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States and Contingencies: How to Understand Savage without Anyone Being Hanged

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Abstract

Models of decision making under uncertainty gain much of their power from the specification of states so as to resolve all uncertainty. However, this specification can undermine the presumed observability of preferences on which axiomatic theories of decision making are based. We introduce the notion of a contingency. Contingencies need not resolve all uncertainty, but preferences over functions from contingencies to outcomes are (at least in principle) observable. In sufficiently simple situations, states and contingencies coincide. In more challenging situations, the analyst must choose between sacrificing observability in order to harness the power of states that resolve all uncertainty, or preserving observability by working with contingencies.

Contents

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1 Introduction

The classical model of decision making under uncertainty involves sets of (i) states of nature (or "states of the world"), S; (ii) outcomes, X; and (iii) acts, A, that are functions from states to outcomes. Choice is made among the acts, and is formally described by a binary relation, a choice function, choice probabilities, or some other model of selection among alternatives. The choice of the act by the decision maker and of a state by "Nature" jointly determine the outcome.

Formal models of choice among functions (from an abstract set, S, to another abstract set, X) can have a variety of interpretations, and, indeed, pop up in various contexts in economic theory. In particular, the formal model of decision under uncertainty can also describe social choice (where S is interpreted as a set of individuals), choices over time (where S represents a set of time periods), or multi-criteria decision making (where S describes different criteria of choice). Decision theory, at least as taught in textbooks, has a set of informal guidelines for successful modeling of choice problems in each of these interpretations. These "user manuals" are not part of the theory itself, and are usually not formally modeled, though in principle they could be modeled.[1]

For decision under uncertainty, the informal guidelines usually include two principles:

1. Each state describes all relevant aspects of reality, and exactly one of the states will transpire (as in the interpretation of the standard model of probability theory). Common exceptions include models in which outcomes are objective ("roulette") lotteries, as in Anscombe-Aumann (1963), where some uncertainty (namely, the result of the spin of the roulette wheel) is not resolved by the choice of a state of nature. This, however, is often viewed as a mathematical simplification that is useful in providing some structure and richness to the set of outcomes. Other exceptions include models of bounded rationality in which the decision maker might not be aware of all

[1] For example, game theory textbooks usually provide a formal construction of the "normal form" of a game based on its "extensive form". This could be viewed as a formal guide to modeling situations of interest (captured by the extensive form) in a given formal structure (the normal form). Note that this involves an additional layer of modeling: both the extensive and the normal form games can be viewed as models of reality. Formally describing how to construct one from the other can be viewed as a description of the work of the game theorist. This is particularly intuitive when one constructs the normal form from the extensive one: the "real" social situation unfolds over time, in a way that is closer to the extensive form, and the modeler who prefers to work with a simpler mathematical object constructs the normal form.

the possible states, or considers only a coarse partition of the state space. By and large, however, the common lore of decision theory suggests that the "right" way to model a problem is by including all that is relevant in a description of a state.

2. The decision maker cares only about the outcome she experiences. Outcomes are therefore expected to include all that is relevant to the decision maker's well-being and choice considerations. This principle, too, has exceptions, often in the behavioral economics literature. For example, the context in which an outcome is experienced, the very fact that a choice has been made, and the counterfactual choices that could have been made could all be relevant to the experience of the decision maker, and could affect her choices. A prominent example is given by models involving regret, in which an outcome could result in different levels of well-being depending on the alternative outcomes that were possible but are not experienced. In a formally similar way, if the decision maker is affected by considerations of responsibility and guilt, the very act of choice can affect the resulting outcome. Being rich when others are poor can affect well-being differently if the decision maker chose this state of affairs than if she isn't responsible for it. Similarly, the decision maker might experience "warm glow" (Andreoni, 1990; Evren and Minardi, 2017) as a result of the very fact that she made a certain choice. Yet, the standard model of decision making is consequentialist, and is applicable if the modeler strives to define outcomes in such a way that they describe all that is relevant to the decision.

These guidelines for using the model of decision making under uncertainty provide neat distinctions between beliefs (about states), tastes (over outcomes), and choices (among acts). Beliefs are defined (only) over states: any relevant question about knowledge or belief boils down to a question about the likelihood of a subset of states (usually referred to as "events"). Tastes are defined (only) over outcomes: any question that has to do with well-being should be reflected in the desirability of outcomes. Finally, choices are made (only) among acts: the decision maker has no control over the states, and cannot choose outcomes directly; any choice of an outcome is mediated by a choice of an act.

The three-way distinction between states/outcomes/acts and, correspondingly, between beliefs/tastes/choices is highly insightful. While it has been discarded by some theories of decision (see Jeffrey, 1965), it has largely been adopted by the field, including its applications to economic theory, game theory, and related areas.[2] Moreover, in cases where real-life phenomena

[2]This distinction predates the formal development of decision theory. To a large extent,

required greater flexibility than these distinctions allow, the common approach, at least in theoretical work, is to re-define the basic concepts so as to retain the neat distinctions. For example, if a given outcome yields different levels of well-being when experienced at different states, the standard theoretical approach is to re-define the outcome so that it includes all aspects of the state that are relevant to well-being. Thus, when Aumann (1971) suggested that the desirability of an outcome "swimsuit" depends on it occurring on a rainy or sunny state of nature, Savage's (1971) response was that a "swimsuit" is not an outcome, and that an appropriately defined outcome could be "sun bathing in a swimsuit on a sunny day on the beach". Similarly, the notion of a "state" can and has often been re-defined, in such a way that all relevant uncertainty boils down to the uncertainty about the state that transpires.

Such re-definitions of primitives of a model are often very powerful, allowing a variety of seemingly different problems to be analyzed by the same principles. A similar approach has been adopted by game theory, which also offers a general method of analysis by identification of basic concepts. On top of the decision-theoretic concepts of states, outcomes, and acts/strategies, game theory also involves players, and this concept can also be re-defined when needed. For example, to cope with dynamic inconsistency it is common to define each agent of a player as a separate player. Even in general equilibrium theory, which is more structured than either game theory or decision theory, one has some freedom in defining a basic concept such as a "good". If the same product is sold at different locations at different prices, the standard theoretical approach is to re-define it in such a way that its location is part of its description. With this definition one finds that there may be many goods in the model, but each good can still have a single price, and the general modeling method can be applied.

At the same time, re-definitions of this nature carry the risk of rendering a theory vacuous. If, for example, at every node in the game a new player is defined, can we predict future play based on the utilities of players we observed in the past? Or, if every time and location define a new good, can we say something meaningful about the reaction of demand to price changes?

Such questions are clearly relevant also when one re-defines the basic concepts of decision theory. We find that they are particularly important in this context for two reasons. First, the notion of "states of nature" has undergone at least five major conceptual re-definitions since it was formally

it is already suggested in Pascal's analysis of the "Wager" (Pascal, 1670).

introduced. Second, decision theory is supposed to lay the foundations for concepts such as "utility" and "probability", and, as such, it takes special care in relying on observable, or at least presumably-observable data. However, the re-definitions of the concepts of states mentioned above make the notion of presumably-observable preferences questionable. The purpose of this note is to spell out the five re-definitions of "states", and to highlight the tension between them and the principle of observability. We hold that sometimes this theoretical tension is too difficult to ignore, and one has to make a choice: adhere to the general "conceptual framework" of decision theory, using states in their utmost generality while sacrificing the presumption of observability, or restrict attention to more limited models in which preferences can be assumed observable (in principle). We refer to the states of nature in the latter models as "contingencies". Thus, we suggest to retain the term "states" for models in which each element $s \in S$ is interpreted to resolve all uncertainty, and to use the term "contingencies" for models in which all preferences among acts in A can be in-principle observed. In many problems of interest the two interpretations are compatible, and the set S can simultaneously be viewed as a set of states and of contingencies. In such problems one can rest assured that all relevant sources of uncertainty are captured in the elements of S, as well as that one can cite axiomatizations that rely on observed preferences. However, in many other problems one has to choose between states or contingencies, that is, whether to insist on resolution of all uncertainty or on observability.

We hold that both practices are valid and useful. When using states, one reaps the benefits of clarity of thought and of exposition, and uses the decision theoretic model as a very powerful analytical tool. When employing contingencies, one can use axiomatic derivations such as Savage's (1954) to identify specific models, whether for normative or descriptive purposes. However, one should be careful and recall that it is not always the case that these theoretical benefits can be enjoyed in tandem.

The rest of this note is structured as follows. Section 2 discusses the five re-definitions of states in the literature, and illustrates how the generality of decision theory relies on the ability to specify states in sufficient detail as to capture all relevant uncertainty. Section 3 argues that in the process of specifying such states, one compromises – if not completely foregoes – the ability to observe all relevant preferences, even in principle. Section 4 concludes by highlighting the tradeoff that sometimes exists between using states and contingencies.

2 What is a State?

2.1 Resolving All Uncertainty

The notion of a "state of the nature" or "state of the world" has roots dating back to Leibniz's "possible worlds" and to the notion of an outcome of an experiment. States are standardly assumed to have three fundamental properties: they are mutually exclusive, exhaustive, and describe all relevant uncertainty. Hence, whatever transpires, there is precisely one realized state, and no two occurrences would be mapped to the same state unless they are indeed equivalent for all intents and purposes. These properties allow us to think of probability as a measure over states, as suggested by Borel (1924b) and Kolmogorov (1956). The state space is the object over which subjective probability is defined and calibrated by behavior, as in the works of de Finetti (1931, 1937) and Savage (1954).[3]

Our focus here is on the requirement that states resolve all uncertainty. The Bayesian approach reinforces the call for states that are sufficiently informative to describe anything that might ever be of interest. A Bayesian adopts a prior probability over the state space and updates it according to Bayes' rule. But Bayesian updating is a rather mechanical form of learning, which allows the generation of no new states and the formulation of no new theories. Zero probability events will never be assigned positive probabilities, and there is no mechanism that could come up with new states that were not included in the original model. A Bayesian reasoner had thus better employ all her powers of imagination when formulating the state space, since it is otherwise too late.[4]

[3] Anscombe and Aumann (1963) depart from this tradition by allowing for uncertainty that is not captured by the states: their objects of choice involve lotteries given states, and the outcome of the lottery is not described in the state. Indeed, they viewed this feature as a merit of their model, because they believe that, from a cognitive perspective, subjective probabilities are more natural when compared to objective ones than in the abstract (Aumann, personal communication on two occasions, a few decades apart). However, most researchers who employ the Anscombe-Aumann framework do so for mathematical convenience, while conceptually preferring a model with only one type of uncertainty, that can be described in terms of which state obtains.

[4] Luce and Raiffa (1989, pp. 301–302) emphasize the importance of formulating one's model: "In practice, however, one's choices for a series of problems—no matter how simple—usually are not consistent. [...] Once confronted with such inconsistencies, one should, so the argument goes, modify one's initial decisions in such a manner as to be consistent. Let us assume that this jockeying—making snap judgments, checking on their consistency, modifying them, again checking on consistency, etc.—leads ultimately to a bona fide a priori distribution. [...] then we are committed to a criterion which selects as optimal only acts that are best against this a priori distribution."

2.2 The Power of Resolution

A notion of a state complete enough to resolve all relevant uncertainty is not simply a matter of technical convenience, but brings tangible gains in terms of the usefulness of the model. A sufficiently detailed state space yields a more powerful language that in turn expands the scope of the theory and resolves confusion. The following examples illustrate.

1. Static, one-shot decision problems are analytically convenient, but uncomfortably limiting. Many decision problems involve sequences of decisions, interspersed with choices by Nature, to which static models seemingly do not apply.

   However, we can think of dynamic decision problems as games played between the decision maker and Nature. A state is a strategy for Nature, and a strategy for the decision maker induces an act. This allows us to collapse the decision problem into a single-stage problem. The construction is identical to the "normal form" representation of an "extensive form", and consists of defining strategies as functions from all decision nodes at which players might find themselves into the available moves at these nodes. The equivalence between the extensive and the normal forms of a decision tree (or a game) requires a variety of rationality assumptions: the decision maker is assumed to know the decision tree; to correctly predict her emotional reactions; to be dynamically consistent; and to be unfazed by the size of the decision matrix (which will typically be exponential in the size of the decision tree). However, this construction of the state space simplifies the decision problem conceptually, and leaves us with a canonical decision matrix.[5]

2. One of the most important examples of appropriately specifying the state space is Harsanyi's (1967, 1968) modeling of incomplete information games. Before his contribution, game theory was limited to cases in which players' utilities were known. Harsanyi's innovation can be summarized by saying, "Well, if utilities and/or beliefs are not known, let's put them in the description of the state." In a mental exercise that mirrors his own treatment of social choice problems (the exercise that Rawls (1971) later referred to as going "behind the veil of ignorance"), Harsanyi expanded the state space to describe anything

[5] Savage (1954, pp. 15–17) explains this construction, describing the choice of an act as a single choice of a strategy made, as it were, before the decision maker is born.

that *any* player might not know, including their own attributes. At the risk of being overly dramatic, one can say that this re-definition of the state space opened the door to the conquest of economic theory by strategic game theory.

3. The application of decision theory to game theory, and, in particular, to interactive epistemology, required that a state of nature—often referred to as "a state of the world" in this context—be rich enough to describe the uncertainty as viewed from the perspective of *each* of the players involved. Because each player considers other players' moves as a source of uncertainty, a state needs to describe the choices made by all players. Moreover, to reason about these, a state needs to describe what a player knows, and therefore also what all players know. Thus, states were upgraded to describe actions, information, information about others' information, and so on. The encapsulation of these hierarchies of information, or beliefs, in a single state space, pioneered by Aumann (1976), gave rise to powerful results. Among these are the demonstration that Bayesian rationality implies correlated equilibrium if players share a common prior (Aumann, 1987), and that Bayesian rationality gives rise to rationalizable play if their priors differ (Brandenburger and Dekel, 1987). It also raised fundamental problems, in the form of results that were (and are) considered counterintuitive, such as the Impossibility of Agreeing to Disagree, or unrealistic, such as no trade and no betting theorems. (See Aumann, 1976, Milgrom and Stokey, 1982, Sebenius and Geanakoplos 1983.)

4. Newcomb's paradox (see Nozick, 1969) challenges the rationality of choosing dominant strategies. While the original paradox involves the abstruse and metaphysical notion of an "omniscient predictor", the paradox has a rather mundane version: the decision maker is asked to choose either only the contents of an opaque box, or the contents of both the opaque and a transparent box. The decision maker can see that the transparent box contains \$1,000, and knows that the opaque may or may not contain \$1,000,000.

   The seemingly obvious representation of this problem has two states, intuitively described as "the opaque box is empty" and "the opaque box contains \$1,000,000". It then seems obvious that taking both boxes is a dominant strategy. The paradox is then created by supposing that many people have been faced with this choice in the past. All those who were greedy (chose both boxes) found that the opaque box was empty, whereas all the modest ones (who chose only the opaque) ended up millionaires. We are thus faced with the discomforting picture of a seemingly rational decision maker who is sorely tempted to choose a dominated strategy.

The simplest way out of this paradox is to reformulate the states more carefully (see Gibbard and Harper, 1978). The formulation of a decision problem implicitly assumes that one's choice of act has no influence over the states of the world, or over their probabilities. One can well imagine the decision maker facing Newcomb's boxes coming to believe that her choice of act somehow affects whether the opaque box is bristling with cash. We cannot simply inject this effect into the existing two-state model, since a "state" whose occurrence partly depends on the decision maker's act is not a state. However, we can instead think of a state as a function from acts to outcomes. This allows us to re-phrase Newcomb's problem with (at least) four states, describing the content of the opaque box *as a function of* the decision maker's choice.[6] In such a formulation, the dominance argument breaks down, as there exists a state in which the decision maker is rewarded for modesty and penalized for greediness. Moreover, the evidence cited suggests that this state of the world should have non-negligible probability.

Defining the states of the world to be all the functions from acts to outcomes (see Karni and Schmeidler, 1991, Karni, 2017) is akin to the definition of a state space as the semantic of a propositional logic model: in the latter, a state is a truth function, assigning "true" or "false" values, in a consistent way, to each proposition. In the context of decision making, the propositions of interest are "if I choose a the outcome is r". An assignment of truth values to all these, which is consistent with the principle that the outcomes are mutually exclusive and exhaustive, is equivalent to a function that assigns to each act a unique outcome.

5. Monty Hall's three-door riddle is, by now, well known and well understood (Gilboa, 2009 (p. 120–122), Rosenhouse, 2009). Yet, at the time it posed a puzzle, and even many mathematicians tended to give

[6]Formally, there are four possible outcomes and two acts, so that there should be 16 states in the problem. But to convey the main point one may retain the assumption that the decision maker knows the *possible* contents of each box (either \$0 or \$1,000,000 for the opaque box; only \$1,000 for the transparent box), allowing a reduction to four states.

the wrong answer. Clearly, the riddle is not one of mathematics, but of modeling.

People often argue that there are three states, corresponding to which door contains the coveted prize, with a prior probability $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ that makes each door equally likely. Once the middle door has been opened to reveal that it contains no prize, the prior probability is updated to $(\frac{1}{2}, 0, \frac{1}{2})$. This is a perfectly correct mathematical calculation, only it is conducted in the wrong model. In particular, the decision maker needs to model not only the location of the prize, but how she learns that one door does *not* contain the prize. To capture this, we can reformulate the notion of a state to identify which door contains the prize, and which door Monty opens. The probability that the prize lies behind each door remains $\frac{1}{3}$, but the states are not equally likely – Monty never opens the door chosen by the decision maker, never opens a door that contains the prize, and is equally likely to open either door not chosen by the decision maker if neither contains the prize. Once these regularities are built into the prior, the updating trivially leads to the conclusion that one's odds of winning the prize is $\frac{1}{3}$ if sticking with one's original door, but $\frac{2}{3}$ if switching. The key here is that the state of the world should include the underlying uncertainty, the information obtained, and also the process through which it was obtained.

The fact that, beyond the information one obtains, one may learn from who provided this information, what else they could have said, and so forth, is essential to information economics.[7] It is also relevant to the understanding of language, as illustrated by Grice's Principle (Grice, 1957).[8] Further, there are several apparent deviations from classical decision theory that can be explained by reformulating states to include information channels. A few examples are:

(a) A possible resolution of Ellsberg's "paradoxes", that is, a way to reconcile ambiguity non-neutrality with subjective expected utility theory, is to allow for the possibility that decision makers who participate in experiments are suspicious of the experimenters (Morris, 1997, Mukerji, 1997). In particular, the former

[7] This point was made by Roger Myerson, (personal communication, 1991) when discussing the importance of the Monty Hall problem for probability classes.

[8] Grice suggested that, in normal conversations, people tend to say the simplest statement that conveys the message they wish to get across, and if a simpler message has not been sent, the listener can infer that it is probably untrue.

might trust that the latter do not lie, but could conjecture that the composition of the "unknown urn" might be determined by the experimenters after the subjects make their choices, in such a way that minimizes the subjects' payoffs. Thus, the claim is that people who may seem to be susceptible to ambiguity in a model with "naive" states, may not be so in a more refined model, where the states describe not only the bet but also the way that the odds respond to their own choices in the experiment.

(b) Some phenomena that are considered to be "framing effects" (Tversky and Kahneman, 1981) may be explained by learning from the way information is gathered. To consider a simple example,[9] suppose that a patient has to make a decision about a medical procedure, which has only "success" and "failure" as possible outcomes. Assume that the patient tends to make a different decision if his doctor tells him that the chance of success is 97% than if she tells him the chance of failure is 3%. One may argue that the very fact that the doctor chose to highlight one of the two possibilities is informative in and of itself. In particular, getting the information on 3% probability of failure, the patient may ask himself, "Why didn't she say that success had 97% probability? Is she uncomfortable about something here?" We may embed the dialog in a richer model, in which the doctor has some uncertainty about the cited numbers, and the patient has second-order beliefs about the doctor's uncertainty, motives, etc., and different choices given different "frames" would be perfectly in line with standard economic theory.[10]

(c) Along similar lines, some violations of the weak axiom of revealed preference (or the "independence of irrelevant alternatives") can be explained if one considers the possibility that the choice set is informative. For example, the "compromise effect" (see Simonson, 1989) refers to the finding that alternative b may be chosen more often from the choice set $\{a, b, c\}$ than it is from the choice

[9] Ehud Kalai, 1990s, personal communication.

[10] This type of explanation appears rather plausible in one of the most powerful examples of framing effects, namely the choice of a default option in 401K forms (Madrian and Shea, 2001). If we assume that a typical agent considers herself to be similar to the majority, and believes that the form was designed in such a way that the modal choice is the default, choosing the default is akin to ignoring one's private signal and following the herd. As is well known from information cascade models, such behavior is perfectly compatible with subjective expected utility maximization.

set $\{a, b\}$, *if* b can be viewed as a compromise between a and c. Wernerfelt (1995) suggested that this may be a result of rational updating, assuming that the very existence of options on the menu says something about their popularity, and therefore also about their utility to a randomly chosen consumer. Thus, whereas in the presentation of the axiom in classrooms one may think of a choice set as a tool used to observe given preferences, this explanation suggests that uncertainty is involved, and that the states should describe the choice set itself.

These five classes of problems share some similarities, as do their solutions. For example, embedding a dynamic decision problem in its one-shot, "normal" form and the resolution of Newcomb's paradox are quite similar, as both consist of defining states as functions from possible alternatives to consequences. Nonetheless, we have at least five distinct (if not disjoint) classes of conceptual problems that have been resolved by making the state space sufficiently informative: (i) the static, one-shot decision problem model is rather limited; (ii) there may be uncertainty about the nature of a game, players' payoffs and beliefs; (iii) an agent's uncertainty may be intertwined with that of others with whom she interacts; (iv) acts might seem to causally affect states; (v) the very way that information is imparted might be informative. In all five cases, problems, puzzles, and paradoxes are resolved simply and elegantly by appropriately formulating the states. When the states describe *everything* that can be of interest – be it Nature's strategy, the parameters of the game, the decisions of others, causal relationships, the information protocol – conundrums disappear.

3 What is Observable?

This section suggests that the power of the suitably enlarged state spaces described in Section 2 is purchased at a cost, beyond the obvious one of making the model more cumbersome. Defining, or at least imagining a large state space is one thing; assuming that preferences between acts defined on it are observable is quite another.

3.1 Presumably-Observable Beliefs

If the set of consequences X consists of monetary outcomes and we are comfortable assuming the decision maker is risk neutral, then observing preferences is equivalent to observing beliefs. In some cases, we might argue that beliefs can be objectively specified. We are often comfortable asserting that the probability a fair coin produces a head is 1/2, or that a fair die produces each of its sides with probability 1/6. Such conventions are useful for thought experiments, but leave open the gnawing suspicion that it is not obvious how we are to ascertain that the coin or die is fair. A standard response is to take a frequentist approach, appealing to previous tosses of the coin or die, and to use the relative frequencies of these tosses (at least if one has enough of them) as an estimate of the relative probabilities. This type of reasoning expands the ability to form beliefs to i.i.d. trials in general, including many for which there is no a priori idea of what "fair" means. However, it is still uncomfortable to restrict decision theory to cases in which one has an arbitrarily large record of past draws.

Starting with Borel (1924a,b), Ramsey (1926a,b), and de Finetti (1931, 1937), and culminating with Savage (1954) and Anscombe-Aumann (1963), the Bayesian approach relied on the idea that subjective probabilities can be defined and measured by betting behavior. The intuitive dictum, "Put your money where your mouth is" became a principle of measurement. The idea is that a decision maker, when choosing a lottery that pays 1 if state θ occurs over a lottery that pays 1 if state θ' has occurred, has just revealed that she regards state θ as more likely. This principle is backed by compelling intuition. If Sarah insists that Horse 1 is more likely to win the race, but then bets thousands of dollars on Horse 2, we might well suspect that her "true", or at least more relevant subjective probability puts higher weight on Horse 2 winning than on Horse 1.

Given the ability to assess such bets, we say that beliefs are *presumably observable*. But having come this far, we can note that nothing now depends on the presumption that X consists of monetary outcomes, or that the agent is risk neutral. Instead, following the lead of Savage (1954), the same approach allows us to suggest that *preferences* are presumably observable. The next two subsections consider what it means to be "presumably-observable."

3.2 Observability

When saying that an event or magnitude is observable, it is typically assumed that the event or magnitude can be measured, and, furthermore, that the measurement can be repeated. It is further assumed, at least implicitly, that the measurement could be taken by different scientists, who would reach similar conclusions. For example, if we think of a physical quantity $\mu \in \mathbb{R}$, a measurement thereof would be a real-valued random variable X_t, ideally with $E(X_t) = \mu$, and it would be natural to assume that different measurements generate a sequence of $(X_t)_t$ that are i.i.d. given μ. As μ is not known, a Bayesian approach would suggest that the measurements $(X_t)_t$ be exchangeable. In any event—taking a Bayesian or a Classical approach—one can test some implications of the conditional-i.i.d. assumption. For instance, statistics that are computed based on long sequences should have similar distributions, and if we can compute their averages, the resulting values should be similar.

Notice that this notion of observability implicitly assumes that the phenomenon to be measured has a certain degree of *stability* and *causal independence*: it has to be sufficiently stable so that it would not change from one measurement to another, or from the measurement of one attribute to another, and it has to be causally independent of the process of measurement.

3.3 Presumably-Observable Preferences

If preferences are to be presumably observable, then for any choice there exist repeated measures of that choice that are conditionally i.i.d. (or exchangeable).[11] In some cases, we might think of these repeated measures as reflecting repeated choices, whereas in other cases we might interpret the repeated measures as reflecting multiple observers of a single choice.

For example, consider the revealed-preference foundation for utility maximization over a finite set of consumption bundles, X. One assumes that a binary relation over X can be observed, and proves that this relation can be represented by maximization of a numerical function if and only if it is complete and transitive. The assumption that the preference relation between $x, y \in X$ is observable means that there are random variables $xy_t \in \{0,1\}$, for $t \geq 1$, where $xy_t = 1$ is interpreted as "at measurement t, x was found to be at least as desirable as y"(typically, because x and y were both feasible, but x was chosen), and that these variables are conditionally i.i.d. Note that this assumption implicitly requires that the preference one presumably measures would not change from one t to another – neither exogenously, say, because of the passage of time, nor endogenously, say, because of habit formation.[12]

[11] In some cases, such as the notion of "qualitative probability", the observable ranking might be interpreted as a cognitive construct, "at least as likely as". Our discussion applies to these as well as to choice data. Gul and Pesendorfer (2008) argue that choices are the only conceivable economic data.

[12] A more elaborate model would allow the study of changes in preferences due to considerations such as habit formation. An observation would then specify the preferences before and after the change in preference, while presuming that such an observation could

The observability assumption is an empirical assertion about the data that are available. To see this more clearly, it is useful to think of axiomatizations as second-order scientific theories, along the following lines. Let there be an external reality, consisting of people, households, or organizations making decisions. Let there be a working economist, WE, who models these decisions. We can think of the economist as examining the data and describing it by a model. For example, WE may consider a household choosing between pairs of bundles, and attempt to estimate a utility function that would explain the household's past choices and help predict future ones. There is, however, also a "second-order-" or "meta-" theorist, Th, who models WE's work. Th observes WE, and asks, when can he (WE) assume that a utility function exists? Suppose that Th proves the theorem mentioned above, stating that any complete and transitive binary relation can be represented by utility maximization. The theorem is a mathematical result, showing the identity of two sets of binary relations. Binary relations and sets thereof are, obviously, mathematical constructs. In the present context they are part of a model that is designed by Th to describe reality. The reality that Th considers, however, isn't the household's choices, but the scientific work that WE performs in trying to describe these choices. That is, Th considers WE, who collects data and writes models. She (Th) then writes down a formal model, describing WE's work, by defining two classes of binary relations. One binary relation refers to the choice data that WE gathers, and the other refers to the preferences arising out of the utility maximization WE uses in his paper. The theorem that Th proves allows her to say when WE's attempt to describe the household's behavior via utility maximization might be successful. For WE, there is one formal model, that of utility maximization; for Th there are two—WE's model of utility maximization, as well as another model to describe WE's data (by a binary relation).[13]

Clearly, Th's work as a social scientist should be judged not only for its mathematical beauty or depth, but also for its relevance. For example, assume that another theorist, Th′, comes along and says to Th, "Your theorem is really nice, and even correct. But I have to tell you, as a model of reality it's rather poor. I have talked to many empirical economists, and none of them seems to think that a binary relation can be observed in reality. Maybe it can be in experiments. But with empirical data—not really.

be repeated in a conditional-i.i.d. (or exchangeable) manner.

[13]If we liken WE to an artist, who represents reality by a painting, Th is akin to a painter who represents another painter in the act of painting. Th's canvas might have a representation of the canvas that WE draws on, as well as of the reality WE observes.

What's observable is, at best, a choice of a bundle out of a budget set. That is, given prices p, WE can observe a chosen bundle x. His resulting database is, therefore, of the type $\{(p_i, x_i)\}_{i \leq n}$—but no choice among finite subsets of bundles can be assumed observable." Thus, Th$'$ might suggest that Th abandon her analysis of WE's work in favor of an alternative model based on Afriat's Theorem (Afriat, 1967), which characterizes databases $\{(p_i, x_i)\}_{i \leq n}$ that can be rationalized by utility maximization.

From the perspective of Th$'$, Th was using a model that was too highly idealized. However, just as working economists may make idealized assumptions that can be misleading—say, ignoring transaction costs in a particular environment—so might theorists make excessively idealized assumptions about observability, allowing Th to defend her model as usefully sacrificing some realism in order to gain tractability. Moreover, just as a mathematical model of a working economist might be a reasonable approximation of reality in some contexts but not in others, so can the assumptions of observability in the case of the theorist's work. For example, binary relations might well be observable in experiments, but not in empirical work. However every model of decision theory that is committed to the principle of presumably-observable beliefs makes some assumptions about observability.

3.4 A Conflict of Principles

Making states sufficiently detailed as to capture all relevant uncertainty can be at odds with the argument that preferences are at least in principle observable. To begin with, as the state space becomes larger, there are more parameters to be assessed in order to describe preferences. Worse still, often the sources of information on which one could have drawn in order to define frequentist beliefs are no longer available, as they are described in the states. As noted in the preceding subsection, if a decision problem involving some state space Ω is encountered repeatedly, the relative frequencies of the states in past problems can serve as a prior probability measure for the next one. But if one wishes to allow for processes that are not i.i.d. – for example, temporal changes in the underlying distribution, or causal dependency between the realizations in different time periods – one needs to consider a larger state space, such as Ω^∞, describing *sequences* of realizations in Ω. The problem of generating beliefs over this state space is two-fold: first, Ω^∞ is clearly much larger than Ω, demanding many more parameters to be estimated or guessed. Second, the history on which one wished to rely in forming beliefs is no longer there to help. While a current replica of Ω may have been encountered in the past, the entire product Ω^∞ is encountered only once. Historical data have turned against us, so to speak: from being part of the solution they switched into part of the problem.

In each of the examples considered in Section 2.2, the specification of the state space so that it captures all relevant uncertainty requires that we form beliefs not only about more states than those we encounter in a naive state space, but also that we form beliefs about new types of information: probabilities of counterfactual statements, joint distributions over the types of players in the game, others' actions, possible causal relationships, and various conceivable regimes of information dissemination. In each case, the proposition that beliefs are at least in principle observable is called into question:

1. The construction of states of the world as Nature's strategies implies that in many problems a single state cannot be observed even in principle. For example, assume that the decision maker can choose a or b, and, as a result, obtain the outcome 0 or 1. Thus the game against Nature has two stages, and the decision tree has four leaves. In the normal form of the game, or the decision matrix, there will be four states of the world. Each state should specify what would be the outcome of act a as well as act b, *even though only one* of these acts will actually be chosen. Thus, a state can be thought of as a list of conditional statements. The antecedent of exactly one of these conditional statements materializes in any given instance, and the rest remain counterfactual.[14] Importantly, the decision maker's choice also determines her information partition, and she will never get to observe an isolated state. As a result, there are functions defined on the states that cannot be compared in this problem. The problem is attenuated if one can think of it as repeated under more or less the same conditions, as in the case of drawing balls from urns. But when the decision maker has to choose her life path, or a country has to decide whether to go to war, the counterfactual statements needed to define states are hard to assess for their truth values. And the assumption that the decision maker's beliefs can be gleaned from observed bets on states or choices between pairs of acts becomes doubtful.

2. If we take the Harsanyi-Rawls suggestion of considering the uncertainty we faced before we were born seriously, we are led to discuss

[14] We can denote the states by $s_{00}, s_{01}, s_{10}, s_{11}$, where s_{ij} denotes outcome i from act a and outcome j from act b. Note that if the decision maker chooses a she will have the partition $\{\{s_{00}, s_{01}\}, \{s_{10}, s_{11}\}\}$ and if she chooses b – the partition $\{\{s_{00}, s_{10}\}, \{s_{01}, s_{11}\}\}$. She will never observe a single state.

the observed preferences of the decision maker's self before she came into being. But such selves do not make decisions. Clearly, there are many applications of Harsanyi's model that need not take such a literal interpretation of prior beliefs before birth. If a worker might be of a "high" or a "low" type, or a firm can have high or low production costs, they can easily use the population statistics as the prior over their own type, and these beliefs would be reasonable beliefs for others to have over their type. These types of applications typically assume, implicitly, that the entire problem has been repeated in the past, so that empirical frequencies in the past may serve as prior beliefs for the current instantiation.[15] However, if a problem is not repeated in sufficiently similar ways, either because of its unique character or because the entire history is part of the definition of the problem, going "behind the veil of ignorance" is a philosophical exercise that stands at odds with the notion of revealed preference.

3. Demanding that a state resolve all uncertainty for several players simultaneously raises questions about the observability of choices among bets on such states. For example, if a game has two players, and we wish to derive the probability that a player has about the state space, we should first decompose a state into the player's own act, and the "state" as she perceives it, that is, resolving all uncertainty that is external to her. We should then imagine that she is offered bets on such states, depending on the other player's choices. The same exercise should be conducted for the other player. It is not obvious how one can elicit such belief from betting behavior, even in principle. (See Samet and Schmeidler, 2018.) Further, the elicited prior beliefs would be on different spaces that then need to be "patched" together.[16]

4. When we consider all possible causal relationships between acts and outcomes, we again undermine the presumably-observable basis of subjective probabilities. The problem (pointed out in Gilboa and Schmeidler, 1995)[17] is the following: start with the set of acts that the decision maker can choose among, A, and the set of conceivable consequences, X. We are supposed to define the state space as all possible functions from the former into the latter: $S = X^A$. Then, Savage's construction would require a complete preference relation (satisfying P1-P7) on *all*

[15]This can also be the case when a prior is used in Bayesian statistics.

[16]See Samet (1998) and Feinberg (2000) on results that allow the different priors to be derivable from a single prior probability of an external observer.

[17]See also Gilboa (2009), Gilboa, Postlewaite, and Schmeidler (2012).

"conceivable acts", which are all the functions from S to X. That is, while we can only observe choices between pairs in A, we are asked to imagine choices between all pairs in $F = X^S = X^{(X^A)}$, a set that is larger than A by two orders of magnitude.

We might respond by either enriching the set of acts available to the decision maker, or reducing the set of states. Both approaches are problematic. The obvious expanded set of acts is F. But if we take F as our starting point, with no means of limiting the set of states, then we simply put this construction mechanism in motion again, giving us the state space X^F and the set of conceivable acts $X^{(X^F)}$, giving an even greater mismatch. We might instead argue that there are some states we can a priori eliminate from consideration. When discussing Newcomb's paradox in Section 2.2, for example, we took it for granted that only four of the sixteen possible states need be considered. But we are now assuming that we have a priori knowledge of the very beliefs we hope to infer from behavior. This may sometimes be appropriate, but appears to be a rather fragile foundation for decision theory.[18]

5. The fact that information might be part of the state space suggests that the measurement of beliefs might also be intertwined with the beliefs measured. Whatever is our measurement device, such as a lab experiment, it is possible that the decision makers involved suspect that there are some causal relationships between their behavior in the experiment and the outcome they would experience in the future. As suggested by the explanation of ambiguity aversion in Ellsberg's experiment in Section 2.2, it is possible that, when participating in an experiment, people think of it as a game involving the experimenters. Indeed, rationality seems to suggest that they should. To push this idea to a ridiculous extreme, one may explain any observed behavior as maximization of subjective expected utility: all one needs to do is to include a state at which, should the decision maker behave differently than she actually has, she would be kidnapped by malicious aliens. Clearly, the alien threat does not only allow to explain away any anomaly; it also renders prediction impossible, as behavior that was observed under the threat would not be indicative of behavior

[18]Schipper (2016) similarly notes that there is a tension between incorporating information in states and the ability to elicit beliefs about these states from choice behavior. He considers restrictions on the set of states and on the set of acts as possible responses. He recognizes the weakness of either approach, in that it undermines the ability to elicit beliefs from choice behavior.

when the aliens are no longer there. But even when limiting attention to more reasonable conjectures about the outcome of the experiment, the elicitation of beliefs by betting behavior is problematic when the elicitation procedure might be causally intertwined with the beliefs to be elicited.

To conclude, larger state spaces increase the descriptive power of the model, and they can resolve a variety of unpleasant paradoxes. But this enhanced theoretical power comes at a cost. From a psychological point of the view, the task of forming beliefs over the state space becomes more demanding, often attempting to estimate more parameters based on less information. From a philosophical viewpoint, the presumably-observed behavioral foundations of subjective probability become shaky, at times to such a degree that the exercise does not seem to be coherent even "in principle".

We close this section with a casual observation on the sociology of the field. Decision and game theory are more "conceptual frameworks" than specific theories. Even when we couple the framework with a solution concept (such as expected utility maximization, or Nash equilibrium), there is a considerable degree of freedom in defining "players", "states", "acts", and so forth. This freedom is very useful, and it is responsible for many interesting and insightful analogies. But it is also responsible for conceptual pitfalls. For example, we may explain Savage's model using balls drawn from urns, or perhaps an upcoming election. The states are quite naturally defined in these problems, and we derive the notion of a prior belief. Then we move on to incomplete information games, and assume that the unborn players have a prior over their subsequent identities, "as in Savage". Similarly, facing a causal paradox such as Newcomb's, we may re-define the states to deal with it, but, facing the next problem, go back to a simpler definition of the state space. Our claim above is not that Savage's derivation of prior probabilities is logically flawed. The claim is that a construction that makes sense in the classroom examples of the state space may be quite shaky in many of the applications in which economic theory resorts to Savage's model.

4 Contingencies

The examples presented in Section 2 show that it is often tremendously useful to specify states as resolving all uncertainty. However, Section 3 shows that in doing so, we typically lose the ability to observe preferences, even in principle. It is still helpful to work with such states, as a way of organizing one's thoughts and intuition about the problem at hand. One may even assume that the agents being modeled are Bayesian (or may choose to be Bayesian oneself). However, one cannot cite Savage as a justification of such Bayesianism – the preference relation that forms the point of departure for Savage's has drifted into obscurity.

This discussion leads us to suggest the following convention: a set S in a decision theoretic model is referred to as a set of *contingencies* for a decision maker relative to a set of outcomes X if there exists a complete binary relation $\succsim$ such that, for any $f, g \in X^S$, one can in principle observe a sequence of random variables $fg_t \in \{0, 1\}$ that are conditionally i.i.d. That is, we propose to use the term "contingencies" as follows: when presenting a decision model, and referring to S as a set of contingencies, the economist makes an implicit claim about observability of choice. In such cases, one can ask whether certain axioms (such as Savage's) hold or not. However, this question might be meaningless if the set S consists of states that are not necessarily contingencies. In such applications one has to be careful about citing axiomatic systems that presuppose observability of all choices.

To consider a prominent example, let us revisit Savage's model. There is a set of states S, a set of outcomes X, and a set of acts, which are functions from states to outcomes, $A \subset X^S$. A preference relation $\succsim$ is defined on A. One way to obtain such a model is – as described by Savage (1954) – to begin with S and X as primitive sets, and to define A as all the functions from the former to the latter, that is $A \equiv X^S$.[19] If a complete relation $\succsim$ on A is presumably-observable, we can then say that the original set S consists of contingencies. In accordance with our definition, contingencies are elements of a set S with respect to which preferences between any two acts can be deemed observable. Claiming that a given set S is a set of contingencies is an implicit claim about what type of data the working economists can gather.

Another way to define the decision theory model is to start with primitive propositions of interest, including two primitive sets A and X, and to generate the state space by all the consistent truth value assignments to these propositions. In particular, a state in such a model should specify whether "act a results in outcome x" is correct for any $a \in A$ and any $x \in X$, so that the states are at least as rich as all the functions from A to X. The state might have to specify additional information, for example, whether and how preferences depend on the act of measurement. Saying that a set S is a set

[19]This is literally true in Savage's model, but if one selects a subset of acts that are measurable relative to an algebra that is smaller than 2^S the construction is conceptually identical.

of *states* makes an implicit claim about what is of interest in the problem, namely, that what is not specified by states in S is immaterial.

Thus, both the claim that S is a set of contingencies and that it is a set of states are informal claims about the work of an economist who uses the set to model a problem. The first is a claim about observable preferences, the second is a claim about the appropriateness of the model. It is certainly possible that a set S be both a set of states and of contingencies. For example, a small investor in the stock market can consider all possible values of financial assets, and if we define them as the set S we can make a reasonable claim that (i) all that is relevant to the investor is described by any $s \in S$; (ii) we can, in principle, attach any monetary outcome to any state s, generating acts, and observe the investor's choices between such acts. In this example the working economist can be praised for successful modeling, capturing all that might matter in a way that it is presumably-observable. However, in many situations a tension is generated between the two desiderata, and the economist has to choose between allowing enriching the state space, sacrificing observability in the hope of resolving all relevant uncertainty, or vice versa. The five classes of examples given above are viewed as instances in which the standard lore of modeling opted for states that are not contingencies. Clearly, such states provide invaluable theoretical power, but they undermine the claim of observability of preferences, and, with it, the foundations of Bayesianism.

In the exchange between Aumann and Savage (Aumann, 1971; Savage 1971) regarding axioms P3 and P4, Aumann attacked the implicit assumption of state-independent utility, providing as examples the outcomes "swimsuit" and "umbrella", whose utilities change as a function of the weather (described by the state of the world). Savage replied that these goods aren't outcomes, and that an outcome should specify all aspects relevant to the individual's well-being, say, "lying on the beach on a sunny day" or "running for shelter half naked in the rain". He was fully aware of the fact that his model required one to imagine acts that assign an outcome such as "lying on the beach on a sunny day" to the state "Rain". He viewed his own model as asking the reader for some indulgence in imagining that the decision maker had preferences over all such acts and that these preferences satisfied P1-P7.

We would re-phrase Savage's answer by using the term "contingency". We can view rain and sun as contingencies, and elicit beliefs about these events. The exercise is relatively simple if the outcomes, X, are only monetary. In this case, Savage's axioms might even hold.[20] However, as pointed

[20]Obviously, for P6 to hold one would have to enrich the set of contingencies and think

out by Aumann, if X includes the outcomes "swimsuit" and "umbrella", while S remains a set of contingencies, P3 and P4 are violated. As a way to retain the basic structure, Savage suggested to redefine the set of outcomes, to obtain X', such that "swimsuit" and "umbrella" are functions from S to X'. However, S is no longer a set of contingencies relative to X': since *all* functions from states to outcomes are to be considered, the model deals with a complete relation $\succsim$ over $(X')^S$. Such a relation can be imagined, and some decision makers might find it very useful in order to organize their thinking about the problem. But one cannot argue that it is observable. The claim that one can observe repeated choices between acts that involve "lying on the beach on a sunny day" at the state "rain" is hardly tenable.

The definition of contingencies is even more problematic when the states of the world involve uncertainty about one's life or health (physical or psychological). In the example above, one would be hard-pressed to find a database with actual choices between all the acts $(X')^S$, but such hypothetical choices could still be imagined by the decision maker. By contrast, when part of the uncertainty has to do with one's mental or physical ability to experience outcomes, even these hypothetical choices become a stretch of imagination for most people. Savage was aware of these limitations. Ridiculing his own mental exercise, he wrote, "... I should not mind being hung so long as it be done without damage to my health or reputation" (Savage, 1971, p. 307). Being hanged is clearly not a contingency. The assumption that a working economist can gather data about the decision maker's preferences over acts attaching any imaginable outcome to the event in which he is hung is plainly not true. Moreover, such acts become so difficult to imagine, that a complete binary relation over them might even cease to be useful as a way of organizing one's thoughts.

Note that what is considered to be an acceptable idealization of contingencies may depend on the decision maker in question. For example, suppose we wish to elicit the decision maker's beliefs about a financial crisis. If we are dealing with a small investor, we can imagine offering her a collection of financial assets that would match practically any financial outcome to any contingency. In particular, we can offer her a bet that would make her very rich if the DJIA loses 50% of its value. By contrast, if we are considering a very rich decision maker, who owns large shares in many DJIA companies, such an exercise would become harder to execute for us in a lab, and even hard for the decision maker to imagine. Along similar lines, we can ask a resident of Country A to consider various bets on a military coup in Country

of "rain" and "sun" as events in this space.

B, but it may not make sense to assume that the choices of Country A's leader between such bets are observable. In other words, we can think of Savage being hanged as a contingency for us, but probably not for him.

5 Conclusion

If we define states as describing all relevant uncertainty—including all possible causal relationships between acts and states, all possible causal relationships between the measurement procedure and the measured phenomenon, and also the history that preceded the decision maker's birth—then there is little hope for arguing that we can observe beliefs over such states. In response, we propose applying axiomatic decision theories to cases in which states can be viewed as contingencies. We can then at least imagine constructing decision problems that will reveal preferences over actions, allowing us to attach meaningful interpretations to beliefs and preferences. By contrast, in problems involving state-outcome pairs that cannot plausibly appear in any real decision problem, one should be careful in citing Savage as a foundation for Bayesianism. We may still enjoy the beauty and simplicity of this approach, but we cannot refer to it as the only coherent way of decision making.

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