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Entrepreneurship and the Racial Wealth Gap Daniel Albuquerque, Tomer Ifergane

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# Entrepreneurship and the Racial Wealth Gap\*

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#### Abstract

Entrepreneurship promotes wealth accumulation. However, Black households face significant barriers to entrepreneurship, operating fewer and smaller businesses. We formalize a general equilibrium model of entrepreneurship choice and wealth accumulation in which Black households experience adverse distortions as entrepreneurs and as workers. Disciplined by microdata, our model matches well the observed racial wealth gap and the correlation between wealth and entrepreneurship. We find that distortions faced by Black entrepreneurs are the key factor for understanding the racial wealth gap across the wealth distribution. Our analysis also indicates that addressing racial disparities in the U.S. can substantially increase output.

**Keywords:** Racial wealth gap, entrepreneurship, incomplete markets, wealth accumulation, financial frictions, wealth inequality

**JEL Codes:** E21, J15, D31, D52

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# **1** Introduction

Racial wealth inequality in the United States is striking. Figure 1 shows that the average Black household's net worth from 2001 to 2019 was equal to \$133,600, while the average White household held \$811,900. In other words, Black households held 83.6% less wealth. Beyond racial disparities, overall wealth inequality is also high: the top 10% of households hold 73.2% of total wealth. Entrepreneurs, those who own and manage a business, are overrepresented at the top of the wealth distribution and an established literature has highlighted the central role of entrepreneurship in understanding overall wealth inequality (Quadrini, 2000; Castaneda, Diaz-Gimenez, and Rios-Rull, 2003; Cagetti and De Nardi, 2006). At the same time, Black households have three times lower entrepreneurship rates than White ones and the median Black-owned firm is 2.9 times smaller than its White counterpart. This paper asks: what is the contribution of disparities in entrepreneurship to the racial wealth gap?

We begin our analysis with an empirical study of race, wealth and entrepreneurship in the United States. Using households surveys (Panel Survey of Income Dynamics and Survey of Consumer Finance) we document the salient empirical regularities. First, there is a large racial wealth gap both in terms of average wealth and at different percentiles of the wealth distribution. These gaps are stable and unchanged from 1989–2019. Second,

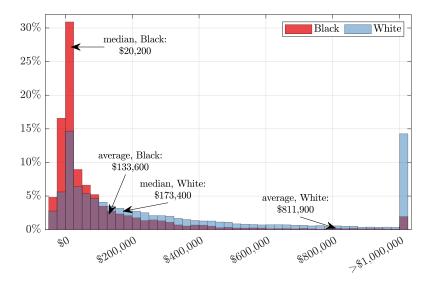


Figure 1: Histogram of wealth for Black and White households, 2001-2019

*Notes*: This figure plots the histogram of wealth for Black and White households, using data from 2001 to 2019. Values are adjusted to 2019 dollars. *Source*: SCF.

these wealth gaps exist alongside sizable gaps in entrepreneurship rates and firm sizes.

Next, we focus on the relationship between the racial gaps in wealth and entrepreneurship that we documented. Regardless of race, entrepreneurship rates are increasing in wealth, and we document that these correlations are remarkably similar for Black and White households. Using non-linear Blinder–Oaxaca decompositions we report that entrepreneurship explains, in the correlational sense, a sizable share of the racial wealth gap and more so at the top of the wealth distribution. Factors affecting the entrepreneurship choice such as education and labor income are also important, but more so for poorer households. Furthermore, we demonstrate how wealth is a vehicle of wealth mobility, improving the probability of Black and White households to move up the wealth distribution. However, while informative, this empirical analysis is not causal, and neglects the dynamic effects of entrepreneurship on wealth accumulation. To overcome this we build a structural model.

Our main contribution is developing an incomplete market, general equilibrium, model of wealth accumulation featuring a dynamic choice between employment and entrepreneurship, along with differences in economic outcomes between Black and White households. In the model Black and White households have identical preferences and beliefs, but Black households are subject to distortions that hinder their income generation both as workers and as entrepreneurs. Households save to smooth income fluctuations and also to alleviate financial constraints faced by entrepreneurs. Since households are infinitely lived, the model generates strong intergenerational persistence of wealth. The model is able to replicate the racial wealth gap and the correlation of wealth, entrepreneurship and race, and allows us to analyze counterfactuals that shed light on the extent that observed differences in entrepreneurial outcomes help us understand the racial wealth gap. We can then use it to analyze the importance of entrepreneurship for the racial wealth gap, and report four main findings.

First, our main result is that current distortions faced by Black entrepreneurs are the major factor accounting for racial wealth differences. Removing these distortions reverses the sign of the average racial wealth gap, making Black households more than 20% wealthier on average than White households. Moreover, the median racial wealth gap would also fall drastically to 21.6% from a baseline of 76.2% in our calibrated model.

Second, we analyze policies targeted at reducing the racial wealth gap by subsidizing Black entrepreneurship and conclude that this would only be effective if it generated much higher entrepreneurship rates for Black households than for White ones. This is due to ongoing distortions in the labor market that push some Black households into entrepreneurship even in the face of distortions, highlighting the importance of analyzing the two markets at the same time, since one is the outside option of the other.

Third, the entrepreneurship distortion has important macroeconomic consequences, as removing it alone increases output by 4.0%. This result is mainly due to a relative reallocation of resources from White-owned to Black-owned firms as Black-owned firms can now realize their full potential.

Last, we show that even under favorable conditions it would take more than 150 years for the racial wealth gap to close, since it takes time for Black households to become entrepreneur, grow their firms, accumulate profits, and break into the top of the wealth distribution. The combined results highlight the centrality of entrepreneurship in explaining the racial wealth gap and in studying policies targeting it.

The key ingredient that allows us to generate difference between ex-ante identical Black and White households in the model is *distortions*. In our model, the desirability of entrepreneurship at any level of assets and labor income is determined both by the expected profits as entrepreneur and the forgone labor income as a worker. Therefore, the model gives rise to endogenous sorting into entrepreneurship. We leverage tools from the misallocation literature and model racial differences affecting this entrepreneurship choice using *distortions*—disciplined by micro-data—that affect both margins. These distortions (i) reduce the wage conditional on employment of Black workers (*labor income distortion*), (ii) lower the attachment rate of Black workers to the labor market and increases their labor income risk (*labor income risk distortion*), and (iii) reduce the size of Black-owned firms, controlling for productivity (*entrepreneurship distortion*). These distortions are reducedform modeling tools which, in reality, map into frictions and institutional barriers such as discrimination, different access to education, different social capital, etc. Thus, our model remains agnostic regarding their root causes, but allows us to separately examine them as distinct margins of influence.

We quantify the model distortions in two steps. First, labor income processes are separately estimated for Black and White households. These processes include permanent and transitory components along with a non-employment shock, all with race-specific parameters which allow us to pin down the labor income and labor income risk distortions. Second, to calibrate the entrepreneurship distortion we use the empirical estimates of Tan and Zeida (2024), who estimate a wedge in the average revenue product of capital and labor directly from firm-level data. Given the estimates for the distortions, we use moments on entrepreneurship, wealth dispersion, and the correlation between labor income and entrepreneurial entry to calibrate the model's steady state. Since the racial gaps in wealth and entrepreneurship have been stable over the last thirty years, we interpret these prevailing gaps as steady-state outcomes. Importantly, we do not target these gaps. The resulting calibrated model does an excellent job in capturing the racial wealth gap, generating an untargeted average racial wealth gap of 78.3%, compared to 83.6% in the data. Our model also replicates the increasing and similar entrepreneurship rates conditional on wealth for Black and White households and is consistent with the gaps in entrepreneurial outcomes both in terms of entrepreneurship rates and firm sizes.

#### **1.1 Related literature**

This work is primarily related to works on drivers of the racial wealth gap in the macroeconomics literature. Aliprantis, Carroll, and Young (2022) and Ashman and Neumuller (2020) model exogenous labor income gaps, and White (2007) does so through differences in human capital accumulation. All of these conclude that observed differences in labor earnings can generate large racial wealth gaps. Entrepreneurs' income includes both dividends and wages paid to the entrepreneur, but labor income only includes the latter. In our calibration, we make sure to clearly separate entrepreneurial and labor income. We find that it is the entrepreneurs' earnings that are crucial for understanding the racial wealth gap and, in particular, differences at the very top of the wealth distribution, although labor earnings differences also matter. In related work, İmrohoroğlu, Kumru, and Lain (2025) find that crime has little bearing on the racial wealth gap precisely because it mainly affects low-income households. Thus, our contribution relative to this literature is to isolate, model, and highlight the importance of entrepreneurship choices.

Closest to our work are Lipton (2022) and Boerma and Karabarbounis (2023), who also model differences in entrepreneurship and firm ownership between Black and White households. Compared to Lipton (2022), we model distortions in the labor market and entrepreneurship jointly and allow for endogenous firm creation, thus allowing us to consider changes in firm ownership over time and to assess the contribution of equilibrium forces to it. Boerma and Karabarbounis (2023) focus on the ability of heterogeneous beliefs and historical exclusion of Black households from markets to generate a persistent racial wealth gap via entrepreneurship choices. Relative to Boerma and Karabarbounis (2023) we model richer and race-specific labor income processes and explicitly target the correlation between labor income and entry into entrepreneurship. Because labor income is positively correlated with entrepreneurial entry, this is an important feature to capture when considering the relative importance of entrepreneurship vs labor income distortions. Moreover, our structural model allows us perform counterfactuals that are not subject to the criticism of Catherine, Lu, and Paron (2024), and complements their analysis of differences in wealth excluding entrepreneurs.

This paper is motivated by the literature documenting barriers faced by Black entrepreneurs. Most of the literature so far has focused on credit barriers for Black entrepreneurs,<sup>1</sup> but recently Tan and Zeida (2024) argue that a demand distortion is what best explains the observed differences between Black and White-owned firms. We show evidence that large Black-owned firms, which might not be credit constrained, are still considerably smaller than their White-owned counterparts, which supports Tan and Zeida (2024) findings.

Our paper also contributes to the growing body of work leveraging tools from the misallocation literature to study disparities in outcomes. The seminal work is Hsieh et al. (2019), and directly related to us are works studying entrepreneurship disparities between groups. The most relevant among these are Bento and Hwang (2022) and Tan and Zeida (2024), who use rich panel data to study the different barriers faced by Black entrepreneurs. Our paper complements these works by mapping their results on entrepreneurship into consequences for wealth accumulation.

Finally, another sector that has received attention is the housing market, including its importance for the racial wealth gap (e.g., Flippen, 2004; Faber and Ellen, 2016; Kermani and Wong, 2021; Gupta, Hansman, and Mabille, 2022; Higgins, 2023). Since housing wealth is more concentrated in the middle of the wealth distribution and Black households are poorer on average, housing represents a higher share of Black-owned wealth, while private businesses represent a higher share of White-owned wealth.<sup>2</sup> Thus, disregarding housing wealth actually increases the average racial wealth gap between Black and White households from 83.6% to 88.5%, which leads us to focus on entrepreneurship.

<sup>&</sup>lt;sup>1</sup>Studies have found that Black entrepreneurs face lower approval rates for credit (Blanchflower, Levine, and Zimmerman, 2003; Blanchard, Zhao, and Yinger, 2008; Cavalluzzo and Wolken, 2005; García and Darity Jr, 2021); face higher interest rates (Dougal et al., 2019; Hu et al., 2011); get access to smaller loans (Atkins, Cook, and Seamans, 2022; Bates and Robb, 2016); have a harder time raising start-up capital and apply for loans less often, fearing they would be denied (Fairlie, Robb, and Robinson, 2022).

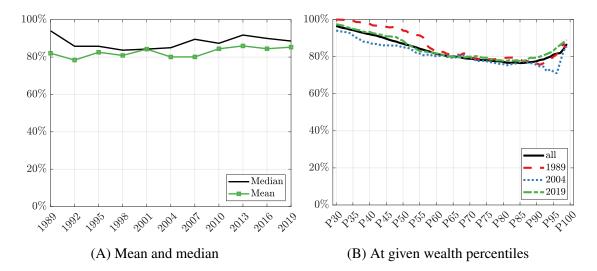
<sup>&</sup>lt;sup>2</sup>From 2001 to 2019, housing wealth (net of housing debt) represented 52.1% and 31.6% of the wealth held by Black and White households, respectively. For private business, these shares were 12.2% and 21.4%.

The paper proceeds as follows. Section 2 presents stylized facts regarding disparities in wealth, and entrepreneurship, in the U.S. and documents empirical evidence in support of our proposed mechanism. Section 3 develops our model. Section 4 calibrates the model and discusses its fit. Section 5 demonstrates the role of gaps in entrepreneurial outcomes in generating the racial wealth gap and its macroeconomic implications, and analyzes the effects of entrepreneurship subsidies. Section 6 analyses counterfactual scenarios about the future of the racial wealth gap and the potential role of wealth transfers. Section 7 concludes.

# 2 Race, wealth, and entrepreneurship in the US: empirical evidence

This section presents empirical evidence that guide our analysis of the relationship between entrepreneurship and the racial wealth gap. We document that (1) there is a substantial and stable racial wealth gap; (2) entrepreneurs are disproportionately represented among the wealthy, both among Black and White households; and (3) Black households are three times less likely to be entrepreneurs and Black-owned firms are significantly smaller than their White-owned counterparts. Using a non-linear Blinder–Oaxaca decomposition, we argue that entrepreneurship choice and factors affecting it account, in the correlational sense, for a large share of the racial wealth gap. We further demonstrate how, from a dynamic perspective, entrepreneurship is a vehicle for upwards wealth mobility. These empirical patterns on the relationship between wealth, entrepreneurship, and race will discipline our analysis and allow us to evaluate the model developed in the following sections. Given the importance of labor market outcomes to entrepreneurship choices we provide additional descriptive statistics in Appendix B.

Our primary data sources are the Survey of Consumer Finances (SCF) and the Panel Survey for Income Dynamics (PSID). We mostly use the SCF to examine wealth and entrepreneurship and use the PSID to examine labor markets, and when a panel structure is necessary. We view these surveys as complementary sources. The SCF oversamples wealthy households to focus on the top of the wealth distribution, while the PSID is wellsuited for the bottom of the income distribution. Importantly, the PSID added an extra sample in the 1990s to better capture the increase in minority households since its inception, which means that the sample size of Black households is substantial. In both surveys, the unit of observation is a household. We restrict our sample to households where the main respondent identifies as Black or White, excluding all households that also identified as Latinx or of Hispanic origin. Since we are interested in fitting our model to current gaps in wealth and entrepreneurship, we focus on the period between 2001 and 2019 to calibrate the model.



#### 2.1 The racial wealth gap

Figure 2: The racial wealth gap

*Notes*: Wealth gaps are defined as  $1 - \hat{w}^B / \hat{w}^W$ , where  $\hat{w}^i$  where denotes a measure of household wealth such as the average, median, or a particular quantile for households of race *i*. Panel (A) shows the average (median) racial wealth gap between Black and White households. Panel (B) shows the racial wealth gap at each percentile of wealth by comparing the distribution of wealth of Black and White households, separately. Panel (B) shows the results using the whole sample over 1989-2019, or for specific years. *Source*: SCF 1989-2019.

We define wealth as total assets minus total liabilities of a household and wealth gaps as  $1 - \hat{w}^B / \hat{w}^W$  with  $\hat{w}^i$  denoting a statistic of the wealth distribution (average, median, etc.) for households of race *i*. The average and median racial wealth gaps are reported in Figure 2A. Since the 1980s, these gaps have been stable and hovered between 80% and 90%, averaging at 83.6% and 88.4% for the average and median gaps, respectively, since 2001.

The finding of a sizable and recently stable racial wealth gap is well documented in the literature, and Figure 2A is consistent with these findings. Kuhn, Schularick, and Steins (2020) extended the SCF further back in time and document that the wealth gap has been

more or less stable in the last seventy years. Derenoncourt et al. (2024) go even further back to the 1860s and report that there was significant progress in closing the gap in the fifty years after the Emancipation, from an extremely high level in 1860, and also some progress from 1920 to 1950. However, progress has stalled since then.

Figure 2B compares the wealth gaps at different percentiles of the separate Black and White wealth distributions. We do this exercising either by pooling together the whole sample from 1989-2019, or by looking at specific years only. The figure shows that the racial wealth gap is higher at the bottom of the wealth distribution, falls until the 80<sup>th</sup>-85<sup>th</sup> percentile, and then starts rising again. Regardless of the wealth statistic one chooses to examine, the following facts emerge: the racial wealth gap is sizable, it is not localized in a particular part of the distribution, and it has remained virtually unchanged for the past three decades.

#### 18%15%median White firm is 2.9 times larger 12%log sales 9% 6%3%Black 0% 2010 1992 1995 2004 2010 2013 1989 1998 2007 2019 2001 White R10 2'50 220 2<sup>30</sup> 260 P70 RHD 280 2090 Owns & manages, Black -Owns, Black Owns & manages, White ---- Owns, White percentile of sales distribution (A) Entrepreneurship rates (B) Firm size distribution

#### 2.2 Racial differences in entrepreneurship

Figure 3: Entrepreneurship rates and outcomes

*Notes*: Panel (A) reports the share of Black and White households that are entrepreneurs according to two definitions: (i) owns a private business; (ii) owns and actively manages a private business. Panel (B) reports the percentiles of the distribution of log revenue of Black and White-owned firms separately. There is no information for the lower percentiles because some firms do not report positive sales. *Source*: SCF.

A long-standing strand of the economic literature has emphasized the importance of entrepreneurship for understanding overall wealth accumulation and wealth inequality (Quadrini, 2000; Castaneda, Diaz-Gimenez, and Rios-Rull, 2003; Cagetti and De Nardi, 2006). To understand whether entrepreneurship is also essential for accounting for the racial wealth gap, i.e., accounting for cross sectional differences between groups, we start by exploring if there are differences in entrepreneurship between Black and White households. As we argue below, the answer is a clear yes.

We define an entrepreneur as a household who owns and actively manages a private business, as documented by the SCF. Households that own a business but do not manage it are not considered entrepreneurs to exclude households that made a portfolio choice of investing in a private business but are otherwise not engaged in entrepreneurial activity. Figure 3A plots the entrepreneurship rates over the last 30 years according to the SCF. It shows that there is also a racial gap in entrepreneurship rates, which has been stable and sizable, around 9 p.p. (5.2% for Black households vs 14.2% for White ones), over the last three decades.<sup>3</sup> That is, Black households are nearly three times less likely to be entrepreneurs than White ones.

Examining the PSID allows us to check if this is true also over a longer time period and different definitions of entrepreneurship. Figure A.1 in the Appendix plots the entrepreneurship rate over time of Black and White households. We report results for three alternative definitions of entrepreneurship: (i) self-employment; (ii) ownership of a business; (iii) ownership of an incorporated business. According to the first two definitions, the racial gap in entrepreneurship rates is shrinking. However, these definitions of entrepreneurship differ from ours as they include individuals who turned to self-employment due to a precarious situation in the labor market (Levine and Rubinstein, 2017; Fairlie and Fossen, 2018). When restricting attention to the owners of incorporated businesses, a definition of entrepreneurship that more closely aligns with our SCF-based definition and relates more closely to wealth accumulation, a similar picture emerges in the PSID and the SCF. Figure A.1 reports a stable racial gap in entrepreneurship rates going back to at least the mid-1970s. It is noteworthy that the transition rate between self-employment and ownership of incorporated businesses is small.<sup>4</sup>

<sup>&</sup>lt;sup>3</sup>Results are also reported for the alternative "owns a business" definition as well and are similar. Our measure is more restrictive and results in a smaller gap in entrepreneurship rates.

<sup>&</sup>lt;sup>4</sup>This is motivated by previous research which has shown that incorporated businesses are those most associated with entrepreneurship activities, are more likely to be present at the top of the wealth distribution, and evidence shows little switching from unincorporated businesses to incorporated ones Levine and Rubinstein (2017). We also find that transition between non-incorporated businesses and incorporate businesses are rare in the PSID. For example, between 1969 and 1998, when the PSID was annual, the transition rate from self-employed to incorporated business was only 5.3%, and from 1999 to 2019 the two-year transition rate was 6.4%.

Our findings align well with those of Fairlie and Meyer (2000), who use census data from 1910 to 1990 to document the longer trends of Black and White self-employment rates. They find: (i) a stable gap in entrepreneurship; (ii) that Black households have a third of the rate of entrepreneurship of White households; and (iii) that entrepreneurship rates were 4.1% and 11.4% in 1990 for Black and White households, respectively. These results are qualitatively similar to ours, while using a different data source and definition of entrepreneurship. Bento and Hwang (2022), find a closing racial gap in entrepreneurship rates. However, they do so while defining entrepreneurship as self-employment using data from the Survey of Business Owners and the Current Population Survey. Our above discussion on the different definitions of entrepreneurship in the PSID, reconciles the two sets of results. We conclude that the notion of entrepreneurship that is relevant for our purposes has indeed shown a stable gap between Black and White households and is consistent with existing works.

On top of differences in entrepreneurship rates, Figure 3B shows that there are also differences in outcomes conditional on being an entrepreneur. Using information from firm owners in the SCF we can calculate the implied revenue distribution for firms owned by Black and White households. We find that the median White-owned firm is 2.9 times larger than the median Black-owned firm, and the difference seems relatively stable throughout the firm-size distribution.

This large difference in revenue is unsurprising in light of a long literature on the different outcomes and barriers faced by Black and White entrepreneurs. Interestingly, most of it has focused on the barriers in access to credit for Black entrepreneurs. However, recent evidence from Tan and Zeida (2024) suggest that, while extra financial constraints play a role, the most important barrier is lower consumer demand, or a markup wedge, for products of Black-owned firms. Figure 3B is in line with this finding. This is because the distribution of revenue of Black-owned firms is shifted downwards compared to that of White-owned firms. Lower demand for all Black-owned firms or other factors that limit Black entrepreneurs from realizing their full potential across the board can explain this permanently smaller size across the distribution. Additional credit constraints, in contrast, would not affect larger, better-capitalized, Black-owned firms, and these firms would be able to catch up with their White-owned counterparts. This does not seem to be the case, and our structural model will allow us to match the patterns in Figure 3B.

#### **2.3** Entrepreneurship and wealth

Having established that there are large differences in wealth and entrepreneurship between Black and White households, we investigate whether the latter can help explain the former. This is not a straightforward question since entrepreneurship is correlated with many other variables, like labor income, education and innate ability.

We start by establishing a simple fact; entrepreneurship is positively correlated with wealth. Figure 4, reports that the correlation between entrepreneurship and wealth is strong regardless of race. Furthermore, entrepreneurship rates conditional on wealth are surprisingly similar between Black and White households.<sup>5</sup> More than 60% of Black or White households in the top 1% of the overall wealth distribution are classified as entrepreneurs, while in the bottom half the corresponding figure is less than 10%. Interestingly, the average racial wealth gap between Black and White workers (75.6%) and between Black and White entrepreneurs (79.4%) are quite similar to the overall racial wealth gap of 83.6%. However, White entrepreneurs hold 45.3% of White-owned wealth, while Black entrepreneurs hold only 25.3% of Black-owned wealth, which is explained by a lower entrepreneurship rate among Black households. As entrepreneurs are wealthier than the average population and Black households are less likely to become entrepreneurs, this creates a phenomenon of missing Black entrepreneurship wealth that can help explain the large racial wealth gap.

#### 2.3.1 Non-linear Blinder–Oaxaca decompositions

Given this correlation, we now turn our attention to unpacking the statistical relationship between entrepreneurship and wealth in the data by employing a non-linear Blinder (1973) – Oaxaca (1973) (BO) decomposition. For comparison, we also examine the contribution of labor income and education towards the racial wealth gap.

We are interested in explaining differences in the outcome variable  $y_j$  = wealth of household *j* between Black and White households. The original BO framework can be limiting because it assumes a linear relationship between the explanatory variables  $X_j$ and the outcome variable  $y_j$  (Barsky et al., 2002). However, it could be that differences at the top and the bottom of the wealth distribution have different causes. A non-linear BO decomposition allows one to capture that. The idea is to re-weight the population of White households to match the conditional distribution of a set of explanatory variables for

<sup>&</sup>lt;sup>5</sup>Figure A.2 presents a similar pattern for other definitions of entrepreneurship in the SCF and the PSID.

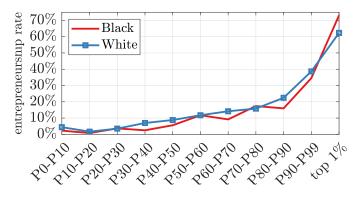


Figure 4: Entrepreneurship rates by wealth fractiles

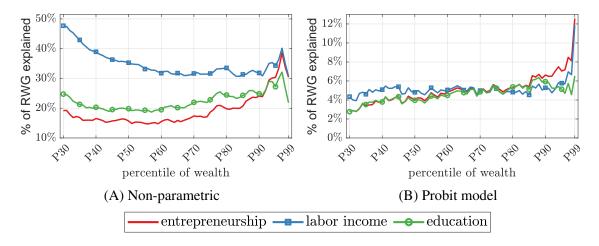


Figure 5: Blinder-Oaxaca non-linear decomposition - importance for explaining the RWG

*Notes*: Figure 4 shows the share of Black and White households that are classified as entrepreneurs in different fractiles of the overall wealth distribution. Entrepreneurs are defined as those that own and manage a private business and "P10-P20", for example, denotes those in between the 10<sup>th</sup> and 20<sup>th</sup> percentiles. Figure 5 displays how much each variable can explain the racial wealth gap at each percentile of the Black vs White wealth distributions, according to non-linear Oaxaca-Blinder decompositions that re-weight the sample of White households. Panel (A) shows the percentage of the RWG that closes when re-weighting non-parametrically one variable at a time according to Equation (1). Panel (B) shows how much the RWG widens when removing the highlighted variable from the Probit in the re-weighting of Equation (2). "Entrepreneurship" is a dummy for being an entrepreneur, "education" are dummies for high-school and college completion, and "labor income" is wage income. *Source*: SCF, 2001-2019.

Black households.<sup>6</sup> This type of exercise follows the methodology of DiNardo, Fortin, and Lemieux (1995) and has been used extensively in the literature on racial differences, e.g., Barsky et al. (2002), Altonji, Bharadwaj, and Lange (2012), Luo (2021), and Sabelhaus and Thompson (2023) (see Fortin, Lemieux, and Firpo, 2011, for a comprehensive discussion on decomposition methods).

**Non-parametric decomposition** When isolating the effect of a single variable, one can do the re-weighting non-parametrically. Let  $\omega_j^W$  be the weight of White household *j* in the sample. For the effect of entrepreneurship, we re-weight every White household by

$$\widehat{\omega}_{j}^{W} = \omega_{j}^{W} \times \begin{cases} \mathbb{P}(\text{entrep} = 1|B) / \mathbb{P}(\text{entrep} = 1|W), \text{ if } j \text{ is an entrepreneur} \\ \mathbb{P}(\text{entrep} = 0|B) / \mathbb{P}(\text{entrep} = 0|W), \text{ otherwise} \end{cases}$$
(1)

where  $\mathbb{P}(\text{entrep} = 1|B)$  is the entrepreneurship rate of Black households, for example. Thus, according to the new weights  $\widehat{\omega}_j^W$  the entrepreneurship rate among White households is the same as that of Black households. A similar logic applies to the education variable, which is categorical and can have three different values: high-school completed, college completed, and neither. For labor income, a continuous variable, we calculate percentiles  $\{5, 10, \dots, 95, 100\}$  of the Black labor income distribution and re-weight White households such that the percentiles of their labor income distribution is the same.<sup>7</sup>

After the re-weighting in Equation (1) we can then compare the racial wealth gap at each percentile of the White and Black distributions with the original one shown in Figure 2B. Results from this exercise are reported in Figure 5A, which shows which percentage of the racial wealth gap is closed by the re-weighting of each variable. It demonstrates that each variable, in isolation, can explain between 16-50% of the RWG at the bottom of the

<sup>&</sup>lt;sup>6</sup>We do the re-weighting on the White distribution for two main reasons. First, for variables like labor income, in many datasets there would be several White households with higher labor income than the highest labor income for Black households. Thus, it would not be possible to re-weight the Black distribution to match the highest percentiles of the White distribution. Second, one potential reason why the joint distribution of  $(X_j, y_j)$  is different for Black and White households is discrimination, which can keep, for example, the returns to education low for Black households lower. Thus, in this scenario one can interpret the White distribution as the "normal" estimate, to which the Black distribution would converge to in the absence of discrimination.

<sup>&</sup>lt;sup>7</sup>We do this with weights  $\widehat{\omega}_{j}^{W} = \omega_{j}^{W} \times 5\%/m_{j}$  where  $m_{j}$  is the mass of White households in household's *j* fractile of labor income. For example, if 10% of White households have a labor income within percentiles 95 and 100 of the Black labor income distribution, then for them  $\widehat{\omega}_{j}^{W} = \omega_{j}^{W} \times 5/10$ . We also discard all White households with labor income above the maximum for Black households, and re-weight those with zero labor income separately.

wealth distribution, and between 19-40% at the top. Moreover, labor income explains more of the racial wealth gap at the bottom of the wealth distribution, indicating that households with higher labor income in that region are significantly wealthier than those with lower labor income. However, at the top of the wealth distribution the importance of labor income decreases and that of entrepreneurship increases, eventually catching up at the 95<sup>th</sup> percentile and then surpassing it. Even though entrepreneurship itself only becomes the most important variable only at the very top percentiles, factors affecting the entrepreneurship choice like labor income and education account for a significant shares of the racial wealth gap across the distribution. However, because wealth is highly concentrated, ultimately, the differences at the top have a higher effect on the average racial wealth gap, as we will see below.

**Parametric (Probit) decomposition** While the strategy above allowed us to have a nonliner relationship between each explanatory variable and wealth, if we wanted to control for many variables at the same time that would mean creating a multi-dimensional bin structure which can easily become unfeasible due to the large number of bins and the relatively small sample size. Thus, to be able to control for many variables at the same time we rely on a Probit model Fortin, Lemieux, and Firpo (2011). The idea is to estimate the probability of a given household to be White or Black given a set of variables, and use this predicted probability to re-weight the White sample. Therefore, first we estimate a Probit of an indicator of whether a household is Black on a set  $X_j$  of variables, which include labor income, a dummy for entrepreneurship, education dummies, and also controls for gender and age. Then the Probit re-weighting, controlling for several variables at the same time, is given by

$$\widehat{\omega}_{j}^{W} = \omega_{j}^{W} \times \frac{\widehat{p}_{j}}{1 - \widehat{p}_{j}}, \text{ with } \widehat{p}_{j} = \Phi(\widehat{\beta}' X_{j})$$
<sup>(2)</sup>

with  $\hat{p}_j$  equal to the estimated probability of household *j* being Black given controls  $X_j$ , and  $1 - \hat{p}_j$  of it being White, and where  $\Phi(x)$  is the cumulative density function of the standard normal distribution.

To isolate the impact of a single variable we remove one explanatory variable at a time from the Probit, perform the re-weighing in Equation (2), and calculate how the RWG at each percentile changes, while still controlling for all the other variables. Figure 5B shows the change in the percentage of the racial wealth gap that is explained when removing the highlighted variable. We can see that the percentages are significantly smaller than those in Figure 5A since all variables are highly correlated with each other. Because all the variables together explain a large share of the racial wealth gap (70.7% of the mean, 66.9% of the median), here one should focus on the relative ranking only. Viewed in isolation, labor income again is the variable that explains the most of the racial wealth gap at the bottom of the wealth distribution, up to the 65<sup>th</sup> percentile. From that point up to the 80<sup>th</sup> all three variables – entrepreneurship, labor income, education – have equal importance. At the top, we see again that entrepreneurship becomes the most important variable, from the 85<sup>th</sup> to the 97<sup>th</sup> percentile. Somewhat surprisingly, labor income becomes almost as important as entrepreneurship at the 98<sup>th</sup> and 99<sup>th</sup> percentiles as well.

Overall, the picture that emerges from the BO decompositions is that labor income is more important than entrepreneurship for explaining gaps at the bottom, but that entrepreneurship has a more significant role at the top percentiles. In the end, it means that entrepreneurship is more important in accounting for the average racial wealth gap as well: without entrepreneurship our Probit decomposition explains 8.6% less of the average racial wealth gap, compared to only 7.9% less without labor income.

#### 2.3.2 Entrepreneurship and wealth mobility

The framework above, while informative, has two possible drawbacks. First, it is only correlational, and uninformative on causality. Second, it misses dynamic effects. For example, if entrepreneurs in the bottom 50% of wealth are not wealthier than the non-entrepreneurs in the same part of the distribution, but entrepreneurship enables them long-term upward mobility. The structural model developed in the next section is able to deal with both issues when performing counterfactuals.

But first, we investigate whether entrepreneurship is correlated with wealth mobility in the data. To do so, we use PSID data to estimate the following regression

$$\mathbb{1}\{g \to \tilde{g}\}_{j,t,t+h}^{i} = \alpha_{t,g}^{i} + \gamma_{g}^{i} \times \text{entrep}_{t,g}^{i} + \beta_{g}^{i} X_{j,t,g}^{i} + \varepsilon_{j,t,g}^{i}$$
(3)

where  $\mathbb{1}\{g \to \tilde{g}\}_{j,t,t+h}^{i}$  is an indicator function that equals one if household *j* of race *i* was in wealth group *g* at time *t* and group  $\tilde{g}$  at time *t* + *h*. Figure 6 plots the estimated coefficients  $\hat{\gamma}$  for  $g \in \{\text{bottom 50\%, P50-P90, top 10\%}\}$ . Panels (C) and (D) show that entrepreneurship is associated with an increase in upward mobility for Black and White households, both from the Bottom 50% to P50-P90, and from the P50-P90 to the top 10%. Panels (A) and (B) tell a slightly different story. Black entrepreneurs have lower downward mobility rates

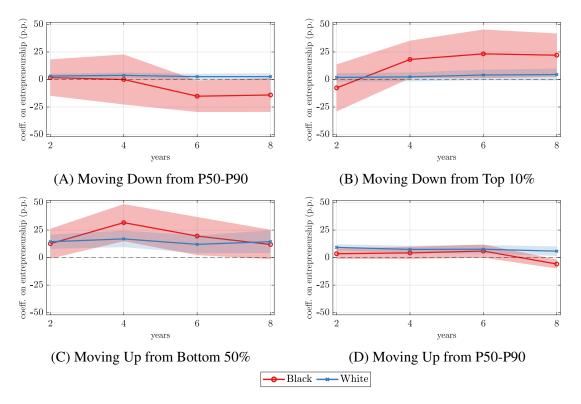


Figure 6: Entrepreneurship and wealth mobility

*Notes*: This figure plots the estimated  $\hat{\gamma}_g^i$  coefficients on entrepreneurship in Equation (3), i.e., the correlation of entrepreneurship with the probability of a household moving up/down from its current wealth group to the next one, in the horizon given by the *x* axis. The wealth groups are the bottom 50%, P50-P90, and the top 10%. For example, in Panel 6C being an entrepreneur for Black households is correlated with a higher transition rate of 12.4p.p. from the bottom 50% to P50-P90 at a 2-year horizon. All the point estimates and average transition rates are reported in Table A.2. Other controls include age, gender, education, employment status, wealth percentile, labor income percentile, and year fixed effects. Confidence intervals are 95% and standard errors are clustered at the household level.

from the P50-P90, but higher ones from the top 10% at longer horizons compared to nonentrepreneurs. White entrepreneurs also have higher downward mobility both from the P50-P90 and the top 10%. Some of the estimated coefficients are quite large. For example, being an entrepreneur for Black households is correlated with a higher transition rate of 12.4p.p. from the bottom 50% to P50-P90 at a 2-year horizon, when the average transition rate for Black households between those groups is of 10.2%. However, notice the wide confidence intervals, specially for Black households, given their lower entrepreneurship rate and lower sample size at the top of the wealth distribution. All point estimates and average transition rates are reported in Table A.2.

We interpret the combined results as suggestive that entrepreneurship is associated with higher upward wealth mobility for both Black and White households across the wealth distribution, which is something the BO decomposition did not capture, and thus its importance could be even higher for explaining wealth differences. Moreover, there is some evidence that entrepreneurship within the top 10% might be riskier for Black entrepreneurs than for White ones, which could be indicative of additional barriers to entrepreneurship which, if removed, would further increase the importance of entrepreneurship for Black wealth accumulation. To explore these questions, we turn to our model next.

## 3 Model

Our model utilizes the workhorse incomplete market model à la Bewley-Imrohoroglu-Hugget-Aiyagari set in general equilibrium. We augment it with a dynamic discrete entrepreneurship choice under a financial friction and decreasing returns to scale production technology, as in Evans and Jovanovic (1989). This allows for a non-degenerate distribution of firms, and positive profits for an owner-manager entrepreneur. Our modeling approach is motivated by the works of Quadrini (2000), Castaneda, Diaz-Gimenez, and Rios-Rull (2003), and Cagetti and De Nardi (2006), which show how entrepreneurship can be used to model wealth concentration and mobility. We also include a rich, estimated, labor income process featuring permanent, transitory, and non-employment shocks.

#### 3.1 Environment

Time *t* is continuous. There exists a unit mass of households that differ in race  $i \in \{B, W\}$ , where *B* denotes Black and *W* denotes White. The mass of households of each race is denoted by  $m^i$ , which is exogenous and fixed. Households are identical ex-ante, but Black household are subject to race-specific distortions that we explain in detail below. In the end, these distortions make Black households worse-off both as entrepreneurs and as workers, when compared to White households. Our entrepreneurship choice model is consistent with recent empirical evidence by Bhandari et al. (2024) who use administrative data for the U.S. and highlight the importance of pecuniary incentives in explaining entrepreneurial entry, especially so for larger businesses. Thus, pointing away from preference-heterogeneity based explanations.

Households are infinitely lived, and we interpret them as dynasties. This choice is equivalent to households having perfect "warm glow" motives towards their offspring and leaving bequests, which in our setting generates intergenerational transmission of wealth and high persistence of racial wealth inequality. All households in the model can save and accumulate wealth *a* subject to a borrowing constraint  $a \ge \underline{a}$ . Asset positions can be either positive or negative, and negative positions are debt owed to other households. Assets can also be rented out to firms as capital. We assume that capital and debt yield the same net return.

Households are either entrepreneurs or workers. Workers face uninsurable idiosyncratic shocks to their labor productivity  $z_L$ , and the per-productivity-unit wage rate w is determined in general equilibrium. Entrepreneurs hire labor and capital to produce a homogeneous final consumption numeraire good. Each entrepreneur operates a firm with a decreasing returns to scale technology, and its capital choice is subject to a collateral constraint such that they cannot utilize more capital than a multiple  $\lambda_{CC}$  of their assets a. Additionally, entrepreneurs are heterogeneous with respect to their idiosyncratic productivity  $z_F$ , which evolves stochastically. Idiosyncratic shocks to  $z_F$  influence the entrepreneur's flow profit income and generate an uninsurable consumption risk. The share of entrepreneurs is endogenously determined by workers, who have a business idea with rate  $\eta$ , and then decide whether to start a firm or not. Entrepreneurs exit at an exogenous rate  $\lambda_D$  and return to the worker pool.

#### **3.2 Racial disparities**

**Race-specific distortions.** Following the misallocation literature, we treat racial disparities or *distortions* as fundamentals that distort otherwise identical household problems.<sup>8</sup> This lets us quantify how each distortion shapes steady-state wealth gaps without committing to a single root cause (e.g. discrimination, network externalities, or barriers to education). We set all White households' distortions to zero and calibrate Black households' distortions from micro data.

<sup>&</sup>lt;sup>8</sup>Similar distortions-based modeling strategies were used to study the role of misallocation in determining cross-group differences. Recent examples include the seminal work of Hsieh et al. (2019) studying occupational sorting in the U.S.; Bento and Hwang (2022) in the context of the Black-White entrepreneurship gap; Morazzoni and Sy (2022) who study barriers faced by female entrepreneurs; and Goraya (2023) who analyzes barriers to entrepreneurship among members of different casts in India.

(1) Labor-income level distortion  $\tau_L^B$ . A proportional labor productivity distortion lowers the effective labor supplied by a worker with innate labor productivity  $z_L$ :

$$\underbrace{z_L}_{\text{true productivity}} \longrightarrow (1 - \tau_L^i) z_L, \qquad i \in \{B, W\}.$$

Labor income equals  $w(1 - \tau_L^i)z_L$ . We later calibrate  $\tau_L^B \in [0, 1]$  to the Black–White median wage ratio, conditional on employment. This implies that eliminating the distortion raises both labor income and labor supplied.

(2) Labor-income risk distortion. Black and White households face Markov income processes with distinct transition rates and shock variances for all components (permanent, transitory, and non-employment). Panel estimates we discuss below imply higher earnings volatility and longer non-employment spells for Black workers, strengthening precautionary saving motives.

(3) Entrepreneurship distortion  $\tau_y^B$ . Entrepreneurs with technology  $y = z_F k^{\alpha} h^{\beta}$  and  $\alpha + \beta < 1$ , where y denotes revenue, k capital, h labor, and  $z_F$  productivity, hires factors as-if maximizing perceived profits

$$\pi^i = (1 - \tau_v^i) z_F k^\alpha h^\beta - wh - rk,$$

subject to a borrowing limit discussed below. We calibrate  $\tau_y^B \in [0, 1]$  to the wedge estimated by Tan and Zeida (2024), encompassing factors such as customer discrimination and market access. Lower profits endogenously slow capital accumulation and keep Black entrepreneurs closer to the borrowing limit.

#### 3.3 Workers

Workers choose how much to consume c and save subject to a borrowing limit. They receive a business idea allowing them to start a firm at an exogenous rate  $\eta$ . When the idea arrives, workers face the discrete choice of whether to use the idea to start a firm or not. Ideas are assumed to be non-tradeable and cannot be stored. For clarity, we state the value functions in their steady-state forms, referring to constant prices and omitting time derivatives.

Let  $V(a, z_L, i)$  denote the value of being a worker with asset level *a*, labor productivity  $z_L$ , and race *i*. The worker faces the following problem:

$$\rho V(a, z_L, i)$$

$$= \max_{c} \left\{ u(c) + V_a s_V(a, z_L, i) + \eta \max \left\{ \mathbb{E} \left[ F(a, \psi(z_L), i) \right] - V(a, z_L, i), 0 \right\} + A^i_{z_L} V(a, z_L, i) \right\}$$
(4)

subject to the borrowing constraint  $a \ge \underline{a}$ , where  $u(c) = c^{1-\gamma}/(1-\gamma)$  denotes flow utility from consumption,  $\gamma$  is the coefficient of relative risk aversion;  $\rho$  is the common discount rate;  $\mathbb{E}[F(a, \psi(z_L), i)]$  is the worker's expected value from becoming an entrepreneur after receiving an idea;  $V_a = \partial V(a, z_L, i)/\partial a$  denotes the partial derivative; and  $A_{z_L}^i$  is the generator for the stochastic process governing  $z_L$ . This race-dependent generator, encodes information about expected changes in the three components governing the evolution of  $z_L$ over time: permanent productivity  $z_P$ , transitory productivity  $z_T$ , and employment status  $l_t$ , all explained in detail in the next section. The law of motion for assets  $\dot{a} = s_V(\cdot)$  is

$$s_V(a, z_L, i) = w z_L^i (1 - \tau_L^i) (1 - t_w) + (1 - t_a I_{a>0}) (r - \delta) a - c + T,$$
(5)

where w denotes the wage per distortion-adjusted labor productivity,  $r - \delta$  the net return for asset holdings, with r being the rental rate of capital and  $\delta$  its depreciation rate. All households face a proportional tax rate of  $t_w$  on their labor income and a tax rate of  $t_a$ on their positive capital income. Thus,  $I_{a>0}$  is an indicator that equals one if  $a \ge 0$ , and zero otherwise. Households receive a lump-sum transfer benefit of T, which generates an income floor in our model.

The quality of a prospective business idea is governed by  $\psi(z_L)$ , which maps labor productivity  $z_L$  into the initial productivity of an entrepreneur such that the entrant firm has  $z_F = \psi(z_L)$ . This mapping is used to capture the correlation between labor income and entry, possibly driven by education and overall ability, and is discussed as part of the calibration strategy.

#### 3.4 Labor income

Idiosyncratic labor productivity  $z_{L,t}$  is modeled similar to the jump-drift process of Kaplan, Moll, and Violante (2018), augmented with employment and non-employment status. All parameters are race-dependent. The process for labor productivity is given by

$$z_{L}^{i}(l^{i}, z_{P}^{i}, z_{T}^{i}) = l^{i} \times e^{z_{P}^{i} + z_{T}^{i}},$$
(6)

where  $l^i \in \{0, 1\}$  is the employment status,  $z_P^i$  is the permanent component of log income, and  $z_T^i$  is the transitory component, all of which are idiosyncratic. We assume  $l^i$  is a Poisson jump process where  $\lambda_{ll'}^i$  denotes the rate at which households of race *i* switch from employment status  $l \in \{0, 1\}$  to  $l' \in \{1, 0\}$ ..

The permanent and transitory components follow a jump-drift process given by:

$$dz_{P,t}^{i} = -\mu_{P}^{i} z_{P,t}^{i} dt + dJ_{P,t}^{i},$$

$$dz_{T,t}^{i} = -\mu_{T}^{i} z_{T,t}^{i} dt + dJ_{T,t}^{i},$$
(7)

where  $dJ_{j,t}^i$  is an idiosyncratic jump process with an arrival rate of  $\lambda_j^i$ , in which case  $z_{j,t}$  is redrawn from a normal distribution with mean zero and variance  $(\sigma_j^i)^2$ ,  $j = \{P, T\}$ .

#### **3.5** Entrepreneurs

The entrepreneurs' optimization problem is:

$$(\rho + \lambda_D)F(a, z_F, i)$$
(8)  
=  $\max_{c} \left\{ u(c) + F_a s_F(a, z_F, i) + \lambda_D \mathbb{E}_{z_L}[V(a, z_L, i)] + F_{z_F}(\mu_F z_F) + \frac{(z_F \sigma_F)^2}{2} F_{z_F z_F} \right\},$ 

with the associated law of motion of assets  $\dot{a} = s_F(\cdot)$  given by

$$s_F(a, z_F, i) = (1 - t_\pi) \pi(a, z_F, i) + (1 - t_a I_{a>0}) (r - \delta) a - c,$$
(9)

where  $t_{\pi}$  is a business-income tax. Entrepreneurs are subject to the same borrowing constraint  $a \ge \underline{a}$  as workers.

Firms die with rate  $\lambda_D$ , in which case the household becomes a worker again, and  $\mathbb{E}_{z_L}[V(a, z_l, i)]$  is the expected value of this transition.<sup>9</sup> Let  $n^i(z_L)$  denote the PDF of the

<sup>&</sup>lt;sup>9</sup>The model does not allow for endogenous firm exit, thus it does not capture the option value of closing a firm. However, the continuation value after firm exit is significantly higher for White than for Black entrepreneurs, since they return to a better labor market. This influences the decision to start a firm as a worker, consistent with the findings of Catherine (2022).

stationary distribution of the process described in Equation (6). We assume that following the exogenous exit, entrepreneurs get reintroduced into the labor force with labor productivity and employment status re-drawn from  $n^i(z_L)$ .<sup>10,11</sup>

#### 3.6 Production

Firms are each owned by a single entrepreneur and differ in their productivity  $z_F$ . Conditional on staying in business,  $z_F$  follows a random growth process with average growth rate  $\mu_F$  and variance  $\sigma_F^2$  given by:

$$dz_{F,t} = \mu_F z_{F,t} dt + \sigma_F dB_t, \tag{10}$$

on the support  $z_F \in [\underline{z}_F, \infty)$ , where  $dB_t$  denotes a Brownian motion process. Note that while this process governs productivity, it does not govern firm size if the collateral constraint binds. Thus, a new firm can grow due to productivity shocks or by the owner saving to relax the collateral constraint.

While it is not possible to get an analytical solution to the exact distribution of  $z_F$ , we can use an asymptotic result (Gabaix, 2009) stating that as  $z_F \rightarrow \infty$  its stationary distribution  $f(z_F)$  has a Pareto right tail with parameter  $\zeta$  that depends on  $\mu_F$ ,  $\sigma_F^2$ ,  $\lambda_D$  (see details in Appendix E.1). We later use  $\zeta$  to calibrate the dispersion of top wealth in the economy.

Firms produce a single homogeneous final consumption good y by renting physical capital k and distortion-adjusted labor h from households using a production function  $y = z_F k^{\alpha} h^{\beta}$ , with  $\alpha + \beta < 1$ . The entrepreneurship distortion,  $\tau_y^i$ , reduces the firm's perception of its own productivity or, alternatively, the firm's perception of output prices. Profits are given by:

$$\pi(a, z_F, i) = z_F k(a, z_F, i)^{\alpha} h(a, z_F, i)^{\beta} - wh(a, z_F, i) - rk(a, z_F, i),$$
(11)

<sup>&</sup>lt;sup>10</sup>The stationary distribution of workers over  $z_L$  is different from  $n^i(z_L)$ , given by the exogenous income process, due to differences in entry decisions. Thus, this assumption simplifies the numerical implementation. Quantitatively, the transition rates across labor statuses within workers dominate those between workers and entrepreneurs making the two distributions approximately the same.

<sup>&</sup>lt;sup>11</sup>One might worry that this assumption on labor productivity after exit incentivizes households to enter entrepreneurship to redraw their  $z_L$ . However, in the calibrated version of the model, this is not a likely concern since high  $z_L$  households are much more likely to enter into entrepreneurship. Furthermore, ideas arrive once every twenty-two years on average, and firms exit once every ten years on average, making this incentive negligible.

where due to the distortion  $\tau_{y}^{i}$ , factors are chosen according to

$$\{h(a, z_F, i), k(a, z_F, i)\} = \underset{\{h, k\}}{\operatorname{arg\,max}} (1 - \tau_y^i) z_F k^{\alpha} h^{\beta} - wh - rk, \quad \text{s.t. } k \leq a \lambda_{CC},$$
(12)

and *a* denotes the asset position of the entrepreneur. For firms with a non-binding collateral constraint, the first order conditions are given by

$$(1 - \tau_y^{\ i})\alpha z_F h^\beta k^{\alpha - 1} = r, \tag{13}$$

$$(1-\tau_y^i)\beta z_F h^{\beta-1}k^{\alpha} = w.$$
(14)

Without the credit constraint and the entrepreneurship distortion  $\tau_y^i$ , these first-order conditions imply that profits are a share  $(1 - \alpha - \beta)$  of the total output of each firm. However, given the financial friction the production decision will reflect lower capital intensity due to its higher shadow price. Let  $\mu_{CC}(a, z_F, i)$  denote the Lagrange multiplier of the collateral constraint. Thus, factor quantities are chosen according to:

$$h(a, z_F, i) = \left(\left(1 - \tau_y^i\right) z_F\right)^{\frac{1}{1 - \alpha - \beta}} \left(\frac{\alpha}{r + \mu_{CC}(a, z_F, i)}\right)^{\frac{\alpha}{1 - \alpha - \beta}} \left(\frac{\beta}{w}\right)^{\frac{1 - \alpha}{1 - \alpha - \beta}}$$
(15)

$$k(a, z_F, i) = \left(\left(1 - \tau_y^i\right) z_F\right)^{\frac{1}{1 - \alpha - \beta}} \left(\frac{\alpha}{r + \mu_{CC}(a, z_F, i)}\right)^{\frac{1 - \mu}{1 - \alpha - \beta}} \left(\frac{\beta}{w}\right)^{\frac{\mu}{1 - \alpha - \beta}}$$
(16)

Note that if the productivity distribution has a tail parameter of  $\zeta$  then the firm size distribution in terms of labor is also a Pareto with a tail parameter equal to  $\zeta(1 - \alpha - \beta)$ .

Finally, observe that the higher is  $\tau_y^B$ , the lower are the quantities of capital and labor demanded by Black-owned firms. However, actual marginal products of those factors would be higher, all else being equal, implying that a reallocation of capital and labor towards Black-owned firms in the model can be output increasing as in standard theories of misallocation.

### 3.7 Equilibrium

The model economy has three markets: assets, labor and goods. In general equilibrium, these three markets clear, with total net assets positions in the economy equal to the firms' capital demand, total distortion-adjusted labor supplied by households equal to the total amount of labor demanded by firms, and total output produced equal to the total amount

of output consumed and invested in capital accumulation. Additionally, the government operates its transfer scheme under a balanced budget. For conciseness, a formal statement of the equilibrium definition and market clearing conditions in the economy is relegated to Appendix C. A detailed solution algorithm is given in Appendix D.

# 4 Calibration

This section details the calibration procedure and reports the model fit and performance. We follow a three-step calibration strategy. First, externally set several parameter values according to literature conventions. Second, use micro-data and empirical estimates to discipline the distortions. Finally, we calibrate the remaining parameters of the model internally to match key moments concerning entrepreneurship, wealth, and labor income in the data. Overall, the calibrated model is consistent with the patterns in the data delivering an average racial wealth gap of 78.3% and a median racial wealth gap of 76.2%, compared to 83.6% and 88.4% in the data, respectively. Importantly, we do not target these racial wealth gaps, rather we obtain them as a result of the, empirically-disciplined distortions and the endogenous forces.

Externally calibrated parameter values. We set the coefficient of relative risk aversion to  $\gamma = 1.5$ , as is conventional in the literature. We follow Hubmer, Krusell, and Smith (2021) in setting the depreciation rate of capital at  $\delta = 4.8\%$ . The exit rate of firms is set to an annual value of  $\lambda_D = 0.1$  which is also conventional. We set the volatility  $\sigma_F$  to target a profit volatility of 12% among the largest, financially unconstrained firms, which is consistent with recent estimates in the literature (e.g., Gabaix, 2011).<sup>12</sup> We also normalize  $\underline{z}_F = 1$ .

#### 4.1 Disciplining the distortions

**Labor income process estimation.** Using data from the PSID from 2001 to 2019 we separately estimate the parameters governing the income processes for Black and White workers. The parameters  $\tau_L^B$ ,  $\lambda_{01}^W$ ,  $\lambda_{10}^W$ ,  $\lambda_{01}^B$  and  $\lambda_{10}^B$  are calculated directly from the data, while the remaining ones are estimated via Simulated Method of Momemnts (SMM). Table 1 reports the estimated parameter values. For a detailed explanation of the estimation

<sup>&</sup>lt;sup>12</sup>Profits or labor demand of the unconstrained firms will be proportional to  $z_F^{1/(1-\alpha-\beta)}$ , thus if the volatility of log  $(z_F)$  is equal to  $\sigma_F$ , the volatility of profits is equal to  $\sigma_F/(1-\alpha-\beta)$ .

procedure see Appendix B.3.

Parameter	Symbol	Black HHs	White HHs
Labor income distortion	$ au_L^B$	50.9%	0.0%
Mean reversion, permanent	$\mu_P$	0.71%	0.01%
Mean reversion, transitory	$\mu_T$	64.2%	83.8%
Volatility of jumps, permanent	$\sigma_P$	0.67	0.68
Volatility of jumps, transitory	$\sigma_T$	0.33	0.22
Jump rate, permanent	$\lambda_P$	0.05	0.04
Jump rate, transitory	$\lambda_T$	0.77	3.67
Jump rate, Employment $\rightarrow$ Non-employment	$\lambda_{10}$	15.4%	10.0%
Jump rate, Non-employment $\rightarrow$ Employment	$\lambda_{01}$	31.5%	44.2%

Table 1: Estimated parameters for the labor productivity process  $z_{L,t}$ 

*Notes:* This table reports the estimated parameters of the processes for the components of labor income productivity  $z_{P,t}$  and  $z_{T,t}$ , and labor status  $l_t$ . All transition rates are at an annual frequency.

The labor income distortion  $\tau_L^B = 50.9\%$  is calculated as the gap between Black and White households in median weekly labor income when employed, without controlling for other variables such as family structure and education, since we do not model these explicitly. We consider both male and female led households, leading our measure to be slightly higher than those reported by Bayer and Charles (2018).

The resulting income process features a more persistent permanent component for White households, where persistence is given by  $1 - \mu_P^i$ . However, the values for volatility and jump rates are similar, with persistent shocks estimated to arrive on average every 20 to 23 years. The estimated transitory process is quite different between races. White households face more frequent shocks, but these are less volatile and dissipate quicker than the transitory shocks faced by Black households. Finally, Black households are estimated to face a lower probability of finding a job when non-employed, and also a higher probability of losing a job when employed, resulting in a higher non-employment rate in labor markets.<sup>13</sup>

The entrepreneurship distortion. Together with the parameters of the labor income process, the entrepreneurship distortion  $\tau_y^B$  is the most important parameter for our calibration exercise. Fortunately, an exact empirical counterpart for  $\tau_y^B$  is available in the work of

 $<sup>\</sup>frac{13}{13}$  The non-employment rate for Black and White households is equal to  $\lambda_{10}^B/(\lambda_{10}^B + \lambda_{01}^B) = 32.8\%$  and  $\lambda_{10}^W/(\lambda_{10}^W + \lambda_{01}^W) = 18.5\%$ , respectively.

Tan and Zeida (2024), who study the differential conditions faced by Black-owned businesses using tools from the misallocation literature. The authors jointly estimate a 'markup wedge', which commonly affects the average revenue product of all factors of production, and factor-specific wedges, affecting the average revenue product of specific factors. Using the Kaufman survey, which is a high-quality firm panel, the authors report that the markup wedge is the single most important driver of differences between Black and White-owned firms, thus validating our modeling choices. We use the estimate for  $\tau_y^B$  obtained by Tan and Zeida (2024) after controlling for industry and year fixed effects and for productivity proxies. This mapping yields that  $\tau_y^B = 0.507.^{14}$ 

#### 4.2 Internal calibration and targeted moments

Parameter	Symbol	Value
Capital share, production function	α	0.32
Labor share, production function	β	0.41
Discount rate	ρ	9.28%
Borrowing limit	<u>a</u>	0.17
Collateral constraint	$\lambda_{CC}$	3.72
Idea arrival rate	η	4.45%
Tail of $z_F$ process	$\zeta (1 - \alpha - \beta)$	1.58
Tax rate	$\overline{t}$	13.34%
Minimum permanent labor income for entry into entrepreneurship	$\Psi_0$	0.68
Elasticity of initial firm productivity to permanent labor productivity	$\Psi_1$	0.35
Entrepreneurship distortion	$ au_y^B$	50.69%

Table 2: Internally calibrated parameters

*Notes:* This table summarizes all internally calibrated parameter values.

The internally calibrated parameters are set to target key moments related to the interaction between wealth, entrepreneurship, income, and race. Most of the moments we target are aggregate ones, independent of race. The only race-dependent moment we targeted is

<sup>&</sup>lt;sup>14</sup>To map the estimates of Tan and Zeida (2024) to our model correctly, observe that the average revenue product of capital in our model is given by  $ARPK = \frac{(1-\tau_y)^i}{(1+\tau_k(a,z_F,i))} \left(\frac{\alpha}{r}\right)$ , and for labor  $ARPL = (1-\tau_y)^i \left(\frac{\beta}{w}\right)$ , where  $\tau_k(a, z_F, i) r = \mu_{CC}(a, z_F, i)$ . The common factor that would influence both the average revenue product of labor and capital is  $\tau_y^i$ . Thus, using Tan and Zeida (2024) notation of  $\delta^{\mu}$  for the markup wedge, we have that  $\log(1-\tau_y^B) = \delta^{\mu}$ . We use their estimate of the markup wedge from Table 2, column (2) which is  $\delta^{\mu} = -0.707$ .

the share wealth held by entrepreneurs of each race. The model has ten internally calibrated parameters summarized in Table 2. These are set to target eleven moments reported in Table 3.<sup>15</sup> Although most parameters affect mainly one or two targeted moments to a first order, we stress that all the targeted moments summarized in Table 3 are jointly determined by all parameters.

Wealth moments. We follow the literature by targeting a net return on wealth of 4% annually and a capital-to-annual-output ratio of 3. The main parameters affecting those two moments are the discount rate  $\rho$ , the share of capital in the production function  $\alpha$ , and the collateral constraint  $\lambda_{CC}$ , as they jointly capture the desire of households to hold assets and the firms' demand for capital. Our model fits those targets well, achieving a net return of 4.07% and a capital-to-output ratio of 3.17.

We also target the share of households with negative assets (10.5%), the share of wealth by the 50<sup>th</sup> to 90<sup>th</sup> percentiles of the wealth distribution (24.9%), and the share held by the top 10% (73.2%). Our model delivers an excellent fit to those targets, as shown in Table 2. The key parameters affecting these include  $\rho$ ,  $\lambda_{CC}$ , the borrowing limit  $\underline{a}$ , <sup>16</sup>  $\alpha$ ,  $\beta$  and  $\zeta$ .

Several notes are in order regarding these parameters. We calibrate the Pareto tail of the productivity distribution to  $\zeta = 5.85$  implying that the firm-size distribution in our model has a tail of  $\zeta (1 - \alpha - \beta) = 1.58$ .<sup>17</sup> Additionally, our calibration internally sets the degree of decreasing returns to scale to  $\alpha + \beta = 0.73$ , in line with the literature, with  $\alpha = 0.32$  and  $\beta = 0.41$ . These values do not map directly to the empirical factor shares since the empirical labor share also includes the CEO and partners' labor income that in our model are labeled as profits instead. Thus, our value for  $\beta$  should be strictly below the empirically observed payroll share (53.3% for the U.S. in 2010-2012 according to Elsby, Hobijn, and Şahin (2013)).

**Entrepreneurship moments.** We target the overall household entrepreneurship rate in the SCF, which is 12.7%. To capture the correlation of entrepreneurship and wealth we also target the share of Black-owned and White-owned wealth held by entrepreneurs, which is

<sup>&</sup>lt;sup>15</sup>Our distance metric is the mean squared relative weighted error such that  $MSRE = \sum_{j=1}^{11} \left[ (S_j^{model} - S_j^{data}) / (S_j^{data})^2 \right] \times \frac{\omega_j}{\sum_{k=1}^{11} \omega_k}$ , where  $S_j^{model}$  and  $S_j^{data}$  correspond to the value of the  $j^{th}$  moment in the model and the data, and  $\omega_j$  is its weight.

<sup>&</sup>lt;sup>16</sup>Our calibration sets  $\underline{a} = 0.17$  which corresponds to 24.33% of the median household labor income in the model.

<sup>&</sup>lt;sup>17</sup>Empirically, the firm-size distribution is considerably more skewed (e.g., Axtell, 2001). However, modeling the firm-size distribution and wealth inequality as joint phenomena requires taking a stance on ownership structures and portfolio choices, which lies beyond the scope of this paper (for an example that includes human capital wealth, see Aoki and Nirei (2017)).

Targeted moment	Source	Data	Model
Net return	literature	4.0%	4.1%
Capital to output ratio	literature	3.0	3.2
Wealth share of those in P50-P90 percentiles	SCF	24.9%	24.6%
Wealth share of the Top 10%	SCF	73.0%	72.0%
Share of households with negative net wealth	SCF	10.5%	11.0%
Entrepreneurship rate	SCF	12.70%	11.7%
Share of wealth held by entrepreneurs, Black HH	SCF	25.6%	26.4%
Share of wealth held by entrepreneurs, White HH	SCF	46.2%	38.5%
$ER_{P50-P90}/ER_{P0-P50}$	PSID	2.6	2.6
$ER_{P90-P100}/ER_{P0-P50}$	PSID	5.8	5.0
Ratio of benefits to median wage	literature	33.0%	32.3%
Untargeted moment	Source	Data	Model
Average racial wealth gap	SCF	83.6%	78.3%
Median racial wealth gap	SCF	88.4%	76.2%
Entrepreneurship rate, Black HH	SCF	5.2%	6.0%
Entrepreneurship rate, White HH	SCF	14.2%	12.8%
Wealth of avg. ent. to avg. HH, Black HH	SCF	4.9	4.4
Wealth of avg. ent. to avg. HH, White HH	SCF	3.2	3.0

Table 3: Summary of targeted moments and model fit

*Notes:* This table summarizes the targeted moments and reports the model's fit with respect to each, as well as the model's overall fit.  $ER_{P50-P90}/ER_{P0-P50}$  denotes the relative entry rate into entrepreneurship of households in the P50-P90 of the labor income distribution, relative to those in the P0-P50, and analogously for  $ER_{P50-P90}/ER_{P0-P50}$ . All the data refers to averages over the 2001-2019 period.

25.3% and 45.3%, correspondingly. These moments are central to our analysis and we assign to them higher weights when evaluating the model fit.<sup>18</sup> Importantly, we do not target the entrepreneurship rate by race or wealth and these correlations are left for validation purposes. The main determinants of overall entrepreneurship rates in the model are: the idea arrival rate  $\eta$ , which mechanically limits the number of potential entrepreneurs; the tail of the productivity process  $\zeta$ ; the entry process parameters  $\Psi_0$ ,  $\Psi_1$  that will be introduced shortly; and  $\beta$ , since it governs the demand for labor and thus the wage which is the outside option to starting a business.

<sup>&</sup>lt;sup>18</sup>All other moments are assigned an equal unit weight  $\omega_j = 1$  in our objective functions while these three moments are weighted  $\omega_j = 2$ .

To match the entrepreneurship rates in the model, we set the idea arrival rate to  $\eta = 4.45\%$ . Under this parameter value and  $\lambda_D = 10\%$ , the maximum possible rate of entrepreneurs out of the general population is 36.7%.<sup>19</sup> Because many households endogenously choose not to become entrepreneurs, we arrive at a total entrepreneurship rate of 11.65% in the model compared with 12.7% in the data. This understatement of the entrepreneurship rate is mostly due to an understatement of White entrepreneurship rates (12.8% in the model and 14.2% in the data) and an overstatement of the Black entrepreneurship rate (6% in the model and 5.2% in the data). Note that these differences imply that the racial gap of in entrepreneurship rates is understated in our model.

The model matches well the share of Black wealth held by Black entrepreneurs of 26.4% (25.3% in the data), however, it understates the importance of White entrepreneurship. The entrepreneurship rate of White households is 12.8% in the model, and they control 38.5% of White-owned wealth, both lower than in the data. Furthermore, observe that entrepreneur wealth vs average household wealth is in line with the data. Since the model understates the importance of White entrepreneurship in accounting for wealth inequality while matching well Black entrepreneurs' wealth and slightly overstating Black entrepreneurship rates, our model is prone to understating the importance of entrepreneurship in accounting for racial wealth inequality.

**Income and entry decision moments.** Denote the average entry rate into entrepreneurship in labor income fractile *j* by  $ER_j$ . We target the ratio  $ER_{P50-P90}/ER_{P0-P50}$  and also  $ER_{P90-P100}/ER_{P0-P50}$  to capture the correlation between labor income and entry rates. In the data, we observe an positive correlation between income and entry into entrepreneurship with  $ER_{P50-P90}/ER_{-P50} = 2.56$  and  $ER_{P90+}/ER_{-P50} = 5.79$ . In Appendix B.2 we use the PSID and show that entrepreneurial entry is positively correlated with labor income. Furthermore, this correlation disappears once we control for education. Therefore, we conclude that the upward gradient between income and entry originates with differences in unobserved human capital. In the model, these differences are captured by permanent income. We thus specify the following entry process.

We assume an isoelastic mapping between the productivity of entrants and their permanent income  $z_P$  as follows

$$\log\left(z_F - \underline{z}_F\right) = \Psi_1 \log\left(z_P - \Psi_0\right). \tag{17}$$

<sup>&</sup>lt;sup>19</sup>The entry process function  $\psi$  further limits this number. We stress that  $\tau_L^B$  is not allowed to influence idea quality in the model.

Note that this implies  $\psi(z_L) = \underline{z}_F + (z_P - \Psi_0)^{\Psi_1}$ , for  $z_P \ge \Psi_0$ . We preclude workers with  $z_P < \Psi_0$  from becoming an entrepreneur we let  $\tilde{F}(a, 0, i) = -\infty$ . When  $z_P \ge \Psi_0$ , the value of entering entrepreneurship  $\mathbb{E}[F(a, \psi(z_L), i)]$  is equal to the value of being an established entrepreneur with firm productivity  $z_F = \psi(z_L)$ , which is given by  $F(a, \psi(z_L), i)$ . We calibrate  $\Psi_0$  and  $\Psi_1$  to match  $ER_{P50-P90}/ER_{-P50} = 2.56$  and  $ER_{P90+}/ER_{-P50} = 5.79$ . The resulting model delivers a good fit with  $ER_{P50-P90}/ER_{-P50} = 2.55$  and  $ER_{P90+}/ER_{-P50} = 5.24$ .

Our final targeted moment is an income floor of 33% of the median household income, in line with other studies (e.g., see Straub, 2019). To obtain this, we calibrate a simplified tax system with a single parameter  $t_w = t_\pi = t_a = \overline{t} = 13.34\%$ , and achieve an income floor of 32.26% of the median household's pre-tax labor income.

#### 4.3 Model Validation

This section details several validation checks we conduct to make sure our model is suitable for the analysis that follows. Primarily, we verify that our model indeed captures the racial wealth gap, the correlation between entrepreneurship and wealth, and racial differences in business performance.

The model generates a large and untargeted racial wealth gap consistent with the data. The model yields a 78.3% average racial wealth gap, compared to 83.6% in the data, and a median one of 76.2%, compared to 88.4% in the data. These results show that the model is able to account for almost the entirety of the average racial wealth gap, and somewhat undershoots the median. Because wealth in the U.S. is heavily concentrated, this is not surprising. Entrepreneurship is likely to explain most of the right tail of the wealth distribution, and therefore racial differences in entrepreneurship choices and their determinants are able to account well for the average racial wealth gap. However, other factors, not modeled here, might be important in governing the median gap.

The correlation between entrepreneurship and wealth, overall and by race, is untargeted in our setting. Figures 7A and 7B report that our model captures this correlation well for Black and White households separately. This is both crucial and reassuring, as the main goal of the paper is to understand the role that entrepreneurship plays in understanding wealth, specifically, wealth differences across races.

Previously, when discussing Figure 3B we indicated that there is a constant gap in the size of White-owned firms vs Black-owned firms. We conjectured that then that our entrepreneurship distortion  $\tau_v^B$  would be able to deliver a constant difference in size, while

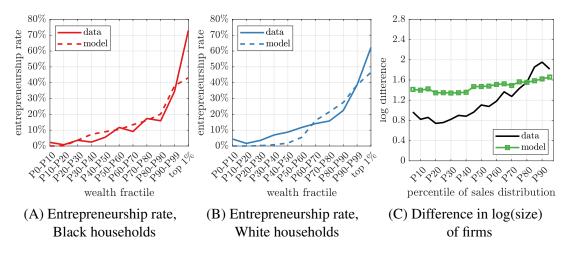


Figure 7: Untargeted entrepreneurship moments - model validation

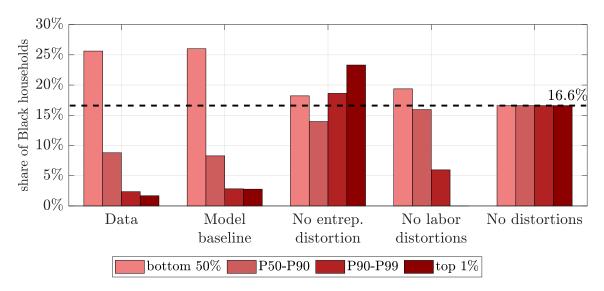
*Notes*: This figure evaluates the model's performance by comparing untargeted entrepreneurship outcomes to the data. Panels (A) and (B) report the share of entrepreneurs among Black and White households within each wealth fractile of the overall wealth distribution. Panel (C) shows the log differences in firm size, measured as revenue, of White relative to Black-owned firms conditional on their revenue fractile. For example, P50 shows the log differences between the median White and Black-owned firms in the data and in the model. Lower percentiles are shown as missing in the data because some firms do not report positive sales. Source: SCF, 2001-2019.

a distortion in collateral constraint would see Black firms owned by wealthy individuals converge to their White counterparts. Figure 7C supports our modeling choices and indicates that the entrepreneurship distortion indeed delivers a constant difference across the size distribution of Black vs White-owned firms. The resulting gap is broadly consistent with the empirically observed gap and its overall magnitude and slope are similar.

## **5** Results

#### 5.1 Decomposing the racial wealth gap

With the quantified model at hand, we now explore the role of entrepreneurship choices in determining wealth outcomes. Recall that the model allows for Black and White house-holds to differ in outcomes due to distortions faced by Black workers (labor income and a labor income risk distortion), and those faced by Black entrepreneurs (entrepreneurship distortion), both affecting the entrepreneurship choice. To disentangle the effects of the two, we conduct a comparative statics exercise where we equalize the conditions faced by



Black households to those faced by White ones, only for workers, only for entrepreneurs, or both.

Figure 8: Racial representation along the wealth distribution

*Notes*: Each bar shows the share of Black households within a fractile of the wealth distribution. The first set of bars is derived from SCF data. The baseline case corresponds to our calibrated model. The next three sets of bars correspond to the two counterfactuals in which the entrepreneurship distortion, labor market distortions, and all distortions are removed.

Before discussing the results, it is instructive to consider the non-trivial effects of removing the distortions on wealth outcomes. Removing the entrepreneurship distortion in partial equilibrium makes Black entrepreneurs richer on impact. These existing entrepreneurs will demand more labor and capital, raising factor prices. Ultimately, these forces alter the sorting patterns into entrepreneurship for both Black and White households. Thus, without entrepreneurship distortions, the continuing labor market distortions imply that Black households are more likely to choose entrepreneurship, which improves wealth outcomes. Note however, that it is harder for these households to accumulate wealth in the labor market.

The labor market distortions are even more complex. Removing the labor income distortion raises the labor income of Black households improving their ability to accumulate wealth. Simultaneously, removing the labor income risk distortion reduces the precautionary savings motive of Black households as it reduces their earnings risk. Both of these changes, all else being equal, make entrepreneurship less desirable to Black households and likely suppress wealth accumulation. Two caveats are in order: first, removing labor market distortions increases the quantity of distortion-adjusted labor available for hire thus lowering wage income overall; second, having households spend more time in high permanent income states enables households to receive more high-quality ideas. Both of these forces make entrepreneurship more desirable so the results are not ex-ante obvious.

	Entrepreneurship rates		Racial wealth gap		
	Black	White	gap (W-B)	Average	Median
Data	5.2%	14.2%	9.0%	83.6%	88.4%
Model	6.0%	12.8%	6.8%	78.3%	76.2%
No entrepreneurship distortion	18.9%	10.7%	-8.2%	-21.4%	21.6%
No labor market distortions	0.0%	13.6%	13.6%	70.9%	31.3%
No distortions	11.7%	11.7%	0.0%	0.0%	0.0%

Table 4: The racial wealth gap and entrepreneurship outcomes

*Notes:* This table reports each counterfactual scenario, the entrepreneurship rate of Black and White households, the gap between them, and the average and median racial wealth gap. The entrepreneurship gap is expressed as the difference in entrepreneurship rates between the groups. For comparison purposes, the baseline model and the SCF data are also reported

Figure 8 and in Table 4 report how counterfactual changes in the distortions influence the racial wealth gap and the representation of Black households along the wealth distribution.

Strikingly, Figure 8 demonstrates that removing the entrepreneurship distortion alone flips the sign of the steady-state racial wealth gap, eliminates the under-representation of Black households at the bottom of the wealth distribution and even creates an overrepresentation of Black households at its top. As Table 4 reports, in this counterfactual scenario, the racial wealth gap is -21.4%. That is, the average Black household is 21.4%wealthier than the White one, and the entrepreneurship rate among Black households is 8.2 p.p. higher than for White households.<sup>20</sup>

Removing the entrepreneurship distortion also lowers the median racial wealth gap from 76.2% to 21.6%. One might wonder: why the median racial wealth gap responds at all to the entrepreneurship distortion, which affects mainly the right tail of the distribution? Two factors contribute to this effect on the median. First, when conditions are equalized, Black entrepreneurship is more profitable all else being equal. Thus, Black workers save

<sup>&</sup>lt;sup>20</sup>Even though Black households are wealthier on average, we cannot conclude that their welfare higher since they still face labor market distortions. See Brouillette, Jones, and Klenow (2021) for a study that measures the welfare gap between Black and White households.

more so they can better capitalize on emerging opportunities. Second, wealth outcomes persist over time. Children with a wealthy entrepreneur parent are more likely to be wealthier even when they are not entrepreneurs themselves.<sup>21</sup>

In comparison, equalizing the conditions faced by Black workers helps alleviate poverty among Black households. However, it also leads to a severe reduction of Black entrepreneurship. Because the entrepreneurship distortion is still in place, Black households do not enter into entrepreneurship and instead stay in the labor market, with virtually no Black households in the top 1%. The median wealth gap declines to 31.3% and the average to 70.9%, the latter hardly changing compared to the baseline. On both statistics, reducing the entrepreneurship distortion is more meaningful for reducing the racial wealth gap than reducing the labor market distortions.

These results illustrate entrepreneurship, a phenomenon concentrated at the top of the wealth distribution, is pivotal in determining racial wealth outcomes. To eliminate the racial wealth gap one must target entrepreneurship outcomes and generate an equal representation of Black households among the very rich. This result is informative for emphasizing which channels are key to close the racial wealth gap. Policy interventions targeting the entrepreneurship distortion are promising, while policies that focus on the labor market outcomes of Black households are less effective in achieving this goal. When concerned with poverty alleviation among Black households, focusing on labor market outcomes is appropriate. However, policies targeting labor market outcomes alone can exacerbate the under-representation of Black households among the wealthiest. Such under-representation could even reduce the political influence of Black households (Bartels, 2009) and ultimately may have unintended social consequences.

#### 5.2 The macroeconomic implications of racial distortions

To understand how significant are these racial distortions to macroeconomic outcomes we utilize the aggregate production function representation of our model economy to decompose the role of the distortions on macroeconomic aggregates. The aggregate production

<sup>&</sup>lt;sup>21</sup>These counterfactual shifts on the relative wealth of Black compared to White households occur without much change in the overall wealth distribution as show in Table A.1 in the Appendix. This happens because the first-order determinant of overall wealth dispersion is the stochastic process governing income dispersion from profits, which remains unchanged in all scenarios.

function is as follows

$$\log Y = \underbrace{\alpha \log K + \beta \log N + (1 - \alpha - \beta) \log m_F}_{\text{factor quantities}} + \underbrace{\beta \log \left( \mathbb{E} \left( z_L \left( 1 - \tau_L^i \right) \right) \right)}_{\text{agg. labor productivity}} + \underbrace{\log \left( \text{TFP} \right)}_{\text{agg. productivity}} .$$
(18)

Thus, aggregate output Y is a constant-returns-to-scale production function of the aggregate capital stock, K, the aggregate number of workers N, and the number of firms  $m_F$ . It also depends on aggregate labor productivity  $\mathbb{E}(z_L(1-\tau_L^i))$  or the distortion-adjusted labor input provided by a worker, and total factor productivity TFP. Labor productivity is affected by the labor income distortion, and by the labor income risk distortion which governs the distribution of  $z_L$  among Black households. TFP is also endogenous in our model since it is not only affected by the entrepreneurship distortion, but aggregates the productivity distribution of all entrepreneurs while accounting for endogenous sorting patterns governing entry into entrepreneurship and their savings choices that affect their collateral constraints and ultimately their capital choice.

Table 5: The effects of distortions on aggregate quantities

Distortions removed	Y	K	Ν	$m_F$	$\mathbb{E}(z_L\left(1-\tau_L{}^i\right))$	TFP
Entrepreneurship	4.0%	4.7%	-0.4%	3.2%	3.1%	0.5%
All	10.2%	9.8%	-0.1%	0.5%	11.8%	2.0%

Notes: This table reports for each variable the percentage deviations with respect to the baseline.

Removing the entrepreneurship distortion increases steady-state GDP by 4.0%. This is primarily due to a higher demand for labor and capital by existing and new entrant Black-owned firms, even though the primitives governing firm productivity are unchanged. Removing the distortion implies a factor reallocation from White-owned to Black-owned firms in relative terms. Higher labor demand also leads some high  $z_L$  White workers to remain in the labor market increasing economy-wide labor productivity.

Furthermore, removing all distortions, raises steady-state GDP by 10.2%. This increase indicates the substantial potential gains from policies aimed at alleviating the root causes of racial differences in outcomes. While this number is large, it is not extraordinary. In a similar exercise, but using different models and distortions, Hsieh et al. (2019) consider removing frictions preventing efficient sorting across occupations, and report an increase in steady-state GDP of 9.9%. The results imply that our estimates are well within the bounds suggested by the literature, and lend further support to the critical role of efficient

occupational sorting to economic conditions.

#### 5.3 Evaluation of policies targeting the racial entrepreneurship gap

	Entreprer	neurship rate	Racial we	Racial wealth gap		Tax	Subsidy
	Black	White	Average	Median	$\frac{E\left[y^B\right]}{E\left[y^W\right]}$	rate	rate
Baseline	6.0%	12.8%	78.3%	76.2%	9.7%	-	-
Profit subsidy	8.0%	12.7%	73.7%	75.6%	9.8%	0.25%	6.37%
Revenue subsidy	9.8%	12.6%	69.7%	74.1%	10.5%	0.25%	3.29%
Capital subsidy	10.6%	12.5%	66.4%	71.9%	10.9%	0.25%	15.11%

Table 6: Policy counterfactuals - subsidizing Black entrepreneurship

*Notes:* This table reports entrepreneurship and wealth outcomes following a subsidy aimed at stimulating Black entrepreneurship. The tax rate represents the additional labor income tax  $t_w$  necessary to fund the subsidy policy, and  $E[y^B]/E[y^W]$  represents the average size of a Black-owned business relative to a White-owned one, as measure by output. Denoting the subsidy on profits, revenue and capital by  $s_{\pi}^i, s_y^i, s_k^i$ , respectively, the firms' problem in Equation (12) with all subsidies becomes  $\{h(a, z_F, i), k(a, z_F, i)\} = \arg \max_{\{h,k\}} [(1+s_y^i)(1-\tau_y^i)z_Fk^{\alpha}h^{\beta} - wh - (1-s_k^i)rk]$ , subject to  $k \leq a\lambda_{CC}$ , and profits are given by  $\pi(a, z_F, i) = (1+s_{\pi}^i) [(1+s_y^i)y(a, z_F, i) - (1-s_k^i)rk(a, z_F, i) - wh(a, z_F, i)]$ .

We now ask, is it possible to affect the racial wealth gap without addressing its root causes? This question is of interest since, while deep and profound social change might be slow, it may be the case that there is scope for policy interventions. We consider subsidies to either profit, revenue, or capital for Black entrepreneurs,<sup>22</sup> all funded with a higher labor income tax on all workers.<sup>23</sup> We compute new steady states in which each of these subsidy programs have an identical fiscal cost fixed at 0.1% of our baseline GDP. The fiscal cost is chosen to be illustrative. Results are reported in Table 6.

The main result from this exercise is that subsidy policies do not have a large impact on the racial wealth gap, even though they are able to increase the Black entrepreneurship rate to as high as 10.6%. Furthermore, as Table 6 shows, the revenue ratio between Black and White owned firms  $E[y^B]/E[y^W]$ , is still such that the average Black-owned business is

<sup>&</sup>lt;sup>22</sup>Boerma and Karabarbounis (2023) also highlight the effectiveness of policies that increase the rate of return of entrepreneurship for Black households.

<sup>&</sup>lt;sup>23</sup>Shifting the funding burden to White workers only does not meaningfully affect the results. If anything, it makes supporting Black entrepreneurship slightly harder since Black workers would not be taxed and therefore, would be less inclined to become entrepreneurs.

between nine to ten times smaller than its White-owned counterpart. Thus, the subsidies are not enough to entirely offset the impact of the entrepreneurship distortion. This is clearest when examining the revenue subsidy, which is the closest to a negative entrepreneurship distortion.<sup>24</sup> Black entrepreneurship rates increased to 9.8% using a revenue subsidy at a rate of only 3.29%, while the entrepreneurship distortion is more than 50%, so Black-owned firms are still inefficiently smaller.

This might seem puzzling at first. However, existing labor market distortions make the outside option of Black entrepreneurs worse thus enabling the policymaker to close the entrepreneurship rate gap while still having a net positive distortion in place. If there were no labor market distortions acting as a countervailing force, the relationship between the racial wealth gap and the gap in entrepreneurship rates would be more direct. Thus, ignoring labor market distortions could lead one to overestimate the impact of policies targeted at equalizing entrepreneurship rates on the racial wealth gap. Once the interaction between labor market and entrepreneurship outcomes is properly considered. As long as the distortions are still in place, closing the racial wealth gap with policies targeted at entrepreneurs is not possible without Black entrepreneurship rates overshooting these of their White counterparts.

Finally, we find that the capital subsidy is the most effective policy among those examined. It causes the largest reduction in both the average and median racial wealth gaps, the largest increase in the relative size of Black-owned firms, and is also the cheapest. Notice that, because collateral-constrained firms cannot increase their capital input even if its cost is reduced by the subsidy, most of the benefits of this policy go towards the larger Blackedowned firms, owned by wealthier individuals. This finding suggests that policies aiming to help larger Black-owned firms catch up with the largest White-owned firms might be more successful in reducing the racial wealth gap than policies targeting small firms.

## 6 The dynamics of the racial wealth gap

So far we have compared different steady states in the model. However, if one is interested in changing economic outcomes, the speed of change also matters. Thus, we now investigate how long would it take to close the part of the racial wealth gap accounted for by our

<sup>&</sup>lt;sup>24</sup>The comparison between the two is not exact because the subsidy affects the choice of capital and labor and the profits given those choices, but the distortion affects only the optimal choice of inputs, as explained in Section 3.

model. To start, we analyze the transition dynamics of the model from the initial steady state calibrated to the U.S. in 2001-2019 to the counterfactual one in which there are no racial distortions, by removing all of them immediately, a very optimistic assumption.<sup>25</sup>

When the distortions are immediately removed, it takes between 150 to 200 years for the average racial wealth gap to close (Panel (A)), and 100 years to close the median racial wealth gap (Panel (B)). The first main result of this exercise is that wealth convergence occurs slowly. Alternatively, the initial conditions play a powerful role in shaping the transition, as from t = 0 onward, there are no exogenous distortions imputed to the model. Still, it takes more than a century and many generations for Black households to catch up with White ones. The second main result is that convergence between Black and White households occurs faster at the bottom of the distribution than at the top. The median racial wealth gap is faster to close than the average. Moreover, Panel (C) shows that Black households only obtain equal representation at the top 1% of wealth and reach their population weight of 16.6% after 150 years, in line with the average racial wealth gap.

Both results above might seem particularly puzzling given the very fast convergence in entrepreneurship rates shown in Panel (D), which happens in less than 100 years. However, even though the removal of distortions increases the current and future profits of existing Black-owned firms and incentivizes the creation of new ones, it takes time for the new entrant firms to grow to their optimal size, equal to that of their White-owned counterparts. Panel (E) reports that the profitability of Black and White-owned firms is equalized only after 150 years. Thus, even though the entrepreneurship rate converges quite quickly, it takes time for newly created Black-owned firms to grow and then generate profits that are comparable to those of White-owned firms. Finally, panel (F) reports the increased share of Black households among top-income earners. Notice that once equal representation among top-income earners is achieved, or even slightly before, the median racial wealth gap closes. For the average racial wealth gap, it is still necessary for firm owners to have time to accumulate profits from their large firms, break into the top 1% of the wealth distribution, and only then accumulate enough wealth to close the average racial wealth gap.

#### 6.1 The effect of wealth transfers

Is it possible to undo the average racial wealth gap with a wealth transfer at time t = 0? To answer this question we implement a wealth transfer using a one-time only proportional

<sup>&</sup>lt;sup>25</sup>Model transition dynamics are solved under perfect foresight and holding population composition constant. The results are depicted in the solid black line in Figure 9.

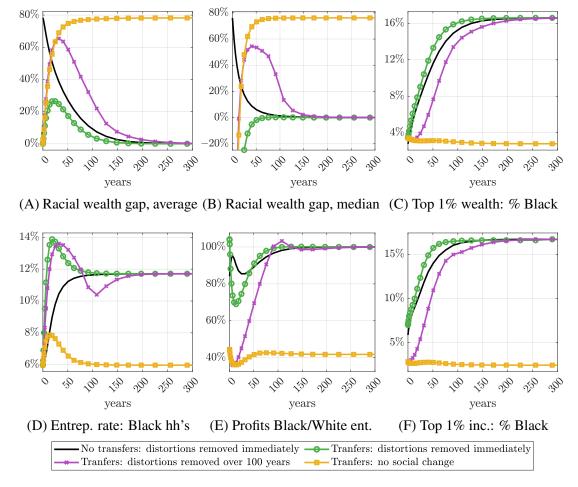


Figure 9: Closing the racial wealth gap

*Notes*: The solid black line shows the transition path from the steady with distortions to a steady without distortions, when all the distortions are removed immediately. The other three cases involve wealth transfers that close the average racial gap assuming that: (i) all distortions are removed immediately; (ii) the distortions close linearly over the next 100 years; (iii) there is no social change and all distortions remain as they were in the initial steady state. The panels show: (A) the average racial wealth gap; (B) the median racial wealth gap; (C) the share of Black households in the top 1% of the wealth distribution; (D) the entrepreneurship rate for Black households; (E) the average profits of a Black entrepreneur relative to that of a White one; (F) the share of Black households in the top 1% of the total income distribution.

wealth tax imposed on White households, redistributed lump-sum to all Black households, independent of their wealth or other characteristics. We also report the results of this wealth transfer in Figure 9 for three scenarios: (i) assuming that all distortions are eliminated immediately; (ii) distortions close linearly over 100 years; and (iii) no social change occurs,

which means that distortions remain unchanged.

The scope of the wealth transfer involved is large. Each White household with a positive net asset position faces a 13.0% tax on their wealth. At the same time, each Black household receives a lump sum transfer of 6.1 times the median household's annual labor income in the model, which is 300% of the total Black household wealth. The total wealth transfers amount to 39.5% of the annual GDP in the model. Thus, compared to U.S. GDP in 2019, this would result in a transfer of \$8.5 trillion.<sup>26</sup>

Panel (A) of Figure 9 illustrates that, in all scenarios, the average racial wealth gap falls to 0% on impact by construction. It reopens shortly afterwards as Black households consume a good portion of the transferred wealth, and income parity is not yet achieved. In the case of transfers combined with immediate removal of the distortions, the average racial wealth gap closes completely after 150 years, or approximately the same horizon in which there were no transfers, albeit with an overall lower level throughout the transition period. Our main result in this section is that, while transfers keep the average racial wealth gap lower throughout the transition, they do not significantly affect the speed at which it converges to zero.

Panel (F) shows why this is the case: even though average wealth is equalized on impact, the income of Black households is still lower than that of White households, even if the exogenous distortions are removed immediately. In the less extreme case of distortions closing slowly, the profitability of Black-owned firms is still significantly lower than that of White ones (Panel (E)), and it takes 100 years for them to converge. Moreover, Black households have just received a transfer of wealth and anticipate a future income rise. Thus, they smooth out the wealth shock and consume more than their income, causing the average racial wealth gap to increase again.

Several notes are in order. First, the median racial wealth gap reverses on impact because the transfers are proportional to White households, but lump-sum to Black households. Second, without social change, i.e., as long as the distortions are in place, wealth transfers cannot, by construction, change long-term wealth inequality, and the racial wealth gap reopens to its original magnitude, with most of the progress undone quickly within the first 50 years.

Finally, in our exercise the distortions are fully exogenous to the dynamics of the racial

<sup>&</sup>lt;sup>26</sup>This number is in line with other estimates: Boerma and Karabarbounis (2023) report a corresponding number of \$10 trillion, Darity Jr and Mullen (2020) of \$8 trillion. Given the approximately 20.1 million Black households in 2019, this would amount to a transfer of approximately \$419,000 per household.

wealth gap. Arguably, it is possible that wealth transfers to Black households could cause a reduction in distortions (e.g., through more investment in education and lower discrimination), which would then help even more the reduction of the racial wealth gap. Given this notion, one can consider the case in which all distortions close upon the transfer and the case in which the transfer delivers no social change as two extreme cases. In the former, transfers help alleviate all distortions right away, the best case scenario, and in the latter, transfers have no effect on the distortions, the worst case scenario. The case of slowly removing distortions in Figure 9 provides insight for a potential intermediate scenario in which social change occurs after the transfer but very gradually. Notice that even with an exogenous downward path towards zero for distortions over 100 years, the average racial wealth gap rises quickly again and, in less than 50 years, it is almost back at its original level. Thus, unless the impact of wealth transfers is such that all distortions are removed immediately, the model suggests that it is unlikely that wealth transfers could generate a virtuous cycle of reduction of inequality and reduction of distortions.

### 7 Conclusion

We develop a model of entrepreneurship and wealth accumulation featuring incomplete markets and a dynamic discrete entrepreneurship choice. In the model, Black households face adverse distortions, as workers and as entrepreneurs. We use U.S. microdata to discipline the model.

Quantifying the impact of each distortion reveals that removing the entrepreneurship distortion would reverse the average racial wealth gap and almost halve the median racial wealth gap. In comparison, we show that addressing labor market distortions has a large impact on the median racial wealth gap, but it can have a negative impact on the representation of Black households at the top of the wealth distribution due to its effects on entrepreneurship choice. Our analysis highlights the crucial role of sorting via the entrepreneurial entry choice on racial disparities.

Our analysis suggests three lessons to inform the future policy debate. First, removing the entrepreneurship distortions increases output by 5.4%, mainly due to factor reallocation towards Black-owned firms, indicating a large potential gain from policies targeting this distortion. Second, subsidy policies aimed at equalizing the entrepreneurship rates are effective at closing the entrepreneurship gap, but they are not enough to close the racial wealth gap, as Black-owned businesses are still smaller than their White counterparts. Last,

in all scenarios explored the racial wealth gap is slow to close and wealth transfers are not effective at increasing the speed of convergence.

The results point to the centrality of entrepreneurship for understanding the racial wealth gap, and the potential for policies that reduce barriers to Black entrepreneurship.

## References

- Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll (2021). "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach". *The Review of Economic Studies*.
- Aliprantis, Dionissi, Daniel Carroll, and Eric R Young (2022). "The dynamics of the racial wealth gap".
- Althoff, Lukas and Hugo Reichardt (2024). "Jim Crow and Black economic progress after slavery". *The Quarterly Journal of Economics* 139.4, pp. 2279–2330.
- Altonji, Joseph G, Prashant Bharadwaj, and Fabian Lange (2012). "Changes in the characteristics of American youth: Implications for adult outcomes". *Journal of Labor Economics* 30.4, pp. 783–828.
- Aoki, Shuhei and Makoto Nirei (2017). "Zipf's law, Pareto's law, and the evolution of top incomes in the United States". *American Economic Journal: Macroeconomics* 9.3, pp. 36–71.
- Ashman, Hero and Seth Neumuller (2020). "Can income differences explain the racial wealth gap? A quantitative analysis". *Review of Economic Dynamics* 35, pp. 220–239.
- Atkins, Rachel MB, Lisa Cook, and Robert Seamans (2022). "Using Technology to Tackle Discrimination in Lending: The Role of Fintechs in the Paycheck Protection Program". *AEA Papers and Proceedings* 112, pp. 296–298.
- Axtell, Robert (2001). "Zipf Distribution of U.S. Firm sizes". Science 293, pp. 1818–20.
- Barsky, Robert, John Bound, Kerwin Ko' Charles, and Joseph P Lupton (2002). "Accounting for the black–white wealth gap: a nonparametric approach". *Journal of the American statistical Association* 97.459, pp. 663–673.
- Bartels, Larry M. (2009). "167Economic Inequality and Political Representation". *The Unsustainable American State*. Oxford University Press.
- Bates, Timothy and Alicia Robb (2016). "Impacts of owner race and geographic context on access to small-business financing". *Economic Development Quarterly* 30.2, pp. 159– 170.

- Bayer, Patrick and Kerwin Kofi Charles (2018). "Divergent paths: A new perspective on earnings differences between black and white men since 1940". *The Quarterly Journal of Economics* 133.3, pp. 1459–1501.
- Bento, Pedro and Sunju Hwang (2022). "Barriers to black entrepreneurship: Implications for welfare and aggregate output over time". *Journal of Monetary Economics*.
- Bhandari, Anmol, Tobey Kass, Thomas J May, Ellen McGrattan, and Evan Schulz (2024). *On the nature of entrepreneurship*. Tech. rep. National Bureau of Economic Research.
- Blanchard, Lloyd, Bo Zhao, and John Yinger (2008). "Do lenders discriminate against minority and woman entrepreneurs?" *Journal of Urban Economics* 63.2, pp. 467–497.
- Blanchflower, David G, Phillip B Levine, and David J Zimmerman (2003). "Discrimination in the small-business credit market". *Review of Economics and Statistics* 85.4, pp. 930– 943.
- Blinder, Alan S (1973). "Wage discrimination: reduced form and structural estimates". *Journal of Human resources*, pp. 436–455.
- Boerma, Job and Loukas Karabarbounis (2023). "Reparations and persistent racial wealth gaps". *NBER Macroeconomics Annual* 37.1, pp. 171–221.
- Brouillette, Jean-Felix, Charles I Jones, and Peter J Klenow (2021). *Race and economic well-being in the United States*. Tech. rep. National Bureau of Economic Research.
- Cagetti, Marco and Mariacristina De Nardi (2006). "Entrepreneurship, frictions, and wealth". *Journal of political Economy* 114.5, pp. 835–870.
- Carvalho, Vasco M. and Basile Grassi (2019). "Large Firm Dynamics and the Business Cycle". *American Economic Review* 109.4, pp. 1375–1425.
- Castaneda, Ana, Javier Diaz-Gimenez, and Jose-Victor Rios-Rull (2003). "Accounting for the US earnings and wealth inequality". *Journal of political economy* 111.4, pp. 818–857.
- Catherine, Sylvain (2022). "Keeping options open: What motivates entrepreneurs?" *Journal of Financial Economics* 144.1, pp. 1–21.
- Catherine, Sylvain, Ellen Jiayang Lu, and James D Paron (2024). "What Explains Wealth and Portfolio Differences between Black and White Americans?" *Available at SSRN*.
- Cavalluzzo, Ken and John Wolken (2005). "Small business loan turndowns, personal wealth, and discrimination". *The Journal of Business* 78.6, pp. 2153–2178.
- Chandra, Amitabh (2003). *Is the convergence of the racial wage gap illusory?* Tech. rep. National Bureau of Economic Research.

- Darity Jr, William A and A Kirsten Mullen (2020). From here to equality: Reparations for Black Americans in the twenty-first century. UNC Press Books.
- Derenoncourt, Ellora, Chi Hyun Kim, Moritz Kuhn, and Moritz Schularick (2024). "Wealth of two nations: The US racial wealth gap, 1860–2020". *The Quarterly Journal of Economics* 139.2, pp. 693–750.
- Derenoncourt, Ellora and Claire Montialoux (2021). "Minimum wages and racial inequality". *The Quarterly Journal of Economics* 136.1, pp. 169–228.
- DiNardo, John, Nicole Fortin, and Thomas Lemieux (1995). *Labor market institutions and the distribution of wages, 1973-1992: A semiparametric approach.*
- Dougal, Casey, Pengjie Gao, William J Mayew, and Christopher A Parsons (2019). "What's in a (school) name? Racial discrimination in higher education bond markets". *Journal of Financial Economics* 134.3, pp. 570–590.
- Elsby, Michael WL, Bart Hobijn, and Ayşegül Şahin (2013). "The decline of the US labor share". *Brookings papers on economic activity* 2013.2, pp. 1–63.
- Evans, David S. and Boyan Jovanovic (1989). "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints". *Journal of Political Economy* 97.4, pp. 808–827.
- Faber, Jacob W and Ingrid Gould Ellen (2016). "Race and the housing cycle: Differences in home equity trends among long-term homeowners". *Housing Policy Debate* 26.3, pp. 456–473.
- Fairlie, Robert, Alicia Robb, and David T Robinson (2022). "Black and white: Access to capital among minority-owned start-ups". *Management Science* 68.4, pp. 2377–2400.
- Fairlie, Robert W and Frank M Fossen (2018). "Opportunity versus necessity entrepreneurship: Two components of business creation".
- Fairlie, Robert W and Harry A Krashinsky (2012). "Liquidity constraints, household wealth, and entrepreneurship revisited". *Review of Income and Wealth* 58.2, pp. 279–306.
- Fairlie, Robert W and Bruce D Meyer (2000). "Trends in self-employment among white and black men during the twentieth century". *Journal of human resources*, pp. 643–669.
- Flippen, Chenoa (2004). "Unequal returns to housing investments? A study of real housing appreciation among black, white, and Hispanic households". *Social Forces* 82.4, pp. 1523–1551.
- Fortin, Nicole, Thomas Lemieux, and Sergio Firpo (2011). "Decomposition methods in economics". *Handbook of labor economics*. Vol. 4. Elsevier, pp. 1–102.
- Gabaix, Xavier (2009). "Power laws in economics and finance". Annu. Rev. Econ. 1.1, pp. 255–294.

- Gabaix, Xavier (2011). "The Granular Origins of Aggregate Fluctuations". *Econometrica* 79.3, pp. 733–772.
- García, Raffi E and William A Darity Jr (2021). "Self-Reporting Race in Small Business Loans: A Game-Theoretic Analysis of Evidence from PPP Loans in Durham, NC". *AEA Papers and Proceedings*.
- Goraya, Sampreet Singh (2023). "How does caste affect entrepreneurship? birth versus worth". *Journal of Monetary Economics* 135, pp. 116–133.
- Gupta, Arpit, Christopher Hansman, and Pierre Mabille (2022). *Financial constraints and the racial housing gap*. INSEAD Working Paper.
- Guvenen, Fatih, Fatih Karahan, Serdar Ozkan, and Jae Song (2021). "What Do Data on Millions of U.S. Workers Reveal about Lifecycle Earnings Dynamics?" *Econometrica*.
- Higgins, Brian E (2023). "Racial segmentation in the us housing market". Unpublished manuscript.
- Hsieh, Chang-Tai, Erik Hurst, Charles I. Jones, and Peter J. Klenow (2019). "The Allocation of Talent and U.S. Economic Growth". *Econometrica* 87.5, pp. 1439–1474.
- Hu, Yue, Long Liu, Jan Ondrich, and John Yinger (2011). The Racial and Gender Interest Rate Gap in Small Business Lending: Improved Estimates Using Matching Methods. Working Paper.
- Hubmer, Joachim, Per Krusell, and Anthony A. Smith Jr. (2021). "Sources of US wealth inequality: Past, present, and future". *NBER Macroeconomics Annual* 35.1, pp. 391– 455.
- Hurst, Erik and Annamaria Lusardi (2004). "Liquidity constraints, household wealth, and entrepreneurship". *Journal of political Economy* 112.2, pp. 319–347.
- Ifergane, Tomer (2024). *Concentrated Risk: Misallocation and Granular Business Cycles*. Working Paper.
- İmrohoroğlu, Ayşe, Çağri S Kumru, and Jiu Lain (2025). *Racial Disparities in Crime and Wealth*. working paper.
- Kaplan, Greg, Benjamin Moll, and Giovanni L Violante (2018). "Monetary policy according to HANK". American Economic Review 108.3, pp. 697–743.
- Kermani, Amir and Francis Wong (2021). *Racial disparities in housing returns*. NBER Working Paper.
- Kuhn, Moritz, Moritz Schularick, and Ulrike I Steins (2020). "Income and wealth inequality in america, 1949–2016". *Journal of Political Economy* 128.9, pp. 3469–3519.

- Lang, Kevin and Jee-Yeon K Lehmann (2012). "Racial discrimination in the labor market: Theory and empirics". *Journal of Economic Literature* 50.4, pp. 959–1006.
- Levine, Ross and Yona Rubinstein (2017). "Smart and illicit: who becomes an entrepreneur and do they earn more?" *The Quarterly Journal of Economics* 132.2, pp. 963–1018.
- Lipton, Avi (2022). "The Racial Wealth Gap and the Role of Firm Ownership". *AEA Papers* and *Proceedings* 112, pp. 351–355.
- Luo, Sai (2021). Racial Gaps in the Early Careers of Two Cohorts of American Men. Working Paper.
- Morazzoni, Marta and Andrea Sy (2022). "Female entrepreneurship, financial frictions and capital misallocation in the US". *Journal of Monetary Economics* 129, pp. 93–118.
- Oaxaca, Ronald (1973). "Male-female wage differentials in urban labor markets". *International economic review*, pp. 693–709.
- Quadrini, Vincenzo (2000). "Entrepreneurship, saving, and social mobility". *Review of economic dynamics* 3.1, pp. 1–40.
- Sabelhaus, John and Jeffrey P Thompson (2023). "The Limited Role of Intergenerational Transfers for Understanding Racial Wealth Disparities". *Federal Reserve Bank of Boston Research Paper Series Current Policy Perspectives Paper* 95748.
- Straub, Ludwig (2019). *Consumption, savings, and the distribution of permanent income*. Working Paper.
- Tan, Eugene and Teegawende H. Zeida (2024). "Consumer demand and credit supply as barriers to growth for Black-owned startups". *Journal of Monetary Economics* 143, p. 103543.
- White, T Kirk (2007). "Initial conditions at Emancipation: The long-run effect on blackwhite wealth and earnings inequality". *Journal of Economic Dynamics and Control* 31.10, pp. 3370–3395.

# Online Appendix to "Entrepreneurship and the Racial Wealth Gap"

Daniel Albuquerque and Tomer Ifergane

# A Additional figures and tables

	Share of wealth held by the							
	bottom 50% P50-P90 P90-P99 top							
Baseline	3.4%	24.6%	32.9%	39.2%				
Counterfactual scenario - baseline without								
Entrepreneurship distortion	4.0%	25.3%	32.7%	38.0%				
Labor market distortions	3.6%	24.6%	32.8%	38.9%				
All distortions	3.8%	24.9%	32.8%	38.5%				

Table A.1: Overall wealth inequality

Notes: This table reports the wealth distribution for the counterfactual scenarios in Section 5.1.

	Black households				White households			
horizon (years)	2	4	6	8	2	4	6	8
Panel A: average transition rates								
Move down from P50-P90	31.7	33.6	35.3	33.4	11.2	12.3	12.6	12.7
Move down from top 10%	70.6	72.4	82.5	85.6	25.7	28.0	29.1	31.1
Move up from bottom 50%	10.2	12.8	15.1	17.1	15.8	21.6	26.6	30.8
Move up from P50-P90	3.0	3.1	3.7	3.3	7.2	8.7	10.1	11.6
Panel B: point estimates from mo	bility r	egressi	ons					
Move down from P50-P90	-3.5	-3.6	-20.9	-18.9	-2.0	-1.4	-2.3	-1.7
Move down from top 10%	-8.1	13.8	21.8	20.5	-4.8	-4.8	-3.4	-3.8
Move up from bottom 50%	12.4	31.3	17.6	9.1	16.3	18.7	14.7	16.5
Move up from P50-P90	4.9	5.7	7.6	-4.3	13.1	11.8	12.0	10.5

Table A.2: Average transition rates between wealth groups and regressions results

*Notes:* Panel A reports the average transition rates (in percentages) between wealth groups over different horizons. Panel B reports the point estimates for the dummy of entrepreneurship on the same transition rates (i.e., coefficient  $\hat{\gamma}_g^i$  in Equation (3)). Source: PSID, 2001-2019.

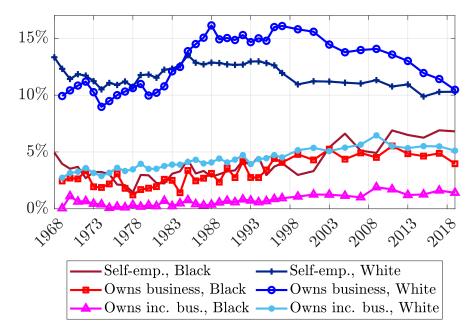


Figure A.1: Entrepreneurship rates, PSID

*Notes*: This figure shows the share of Black and White households over time that are entrepreneurs according to three definitions: (i) self-employed; (ii) owns a business; (iii) owns an incorporated business. Source: PSID.

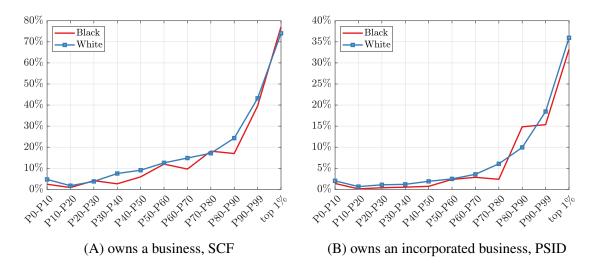
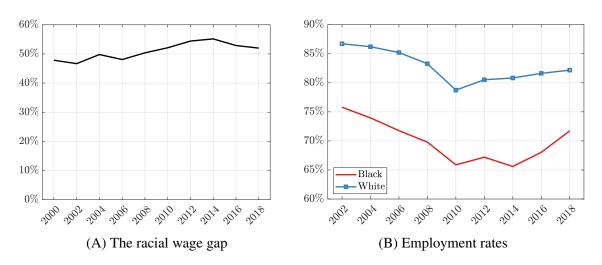


Figure A.2: Entrepreneurship rates by wealth fractiles

*Notes*: This figure shows the share of households of a given race that are classified as entrepreneurs in different fractiles of the overall wealth distribution, where "P10-P20" denotes those in between the 10<sup>th</sup> and 20<sup>th</sup> percentiles of wealth, for example. A household is classified as an entrepreneurs in Panel (A) if it owns a private business, according to the SCF; and in Panel (B) if it owns an incorporated business, according to the PSID. *Source*: SCF and PSID, 2001-2019.

### **B** Labor market outcomes

We highlighted in the main text the differences between Black and White households as entrepreneurs, which is the main focus of our paper. However, there are stark differences in labor market outcomes as well which, as the outside option to entrepreneurship, are also important for our main analysis. In this Appendix we first focus on differences between Black and White households in wages conditional on employment and in employment rates. Second, we show that labor income and entrepreneurship entry are positively correlated in the data, which we argue is due to underlying human capital. Taking this into consideration is important so not to overstate the importance of entrepreneurship for the racial wealth gap. Finally, we describe in detail the estimation of the labor income process used in our model.



#### **B.1** Differences in labor market outcomes

Figure B.1: Differences in labor income and employment rates

*Notes*: Panel (A) shows the racial gap in households' median labor income conditional on employment. Household labor income includes the wages of the main respondent and their spouse, and other sources, such as overtime pay, tips, bonuses, etc. Panel (B) shows the different employment rates for Black and White households. The employment rate is calculated as weeks of employment over the whole year. *Source*: PSID, 2001-2019.

Using PSID data, we calculate the gap in labor income conditional on employment between Black and White households, henceforth the racial wage gap and the gap in employment rates. As the unit of observation is a household, our measure of income includes the total labor income of the survey's main respondent and their spouse, if there is one. We include both male and female heads of household, but restrict the sample to households led by individuals between 25 and 65 years old. We also exclude any individual that reported being self-employed to only include true workers in the sample.

Figure B.1A shows the resulting racial wage gap, measured as the difference between Black and White households in the median wage per worked week in the previous year. The wage gap seems to be slightly increasing from 2000 to 2018, with an average of 50.9%. This is the measure of the racial wage gap that is imputed to the labor income distortion in the model. Notice that this is the unconditional wage gap – it does not control for any other factors, such as differences in education or household composition. Given that we do not model these differences explicitly, this is the appropriate measure to use. Thus, when we perform an exercise in the model where the labor income distortion is closing over time, we interpret it as not just the wage gap conditional on observables closing but also, for example, the convergence of educational attainment leading to convergence in wages.

On top of the gap in labor income conditional on employment displayed in Figure B.1A, we also document a gap in employment rates in Figure B.1B. While the employment rate for Black and White households naturally fluctuates with the business cycle, the gap seems relatively stable. The differences in non-employment rates are due to both a higher unemployment rate and also a higher non-participation rate for Black households. Our income process estimation incorporates this gap in employment rates to capture differences in labor market attachment between Black and White households.

Differences in labor income between Black and White workers have received considerable attention in the literature (e.g., see the review of Lang and Lehmann, 2012), and differences in employment rates have garnered more attention recently (Chandra, 2003; Bayer and Charles, 2018).<sup>27</sup> However, most of the literature focuses on the labor market outcomes of men. Because of the different composition of Black and White households and different employment rates between Black and White women, our headline figures differ from the literature. For example, we document larger gaps than Bayer and Charles (2018), who report a wage gap of around 40% between Black and White male workers since 1980 whereas ours is around 50%. Given the significant share of households led by women, we find it important to use our broader measure. We stress that in doing so we attribute a larger

<sup>&</sup>lt;sup>27</sup>See Derenoncourt and Montialoux (2021) for the impact of minimum wage policies on the decline of the income gap in the 1960s and 1970s; and Althoff and Reichardt (2024) for the long-run effects of being tied geographically to the Deep South.

role to labor market disparities than if we were to use the alternative estimates, which is a conservative assumption.

	(1)	(2)	(3)	(4)	(5)	(6)
	All	All	All	Black	Black	Black
percentile of income	0.012**	0.009*	0.010	0.018*	0.012	0.001
	(0.004)	(0.004)	(0.007)	(0.008)	(0.008)	(0.015)
percentile of wealth	0.043***	0.039***	0.031***	0.019*	0.016	0.001
	(0.004)	(0.004)	(0.006)	(0.010)	(0.009)	(0.009)
education		0.154***	0.187		0.245**	0.304
		(0.036)	(0.159)		(0.080)	(0.224)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Emp. status/age	Yes	Yes	Yes	Yes	Yes	Yes
Individual FE	No	No	Yes	No	No	Yes
R-Squared	0.014	0.014	0.338	0.011	0.013	0.428
Observations	63,453	62,973	60,865	25,255	25,066	24,196

### **B.2** Labor income and entrepreneurship choice

Table B.1: Entrepreneurship entry and income

*Notes*: This table reports the results of estimating Equation (19) either on all households (columns 1-3) or just on Black households (columns 4-6). Entrepreneurship entry is defined as not owning an incorporated business in wave t, but owning one at wave t + 1. The regressors highlighted are the income and wealth percentile groups, and education as measured by years of schooling. Column (1) shows that moving up one percentile group is correlated with a 0.03p.p. increase in the probability of entrepreneurship entry. All specifications include employment status and age as controls, and specifications without individual fixed effects include gender as well. Standard errors are clustered at the individual level. *Source*: PSID, 2001-2019.

In the model, entrepreneurship is an endogenous choice. Therefore, it is important to capture any possible correlation between labor income and the entrepreneurship decision present in the data: if those that start businesses usually have lower wages then increasing entrepreneurship might have a bigger effect on wealth than if those that start businesses were already earning higher incomes.

To investigate such correlations we use data from the PSID and regress the entry choice to become an entrepreneur on a set of observables. Entry  $entry_{i,t+1}$  is a dummy variable indicating that household *i* did not own an incorporated business at wave *t*, but owns one at wave *t* + 1 (there are PSID surveys every other year during this period) and estimate the following specification

$$entry_{i,t+1} = \alpha_t + \alpha_i + \beta_1 income_{i,t} + \beta_2 wealth_{i,t} + \beta_3 education_{i,t} + \Gamma X_{i,t} + \varepsilon_{i,t}, \quad (19)$$

where  $\alpha_t$ ,  $\alpha_i$  denote year and individual fixed effects, and  $X_{i,t}$  is a vector of household-level controls, including employment status, race, gender, and age. Education is measured in years of schooling, and income and wealth are measured as the percentile group of the household, e.g., between P50 and P51. We define an entrepreneur as the owner of an incorporated business in the PSID since it is the closest definition to our measure of choice in the SCF, as discussed in Section 2.2.

Table B.1 reports the result of estimating Equation (19) either using all households (columns 1-3) or just Black households (columns 4-6). Observe that income is an important predictor of entry into entrepreneurship even when controlling for wealth (columns 1 and 4) and a long set of controls. Thus, on average, firms are started by those with higher labor income, even after controlling for wealth.

However, notice that when education is included as a control (columns 2 and 5) the statistical significance of income almost disappears. Furthermore, when individual fixed effects are included (columns 3 and 6), then statistical significance of both income and education disappear. We interpret this set of results as suggestive that underlying human capital generates a positive correlation between labor income and the propensity to become an entrepreneur. This correlation motivates our modeling choice, where we assume a positive correlation between starting entrepreneurial productivity and the permanent component of labor income. This is crucial so not to overstate the importance of entrepreneurship: many entrepreneurs had already good labor market outcomes, so starting a business is not a leap in income as it would be if one had poor labor market outcomes. However, we still find that entrepreneurship is crucial for explaining the racial wealth gap.

Finally, in Table B.2 we document that the same picture on the importance of education emerges when using different measures of wealth, motivated by Hurst and Lusardi (2004) and Fairlie and Krashinsky (2012). We do not report results including individual fixed effects for brevity, but in that case we confirmed that income and education loose their significance as well.

	(1)	( <b>2</b> )	(2)	(4)
	(1)	(2)	(3)	(4)
	All	All	Black	Black
percentile of income	0.013**	0.004	0.012	0.011
	(0.004)	(0.004)	(0.008)	(0.008)
wealth in P50-P95	1.291***		$1.004^{*}$	
	(0.174)		(0.483)	
wealth in top 5%	5.423***		1.352	
	(0.698)		(1.455)	
education	0.145***	0.178***	0.236**	0.248**
	(0.036)	(0.036)	(0.077)	(0.082)
percentile of wealth $\times$ WORK		0.053***		0.019
		(0.005)		(0.014)
percentile of wealth $\times$ NOT WORK		0.016***		0.011
		(0.004)		(0.010)
Year FE	Yes	Yes	Yes	Yes
Empl. status/age	Yes	Yes	Yes	Yes
Individual FE	No	No	No	No
R-Squared	0.016	0.016	0.014	0.014
Observations	62,973	62,973	25,066	25,066

Table B.2: Entrepreneurship entry and income: alternative measures of wealth

*Notes*: This table shows the results of estimating Equation (19) either on all households (columns 1-2) or just on Black households (columns 3-4), under alternative measures of wealth. Entrepreneurship entry is defined as not owning an incorporated business at time t, but owning one at time t + 1. The regressors highlighted are the income percentile group, education (as measured by years of schooling), and two distinct measures of wealth. In columns (1) and (3), wealth is measured non-linearly, with dummies indicating whether a household belongs in the middle of the distribution (P50-P95) or in the top 5% of wealth, as motivated by Hurst and Lusardi (2004). In columns (2) and (4), we interact the wealth percentile group with employment status, as motivated by Fairlie and Krashinsky (2012). Column (1) shows that moving up one percentile group is correlated with a 0.013p.p. increase in the probability of entrepreneurship entry. Standard errors are clustered at the household level. *Source*: PSID, 2001-2019.

#### **B.3** Wage estimation

We now explain in greater detail the estimation of the 17 parameters in the processes of the components of labor income productivity  $z_{P,t}$ ,  $z_{T,t}$  and  $l_t$ : { $\tau_L^B, \mu_P^B, \mu_T^B, \lambda_P^B, \lambda_T^B, \sigma_P^B, \sigma_T^B, \lambda_{01}^B$ ,  $\lambda_{10}^B, \mu_P^W, \mu_T^W, \lambda_P^W, \lambda_T^W, \sigma_P^W, \sigma_T^W, \lambda_{01}^W, \lambda_{10}^W$ }. Overall, we estimate moments from the data, then use Simulated Method of Moments to first estimate the parameters of the processes, and finally optimize over the choice of the grid in which to discretize the process.

The PSID from 2001 to 2019 is the source for our data moments. It is specially suited

for our exercise for three main reasons. First, it is a panel dataset, which allows us to calculate moments based on wage changes over time for a given household. Second, in the 1990s the PSID added an extra sample meant to better capture minorities in the US, which means that the sample size for Black households is similar to those of White households. Finally, the PSID also asks about labor income, weeks worked, and monthly employment dating (since 2003) on the year before the survey, which will be key for the estimation of the racial wage gap conditional on employment, and also for the transition rates between employment and non-employment.

Because our unit of observation is a household, we define as "wage" the total labor income for both the main respondent to the survey and their spouse. We restrict the sample to those in working age between 25 and 65 years old, and consider both male- and female-led households. We exclude anyone that reported being self-employed to only take into account true workers. Most of the moments we calculate are based on changes in wages over time, thus we construct a single dataset with all the qualifying households that appeared in at least two consecutive waves. However, some restrictive moments require us to observe a household twice with a lag of six years (e.g., in 2011 and 2017, but not necessarily in 2013 or 2015). Our smallest sample sizes are for these moments, of 1021 for Black households and 1737 for White households (but we have 7 different combinations of 6-year spans from 2001 to 2019).

The first step in our procedure is to estimate some moments directly from the data. Because we know the labor income of each household in the year before the survey and the number of weeks worked, that allows us to calculate wage conditional on employment. The simple difference on median wage per week worked of Black and White households is our estimate for the racial wage gap, and we find  $\tau_L^B = 50.9\%$ . Because we do not model dimensions such as educational attainment, school quality or household composition, our measure of  $\tau_L^B$  is also influenced by differences in these features between Black and White households. Furthermore, we have monthly dating of employment for households over the course of the year prior to the survey, and we use that to calculate monthly transition rates. With monthly transition rates  $\lambda_m$  in hand, we calculate yearly transition rates for our model with  $\lambda_m = e^{-\lambda_y/12}$ , and find  $\lambda_{10}^B = 15.4\%$ ,  $\lambda_{01}^B = 31.5\%$ ,  $\lambda_{10}^W = 10.0\%$ ,  $\lambda_{01}^W = 44.2\%$ .

Second, we estimate all the other parameters jointly using a Simulated Method of Moments (SMM). The idea is to simulate the processes for  $z_{P,t}$ ,  $z_{T,t}$  and  $l_t$  for a given combination of parameters, and calculate in the model the same moments that we estimated from the data. Then we optimise over the choice of parameters to minimise the sum of squared deviations between the moments simulated from the model and those from the data. We impose the identifying assumption  $\lambda_P \leq \lambda_T$ .

The moments chosen are shown in Table B.3. There is only one moment directly related to the distribution of income across households, and that is the variance of the log of labor income. The other moments are related to the change in log labor income over time for a given household. We target the standard deviation and kurtosis of the changes over 2, 4 and 6 years, and also the fraction of households whose 2 years log changes were smaller than 5%, 10% or 20%. In total, we have 10 moments for both Black and White households for the remaining eight parameters that are left to be estimated, and we weigh all the moments equally.

The simulation involves 5000 households over a period of 1000 years to arrive at the stationary distribution, and six more years to calculate the necessary moments. The simulated process for labor income is annual, but we calculate 2, 4 and 6 years wage changes to match the data.

	В	lack Househ	olds	W	hite Househ	olds
Moments	(1) Data	(2) Model Contin.	(3) Model Discret.	(4) Data	(5) Model Contin.	(6) Model Discret.
var(log(income))	0.67	0.56	0.53	0.64	0.57	0.52
std $\Delta 2y$	0.55	0.64	0.63	0.43	0.54	0.50
std $\Delta 4y$	0.62	0.68	0.72	0.51	0.57	0.59
std Δ6y	0.67	0.78	0.77	0.56	0.66	0.65
kurtosis $\Delta 2y$	7.0	7.5	7.7	9.9	10.5	11.1
kurtosis ∆4y	6.0	6.5	6.1	7.1	8.7	8.4
kurtosis $\Delta 6y$	5.8	5.5	5.5	7.0	7.1	7.2
share( $\Delta 2y < 5\%$ )	16.3%	16.6%	22.3%	20.7%	20.6%	22.0%
share( $\Delta 2y < 10\%$ )	29.3%	28.7%	31.6%	37.5%	36.8%	41.6%
share( $\Delta 2y < 20\%$ )	48.6%	48.3%	47.3%	59.3%	62.0%	66.7%

Table B.3: Labor income moments from data and model

*Notes*: This table shows the moments for Black and White households estimated from the data, simulated by the model without a grid constraint (continuous), and simulated by the model in a specific discretized grid. The moments targeted are: variance of the log of labor income across households; the standard deviation and kurtosis of 2, 4 and 6 year wage changes; and the fraction of households that experience wage changes below 5, 10 and 20% over a 2-year period. *Source*: PSID, 2001-2019.

The estimated parameters were reported in Table 1, and the moments implied by the continuous model are shown in columns (2) and (5) of Table B.3. It shows that the model does an overall great job in matching most moments, including the high kurtosis high-lighted by Guvenen et al. (2021), due to shocks not arriving at every period (Kaplan, Moll, and Violante, 2018). The model seems to undershoot the variance of log income. But, as Figure B.2 shows, the estimated model seems to fit the overall distribution quite well, including the intercept with the share of households that have exactly zero labor income over the course of an year.

Third, with the estimated parameters in hand, we estimate the best grid that, given the parameters, can generate the same moments. We choose 9 grid points for permanent and 3 for the transitory component so as not to burden the numerical solution of the full model. In this step, we construct a grid for percentage deviations from the average wage, where there is a grid point exactly at zero and an equal number of grid points above and below in a symmetric fashion. We then optimise over the width of the grid points furthest away from the average and the curvature of these points (they are not uniformly distributed between zero and the points furthest away from it). The results for the moments constrained to this grid are shown in columns (3) and (6) of Table B.3. One can see that most of the moments are similar to those in columns (2) and (5), suggesting that discretizing the process does not lead to a great loss of accuracy.

### **C** Recursive stationary equilibrium

A recursive stationary equilibrium in the model economy consists of value functions  $V(a, z_L, i)$ and  $F(a, z_F, i)$ ; saving rules  $s_V(a, z_L, i)$ ,  $s_F(a, z_F, i)$  and the corresponding consumption policy function  $c_V(a, z_L, i)$ ,  $c_F(a, z_F, i)$ ; entry choice policies  $I_V(a, z_L, i)$ ;<sup>28</sup> stationary density functions  $g_L(a, z_L, i)$  and  $g_F(a, z_F, i)$ ; a mass of entrepreneurs  $m_F$ ; firm policy functions for capital demand  $k(a, z_F, i)$  and labor demand  $h(a, z_F, i)$ ; firm output and profit functions  $y(a, z_F, i)$  and  $\pi(a, z_F, i)$ ; rental rate of capital r; tax rates  $\tau_a, \tau_{\pi}$  and  $\tau_L$ ; wage rate w; and benefits T which jointly satisfy the following:

1. Consumer optimization - Given prices r and w, transfers T, and the profit functions  $\pi(a, z_F, i)$ , the policy functions  $c_V(a, z_L, i)$ ,  $c_F(a, z_F, i)$  and  $I_V(a, z_L, i)$  solve

 $<sup>{}^{28}</sup>I_V$  is an indicator function that equals one if the worker chooses to become an entrepreneur and zero otherwise for each state in the worker's state space. In the main text this decision rule is replaced by the max operator for readability.

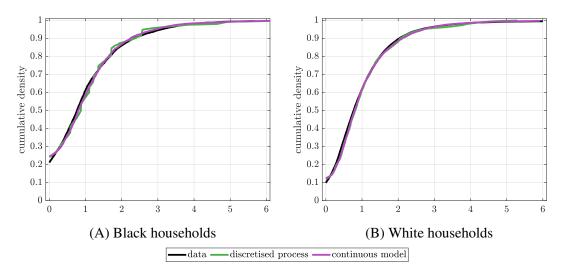


Figure B.2: CDF of log of normalized labor income

*Notes*: This figure shows the CDF of the log of labor income in the PSID and also in the continuous and discretized version of the estimated labor income process. Labor income has been normalised by the average labor income for each race in that year. *Source*: PSID, 2001-2019.

the optimization problems given by problems (4) and (8) that are associated with the value functions  $V(a, z_L, i)$  and  $F(a, z_F, i)$ . The indicator  $I_V(a, z_L, i)$  takes the value of unity if  $F(a, z_F, i) > V(a, z_L, i)$  and zero otherwise. Additionally,  $c_V(a, z_L, i)$  and  $c_F(a, z_F, i)$  induce the saving rules  $s_V(a, z_L, i)$ ,  $s_F(a, z_F, i)$  via Equations (5) and (9).

- 2. Firm optimization Given the rental rate *r* and the wage *w*, the policy functions for capital  $k(a, z_F, i)$  and labor  $h(a, z_F, i)$  are consistent with the firms solving the optimization problem (12). The functions  $k(a, z_F, i)$  and  $h(a, z_F, i)$  govern flow output *y* and profits  $\pi(a, z_F, i)$  via Equation (11) and the relationship  $y = z_F k(a, z_F, i)^{\alpha} h(a, z_F, i)^{\beta}$ .
- 3. Asset market the rental rate r satisfies the asset market clearing condition

$$\sum_{i \in \{B,W\}} \left( \int_{\underline{z}_{L}}^{\overline{z}_{L}} \int_{\underline{a}}^{\infty} a g_{L}(a, z_{L}, i) \, da \, dz_{L} + \int_{\underline{z}_{F}}^{\infty} \int_{\underline{a}}^{\infty} a g_{F}(a, z_{F}, i) \, da \, dz_{F} \right) =$$

$$\sum_{i \in \{B,W\}} \int_{\underline{z}_{F}}^{\infty} \int_{\underline{a}}^{\infty} k(a, z_{F}, i) g_{F}(a, z_{F}, i) \, da \, dz_{F},$$

$$(20)$$

$$(20)$$

$$(20)$$

where  $\underline{z}_L$  and  $\overline{z}_L$  denote the lower and upper bounds for  $z_L$ .

4. Labor market - the wage w clears the labor market as follows

$$\sum_{i \in \{B,W\}} \int_{\underline{z}_L}^{\overline{z}_L} \int_{\underline{a}}^{\infty} \left(1 - \tau_L^i\right) z_L g_L(a, z_L, i) \, da \, dz_L = \sum_{i \in \{B,W\}} \int_{\underline{z}_F}^{\infty} \int_{\underline{a}}^{\infty} h\left(a, z_F, i\right) g_F\left(a, z_F, i\right) \, da \, dz_F$$

$$(21)$$

5. Transfers T are such that the government budget is balanced given the tax rates. This balanced budget rule is given by

$$T(1-m_F) = \underbrace{t_{\pi}\Pi}_{\text{income from profit tax}} + \underbrace{t_w w \sum_{i \in \{B,W\}} \int_{\underline{z_L}}^{\overline{z_L}} \int_{\underline{a}}^{\infty} z_L (1-\tau_L^i) g_L(a, z_L, i) \, da \, dz_L}_{\text{income from labor income tax}} + \underbrace{t_a(r-\delta) \sum_{i \in \{B,W\}} \left( \int_{\underline{z_L}}^{\overline{z_L}} \int_{0}^{\infty} a g_L(a, z_L, i) \, da \, dz_L + \int_{\underline{z_F}}^{\infty} \int_{0}^{\infty} a g_F(a, z_F, i) \, da \, dz_F \right)}_{(22)}$$

income from capital income tax

where  $\Pi$  denotes aggregate profits.<sup>29</sup>

6. Consistency - the population densities  $g_L(a, z_L, i)$  and  $g_F(a, z_F, i)$  have a total mass of unity and have their stationary distributions implied by the saving rules  $s_V(a, z_L, i)$ ,  $s_F(a, z_F, i)$ and decision rule  $I_V(a, z_L, i)$  and is consistent with the following coupled KFEs (time indices are added here to all equilibrium objects)

$$\frac{\partial}{\partial t}g_{L}(a,z_{L},i,t) = -\frac{\partial}{\partial a}[g_{L}(a,z_{L},i,t) \ s_{V}(a,z_{L},i,t)] + A^{*}_{z_{L}}g_{L}(a,z_{L},i,t) - \eta I_{V}(a,z_{L},i,t)g_{L}(a,z_{L},i,t) + \lambda_{D}n(z_{L},i) \int_{\underline{z}_{F}}^{\infty}g_{F}(a,z_{F},i,t)dz_{F}$$
(23)

$$\frac{\partial}{\partial t}g_F(a,z_F,i,t) = -\frac{\partial}{\partial a}[g_F(a,z_F,i,t) s_F(a,z_F,i,t)] + A^*_{z_F}g_F(a,z_F,i,t) 
- \lambda_D g_F(a,z_F,i,t) + \eta \tilde{I}_V(a, \Psi^{-1}(z_F),i,t) \tilde{g}_L(a, \Psi^{-1}(z_F),i,t),$$
(24)

where  $A_{z_L}^*$  and  $A_{z_F}^*$  denote the adjoint operator of the infinitesimal generators of the processes governing  $z_L$  and  $z_F$ . With slight abuse of notation,  $\psi^{-1}(z_F)$  denotes the

 $<sup>\</sup>overline{2^{9} \text{Profits are } \Pi = \sum_{i \in \{B,W\}} \int_{\underline{z}_{F}}^{\infty} \int_{\underline{a}}^{\infty} \pi} (a, z_{F}, i) g_{F}(a, z_{F}, i) dadz_{F}, \text{ where } \pi(a, z_{F}, i) = y(a, z_{F}, i) - wh(a, z_{F}, i) - rk(a, z_{F}, i).$ 

inverse of the mapping in Equation (17) such that it maps the entrant's productivity into the previous value of  $z_L$ , this inverse is only defined for  $z_P \ge \Psi_0$ , for values below  $\underline{z}_F$ , we let  $\Psi^{-1}(z_F) = 0$ . For completeness, we also define  $\tilde{I}_V(a, \Psi^{-1}(z_F), i, t)$ and  $\tilde{g}_L(a, \Psi^{-1}(z_F), i, t)$  as the functions that take the values of  $I_V(a, \Psi^{-1}(z_F), i, t)$ and  $g_L(a, \Psi^{-1}(z_F), i, t)$  when  $z_F \ge \underline{z}_F$  or when  $z_P \ge \Psi_0$  and are otherwise equal to zero.  $n(z_L, i)$  denotes the stationary pdf of the process governing  $z_L$  for group *i*. The mass of entrepreneurs  $m_F$  is given by

$$m_F = \sum_{i=\{B,W\}} \int_{\underline{z}_F}^{\infty} \int_{\underline{a}}^{\infty} g_F(a, z_F, i) \, da \, dz_F.$$
<sup>(25)</sup>

The masses of each race integrate such that

$$m^{B} = \int_{\underline{z}_{F}}^{\infty} \int_{\underline{a}}^{\infty} g_{F}(a, z_{F}, B) da dz_{F} + \int_{\underline{z}_{L}}^{\overline{z}_{L}} \int_{\underline{a}}^{\infty} g_{L}(a, z_{L}, B) da dz_{L}, \qquad (26)$$

$$m^{W} = \int_{\underline{z}_{F}}^{\infty} \int_{\underline{a}}^{\infty} g_{F}(a, z_{F}, W) \, da \, dz_{F} + \int_{\underline{z}_{L}}^{\overline{z}_{L}} \int_{\underline{a}}^{\infty} g_{L}(a, z_{L}, W) \, da \, dz_{L}, \qquad (27)$$

where  $m^B$ , and  $m^W$  are exogenously given numbers such that  $m^B + m^W = 1$ .

Note that for the goods market to clear, the total output produced (given by Equation (37)) must equal the sum of aggregate consumption and investment in capital. This clearing condition is implied by the others since aggregate profits, labor compensations, and capital compensations constitute total income in the economy, and aggregate consumption plus gross investment is total spending.

### **D** Solution algorithm

This appendix details the algorithm used to solve our model. The algorithm builds on the methods of Achdou et al. (2021) for continuous-time and follows along the lines of the definition of the recursive stationary equilibrium in the model economy as given in Appendix C.

The solution algorithm solves a system of three equations (20), (21), and (22), in the three unknowns, r, w, and T. The algorithm follows from the definition of recursive stationary equilibrium.

- 1. **Initialization** Provide a grid for assets, parameter values for the model, and initial guesses for the values of r, w, and T.
- 2. Solve firm block Using the values of *r* and *w* solve for the firms' demand for capital and labor  $k(a, z_F, i)$  and labor  $h(a, z_F, i)$  and for firm profits  $\pi(a, z_F, i)$ .
- 3. **Solve household block** Solve the household optimization problem given the guesses and the calibrated parameters using the algorithm for solving the HJB equations given in Achdou et al. (2021). Given the high dimensionality of the problem, we modify the algorithm as follows:
  - (a) Provide the initial guess that the value function stays put (flow utility is constant) and solve the consumption savings problem as if all the exogenous state variables  $z_L, z_F$  are constant and not subject to exogenous stochastic processes, and the households are not allowed to choose entrepreneurship.
  - (b) Use the solution to the limited problem in step 3a as an initial guess to the consumption savings problem that allows for changes in  $z_L, z_F$ , but still prohibits the entrepreneurship choice.
  - (c) Finally, use the solution to the limited problem in step 3b as the initial guess to the full HJBs given by Equations (4) and (8).

This will allow us to obtain the ergodic stationary distributions  $g_L(a, z_L, i)$  and  $g_F(a, z_F, i)$ , the policy functions  $c_V(a, z_L, i)$ ,  $c_F(a, z_F, i)$  and  $I_V(a, z_L, i)$ , the equilibrium masses, the savings rules, and the mass of entrepreneurs  $m_F$ , the supply of effective labor by households, and the total net aggregate asset supply.

- 4. **Compute capital and labor demand** Combine the masses from step 3 with the capital and labor solutions from step 2 to obtain the aggregate capital and labor demand by the firms given their population composition.
- 5. **Compute government income** Using the tax rates and the total income in the economy, use Equation (22) to compute the government income.
- 6. **Clear markets** Using the results of steps 3, 4, and 5 evaluate Equations (20), (21), and (22). If the system is sufficiently close to zero, stop. Otherwise, update the initial guess accordingly, and repeat from 1 until convergence is achieved.

**Solver** We use a quasi-Newton solver based on the Broyden method and evaluate the Jacobian of the system using finite differences. It is useful to relax the updated solution in the Newton direction such that, at the new guess, the value of  $r - \delta$  lies between zero and the largest discount rate and that w is strictly positive. We use backtracking to choose the largest relaxation parameter from a pre-specified set of values (all less than one), so the new guess is well within these bounds. If the bounds are already violated, which can occur, we use a pre-set relaxation parameter, which, in many cases, leads the algorithm to return to its normal bounds. If the solver is unsuccessful, a new guess is randomized, and the procedure begins anew.

**Stopping criterion and normalizations** A convergence criterion of maximum relative deviation of  $0.5 \times 10^{-2}$  yields fast results and performs well. All equations described in stage 6 are solved after normalization to obtain a meaningful stopping criterion. The labor and capital market clearing conditions are normalized such that they are expressed in percentage deviations of the aggregate supply. The government budget is normalized in such a way that it is expressed as a percentage deviation from the government's total tax revenue.

**Grid for assets** We use n = 200 grid points for assets. The grid is not uniform such that most grid points are concentrated near the borrowing constraint <u>a</u>. The maximum value for assets is set at a = 3,000, corresponding to asset holdings equivalent to around  $3.9 \times 10^3$  unconsumed annual median labor incomes. The asset vector  $\bar{a}$  is set such that it has monotonically increasing increments as follows

$$\bar{a} = (a_{\max} - \underline{a}) \frac{(0, 1, \dots, n-1)^5}{(n-1)^5} + \underline{a}.$$
(28)

This generates monotonically increasing increments with a grid point exactly on the borrowing constraint, which will have a positive mass of households on it.

**Modifications required outside of steady state** To solve for the transition dynamics as in Section 6 and Section 6.1 one needs to solve Equations (20), (21), and (22) in every point in time such that for  $n_t$  periods one is required to solve  $3 \times n_t$  equations given guesses for the paths of r, w and T. As shown in Achdou et al. (2021), the procedure involves solving the HJB in every period backwards from the terminal condition and using the transition

matrices from every period to iterate forward on the distributions  $g_L$  and  $g_F$  from the initial condition and clear the three markets in every period. Since we solve for long horizons, we use a non-uniform grid on time as follows

$$\bar{t} = t_{\max} \frac{(0, 1, \dots, n_t - 1)^3}{(n_t - 1)^3}.$$
(29)

We solve in 30 increments for a total duration of  $t_{max} = 500$  years.

### E Model appendix: Additional derivations

### E.1 Productivity distribution in the model economy

The firm productivity distribution in the economy is given by: (i) the productivity distribution of new entrants; (ii) the exit rate  $\lambda_D$ ; and (iii) the stochastic process in Equation (10) governing the evolution of firm productivity conditional on a firm staying in operation. The distribution of new entrants is influenced by both the stationary distribution of labor income, which affects entrants productivity via  $\psi(z_L)$ , and also the distribution of wealth, which in turn affects the potential profits and ultimately the entry decision of a prospective entrant. We impose an upper bound on the permanent component of labor productivity, which implies an upper bound on the entrant's productivity.

While it is not possible to get an analytical solution to the exact distribution of  $z_F$ , we can use an asymptotic result (Gabaix, 2009) that as  $z_F \rightarrow \infty$  its distribution  $f(z_F)$  has a right tail that satisfies the following Kolmogorov Forward Equation (KFE) in steady state:

$$0 = -\frac{\partial}{\partial z_F} \left[ f(z_F) \mu_F z_F \right] + \frac{1}{2} \frac{\partial^2}{(\partial z_F)^2} \left[ (\sigma_F z_F)^2 f(z_F) \right] - \lambda_D f(z_F).$$
(30)

Through guess-and-verify, one can show that  $f(z_F)$  is a Pareto distribution with tail parameter  $\zeta$ , i.e.  $f(z_F) \propto z_F^{-(\zeta+1)}$ , with:

$$\zeta = \frac{1}{2} - \frac{\mu_F}{\sigma_F^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu_F}{\sigma_F^2}\right)^2 + \frac{2\lambda_D}{\sigma_F^2}}.$$
(31)

Note that a corollary of this tail behavior is that the right tail of the firm size distribution in terms of labor also exhibits a Pareto distribution with tail parameter equal to  $\tilde{\zeta} = \zeta(1 - \alpha - \beta)$ . This behavior results from the following features. Firm size, in terms of labor, is proportional to  $((1 - \tau_y{}^i) z_F)^{\frac{1}{1-\alpha-\beta}}$  for the unconstrained as demonstrated by Equation (15). Thus, in the absence of distortions, the firm-size distribution inherits the tail behavior of  $z_F$  and has a Pareto tail of  $\zeta$ , then  $z_F^{\frac{1}{1-\alpha-\beta}}$ , has a Pareto tail of  $\tilde{\zeta} = \zeta(1-\alpha-\beta)$ .<sup>30</sup> Note that since we consider only two levels of  $\tau_y{}^i$ , the above statement is true within race. The distortions do not affect tail behavior, but scales the productivity distribution.

## E.2 Aggregate production function representation of the model economy

This appendix details the exact derivation of the aggregate properties of the model economy used in Section 5. Let us begin by examining the factor demand functions for firms in Equations (15) and (16)

$$h(a, z_F, i) = \left( \left(1 - \tau_y^i\right) z_F \right)^{\frac{1}{1 - \alpha - \beta}} \left( \frac{\alpha}{r\left(1 + \tau_k\left(a, z_F, i\right)\right)} \right)^{\frac{\alpha}{1 - \alpha - \beta}} \left( \frac{\beta}{w} \right)^{\frac{1 - \alpha}{1 - \alpha - \beta}}, \qquad (32)$$

$$k(a, z_F, i) = \left( \left( 1 - \tau_y^i \right) z_F \right)^{\frac{1}{1 - \alpha - \beta}} \left( \frac{\alpha}{r \left( 1 + \tau_k \left( a, z_F, i \right) \right)} \right)^{\frac{1 - \beta}{1 - \alpha - \beta}} \left( \frac{\beta}{w} \right)^{\frac{\beta}{1 - \alpha - \beta}}, \quad (33)$$

where we have substituted in  $r + \mu_{CC}(a, z_F, i) = r(1 + \tau_k(a, z_F, i))$ . Thus, firm-level output  $y(a, z_F, i) = z_F k^{\alpha}(a, z_F, i) h^{\beta}(a, z_F, i)$  is given by

$$y(a, z_F, i) = \left[ z_F \frac{\left(1 - \tau_y^i\right)^{\alpha + \beta}}{\left(1 + \tau_k(a, z_F, i)\right)^{\alpha}} \right]^{\frac{1}{1 - \alpha - \beta}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1 - \alpha - \beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1 - \alpha - \beta}}.$$
 (34)

It is straightforward to derive aggregate capital K, aggregate effective labor  $Z_L$ , and aggregate output Y by integrating the above three equations along the population measures as follows:

$$K = \left(\frac{\alpha}{r}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\alpha-\beta}} \sum_{i \in \{B,W\}} \int_{\underline{z}_F}^{\infty} \int_{\underline{a}}^{\infty} \left[ z_F \frac{\left(1-\tau_y^{i}\right)}{\left(1+\tau_k(a,z_F,i)\right)^{(1-\beta)}} \right]^{\frac{1}{1-\alpha-\beta}} g_F\left(a,z_F,i\right) \ dadz_F , \quad (35)$$

$$Z_{L} = \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{w}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \sum_{i\in\{B,W\}} \int_{\underline{z}_{F}}^{\infty} \int_{\underline{a}}^{\infty} \left[z_{F} \frac{\left(1-\tau_{y}^{i}\right)}{\left(1+\tau_{k}(a,z_{F},i)\right)^{\alpha}}\right]^{\frac{1}{1-\alpha-\beta}} g_{F}\left(a,z_{F},i\right) \ dadz_{F}, \qquad (36)$$

<sup>&</sup>lt;sup>30</sup>For formal proofs along this line, see Carvalho and Grassi, 2019; Ifergane, 2024.

$$Y = \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\alpha-\beta}} \left[\sum_{i\in\{B,W\}} \int_{\underline{z_F}}^{\infty} \int_{\underline{a}}^{\infty} \left[z_F \frac{\left(1-\tau_y^{\ i}\right)^{\alpha+\beta}}{\left(1+\tau_k(a,z_F,i)\right)^{\alpha}}\right]^{\frac{1}{1-\alpha-\beta}} g_F(a,z_F,i) \ dadz_F \right].$$
(37)

To obtain meaningful terms in the equation for Y, we transform the above equation as follows. First, observe that we can represent Y as

$$Y = K^{\alpha} Z_L^{\beta} \widetilde{TFP}, \qquad (38)$$

where  $\widetilde{TFP}$  is given by

$$\widetilde{TFP} = \frac{\sum_{i \in \{B,W\}} \int_{\widetilde{z}_{F}}^{\infty} \int_{\underline{a}}^{\infty} \left[ z_{F} \frac{(1-\tau_{y}^{i})^{\alpha+\beta}}{(1+\tau_{k}(a,z_{F},i))^{\alpha}} \right]^{\frac{1}{1-\alpha-\beta}} g_{F}(a,z_{F},i) \ dadz_{F}} \left[ \sum_{i \in \{B,W\}} \int_{\widetilde{z}_{F}}^{\infty} \int_{\underline{a}}^{\infty} \left[ \frac{(1-\tau_{y}^{i})}{(1+\tau_{k}(a,z_{F},i))^{1-\beta}} z_{F} \right]^{\frac{1}{1-\alpha-\beta}} g_{F}(a,z_{F},i) \ dadz_{F} \right]^{\alpha} \left[ \sum_{i \in \{B,W\}} \int_{\widetilde{z}_{F}}^{\infty} \int_{\underline{a}}^{\infty} \left[ \frac{(1-\tau_{y}^{i})z_{F}}{(1+\tau_{k}(a,z_{F},i))^{\alpha-\beta}} g_{F}(a,z_{F},i) \ dadz_{F} \right]^{\beta}$$

Second, observe that in this economy, firms are a fixed factor of production. Thus, we can multiply the terms in the integrals composing  $\widetilde{TFP}$  by  $\frac{1}{m_F}$  and multiply the integral itself by  $m_F$  to purge  $\widetilde{TFP}$  from scale effects we obtain

$$Y = K^{\alpha} Z_L^{\beta} m_F^{1-\alpha-\beta} TFP, \qquad (39)$$

with TFP given by

$$TFP = \frac{\sum_{i \in \{B,W\}} \int_{\underline{z_F}}^{\infty} \int_{\underline{a}}^{\infty} \frac{1}{m_F} \left[ z_F \frac{(1-\tau_y^i)^{\alpha+\beta}}{(1+\tau_k(a,z_F,i))^{\alpha}} \right]^{\frac{1}{1-\alpha-\beta}} g_F(a,z_F,i) \ dadz_F} \left[ \frac{\sum_{i \in \{B,W\}} \int_{\underline{z_F}}^{\infty} \int_{\underline{a}}^{\infty} \frac{1}{m_F} \left[ \frac{(1-\tau_y^i)^{\alpha+\beta}}{(1+\tau_k(a,z_F,i))^{1-\beta}} z_F \right]^{\frac{1}{1-\alpha-\beta}} g_F(a,z_F,i) \ dadz_F \right]^{\alpha} \left[ \sum_{i \in \{B,W\}} \int_{\underline{z_F}}^{\infty} \int_{\underline{a}}^{\infty} \frac{1}{m_F} \left[ \frac{(1-\tau_y^i)^{\alpha+\beta}}{(1+\tau_k(a,z_F,i))^{\alpha}} \right]^{\frac{1}{1-\alpha-\beta}} g_F(a,z_F,i) \ dadz_F \right]^{\beta}$$

Last, we wish to separate the notions of labor quality and labor quantity. We can state effective labor input in production as  $Z_L = \mathbb{E}(z_L(1 - \tau_L^i))N$ , where N is the mass of workers, which is incidentally  $1 - m_F$ , and  $\mathbb{E}(z_L)$  is their average quality allowing for the distortions. Observe that average labor quality relates to the distortions as follows

$$E\left(z_L\left(1-\tau_L^i\right)\right) = \sum_{i\in\{B,W\}} \underbrace{\frac{\int_{z_L}^{\overline{z}_L} \int_{\underline{a}}^{\infty} g_L(a, z_L, i) dadz_L}{1-m_F}}_{\text{share of workers belonging to group } i} \times \underbrace{\frac{\int_{z_L}^{\overline{z}_L} \int_{\underline{a}}^{\infty} z_L g_L(a, z_L, i) dadz_L}{\int_{z_L}^{\overline{z}_L} \int_{\underline{a}}^{\infty} g_L(a, z_L, i) dadz_L}}_{\text{average labor productivity in group } i} \times \underbrace{\frac{(1-\tau_L^i)}{(1-\tau_L^i)}}_{\text{distortion on group } i}$$
(40)

We again stress that differences in the average labor productivity emerge endogenously in our model. Ex-ante, without the distortions, Black and White households are endowed with  $z_L$  drawn from the same distributions. However, ex-post, the distortions drive households that differ only in race to be exposed to different shocks and make different entrepreneur-

ship decisions, leading to a steady state where differences in race are predictive of outcomes. Therefore, the aggregate production function in this economy can be represented as

$$Y = K^{\alpha} N^{\beta} m_F^{1-\alpha-\beta} \left( \mathbb{E} \left( z_L \left( 1 - \tau_L^i \right) \right) \right)^{\beta} TFP.$$
(41)

After taking logs, we have

$$\log Y = \underbrace{\alpha \log K + \beta \log N + (1 - \alpha - \beta) \log m_F}_{\text{factor quantities}} + \underbrace{\beta \log \left( \mathbb{E} \left( z_L \left( 1 - \tau_L^i \right) \right) \right)}_{\text{labor efficiency}} + \underbrace{\log \left( TFP \right)}_{\text{aggregate productivity}}$$
(42)