{q_t}{w_t}}$
a	Tobin's Q for K	mean	0.011	0.075	–
		std.	0.002	0.008	–
b	Tobin's Q for N	mean	0.025	–	0.47
		std.	0.005	–	0.08
c	Standard Matching Model	mean	0.068	–	0.64
		std	0.011	–	0.10

Notes:

The tables report mean and standard deviations of time series of the variables in the top rows. The series were generated using the corresponding estimates from Table 2.

Table 3
Summary of Elasticity Findings

elasticity	$\frac{i_t}{k_t}$		$\frac{v_t}{n_t}$	
	mean (std)	*description*	*mean (std)*	*description*
$\frac{\partial \cdot}{\partial \frac{\tilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}} \frac{\frac{\tilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}}{\cdot}$	0.22	positive	0.10	positive
	(0.11)	pro-cyclical	(0.05)	pro-cyclical
$\frac{\partial \cdot}{\partial \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}} \frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}}{\cdot}$	0.78	positive	0.90	positive
	(0.11)	counter-cyclical	(0.05)	counter-cyclical
$\frac{\partial \cdot}{\partial q_t^1} \frac{q_t^1}{\cdot}$	0.10	variable	-0.58	negative
	(0.07)	pro-cyclical	(0.07)	counter-cyclical[5]
$\frac{\partial \cdot}{\partial q_t^2} \frac{q_t^2}{\cdot}$	0.60	positive	0.39	positive
	(0.13)	counter-cyclical	(0.07)	weakly pro-cyclical(?)
$\frac{\partial \cdot}{\partial \tau_t} \frac{\tau_t}{\cdot}$	2.24	positive	1.31	positive
	(0.23)	counter-cyclical	(0.11)	counter-cyclical

Notes:
1. The figure uses estimation results of Table 1a row b.
2. See Section 5 and Appendix D for definitions and derivations of the elasticities.
3. NBER-dated recessions are shaded.
4. The cyclical behavior of the elasticities described in the table is based on the shaded regions.
5. Counter-cyclicality refers to the absolute value.

Table 4
Cyclical Analysis

a. Activity Variables

	static	dynamic		
		TFP shock	IST shock	Matching shock
$\frac{i}{k}$	2.16^{***} (0.14)	0.80^{*} (0.42)	1.61^{***} (0.32)	0.99^{**} (0.37)
$\frac{v_t}{n_t}$	4.33^{***} (0.24)	3.92^{***} (1.11)	1.50^{**} (0.58)	2.59^{***} (0.85)
q_t^1	-4.61^{***} (0.26)	-3.82^{***} (0.83)	-2.11^{**} (0.76)	-2.86^{**} (1.00)
q_t^2	-2.84^{***} (0.26)	-3.47^{**} (1.26)	-0.78^{NS} (0.55)	-1.15^{NS} (0.95)
$\frac{h^1}{n}$	-0.28^{**} (0.12)	-0.02^{NS} (0.53)	-0.46^{NS} (0.31)	-0.75^{NS} (0.45)
$\frac{h^2}{n}$	1.50^{***} (0.17)	0.80^{NS} (1.03)	0.94^{*} (0.53)	1.16^{**} (0.46)

b. Asset Values

	static	dynamic		
		TFP shock	IST shock	Matching shock
$\frac{Q_t^N}{\frac{f_t}{n_t}}$	1.21^{***} (0.28)	2.32^{NS} (1.82)	-0.32^{NS} (0.58)	0.12^{NS} (0.90)
$\frac{\tilde{Q}_t^K}{\frac{f_t}{k_t}}$	20.87^{***} (3.28)	-5.02^{NS} (11.35)	14.33^{**} (6.19)	13.28^{NS} (12.36)
$\frac{Q_t^P}{\frac{f_t}{k_t}}$	-0.80^{***} (0.09)	-0.46^{NS} (0.32)	-0.53^{*} (0.26)	-0.45^{NS} (0.34)
$\frac{Q_t}{f_t}$	-0.36^{***} (0.10)	-0.32^{NS} (0.31)	-0.39^{*} (0.22)	-0.29^{NS} (0.32)

c. Elasticities

	static	**dynamic**		
		TFP shock	**IST shock**	**Matching shock**
$\frac{\partial \frac{i_t}{k_t}}{\partial \frac{\tilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}} \frac{\frac{\tilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}}{\frac{i_t}{k_t}}$	2.67^{***}	0.59^{NS}	2.23^{***}	1.64^{**}
	(0.26)	(0.90)	(0.74)	(0.76)
$\frac{\partial \frac{i_t}{k_t}}{\partial \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}} \frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}}{\frac{i_t}{k_t}}$	-2.67^{***}	-0.59^{NS}	-2.23^{***}	-1.64^{**}
	(0.26)	(0.90)	(0.74)	(0.76)
$\frac{\partial \frac{i_t}{k_t}}{\partial q_t^1} \frac{q_t^1}{\frac{i_t}{k_t}}$	1.16^{***}	1.35^{**}	0.29^{NS}	0.40^{NS}
	(0.11)	(0.61)	(0.21)	(0.41)
$\frac{\partial \frac{i_t}{k_t}}{\partial q_t^2} \frac{q_t^2}{\frac{i_t}{k_t}}$	-0.85^{***}	-0.89^{NS}	-0.28^{NS}	-0.37^{NS}
	(0.14)	(0.56)	(0.41)	(0.46)
$\frac{\partial \frac{i_t}{k_t}}{\partial \tau_t} \frac{\tau_t}{\frac{i_t}{k_t}}$	-7.00^{***}	-4.68^{***}	-3.69^{**}	-5.21^{***}
	(0.34)	(0.94)	(1.42)	(1.45)
$\frac{\partial \frac{v_t}{n_t}}{\partial \frac{\tilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}} \frac{\frac{\tilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}}{\frac{v_t}{n_t}}$	1.29^{***}	0.20^{NS}	1.19^{***}	0.87^{**}
	(0.14)	(0.48)	(0.38)	(0.39)
$\frac{\partial \frac{v_t}{n_t}}{\partial \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}} \frac{\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}}{\frac{v_t}{n_t}}$	-1.29^{***}	-0.20^{NS}	-1.19^{***}	-0.87^{**}
	(0.14)	(0.48)	(0.38)	(0.39)
$\frac{\partial \frac{v_t}{n_t}}{\partial q_t^1} \frac{q_t^1}{\frac{v_t}{n_t}}$	1.84^{***}	1.62^{***}	0.74^{**}	1.01^{**}
	(0.12)	(0.45)	(0.28)	(0.44)
$\frac{\partial \frac{v_t}{n_t}}{\partial q_t^2} \frac{q_t^2}{\frac{v_t}{n_t}}$	0.00^{NS}	-0.60^{NS}	0.26^{NS}	0.16^{NS}
	(0.07)	(0.34)	(0.17)	(0.22)
$\frac{\partial \frac{v_t}{n_t}}{\partial \tau_t} \frac{\tau_t}{\frac{v_t}{n_t}}$	-2.83^{***}	-2.60^{**}	-0.79^{NS}	-1.97^{***}
	(0.21)	(1.06)	(0.61)	(0.66)

Notes:

1. $* * * / * * / * / NS$ denote significance at 1%, 5%, 10% or not significant, respectively.

2. Static column reports point estimates and standard errors in parantheses for b_j in equation (**??**).

3. Dynamic columns report point estimates and standard errors in parantheses for b_{ij} in equation (46).

4. All variables are logged and HP-filtered.

Table 5
Dynamic Correlations of Asset Values with GDP
all logged and HP filtered.

	$\rho(f_t, \frac{Q_{t+/-i}}{f_{t+/-i}})$	
i	lag	lead
0	-0.38	-0.38
1	-0.48	-0.09
2	-0.50	0.18
3	-0.48	0.37
4	-0.44	0.52

	$\rho(f_t, \frac{Q^K_{t+/-i}}{\frac{f_{t+/-i}}{K_{t+/-i}}})$			$\rho(f_t, \frac{Q^P_{t+/-i}}{\frac{f_{t+/-i}}{K_{t+/-i}}})$	
i	lag	lead	i	lag	lead
0	-0.45	-0.45	0	-0.70	-0.70
1	-0.54	-0.16	1	-0.73	-0.45
2	-0.55	0.11	2	-0.67	-0.18
3	-0.53	0.31	3	-0.58	0.03
4	-0.48	0.48	4	-0.46	0.22

	$\rho(f_t, \frac{\tilde{Q}^K_{t+/-i}}{\frac{f_{t+/-i}}{K_{t+/-i}}})$	
i	lag	lead
0	0.58	0.58
1	0.48	0.62
2	0.36	0.64
3	0.30	0.58
4	0.17	0.48

	$\rho(f_t, \frac{Q^N_{t+/-i}}{\frac{f_{t+/-i}}{N_{t+/-i\ t}}})$	
i	lag	lead
0	0.43	0.43
1	0.49	0.34
2	0.53	0.15
3	0.58	0.02
4	0.58	-0.13

9 Appendices

9.1 Appendix A: Derivation of Aggregate Q

I derive a general Q_t which is a function of Q_t^K and Q_t^N. The following derivations are based on Hayashi (1982).

9.1.1 Firm Profits, Cash Flows, and Values

Define firm profits π_t :

$$\pi_t = [f(k_t, n_t) - g(i_t, k_t, v_t, n_t)] - w_t n_t \,. \tag{49}$$

Define cash flow payments to firm owners as cf_t equal to profits after tax minus purchases of investment goods plus investment tax credits and depreciation allowances for new investment goods:

$$\begin{aligned} cf_t &= (1-\tau_t)\pi_t - (1-\chi_t - \tau_t D_t)\, \widetilde{p}_t^I\, i_t \\ &= (1-\tau_t)\left(f(k_t, n_t) - g(i_t, k_t, v_t, n_t) - w_t n_t - p_t^I i_t\right) \end{aligned} \tag{50}$$

The representative firm's value in period t, Q_t, is defined as follows:

$$Q_t = E_t\left[\rho_{t+1}\left(Q_{t+1} + cf_{t+1}\right)\right]. \tag{51}$$

This can be split into capital ϑ_t^k and labor values ϑ_t^n as follows:

$$Q_t = \vartheta_t^k + \vartheta_t^n = E_t\left[\rho_{t+1}\left(\vartheta_{t+1}^k + cf_{t+1}^k\right)\right] + E_t\left[\rho_{t+1}\left(\vartheta_{t+1}^n + cf_{t+1}^n\right)\right], \tag{52}$$

Using the constant returns-to-scale properties of the production function f and of the cost function, g, and equation (50), decompose the stream of maximized cash flow payments as follows:

$$\begin{aligned} cf_t &= (1-\tau_t)\left(f_{k_t}k_t + f_{n_t}n_t - w_t n_t - p_t^I i_t - g_{k_t}k_t - g_{i_t}i_t - g_{v_t}v_t - g_{n_t}n_t\right) \\ &= (1-\tau_t)\left[\left(f_{k_t}k_t - p_t^I i_t - g_{k_t}k_t - g_{i_t}i_t\right) + \left(f_{n_t}n_t - w_t n_t - g_{v_t}v_t - g_{n_t}n_t\right)\right] \\ &\equiv cf_t^k + cf_t^n. \end{aligned} \tag{53}$$

9.1.2 Optimality Equations and Asset Values

Multiply throughout the FOC with respect to investment (9) by i_t, the FOC with respect to capital (8) by k_{t+1}, the FOC with respect to vacancies (11) by v_t, and the one with respect to employment (10) by n_{t+1} to get

$$\begin{aligned}
(1-\tau_t)\left(p_t^I+g_{i_t}\right)i_t &= i_t Q_t^K && (54)\\
(1-\tau_t)\,g_{v_t}v_t &= v_t q_t Q_t^N && (55)\\
k_{t+1}Q_t^K &= k_{t+1}E_t\left\{\rho_{t+1}[(1-\tau_{t+1})\left(f_{k_{t+1}}-g_{k_{t+1}}\right)+(1-\delta_{t+1})Q_{t+1}^K]\right\} && (56)\\
n_{t+1}Q_t^N &= n_{t+1}E_t\left\{\rho_{t+1}\left[(1-\tau_{t+1})\left(f_{n_{t+1}}-g_{n_{t+1}}-w_{t+1}\right)+(1-\psi_{t+1})Q_{t+1}^N\right]\right\} && (57)
\end{aligned}$$

Capital Insert the law of motion for capital (3) into equation (54), roll forward all expressions one period, multiply both sides by ρ_{t+1} and take conditional expectations on both sides:

$$E_t\left[\rho_{t+1}(1-\tau_{t+1})\left(p_{t+1}^I+g_{i_{t+1}}\right)i_{t+1}\right]=E_t\left\{\rho_{t+1}\left[k_{t+2}-(1-\delta_{t+1})k_{t+1}\right]Q_{t+1}^K\right\}. \quad (58)$$

Rearranging:

$$E_t\left[\rho_{t+1}(1-\delta_{t+1})\left(k_{t+1}Q_{t+1}^K\right)\right]=E_t\left\{\rho_{t+1}\left[\left(k_{t+2}Q_{t+1}^K-(1-\tau_{t+1})\left(p_{t+1}^I+g_{i_{t+1}}\right)i_{t+1}\right)\right]\right\} \quad (59)$$

Combining equations (53), (54), (56), and (59) yields:

$$k_{t+1}Q_t^K=E_t\left(\rho_{t+1}\left(cf_{t+1}^k+k_{t+2}Q_{t+1}^K\right)\right) \quad (60)$$

Rearranging:

$$E_t\left(\rho_{t+1}cf_{t+1}^k\right)=k_{t+1}Q_t^K-E_t\left(\rho_{t+1}k_{t+2}Q_{t+1}^K\right). \quad (61)$$

It follows from the definition of the firm's market value in equation (52) that

$$\vartheta_t^k-E_t\left(\rho_{t+1}\vartheta_{t+1}^k\right)=E_t\left(\rho_{t+1}cf_{t+1}^k\right). \quad (62)$$

Thus,

$$\vartheta_t^k-E_t\left(\rho_{t+1}\vartheta_{t+1}^k\right)=k_{t+1}Q_t^K-E_t\left(\rho_{t+1}k_{t+2}Q_{t+1}^K\right), \quad (63)$$

which implies

$$\vartheta_t^k=k_{t+1}Q_t^K. \quad (64)$$

Labor Derive a similar expression for the case of labor. Inserting the law of motion for labor from equation (4) into equation (55), multiplying both sides by ρ_{t+1}, rolling forward all expressions by one period, taking conditional expectations, and combining with equations (53) and (57) get

$$E_t\left(\rho_{t+1}cf_{t+1}^n\right)=n_{t+1}Q_t^N-E_t\left(\rho_{t+1}n_{t+2}Q_{t+1}^N\right). \quad (65)$$

The definition of the firm's value in equation (52) implies that

$$\vartheta_t^n - E_t\left(\rho_{t+1}\vartheta_{t+1}^n\right) = E_t\left(\rho_{t+1} cf_{t+1}^n\right). \tag{66}$$

Thus,

$$\vartheta_t^n - E_t\left(\rho_{t+1}\vartheta_{t+1}^n\right) = n_{t+1}Q_t^N - E_t\left(\rho_{t+1}n_{t+2}Q_{t+1}^N\right). \tag{67}$$

This implies the following expression for the asset value of employment:

$$\vartheta_t^n = n_{t+1}Q_t^N. \tag{68}$$

Aggregation Hence, the total value of a firm, Q_t, equals:

$$Q_t = \vartheta_t^k + \vartheta_t^n = k_{t+1}Q_t^K + n_{t+1}Q_t^N. \tag{69}$$

where the components are defined in equations (56) and (57), respectively.

9.2 Appendix B: The Data

9.2.1 Sample Statistics

Table B1 presents key sample statistics.

Table B1

Descriptive Sample Statistics
Quarterly, U.S. data

a. 1976:1-2016:4 ($n = 168$)

Variable	$\frac{f}{k}$	τ	$\frac{i}{k}$	δ	$\frac{wn}{f}$	$\frac{h^1}{n}$	ψ^1	β
Mean	0.14	0.37	0.024	0.02	0.62	0.126	0.125	0.99
Standard Deviation	0.01	0.05	0.003	0.003	0.03	0.010	0.011	0.005

b. 1994:1-2016:4 ($n = 92$)

Variable	$\frac{f}{k}$	τ	$\frac{i}{k}$	δ	$\frac{wn}{f}$	$\frac{h}{n} = \frac{h^1+h^2}{n}$	$\psi = \psi^1 + \psi^2$	β
Mean	0.15	0.34	0.026	0.02	0.61	0.177	0.176	0.99
Standard Deviation	0.01	0.005	0.002	0.002	0.03	0.012	0.012	0.004

9.2.2 Sources and Definitions

variable	**symbol**	**definition**
GDP	f	gross value added of NFCB
GDP deflator	p^f	price per unit of gross value added of NFCB
wage share	$\frac{wn}{f}$	numerator: compensation of employees in NFCB
discount rate	r	the rate of non-durable consumption growth minus 1
employment	n	employment in nonfinancial corporate business sector
hiring	h	gross hires
separation rate	ψ	gross separations divided by employment
vacancies	v	adjusted Help Wanted Index
investment	i	gross investment in NFCB sector
capital stock	k	stock of private nonresidential fixed assets in NFCB sector
depreciation	δ	depreciation of the capital stock
price of capital goods	p^I	real price of new capital goods

variable	symbol	source
GDP	f	NIPA accounts, table 1.14, line 41
GDP deflator	p^f	NIPA table 1.15, line 1
wage share	$\frac{wn}{f}$	NIPA; see note 7
discount rate	r	NIPA Table 2.3.3, lines 3, 8, and 13; see note 1
employment	n	CPS; see note 2
hiring	h	CPS; see note 3
separation rate	ψ	CPS; see note 3
vacancies	v	Conference Board; see note 4
investment	i	BEA and Fed Flow of Funds; see note 5
capital stock	k	BEA and Fed Flow of Funds; see note 5
depreciation	δ	BEA and Fed Flow of Funds; see note 5
price of capital goods	p^I	NIPA and U.S. tax foundation; see note 6

The sample period is 1976:1-2016:4 unless noted otherwise; all data are quarterly.

Notes:

1. *The discount rate and the discount factor*

The discount rate is based on a DSGE-type model with logarithmic utility $U(c_t) = \ln c_t$. Define the discount factor as $\rho_t \equiv \frac{1}{1+r_t}$

In this model:

$$U'(c_t) = U'(c_{t+1}) \cdot (1 + r_t) \tag{70}$$

Hence:

$$\rho_t = \frac{c_t}{c_{t+1}} \tag{71}$$

where c is non-durable consumption (goods and services) and 5% of durable consumption.

2. *Employment*

As a measure of employment in the nonfinancial corporate business sector (n) I take wage and salary workers in non-agricultural industries (series ID LNS12032187) less government workers (series ID LNS12032188), less self-employed workers (series ID LNS12032192). All series originate from CPS databases. I do not subtract workers in private households (the unadjusted series ID LNU02032190) from the above due to lack of sufficient data on this variable.

3. *Hiring and Separation Rates*

The aggregate flow from non-employment – unemployment (U) and out of the labor force (O) – to employment is to be denoted $OE + UE$ and the separation rate ψ_t is rate of the flow in the opposite direction, $EU + EO$. Worker flows within employment – i.e., job to job flows – are to be denoted EE.

I denote:

$$\begin{aligned}\frac{h}{n} &= \left(\frac{h^1}{n}\right)+\left(\frac{h^2}{n}\right) \\ \frac{h^1}{n} &= \frac{OE+UE}{E} \\ \frac{h^2}{n} &= \frac{EE}{E}\end{aligned} \tag{72}$$

Hence h^1 and h^2 denote flows from non-employment and from other employment, respectively.

Separation rates are given by:

$$\begin{aligned}\psi &= \psi^1+\psi^2 \\ \psi^1 &= \frac{EO+EU}{E} \\ \psi^2 &= \frac{EE}{E}=\frac{h^2}{n}\end{aligned} \tag{73}$$

Employment dynamics now satisfies:

$$\begin{aligned}n_{t+1} &= (1-\psi_t^1-\psi_t^2)n_t+h_t^1+h_t^2 \\ &= (1-\psi_t)n_t+h_t, \quad 0\leq\psi_t\leq 1 \\ h_t^2 &= \psi_t^2\end{aligned} \tag{74}$$

To calculate hiring and separation rates for the whole economy I use the following:

a. *The h_t^1 and ψ_t^1 flows.* I compute the flows between E (employment), U (unemployment) and O (not-in-the-labor-force) that correspond to the E,U,O stocks published by the CPS. The methodology of adjusting flows to stocks is taken from BLS, and is presented in Frazis et al (2005).[8]The data till 1990:Q1 were kindly provided by Ofer Cornfeld. The data from 1990:Q2 onwards were taken from the CPS (http://www.bls.gov/cps/cps_flows.htm). Employment is the quarterly average of the original seasonally adjusted total employment series from BLS (LNS12000000).

b. *The h_t^2 and ψ_t^2 flows.* The data on EE, available only from 1994:Q1 onward, were computed by multiplying the percentage of people moving from one employer to another using Fallick and Fleischman (2004)'s[9]data by the NSA population series LNU00000000, taken from the CPS, completing several missing observations and performing seasonal adjustment.

[8]Frazis, Harley J., Edwin L. Robison, Thomas D. Evans and Martha A. Duff, 2005. Estimating Gross Flows Consistent with Stocks in the CPS, **Monthly Labor Review,** September, 3-9.

[9]Fallick and Fleischman, 2004. "Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows," FEDS #2004-34.

4. *Vacancies*

I use the vacancies series based on the Conference Board Composite Help-Wanted Index that takes into account both printed and web job advertisements, as computed by Barnichon (2012). The updated series is available at

https://sites.google.com/site/regisbarnichon/research/publications.

This index was multiplied by a constant to adjust its mean to the mean of the JOLTS vacancies series over the overlapping sample period (2001:Q1–2013:Q4). As this series is based on indices, in estimation I estimate a scaling parameter a.

5. *Investment, capital and depreciation*

The goal here is to construct the quarterly series for real investment flow i_t, real capital stock k_t , and depreciation rates δ_t. I proceed as follows:

- Construct end-of-year fixed-cost net stock of private nonresidential fixed assets in NFCB sector, K_t . In order to do this I use the quantity index for net stock of fixed assets in NFCB (FAA table 4.2, line 37, BEA) as well as the 2009 current-cost net stock of fixed assets (FAA table 4.1, line 37, BEA).

- Construct annual fixed-cost depreciation of private nonresidential fixed assets in NFCB sector, D_t . The chain-type quantity index for depreciation originates from FAA table 4.5, line 37. The current-cost depreciation estimates (and specifically the 2009 estimate) are given in FAA table 4.4, line 37.

- Calculate the annual fixed-cost investment flow, I_t:

$$I_t = K_t - K_{t-1} + D_t$$

- Calculate implied annual depreciation rate, δ_a:

$$\delta_a = \frac{I_t - (K_t - K_{t-1})}{K_{t-1} + I_t/2}$$

- Calculate implied quarterly depreciation rate for each year, δ_{qt}:

$$\delta_q + (1-\delta_q)\delta_q + (1-\delta_q)^2\delta_q + (1-\delta_q)^3\delta_q = \delta_a$$

- Take historic-cost quarterly investment in private non-residential fixed assets by NFCB sector from the Flow of Funds accounts, atabs files, series FA105013005).

- Deflate it using the investment price index (the latter is calculated as consumption of fixed capital in domestic NFCB in current dollars (NIPA table 1.14, line 18) divided by consumption of fixed capital in domestic NFCB in chained 2009 dollars (NIPA table 1.14, line 42). This procedure yields the implicit price deflator for depreciation in NFCB. The resulting quarterly series, i_t_unadj, is thus in real terms.

- Perform Denton's procedure to adjust the quarterly series i_t_unadj from the Federal Flow of Funds accounts to the implied annual series from BEA I_t, using the depreciation rate δ_{qt} from above. I use the simplest version of the adjustment procedure, when the discrepancies between the two series are equally spread over the quarters of each year. As a result of adjustment I get the fixed–cost quarterly series i_t.

- Simulate the quarterly real capital stock series k_t starting from k_0 (k_0 is actually the fixed-cost net stock of fixed assets in the end of 1975, this value is taken from the series K_t) , using the quarterly depreciation series δ_{qt} and investment series i_t from above:

$$k_{t+1} = k_t \cdot (1 - \delta_{qt}) + i_t$$

6. *Real price of new capital goods*

In order to compute the real price of new capital goods, p^I, I use the price indices for output and for investment goods.

Investment in NFCB Inv consists of equipment Eq and structures St as well as intellectual property, which I do not include. I define the time-t price-indices for good $j = Eq, St$ as $\widetilde{p}_t^j$. The data are taken from NIPA table 1.1.4, lines 10, 11.

I take from http://www.federalreserve.gov/econresdata/frbus/us-models-package.htm the following tax -related rates:

a. The parameter τ – the statutory corporate income tax rate as reported by the U.S. Tax Foundation.

b. The investment tax credit on equipment and public utility structures, to be denoted ITC.

c. The percentage of the cost of equipment that cannot be depreciated if the firm takes the investment tax credit, denoted χ.

d. The present discounted value of capital depreciation allowances, denoted $ZPDE^{St}$ and $ZPDE^{Eq}$.

I then apply the following equations:

$$\begin{aligned} p^{Eq} &= \widetilde{p}^{Eq}\left(1 - \tau_{Eq}\right) \\ p^{St} &= \widetilde{p}^{St}\left(1 - \tau_{St}\right), \end{aligned}$$

$$\begin{aligned} 1 - \tau^{S_t} &= \frac{\left(1 - \tau\, ZPDE^{St}\right)}{1 - \tau} \\ 1 - \tau^{Eq} &= \frac{1 - ITC - \tau ZPDE^{Eq}\left(1 - \chi ITC\right)}{1 - \tau} \end{aligned}$$

Subsequently I compute their change between $t-1$ and t (denoted by Δp_t^j) :

$$\frac{\Delta p_t^{Inv}}{p_{t-1}^{Inv}} = \omega_t \frac{\Delta p_t^{Eq}}{p_{t-1}^{Eq}} + (1-\omega_t)\,\frac{\Delta p_t^{St}}{p_{t-1}^{St}}$$

where

$$\omega_t = \frac{\begin{array}{c}(\text{nominal expenditure share of } Eq \text{ in } Inv)_{t-1}\\ +\ (\text{nominal expenditure share of } Eq \text{ in } Inv)_t\end{array}}{2}.$$

The weights ω_t are calculated from the NIPA table 1.1.5, lines 9,11.

I divide the series by the price index for output, p_t^f, to obtain the real price of new capital goods, p^I.

As all of these prices are indices, in estimation I estimate a scaling parameter e^a.

7. *Labor share*

NIPA table 1.14, line 20 (compensation of employees in NFCB) divided by line 17 in the same table (gross value added in NFCB).

9.3 Appendix C: GMM Estimation of the FOC

This Appendix elaborates on the GMM estimation discussed in Section 4.

9.3.1 The Cost Function and its Derivatives

$$g(\cdot) = \left[\begin{array}{c} \frac{e_1}{\eta_1}(\frac{i_t}{k_t})^{\eta_1} \\ +\frac{e_2}{\eta_2}\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2} \\ +\frac{e_{31}}{\eta_{31}}\left(\frac{i_t}{k_t}\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}}\left(\frac{i_t}{k_t}\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}} \end{array} \right] f(z_t, n_t, k_t). \tag{75}$$

$$\frac{g_{i_t}}{\frac{f_t}{k_t}} = \left[\begin{array}{c} e_1(\frac{i_t}{k_t})^{\eta_1-1} \\ +e_{31}\left(\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}}(\frac{i_t}{k_t})^{\eta_{31}-1} + e_{32}\left(\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}}(\frac{i_t}{k_t})^{\eta_{32}-1} \end{array} \right] \tag{76}$$

$$\frac{g_{v_t}}{\frac{f_t}{n_t}} = \left[\begin{array}{c} e_2\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2-1}\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right] \\ +e_{31}q_t^1\left(\frac{i_t}{k_t}\right)^{\eta_{31}}\left(\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}-1} \\ +e_{32}q_t^2\left(\frac{i_t}{k_t}\right)^{\eta_{32}}\left(\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}-1} \end{array} \right] \tag{77}$$

$$\begin{aligned} \frac{g_{k_t}}{\frac{f_t}{k_t}} &= -\left[e_1(\frac{i_t}{k_t})^{\eta_1} + e_{31}\left(\frac{q_t^1 v_t}{n_t}\frac{i_t}{k_t}\right)^{\eta_{31}} + e_{32}\left(\frac{q_t^2 v_t}{n_t}\frac{i_t}{k_t}\right)^{\eta_{32}}\right] \\ &\quad +(1-\alpha)\left[\begin{array}{c} \frac{e_1}{\eta_1}(\frac{i_t}{k_t})^{\eta_1} \\ +\frac{e_2}{\eta_2}\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2} \\ +\frac{e_{31}}{\eta_{31}}\left(\frac{i_t}{k_t}\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}}\left(\frac{i_t}{k_t}\frac{q2 v_t}{n_t}\right)^{\eta_{32}} \end{array} \right] \end{aligned} \tag{78}$$

$$\begin{aligned} \frac{g_{n_t}}{\frac{f_t}{n_t}} &= -\left[\begin{array}{c} e_2\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2} \\ +e_{31}\left(\frac{q_t^1 v_t}{n_t}\frac{i_t}{k_t}\right)^{\eta_{31}} + e_{32}\left(\frac{q_t^2 v_t}{n_t}\frac{i_t}{k_t}\right)^{\eta_{32}} \end{array} \right] \\ &\quad +\alpha\left[\begin{array}{c} \frac{e_1}{\eta_1}(\frac{i_t}{k_t})^{\eta_1} \\ +\frac{e_2}{\eta_2}\left[\frac{(1-\lambda_1-\lambda_2)v_t+\lambda_1 q_t^1 v_t+\lambda_2 q_t^2 v_t}{n_t}\right]^{\eta_2} \\ +\frac{e_{31}}{\eta_{31}}\left(\frac{i_t}{k_t}\frac{q_t^1 v_t}{n_t}\right)^{\eta_{31}} + \frac{e_{32}}{\eta_{32}}\left(\frac{i_t}{k_t}\frac{q_t^2 v_t}{n_t}\right)^{\eta_{32}} \end{array} \right] \end{aligned} \tag{79}$$

9.3.2 The Estimating Equations

Replacing expected variables by actual ones and a rational expectations forecast error, the estimating equations are:

$$(1-\tau_t)\left(g_{i_t}+p_t^I\right)=\rho_{t+1}\left(1-\tau_{t+1}\right)\left[\begin{array}{c} f_{k_{t+1}}-g_{k_{t+1}} \\ +(1-\delta_{t+1})(g_{i_{t+1}}+p_{t+1}^I) \end{array}\right]+j_t^k \tag{80}$$

I estimate this equation after dividing throughout by $\frac{f_t}{k_t}$.

$$(1-\tau_t)\frac{g_{v_t}}{q_t}=\rho_{t+1}\left(1-\tau_{t+1}\right)\left[\begin{array}{c} f_{n_{t+1}}-g_{n_{t+1}}-w_{t+1} \\ +(1-\psi_{t+1})\frac{g_{v_{t+1}}}{q_{t+1}} \end{array}\right]+j_t^n \tag{81}$$

I estimate this equation after dividing throughout by $\frac{f_t}{n_t}$.

As explained in the text, estimation pertains to $\alpha, e_1, e_2, e_{31}, e_{32}, \eta_1, \eta_2, \eta_{31}, \eta_{32}, \lambda_1, \lambda_2$.

Tobin's Q Approach This approach ignores the other factor of production (i.e., assumes no adjustment costs for it), in the current case investment in capital. Hence in this case $e_1 = e_{31} = e_{32} = 0$ and $\eta_2 = 2$ and only equation (81) is estimated.

The Standard Search and Matching Model In this case $e_1 = e_{31} = e_{32} = 0, \eta_2 = 1, \lambda_1 = \lambda_2 = 0$ and there is only the hiring equation given by:

$$(1-\tau_t)\frac{e_2}{q_t}=\left[\rho_{t+1}\left(1-\tau_{t+1}\right)\frac{\frac{f_{t+1}}{n_{t+1}}}{\frac{f_t}{n_t}}\left[\alpha-\frac{w_{t+1}}{\frac{f_{t+1}}{n_{t+1}}}+(1-\psi_{t+1})\frac{e_2}{q_{t+1}}\right]\right]+j_t \tag{82}$$

This is estimated for e_2 and α.

Instruments The instrument set consists of 8 lags of the following variables – the hiring rate ($\frac{h}{n}$) and the investment rate ($\frac{i}{k}$) for both equations; the rate of growth of output per unit of capital ($\frac{f}{k}$) and the depreciation rate (δ) for equation (12); and the labor share ($\frac{wn}{f}$) and rate of separation (ψ) for equation (13).

9.4 Appendix D: Solving for the Decision Variables

9.4.1 Solution for the Investment Rate and Vacancy Rate

Use the FOC, the estimates of Table 1, and the derivatives of the cost function g in Appendix C to solve for the decision variables as follows:

$$\begin{aligned}
\frac{Q_t^K}{\frac{f_t}{k_t}} &= (1-\tau_t)\left(\left[e_1(\frac{i_t}{k_t}) + e_{31}\left(\frac{q_t^1 v_t}{n_t}\right) + e_{32}\left(\frac{q_t^2 v_t}{n_t}\right)\right] + \frac{p_t^I}{\frac{f_t}{k_t}}\right) \\
\frac{i_t}{k_t} &= \frac{1}{e_1}\left[\frac{\frac{Q_t^K}{\frac{f_t}{k_t}}}{(1-\tau_t)} - \frac{p_t^I}{\frac{f_t}{k_t}} - \left[e_{31}\left(\frac{q_t^1 v_t}{n_t}\right) + e_{32}\left(\frac{q_t^2 v_t}{n_t}\right)\right]\right]
\end{aligned} \tag{83}$$

$$\begin{aligned}
\frac{Q_t^N}{\frac{f_t}{n_t}} &= \frac{(1-\tau_t)\left[\begin{array}{c} e_2\frac{v_t}{n_t}\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2 \\ +e_{31}q_t^1\left(\frac{i_t}{k_t}\right)+e_{32}q_t^2\left(\frac{i_t}{k_t}\right)\end{array}\right]}{q_t} \\
\frac{v_t}{n_t} &= \frac{1}{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2\right]^2}\left[\frac{q_t\frac{Q_t^N}{\frac{f_t}{n_t}}}{(1-\tau_t)} - e_{31}q_t^1\left(\frac{i_t}{k_t}\right) - e_{32}q_t^2\left(\frac{i_t}{k_t}\right)\right]
\end{aligned} \tag{84}$$

Denote:

$$\begin{aligned}
\Lambda_t &\equiv (1-\lambda_1-\lambda_2)+\lambda_1 q_t^1+\lambda_2 q_t^2 \\
\Omega_t &= e_{31}q_t^1 + e_{32}q_t^2
\end{aligned} \tag{85}$$

Thus:

$$\frac{i_t}{k_t} = \frac{1}{e_1}\left[\frac{\frac{Q_t^K}{\frac{f_t}{k_t}}}{(1-\tau_t)} - \frac{p_t^I}{\frac{f_t}{k_t}} - \Omega_t\frac{v_t}{n_t}\right] \tag{86}$$

$$\frac{v_t}{n_t} = \frac{1}{e_2\Lambda_t^2}\left[\frac{q_t\frac{Q_t^N}{\frac{f_t}{n_t}}}{(1-\tau_t)} - \Omega_t\frac{i_t}{k_t}\right] \tag{87}$$

Therefore:

$$
\begin{aligned}
\frac{i_t}{k_t} &= \frac{1}{e_1}\left[\frac{\frac{Q_t^K}{\frac{f_t}{k_t}}}{(1-\tau_t)} - \frac{p_t^I}{\frac{f_t}{k_t}} - \Omega_t\left[\frac{1}{e_2\Lambda_t^2}\left[\frac{q_t\frac{Q_t^N}{\frac{f_t}{n_t}}}{(1-\tau_t)} - \Omega_t\frac{i_t}{k_t}\right]\right]\right] \\
&= \frac{1}{e_1}\left(\frac{\frac{Q_t^K}{\frac{f_t}{k_t}}}{(1-\tau_t)} - \frac{p_t^I}{\frac{f_t}{k_t}}\right) - \frac{1}{e_1}\Omega_t\frac{1}{e_2\Lambda_t^2}\frac{q_t\frac{Q_t^N}{\frac{f_t}{n_t}}}{(1-\tau_t)} + \frac{1}{e_1}\frac{\Omega_t^2}{e_2\Lambda_t^2}\frac{i_t}{k_t} \\
&= \frac{\frac{1}{e_1}\left(\frac{\frac{Q_t^K}{\frac{f_t}{k_t}}}{(1-\tau_t)} - \frac{p_t^I}{\frac{f_t}{k_t}}\right)\frac{1}{e_1}\Omega_t\frac{1}{e_2\Lambda_t^2}\frac{q_t\frac{Q_t^N}{\frac{f_t}{n_t}}}{(1-\tau_t)}}{1-\frac{1}{e_1}\frac{\Omega_t^2}{e_2\Lambda_t^2}} \\
&= \frac{e_1e_2\Lambda_t^2}{e_1e_2\Lambda_t^2-\Omega_t^2}\left[\frac{1}{e_1}\left(\frac{\frac{Q_t^K}{\frac{f_t}{k_t}}}{(1-\tau_t)} - \frac{p_t^I}{\frac{f_t}{k_t}}\right) - \frac{1}{e_1}\Omega_t\frac{1}{e_2\Lambda_t^2}\frac{q_t\frac{Q_t^N}{\frac{f_t}{n_t}}}{(1-\tau_t)}\right] \\
&= \frac{1}{e_1e_2\Lambda_t^2-\Omega_t^2}\left[e_2\Lambda_t^2\left(\frac{Q_t^K}{(1-\tau_t)\frac{f_t}{k_t}} - \frac{p_t^I}{\frac{f_t}{k_t}}\right) - q_t\Omega_t\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right] \\
&= \frac{1}{e_1e_2\Lambda_t^2-\Omega_t^2}\left[e_2\Lambda_t^2\left(\frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}\right) - q_t\Omega_t\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right] \qquad (88)
\end{aligned}
$$

where I have used

$$
\begin{aligned}
\frac{Q_t^K}{\frac{f_t}{k_t}} &= (1-\tau_t)\left(\frac{g_{i_t}}{\frac{f_t}{k_t}} + \frac{p_t^I}{\frac{f_t}{k_t}}\right) \\
\frac{g_{i_t}}{\frac{f_t}{k_t}} &= \frac{Q_t^K}{(1-\tau_t)\frac{f_t}{k_t}} - \frac{p_t^I}{\frac{f_t}{k_t}} \\
&= \frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}
\end{aligned}
$$

Similarly for vacancy creation:

$$
\begin{aligned}
\frac{v_t}{n_t} &= \frac{1}{e_2\Lambda_t^2}\left[\frac{q_t\frac{Q_t^N}{\frac{f_t}{n_t}}}{(1-\tau_t)} - \Omega_t\left[\frac{1}{e_1}\left[\frac{\frac{Q_t^K}{\frac{f_t}{k_t}}}{(1-\tau_t)} - \frac{p_t^I}{\frac{f_t}{k_t}} - \Omega_t\frac{v_t}{n_t}\right]\right]\right] \\
&= \frac{1}{1-\frac{\Omega_t^2}{e_1e_2\Lambda_t^2}}\left[\frac{1}{e_2\Lambda_t^2}q_t\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}} - \frac{1}{e_2\Lambda_t^2}\Omega_t\left[\frac{1}{e_1}\left[\frac{Q_t^K}{(1-\tau_t)\frac{f_t}{k_t}} - \frac{p_t^I}{\frac{f_t}{k_t}}\right]\right]\right] \\
&= \frac{1}{e_1e_2\Lambda_t^2-\Omega_t^2}\left[e_1q_t\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}} - \Omega_t\frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}\right] \qquad (89)
\end{aligned}
$$

9.4.2 Sensitivities

Investment

$$
\frac{i_t}{k_t} = \frac{1}{e_1e_2\Lambda_t^2-\Omega_t^2}\left[e_2\Lambda_t^2\left(\frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}\right) - q_t\Omega_t\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}\right] \qquad (90)
$$

$$
\frac{i_t}{k_t} = \frac{e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1q_t^1+\lambda_2q_t^2\right]^2 P_t^K - (q_t^1+q_t^2)\left[e_{31}q_t^1+e_{32}q_t^2\right]P_t^N}{e_1e_2\left[(1-\lambda_1-\lambda_2)+\lambda_1q_t^1+\lambda_2q_t^2\right]^2 - \left[e_{31}q_t^1+e_{32}q_t^2\right]^2} \qquad (91)
$$

where:

$$
\begin{aligned}
P_t^K &\equiv \frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}} > 0 \\
P_t^N &\equiv \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}} > 0
\end{aligned}
$$

$$\frac{\partial \frac{i_t}{k_t}}{\partial \frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}} = \frac{e_2\Lambda_t^2}{e_1e_2\Lambda_t^2 - \Omega_t^2} > 0 \tag{92}$$

$$\frac{\partial \frac{i_t}{k_t}}{\partial \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}} = \frac{-q_t\Omega_t}{e_1e_2\Lambda_t^2 - \Omega_t^2} > 0 \tag{93}$$

$$\begin{aligned}
\frac{\partial \frac{i_t}{k_t}}{\partial q_t^1} &= \frac{1}{\left[e_1e_2\Lambda_t^2 - \Omega_t^2\right]^2} (\\
&\left(2e_2\lambda_1\Lambda_t P_t^K - P_t^N \left[2e_{31}q_t^1 + e_{32}q_t^2 + e_{31}q_t^2\right]\right) \left[e_1e_2\Lambda_t^2 - \Omega_t^2\right] \\
&- \left[e_2\Lambda_t^2 P_t^K - (q_t^1 + q_t^2)\left[e_{31}q_t^1 + e_{32}q_t^2\right] P_t^N\right] \left[2e_1e_2\lambda_1\Lambda_t - 2e_{31}\left[e_{31}q_t^1 + e_{32}q_t^2\right]\right] \\
&) \\
&= \frac{2e_2\lambda_1\Lambda_t P_t^K - P_t^N\left[2e_{31}q_t^1 + q_t^2(e_{31}+e_{32})\right]}{e_1e_2\Lambda_t^2 - \Omega_t^2} \\
&- \frac{\left[e_2\Lambda_t^2 P_t^K - \Omega_t q_t P_t^N\right]\left[2(e_1e_2\lambda_1\Lambda_t - e_{31}\Omega_t)\right]}{\left[e_1e_2\Lambda_t^2 - \Omega_t^2\right]^2}
\end{aligned} \tag{94}$$

$$\begin{aligned}
\frac{\partial \frac{i_t}{k_t}}{\partial q_t^2} &= \frac{1}{\left[e_1e_2\Lambda_t^2 - \Omega_t^2\right]^2} (\\
&\left(2e_2\lambda_2\Lambda_t P_t^K - P_t^N \left[2e_{32}q_t^2 + e_{32}q_t^1 + e_{31}q_t^1\right]\right) \left[e_1e_2\Lambda_t^2 - \Omega_t^2\right] \\
&- \left[e_2\Lambda_t^2 P_t^K - (q_t^1 + q_t^2)\left[e_{31}q_t^1 + e_{32}q_t^2\right] P_t^N\right] \left[2e_1e_2\lambda_2\Lambda_t - 2e_{32}\left[e_{31}q_t^1 + e_{32}q_t^2\right]\right] \\
&) \\
&= \frac{2e_2\lambda_2\Lambda_t P_t^K - P_t^N\left[2e_{32}q_t^2 + q_t^1(e_{31}+e_{32})\right]}{e_1e_2\Lambda_t^2 - \Omega_t^2} \\
&- \frac{\left[e_2\Lambda_t^2 P_t^K - \Omega_t q_t P_t^N\right]\left[2(e_1e_2\lambda_2\Lambda_t - e_{32}\Omega_t)\right]}{\left[e_1e_2\Lambda_t^2 - \Omega_t^2\right]^2}
\end{aligned} \tag{95}$$

The estimates of Table 1 indicate that $e_1e_2\Lambda_t^2 - \Omega_t^2 > 0$ and that $\Omega_t < 0$. Hence $\frac{i_t}{k_t}$ is a positive function of $\frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}$ and $\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$.

Vacancy Creation

$$\frac{v_t}{n_t} = \frac{1}{e_1e_2\Lambda_t^2 - \Omega_t^2}\left[e_1 q_t \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}} - \Omega_t \frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}\right] \tag{96}$$

$$\frac{v_t}{n_t} = \frac{\left[e_1\left(q_t^1 + q_t^2\right)P_t^N - \left[e_{31}q_t^1 + e_{32}q_t^2\right]P_t^K\right]}{e_1e_2\left[(1-\lambda_1-\lambda_2) + \lambda_1 q_t^1 + \lambda_2 q_t^2\right]^2 - \left[e_{31}q_t^1 + e_{32}q_t^2\right]^2} \tag{97}$$

$$\frac{\partial \frac{v_t}{n_t}}{\partial \frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}} = \frac{-\Omega_t}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} > 0 \tag{98}$$

$$\frac{\partial \frac{v_t}{n_t}}{\partial \frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}} = \frac{e_1 q_t}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} > 0 \tag{99}$$

$$\begin{aligned}
\frac{\partial \frac{v_t}{n_t}}{\partial q_t^1} &= \frac{1}{\left[e_1 e_2 \Lambda_t^2 - \Omega_t^2\right]^2} (\\
&\quad \left(e_1 P_t^N - e_{31} P_t^K\right) \left[e_1 e_2 \Lambda_t^2 - \Omega_t^2\right] \\
&- \left[e_1 q_t P_t^N - \left(e_{31} q_t^1 + e_{32} q_t^2\right) P_t^K\right] \left[2 e_1 e_2 \lambda_1 \Lambda_t - 2 e_{31} \left(e_{31} q_t^1 + e_{32} q_t\right)\right] \\
&\quad) \\
&= \frac{e_1 P_t^N - e_{31} P_t^K}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} - \frac{\left[e_1 q_t P_t^N - \Omega_t P_t^K\right] \left[2 e_1 e_2 \lambda_1 \Lambda_t - 2 e_{31} \Omega_t\right]}{\left[e_1 e_2 \Lambda_t^2 - \Omega_t^2\right]^2} \qquad (100)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \frac{v_t}{n_t}}{\partial q_t^2} &= \frac{1}{\left[e_1 e_2 \Lambda_t^2 - \Omega_t^2\right]^2} (\\
&\quad \left(e_1 P_t^N - e_{32} P_t^K\right) \left[e_1 e_2 \Lambda_t^2 - \Omega_t^2\right] \\
&- \left[e_1 q_t P_t^N - \left(e_{31} q_t^1 + e_{32} q_t^2\right) P_t^K\right] \left[2 e_1 e_2 \lambda_2 \Lambda_t - 2 e_{32} \left(e_{31} q_t^1 + e_{32} q_t\right)\right] \\
&\quad) \\
&= \frac{e_1 P_t^N - e_{32} P_t^K}{e_1 e_2 \Lambda_t^2 - \Omega_t^2} - \frac{\left[e_1 q_t P_t^N - \Omega_t P_t^K\right] \left[2 e_1 e_2 \lambda_2 \Lambda_t - 2 e_{32} \Omega_t\right]}{\left[e_1 e_2 \Lambda_t^2 - \Omega_t^2\right]^2} \qquad (101)
\end{aligned}$$

Hence $\frac{v_t}{n_t}$ is a positive function of $\frac{\widetilde{Q}_t^K}{(1-\tau_t)\frac{f_t}{k_t}}$ and $\frac{Q_t^N}{(1-\tau_t)\frac{f_t}{n_t}}$.

9.5