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Learning in the Marriage Market: The Economics of Dating*

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Abstract

We develop a nontransferable utility model of matching with search and learning frictions. Agents search for a partner and, upon meeting, may *date* – i.e., gradually learn about their compatibility – rather than decide immediately whether to marry. Dating introduces a tradeoff between learning about a specific potential partner and searching for more promising ones. In (steady-state) equilibrium, agents date for longer than is socially optimal and block-segregation fails. Nevertheless, complementarity between agent types is sufficient for a *probabilistic* form of positive assortative matching. Finally, we show that recent technological advances in matching markets reduce agents' investment in dating each potential partner.

Keywords: Search and Matching, Learning, Dating, Probabilistic Sorting.

1 Introduction

Choosing the right partner – arguably one of the most important decisions in one's life – typically involves a great deal of uncertainty: no matter how promising a potential partner may appear, it is often difficult to immediately assess whether s/he will indeed make a suitable partner. As a result, before committing to a serious relationship, prospective couples often attempt to reduce such uncertainty by *dating*, that is, by spending time getting to know one

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another and learning about each other's suitability as a partner. If dating is, at least to some extent, exclusive, dating decisions – whom to begin dating and when to separate – reflect a tradeoff between acquiring more information about a prospective partner's suitability and searching for more promising potential partners.

This paper develops a nontransferable utility model of matching with both search *and learning* frictions in which – in a departure from the existing literature on matching with search frictions[1] – potential partners need not immediately (and irreversibly) decide whether to accept or reject their match, but instead may date in order to gradually learn about its merits. Introducing dating into the classic search and matching marriage-market paradigm opens the door to new questions, such as the (in)efficiency of equilibrium dating and marriage decisions, and whether or not – and in what sense – dating gives rise to assortative matching. The model also allows us to study how dating and marriage patterns are affected by recent advances in search and learning technologies (e.g., the introduction of dating applications such as Tinder, Bumble, and Hinge), which have drastically altered the dating market in recent years by facilitating dating and thickening traditional matching markets. For example, the model can be used to study how changes in search and/or learning frictions affect the amount of time agents invest in each dating partner, and the number of partners individuals date before marrying.

In our model, every pair of potential partners are either compatible as a couple or not: the agents derive positive utility from marrying compatible partners, and incur a cost if they marry incompatible ones. As in the classic matching-with-frictions framework, individuals are characterized by a single characteristic that, following Burdett and Coles (1997), we refer to as their *pizzazz*. Unlike in Burdett and Coles (1997), in our model, the higher an agent's pizzazz, the larger the share of singles on the other side of the market with which the agent is compatible.[2] That is, the probability that a couple is compatible is increasing in the pizzazz of both agents.

The agents in our model engage in a time-consuming and random search for partners. When a pair of potential partners meet, they immediately observe each other's pizzazz. However, while pizzazz is observable, actual compatibility is not. The main novelty in our model is that when a pair of agents meet, they may date for a while and learn about their compatibility before deciding whether to marry or separate. Dating is exclusive and requires mutual consent; i.e., it may be broken off unilaterally by either one of the partners, at any

[1] See Chade, Eeckhout and Smith (2017) for a comprehensive review of this literature.

[2] In Burdett and Coles (1997) there is no uncertainty and pizzazz reflects actual payoffs: when a woman with pizzazz x marries a man with pizzazz y, the woman (resp., man) obtains a payoff of y (resp., x).

time. Borrowing the now canonical learning technology introduced in Keller, Rady and Cripps (2005) in the context of strategic experimentation, we assume that while a pair of compatible agents date they *click* at a rate λ, whereas incompatible couples never click. Thus, once a couple clicks, they infer that they are compatible, and marry. On the other hand, the longer a couple dates without clicking, the more pessimistic they become about their compatibility until, at some point in time, one of them decides to break up and both agents return to the singles market to search for new partners.[3]

We analyze the steady-state equilibrium of the model. In a steady-state equilibrium, agents' dating decisions – whether and for how long to date each potential partner – are optimal, and the induced flows of agents between the single, dating, and married populations are balanced. Thus, both the size and the composition of the population of singles are endogenous.

We start by analyzing a simple version of the model in which all couples are equally likely to be compatible; i.e., all agents have the same pizzazz. This symmetry allows us to obtain a closed form solution for the (unique) steady-state equilibrium and to highlight certain features of our model and, in particular, the agents' dating decisions. First, we derive comparative statics with respect to the speed of search and the speed of learning and show that these two rates have an opposite effect on dating patterns, such as the expected number of people one dates before marriage and the expected time spent dating each potential partner. Second, we show that, in equilibrium, agents date for longer than is socially optimal. Finally, motivated by the wide range of social norms governing the (a)symmetry of dating costs between men and women, we show that a small amount of asymmetry in dating costs alleviates such over-dating and is beneficial to social welfare.

We then study the implications of dating on sorting patterns. To do so, we revert to the general version of our model in which agents differ in their pizzazz; that is, they are vertically heterogeneous. The existence of a steady-state equilibrium in this setting is not immediate. Following Shimer and Smith (2000), we establish existence in the value function space. While existing proof methods are tailored to the binary nature of agents' decisions – whether to accept or reject each match – in our model agents make richer decisions, choosing whether and for how long to date their potential partners. Our proof methodology must therefore differ from existing ones. Another challenge stems from the fact that, in our model, agents transition between three different states – single, dating, and married – rather than two. Furthermore, as noted in Smith (2006), a critical distinction between the existence

[3]We discuss alternative learning technologies after presenting the model in Section 2.

proof methodologies of transferable-utility and nontransferable-utility search models lies in the discontinuity of the value functions. In our model, while utility is nontransferable, value functions are in fact continuous. In this sense, agents' continuous choice of dating times replaces the role of continuous surplus division in smoothing the value functions.

In equilibrium, the time cost of dating is higher for high-pizzazz agents, who are therefore more selective. In particular, we show that an agent with pizzazz x will date a potential partner until either a click occurs or the agent's belief about the couple's compatibility falls below a *breakup threshold* that is strictly increasing in x. Thus, the higher an agent's pizzazz is, the more agents there are who are willing to date her/him. This behavior has implications for equilibrium sorting patterns.

We begin our investigation of sorting patterns by establishing that the classic block segregation result (see, e.g., McNamara and Collins, 1990; Burdett and Coles, 1997; Eeckhout, 1999; Bloch and Ryder, 2000; Chade, 2001; Smith, 2006) – whereby agents are partitioned into classes and marry only agents who belong to the same class as themselves – does not hold in our setting. To see why block segregation fails, note that while in the existing literature equilibrium matching depends on agents' accept/reject decisions, in our setting it is important not only whom agents are willing to date, but also for how long they are willing to do so. If agents were indeed partitioned into classes – with agents willing to date only within their class – different agents within the same class would be willing to date one another for different amounts of time. Hence, the probabilities with which meetings lead to marriage would not need to be identical across agents within a class. In fact, for agents at the bottom of a class, the probability with which dating leads to marriage would be so small that such agents would be better off dating agents from lower classes.

The probabilistic nature of marriage in our setting gives rise to a new notion of assortative matching, which we refer to as *single-crossing in marriage probabilities*. This property is satisfied if, roughly speaking, for any two agents with pizzazz x and $x' > x$, there exists a critical pizzazz level $y^\star$ such that the agent with pizzazz x' has a higher probability of marrying agents with pizzazz $y > y^\star$ and the agent with pizzazz x has a higher probability of marrying agents with pizzazz $y < y^\star$. In particular, this property implies that high-pizzazz agents are more likely to marry other high-pizzazz agents, but (as in reality) on occasion may marry low-pizzazz agents.

We show that, in equilibrium, the single-crossing property is satisfied if the probability that two agents are compatible is supermodular in their pizzazz. Note that this probability can be interpreted as the production function of a match in the canonical model without dating, with payoffs interpreted as reflecting the expected value of a match. Thus, while in a model with time-consuming search but without dating, supermodularity is insufficient for positive assortative matching in equilibrium (Smith, 2006), in our model, it is indeed sufficient.

In our model, positive assortative matching need not be efficient even in the presence of complementarities. We show that even if the probability that agents x and y are compatible is supermodular (for example, xy), the social optimum may require negative assortative matching. Roughly speaking, a social planner would like to reduce the number of partners agents date before marriage in order to reduce the average time it takes them to marry. The number of partners that an agent dates before marrying is inversely related to the probability with which dating leads to marriage, which creates an incentive for a social planner to induce similar marriage probabilities for all potential partners. Since a couple of high-pizzazz agents are more likely to be compatible with one another than a couple of low-pizzazz agents, equating the probabilities with which dating leads to marriage may require negative sorting.

Finally, we show that the search and learning frictions have important – but distinct – effects on sorting. As search frictions vanish, agents date only agents of their own pizzazz. By contrast, as learning frictions vanish, agents are willing to date all potential partners. Thus, learning frictions and search frictions have opposite effects on the resolution of the tradeoff between dating and finding a more promising partner. This suggests that, on the one hand, advances in search technologies that facilitate meeting new potential partners (e.g., dating apps) could make singles more picky and less willing to invest in each given potential partner. On the other hand, social norms that facilitate more informative dating make partners more willing to invest time in dating, even when a potential partner is less promising at first sight.

Related literature

The paper contributes to the matching-with-search-frictions literature by introducing a model in which, before committing to a potential match, agents may acquire information about its prospects. This literature explores the properties of equilibrium matching under various assumptions on the search technology, match payoffs, search costs, the ability to transfer utility, and the agents' rationality. Closely related papers include McNamara and Collins (1990), Smith (1992), Bergstrom and Bagnoli (1993), Morgan (1996), Burdett and Coles (1997), Eeckhout (1999), Bloch and Ryder (2000), Shimer and Smith (2000), Chade (2001, 2006), Adachi (2003), Atakan (2006), Smith (2006), Lauermann and Nöldeke (2014), Bonneton and Sandmann (2022, 2021), Coles and Francesconi (2019), Lauermann, Nöldeke and Tröger

(2020), and Antler and Bachi (2022). To our knowledge, existing models in this literature preclude dating – typically by assuming that agents decide whether to marry *immediately* (and irreversibly) upon meeting.

With the exception of Chade (2006), the above papers also assume that when an agent meets a potential partner, s/he is immediately fully informed about the merits of a match between them. Chade incorporates information frictions into this framework by assuming that agents receive only a noisy signal about the payoffs from marrying a potential partner before making an irreversible decision whether or not to marry that partner. This leads to an *acceptance curse*: the merits of a marriage with a partner, conditional on the latter agreeing to the marriage, are lower than the unconditional merits of such a marriage.

Information frictions have been incorporated into search-and-matching models in the related context of the labor market.[4] Building on Jovanovic (1984), Moscarini (2005) develops a theory of job turnover and wage dynamics in which employers and workers make inferences about the productivity of their match from the produced output. Besides the different context and questions studied, the main difference between Moscarini's (2005) model and ours is that, in the former, new matches are always accepted (workers and firms are symmetric ex ante) and learning occurs entirely ex post, whereas we focus on prematching learning as it is central to the marriage market context.

Das and Kamenica (2005) study the connection between dating and the asymptotic properties of matching using a two-sided bandit model. In particular, they numerically study the asymptotic stability and regret properties of three matching mechanisms – Gale–Shapley matching, simultaneous offers, and sequential offers – when agents use ϵ-greedy learning algorithms. The tradeoff between learning about an alternative and searching for other alternatives is also central to Fershtman and Pavan (2020). That paper focuses on the problem of a *single* decision maker who sequentially chooses between exploring existing alternatives and searching for additional options to explore.

[4]Information frictions have been incorporated into decentralized matching models in other contexts as well. For example, Lauermann and Wolinsky (2016), Lauermann, Merzyn and Virág (2018), and Mauring (2017) study two-sided search models in which buyers and/or sellers make inferences about an *aggregate state* from the terms of trade they encounter. Anderson and Smith (2010) show that information frictions can upset equilibrium sorting in a model without search frictions. In their model, agents choose a partner not only to maximize their current production, but also to signal their productivity to future partners.

2 The Model

We consider a marriage market with nontransferable utility. There is a set of men and a set of women, each containing a unit mass of agents. Each agent is characterized by a single characteristic $x \in X \equiv [0, 1]$. Following Burdett and Coles (1997), we refer to this characteristic as *pizzazz*. We often identify an agent by her/his pizzazz. To simplify the exposition, we assume that the distribution of pizzazz on both sides of the market is the same.[5] In Section 3, we start by analyzing a simpler version of our model in which all agents have the same pizzazz, whereas in Section 4 we analyze the general model in which agents are vertically heterogeneous. In the heterogeneous model, agents' pizzazz is distributed according to a continuous density $g(x)$ that is bounded in $[\underline{g}, \overline{g}]$, with $0 < \underline{g} < \overline{g} < \infty$.

The market operates in continuous time and agents discount the future at a rate of $r > 0$. Agents transition between three states: singlehood, dating, and marriage. Following Shimer and Smith (2000), we assume that while an agent is single, s/he meets singles on the other side of the market according to a quadratic search technology with parameter $\mu > 0$. That is, for any subset $Y \subseteq X$, if the measure of agents with pizzazz in Y in the singles pool is ν, then agent x meets such agents at a rate of $\mu\nu$.

Any pair of potential partners are either compatible or not. We denote the prior probability that agents x and y are compatible by $q_0(x, y)$, and assume that $q_0(\cdot, \cdot)$ is strictly increasing in each of its arguments, symmetric, differentiable, and has bounded derivatives. Furthermore, we assume that $\underline{q} \equiv q_0(0, 0) > 0$ and that $\overline{q} \equiv q_0(1, 1) < 1$. The compatibility (or lack thereof) of a couple determines the payoff each agent receives while the couple is married; in particular, the flow payoff while married to a compatible partner is normalized to 1, whereas the flow payoff while married to an incompatible partner is given by $-z < 0$. We assume that $z(1 - \overline{q}) > \overline{q}$, which implies that no couple will marry without first receiving (positive) information about their compatibility.

When a pair of potential partners meet, they immediately observe each other's pizzazz and can either begin dating or reject the match and return to the singles market. While dating, a couple gradually learn about their compatibility. We make the following assumptions about the process of dating. First, dating requires mutual consent; that is, at any point in time each agent can unilaterally *break up* with her/his partner. Following a breakup, both agents immediately return to the singles market to search for new potential partners. Second, dating is exclusive; that is, while a couple are dating they do not meet other potential partners. Finally, dating entails a flow cost of $c > 0$ to each agent.

[5] None of our results hinge on this symmetry assumption.

While dating, couples receive information about their compatibility according to the following classic learning technology (e.g., Keller, Rady and Cripps, 2005). If the partners are compatible, they *click* according to an exponential distribution with arrival rate $\lambda > 0$, whereas if they are incompatible, they never click. Consequently, once a couple click, they infer that they are compatible and marry immediately. In the absence of a click, the couple *gradually become more pessimistic* about their compatibility. Thus, after dating for a sufficiently long period of time, one of the partners will choose to break up, at which point both partners return to the singles market.

Following Shimer and Smith (2000) and Smith (2006), we assume that married couples are subject to exogenous dissolution shocks that arrive according to an exponential distribution with arrival rate $\delta > 0$. Once a dissolution shock occurs, both agents return to the singles market. Moreover, we assume that $c(r + \delta) < \lambda \underline{q}$, which prevents the cost of dating from excluding agents from the marriage market.

We analyze the steady state of this model. The strategy of each agent is a function that specifies the maximal amount of time for which s/he is willing to date each potential partner. As the two sides of the market are assumed to be symmetric, we focus on symmetric profiles of strategies, i.e., profiles in which man x and woman x use the same strategy. Thus, agent x's strategy is a mapping $T_x : X \to \mathbb{R}_+$, where $T_x(y)$ is the maximal time that agent x is willing to date an agent with pizzazz y. Note that by setting $T_x(y) = 0$, agent x effectively rejects agent y immediately. Since dating requires mutual consent, after agents x and y meet, they will date for at most $\min\{T_x(y), T_y(x)\}$ units of time: if they click beforehand, they will marry, and otherwise they will separate after dating for $\min\{T_x(y), T_y(x)\}$ units of time. In a steady-state equilibrium, (i) the agents' decisions – whether and for how long to date each potential partner – are optimal given the endogenous composition of the singles pool, and (ii) the flows of individuals into and out of each of the three states – singlehood, dating, and marriage – are balanced. That is, the distributions of singles, dating couples, and married couples are stationary.

Discussion of modeling assumptions

Alternative learning technologies. The learning technology we use is meant to simplify the analysis, and rules out the possibility of a "relationship breakdown" (i.e., a conflict that reveals to partners that they are incompatible) while dating. It is possible to incorporate such breakdowns into the model by assuming that while an incompatible couple is dating, they experience a breakdown according to an exponential distribution with arrival rate $\psi > 0$.

A key qualitative feature of our model is that no news while dating is bad news (i.e., leads agents to become more pessimistic over time). If $\psi < \lambda$, then no news remains bad news, and all of the results go through.

Mutually exclusive dating. There are several ways in which we could relax our assumption that dating is mutually exclusive. First, we could assume that while an agent dates, s/he continues to meet new partners, but at a lower rate than when s/he is single. This assumption is analogous to the standard assumption in models of on-the-job search, in which job offers arrive at a lower rate while agents are employed. Alternatively, we could assume that after two agents meet, each of them can allocate her/his time between learning about the other and searching for new partners. As long as there is a tradeoff between learning about the current partner and searching for a new one, our qualitative results remain unchanged.

The prior $q_0(\cdot,\cdot)$ as a "production function." The prior $q_0(\cdot,\cdot)$ in our model is analogous to the production function in the conventional marriage model. To see this, note that in the conventional model it is assumed that agents x and y produce and share a payoff of $f(x,y)$ when married. This payoff can be interpreted as the expected value of the output produced by the two agents while married, which is unknown at the time the marriage is formed. Under this interpretation, the production function is equivalent to the prior in our model.

Preliminary analysis

Continuation values and capital gains. Let $W_s(x)$ denote the steady-state continuation value of agent x when s/he is single. While searching for a partner, the agent obtains a flow payoff of zero. Hence, $W_s(x)$ is derived from the possibility of meeting partners that the agent will date and, eventually, marry. When agent x starts dating a potential partner y, the agent derives a capital gain from the possibility that dating will lead to marriage with y. Since the probability of successful dating depends on the partners' pizzazz, the capital gain from dating also depends on their pizzazz. We denote agent x's capital gain from dating agent y by $V_d(x;y)$.[6] The flow value of singlehood, $rW_s(x)$, is related to the capital gain from dating as follows

$$rW_s(x) = \mu \int_X V_d(x;y)u(y)dy, \tag{1}$$

where $u(y)$ denotes the (steady-state) measure of agents with pizzazz y in the singles pool. This representation of the continuation value of single agents is analogous to the standard representation in the literature (e.g., Smith, 2006), in which the continuation value of single agents is expressed as their expected flow gain from marrying partners in their "acceptance

[6]See the Appendix for an explicit derivation of the capital gain from dating.

set," where the capital gain from a match is replaced with the capital gain from dating.

Optimal dating choices. To understand the agents' dating choices, consider agent x's marginal (net) value of dating agent y for an additional dt units of time.[7] On the one hand, the dating couple $\langle x, y\rangle$ click with probability $\lambda q_t(x, y)dt$, where $q_t(x, y)$ denotes the couple's joint belief about their compatibility after having dated for $t \geq 0$ units of time. Should such a click occur, agent x will enjoy the capital gain from marrying a compatible partner, which is given by $(1 - rW_s(x))/(r + \delta)$. On the other hand, while dating, agent x must forgo the flow value of singlehood, $rW_s(x)$, and incur the flow cost of dating, c. Hence, the *marginal value of dating* is

$$\lambda q_t(x, y)\frac{1 - rW_s(x)}{r + \delta} - (rW_s(x) + c). \tag{2}$$

Standard arguments (e.g., Keller, Rady and Cripps, 2005) show that $\dot{q}_t = -\lambda q_t(1-q_t)$, which implies that the marginal value of dating decreases over time.

Agent x's choice of whether or not to continue dating agent y is relevant only if the latter chooses to continue dating agent x. As in many other two-sided matching models, the mutual consent requirement can sustain an equilibrium in which any two agents reject one another. The matching-with-search-frictions literature typically assumes that an agent accepts any match with other agents whose pizzazz is strictly greater than her/his reservation value – an assumption that precludes this type of equilibrium. In this paper, we make the analogous assumption that an agent x chooses to continue dating an agent y as long as her/his marginal value of dating is positive.[8]

The above assumption and the fact that $\dot{q}_t < 0$ jointly imply that agent x's optimal choice of dating times – as a function of $W_s(x)$ – is characterized by a cutoff belief over her/his compatibility with the current partner, $q^\star(x)$, that is independent of that partner's pizzazz. We refer to $q^\star(x)$ as agent x's *breakup threshold*. From (2) it follows that

$$q^\star(x) \equiv \frac{rW_s(x) + c}{1 - rW_s(x)} \times \frac{r + \delta}{\lambda}. \tag{3}$$

Note that agent x will immediately reject a potential partner y for which $q_0(x, y) < q^\star(x)$.

[7]In the Appendix, we formally set up agent x's problem when s/he meets a potential partner y, solve it, and derive the dating strategies that correspond to its solution.

[8]In our setting, it is irrelevant how agents resolve their indifference when the marginal value of dating is zero.

Hence, the use of this breakup threshold induces the dating strategy

$$T_x^\star(y) = \max\left\{0, \frac{1}{\lambda}\log\left(\frac{q_0(x,y)(1-q^\star(x))}{(1-q_0(x,y))q^\star(x)}\right)\right\}. \tag{4}$$

The choice of dating times determines the probability that any couple $\langle x, y\rangle$ that have just met will marry. We refer to this probability as the *conversion rate* and denote it by

$$\alpha(x,y) = q_0(x,y)(1 - e^{-\lambda(\min\{T_x(y),T_y(x)\})}).$$

Balanced flow. We now characterize the balanced flow condition, which guarantees that the distributions of singles, dating couples, and married couples are stationary.[9] At each moment in time, the outflow of agents with pizzazz x from the singles pool is given by the measure of such agents who meet a partner y such that both x and y are willing to date one another for a strictly positive amount of time. Note that agents date for a positive amount of time if and only if they marry with positive probability. Thus, the outflow of agents with pizzazz x from the singles pool is given by

$$\mu u(x) \int_{\{y:\alpha(x,y)>0\}} u(y)dy.$$

There are two cases in which an agent with pizzazz x returns to the singles pool. First, s/he may break up with an agent that s/he has been dating. The couples $\langle x, y\rangle$ that break up at a given point in time are those that met exactly $\min\{T_x(y), T_y(x)\}$ units of time ago and did not click while dating. The probability that such a couple does not click while dating is $1 - \alpha(x, y)$, and hence the flow of agents with pizzazz x into the singles pool due to failed dating is given by

$$\mu u(x) \int_{\{y:\alpha(x,y)>0\}} u(y)(1-\alpha(x,y))dy.$$

Second, while agent x is married, s/he may return to the singles pool due to a dissolution shock. Denote by $d(x, y)$ the measure of agents with pizzazz x who are dating an agent with pizzazz y. The measure of agents with pizzazz x who are dating is then

$$d(x) = \int_{\{y:\alpha(x,y)>0\}} d(x,y)dy.$$

[9] We focus on the case where the distribution of pizzazz is continuous; the balanced flow condition in the version of the model in which all agents have the same pizzazz is a special case of the one derived below.

Thus, the measure of agents with pizzazz x that are married is $g(x) - u(x) - d(x)$, and the inflow of agents with pizzazz x that return to the singles pool due to dissolution shocks is

$$\delta(g(x) - d(x) - u(x)).$$

By equating the inflow and outflow derived above and rearranging, it follows that the flow of agents with pizzazz x into and out of singlehood is balanced if

$$\mu u(x) \int_{\{y:\alpha(x,y)>0\}} \alpha(x,y)u(y)dy = \delta(g(x) - d(x) - u(x)). \tag{5}$$

Note that the LHS of (5) is the inflow of agents with pizzazz x into marriage, and the RHS of (5) is the outflow of such agents from marriage. Hence, (5) guarantees that the flows into and out of marriage are also balanced. Thus, when (5) holds for every x, the distributions of singles, dating couples, and married couples are stationary.

Steady-state equilibrium. In a steady-state equilibrium, (i) agents set their breakup thresholds optimally given the (endogenous) size and composition of the singles pool, and (ii) the flows between singlehood, dating, and marriage are balanced.

Definition 1 *A* steady-state equilibrium *is a tuple* $\langle W_s(\cdot), q^\star(\cdot), u(\cdot)\rangle$ *that consists of the value functions of single agents, the agents' breakup thresholds, and the measure of single agents, such that* (1), (3), *and* (5) *hold.*

3 Homogeneous Pizzazz

In this section we analyze a simple version of our model in which all agents have the same pizzazz x_0. We denote $q_0 \equiv q_0(x_0, x_0)$, and throughout this section omit the dependence of equilibrium objects on agents' pizzazz. The ex-ante symmetry enables us to obtain a closed-form solution for the unique steady-state equilibrium, which is useful in highlighting several features of our model and, in particular, the agents' dating choices.

We start by showing that in the unique equilibrium dating times are excessively long. To gain intuition, consider an agent who contemplates breaking up with the agent whom s/he is currently dating. The agent takes into account the exogenous cost of dating and the option value of being single (i.e., forgoing the possibility of meeting other singles). However, s/he does not internalize the fact that while s/he is dating her/his current partner s/he is unavailable to other potential partners for whom s/he may be a good match. This consideration is taken into account by a social planner whose objective is to maximize aggregate welfare. The next result establishes that such a planner – who does not know which couples are compatible and chooses the agents' breakup thresholds – would choose higher breakup thresholds (resp., shorter dating times) than the thresholds (resp., dating times) obtained in the unique equilibrium.

Proposition 1 *There exists a unique steady-state equilibrium. Moreover, the equilibrium dating time $T^\star$ is longer than the socially optimal dating time. Furthermore, the pool of singles in equilibrium is smaller than it is under the socially optimal outcome.*

The challenge in the proof is due to the endogeneity of the mass of agents in the singles pool. To solve for the equilibrium in closed form, we make use of the fact that in the homogeneous model, all agents choose the same dating time and, hence, the equilibrium can be analyzed *as if* agents are unaffected by their potential partners' breakup thresholds. This independence allows us to represent the continuation value of single agents, W_s, the capital gain from dating, V_d, and the mass of agents in the singles pool, u, as simple functions of the agents' breakup threshold $q^\star$, and to obtain the closed-form characterization of $q^\star$ (see (A.1)).

The externality underlying the second part of Proposition 1 is reminiscent of the "thick market" externality that emerges in other search-and-matching models (e.g., Shimer and Smith, 2001): when an agent does not search s/he recognizes that s/he forgoes the opportunity of matching with others, but fails to take into account the fact that others cannot match with her/him. However, unlike in these models, the magnitude of this externality in our model follows from information acquisition. To see that information acquisition is the source of the externality, note that as we show below (Proposition 4), the equilibrium outcome becomes efficient as the learning friction vanishes: compatible agents marry instantaneously and incompatible agents break up instantaneously.

We now derive comparative statics that shed light on how the search and learning frictions shape agents' dating decisions. First, consider the effects of an increase in μ. On the one hand, holding the size of the singles pool fixed, this reduces the amount of time it takes a single agent to meet a new potential partner, which in turn makes agents more picky and creates an incentive for each agent to raise her/his breakup threshold. On the other hand, holding the agents' strategies fixed, an increase in μ hastens the exit of agents from the singles pool, which in turn reduces its size, and makes agents less picky, creating an incentive to lower their breakup threshold. Similarly, an increase in λ induces two opposing effects. On the one hand, holding the size of the singles pool fixed, the marginal value of dating a current partner increases, which in turn makes agents less picky in the sense that they are willing to spend more time dating a partner without a click occurring. On the other hand, holding the agents' breakup threshold fixed, agents spend less time dating, which increases the size of the singles pool and makes agents more picky.

We make use of our closed-form characterization to determine which of these forces dominates. For changes in the λ, we do so under the assumption that the dissolution rate δ is low, which is consistent with empirical findings suggesting that, by and large, the length of time for which a couple are married is of a larger order of magnitude than the length of time for which they were dating before marrying (see Browning, Chiappori and Weiss, 2014).

Proposition 2

1. *The breakup threshold $q^\star$ is strictly increasing μ.*
2. *If δ is sufficiently small, the breakup threshold $q^\star$ is strictly decreasing in λ.*

Proposition 2 implies that improvements in search technologies and in learning technologies have markedly different effects on observable dating and marriage patterns. To see this, note that the equilibrium breakup threshold pins down the equilibrium conversion rate (see (A.3)):

$$\alpha^\star = (1 - e^{-\lambda T^\star})q_0 = \frac{q_0 - q^\star}{1 - q_0}.$$

Moreover, the expected number of partners that an agent dates before marriage is $1/\alpha^\star$. Thus, as the speed of search increases – e.g., due to the emergence of dating apps – each couple dates for a shorter amount of time (and is less likely to marry), and the number of partners that an agent is expected to date before marrying increases. This finding is consistent with recent trends in dating markets. On the other hand, as the speed of learning increases, dating is more likely to be fruitful, and agents will, on average, date fewer partners before marrying.

Discussion: Gender Asymmetry in Dating Costs.

The inefficiency of dating choices established in Proposition 1 rests on the assumption that men and women have the same (direct) dating costs. We illustrate this by briefly considering a variant of our baseline model in which on one side of the market agents have a dating cost of $c + \Delta/2$ and on the other side of the market agents have a dating cost of $c - \Delta/2$. In this variant, Δ is a measure of the asymmetry in dating costs between men and women, which may reflect social norms about dating (e.g., cost of courtship and cultural views on premarital dating).[10] Using this modification of the model, we demonstrate that aggregate welfare is hump-shaped in Δ, and that for some $\Delta^\star > 0$, the efficient outcome (under symmetric dating costs) is attained in equilibrium.

Asymmetric dating costs create distinct dating incentives for different sides of the market: the agents for whom dating is more costly are more picky than their counterparts on the other side of the market.[11] Since dating requires mutual consent, an increase in Δ, just like an increase in c, reduces the dating time of every couple. For small Δ, such an increase brings the equilibrium dating patterns closer to the social optimum, whereas for large Δ it moves them further away from it. This is illustrated in Figure 1, which depicts social welfare (left-hand panel) and the continuation value of single agents with high and low dating costs (right-hand panel) as a function of Δ. Note that the continuation value of singles on the side of the market with the higher dating costs is decreasing in Δ, but for low values of Δ, this is more than compensated for by the increase in the continuation value of singles on the other side of the market.

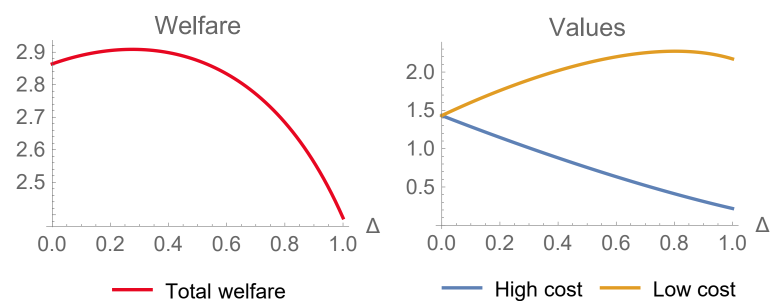

Figure 1: Welfare and continuation value of singles as a function of dating-cost asymmetry Δ, for $c = \mu = \lambda = 1$, $q_0 = 1/2$, and $r = \delta = 1/10$.

Asymmetric dating costs may enhance efficiency by making agents more picky. There are other mechanisms that may lead to such behavior. For example, if agents are slightly overoptimistic about their outside option (e.g., slightly overestimate μ), then their equilibrium behavior will also be closer to the social optimum. Alternatively, the asymmetry in costs may emerge as a result of a social "dating tax" levied on individuals on one side of the market.

[10] Fisman et al. (2006) document differences in the behavior of men and women in dating markets.
[11] That $q^\star$ is increasing in c can be established formally by implicit differentiation of (A.1).

4 General Model

We now turn to the case in which agents' pizzazz is distributed continuously over the interval X. Such heterogeneity allows us to study classic questions regarding equilibrium sorting that are moot when all agents have the same pizzazz. We begin this section by establishing the existence of a steady-state equilibrium and then turn to examine whether (and in what sense) equilibrium outcomes feature assortative matching. Finally, we investigate how changes in search and learning technologies affect equilibrium sorting and the distribution of unmarried agents.

4.1 Equilibrium Existence

We start by deriving a number of properties that any steady-state equilibrium must satisfy. The first property is the strict monotonicity of the continuation value of single agents $W_s(\cdot)$. Since agents' breakup thresholds are independent of their potential partner's pizzazz, a high-pizzazz agent can mimic the *dating times* of a low-pizzazz agent with *every* potential partner. As a high-pizzazz agent is (strictly) more likely to be compatible with any given partner than a low-pizzazz agent, the probability of a click with any given partner under such a mimicking strategy is greater for the high-pizzazz agent than for the low-pizzazz agent. It follows that $W_s(\cdot)$ is strictly increasing, which by (3) implies that $q^\star(\cdot)$ is also strictly increasing.

The second property is the continuity of $W_s(\cdot)$, which follows from a similar mimicking argument. Consider agents $\tilde{x}$ and $\hat{x}$, and a potential partner with pizzazz y. In equilibrium, agent y uses a breakup threshold. Furthermore, the prior belief that a couple $\langle x, y \rangle$ is compatible, $q_0(x, y)$, is continuous in x. As a result, if agents $\tilde{x}$ and $\hat{x}$ have similar pizzazz, the amount of time that agent y is willing to date each of these two agents is similar as well. This means that by mimicking one another's dating strategy (i.e., breakup threshold), agents $\tilde{x}$ and $\hat{x}$ obtain similar continuation payoffs. The continuity of $q^\star(\cdot)$ then follows from the continuity of $W_s(\cdot)$ by (3). In the Appendix, we show that $W_s(\cdot)$ and $q^\star(\cdot)$ are in fact Lipschitz continuous, which will be useful for the analysis that follows.

Lemma 1 *$W_s(\cdot)$ and $q^\star(\cdot)$ are strictly increasing and Lipschitz continuous.*

As noted in Smith (2006), a critical distinction between the existence proof methodologies of transferable-utility and nontransferable-utility search models lies in the discontinuity of the value functions. In our model, while utility is nontransferable, value functions are in fact continuous. Agents' continuous choice of dating times replaces the role of continuous surplus division in smoothing the value functions.

Following the approach pioneered by Shimer and Smith (2000), we establish the existence of a steady-state equilibrium in the value function space. While existence proofs in the literature typically rely on matching and acceptance sets, which reflect binary accept/reject choices, in our model the agents make richer decisions, choosing for how long, if at all, to date each potential partner. Our proof methodology must therefore differ from existing ones. Another challenge stems from the fact that, in our model, agents transition between three states – singlehood, dating, and marriage – rather than just two states.

We prove the existence of a steady-state equilibrium by invoking a fixed point argument. This requires establishing that the mappings (i) from value functions to the probabilities with which each pair of agents marry, and (ii) from these marriage probabilities to the distribution of singles, are continuous and well defined. The latter is the analog of Shimer and Smith's (2000) fundamental matching lemma. Equilibria are then fixed points of an appropriately defined mapping. Using Schauder's fixed point theorem, we establish that such a fixed point indeed exists.

Theorem 1 *A steady-state equilibrium exists.*

4.2 Assortative Matching

Since Becker (1973), the question of who marries whom and, in particular, whether or not matching is assortative has become central to the matching literature. The typical sorting outcome in models of matching with search frictions and nontransferable utility is block segregation: the set of agents is partitioned into classes, and agents only marry agents within their class.[12] In our setting, block segregation – whereby agents are unwilling to date below their class – fails. To see why, suppose that there is a class $[\underline{y}, \overline{y}]$; that is, agents whose pizzazz is $x \in [\underline{y}, \overline{y}]$ date all agents within this class and only such agents. This means that $q^\star(x) = q_0(\underline{y}, x)$ for every $x \in [\underline{y}, \overline{y}]$. Thus, agents at the lower end of their class would date agents within their class for an arbitrarily small amount of time, which would lead to prolonged singlehood. As a result, such agents would find it optimal to date below their class and block segregation fails.

Proposition 3 *There is no equilibrium in which agents are segregated into $n \geq 2$ classes.*

The failure of block segregation emphasizes that, in the realm of pre-matching information acquisition, it matters not only who dates whom, but also with what probability such dating

[12]This result has been established by multiple authors in various settings, as discussed in the Introduction.

leads to marriage. We therefore define a new, probabilistic, notion of assortative matching: *single-crossing in marriage probabilities.* Under this notion, for any $x', x'' \in X$ such that $x' < x''$, the difference $\alpha(x'', y) - \alpha(x', y)$ satisfies a single-crossing property in y (illustrated in Figure 2). Namely, if x' and x'' are equally likely to marry an agent y, then x' is more likely than x'' to marry agents with pizzazz lower than y, whereas x'' is more likely than x' to marry agents with pizzazz higher than y.

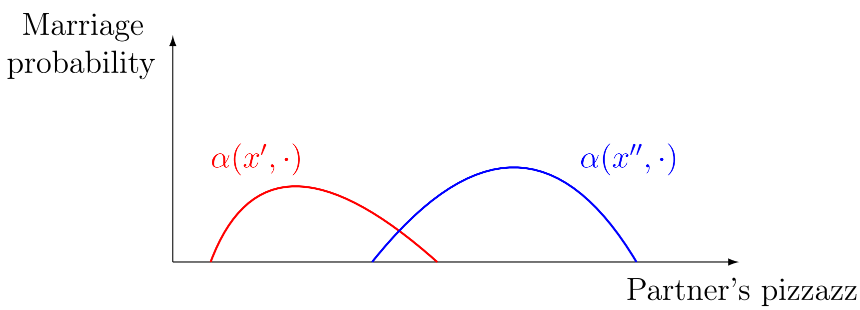

Figure 2: Illustration of single-crossing in marriage probabilities for $x' < x''$.

Formally, for any matching probability function $\alpha(\cdot, \cdot)$ and agents $x', x'' \in X$, denote the set of agents who marry x' or x'' with positive probability by

$$A_\alpha(x', x'') = \{y : \alpha(x', y) > 0 \text{ or } \alpha(x'', y) > 0\}.$$

Definition 2 (Single-crossing in marriage probabilities) *A marriage-probability function $\alpha(\cdot, \cdot)$ satisfies the single-crossing property if for every $x', x'' \in X$ such that $x' < x''$, there exists $y^\star(x', x'') \in X$ such that for any $y \in int(A_\alpha(x', x''))$, $\alpha(x'', y) > \alpha(x', y)$ if and only if $y > y^\star(x', x'')$.*

Theorem 2 *If $q_0(\cdot, \cdot)$ is supermodular, then every steady-state equilibrium marriage-probability function satisfies single-crossing in marriage probabilities.*

Theorem 2 establishes that the equilibrium matching is probabilistically assortative if there is complementarity in $q_0(\cdot, \cdot)$. When conversion rates are binary, as in the conventional marriage market setting, single-crossing in marriage probabilities (with weak inequalities) reduces to Smith's (2006) notion of assortative matching. In Smith's model, agents engage in time-consuming search and utility is nontransferable as in our model. To shed light on the implications of dating on sorting patterns, we interpret $q_0(\cdot, \cdot)$ as the production function and compare the sufficient condition for assortative matching established above and Smith's

(2006) sufficient condition for assortative matching.[13] Smith shows that log-supermodularity of the production function is sufficient for assortative matching, and that supermodularity may not suffice for the induced matching to be assortative. Thus, Theorem 2 shows that when agents can learn about the productivity of the match before agreeing to it, weaker conditions suffice to ensure positive assortative matching.

In Section 3 we showed that equilibrium dating times are inefficient from a social perspective. We now show that the equilibrium sorting patterns may also be inefficient: while there is positive assortative matching in equilibrium, there is negative assortative matching (in the sense of Shimer and Smith, 2000) in the socially efficient outcome. The following example illustrates that a social planner who chooses the agents' dating times without knowing their compatibility may instruct high-pizzazz agents to date only low-pizzazz agents.

Example 1 (Negative Assortative Matching) *Suppose that there are only two levels of pizzazz, $x_h = 1/2$ and $x_l = 2/5$, where $Pr(x = x_l) = 3/4$, and that $q_0(x, y) = xy$. Moreover, suppose that $c = 1/2, \delta = 1/100, r = 1/10$, and $\mu = \lambda = 1$. Under the socially optimal allocation, $\alpha(x_l, x_l) = 3/25$, $\alpha(x_l, x_h) = 4/25$, and $\alpha(x_h, x_h) = 0$.*

To gain intuition as to why efficient allocations may exhibit negative assortative matching, note that the conversion rate from meetings to marriages, $\alpha(\cdot, \cdot)$, is inversely related to the average number of agents one must date before marrying. When low-pizzazz agents are unlikely to be compatible with one another, the social planner has an incentive to direct them to date higher-pizzazz agents in order to shorten their (socially costly) singlehood episode. As illustrated in Example 1, to make sure that there are sufficiently many high-pizzazz agents available to date the low-pizzazz agents, the social planner may need to prevent high-pizzazz agents from dating one another. Thus, negative assortative matching may maximize social welfare. Note that this phenomenon is intrinsically related to dating as, in the absence of dating, the conversion rate in the acceptance set is 1 regardless of the agents' pizzazz.

4.3 Search and Learning Frictions

In recent years, the dating market has undergone drastic changes due to the introduction of new dating apps that have replaced more traditional matching channels.[14] These changes

[13] Various authors have studied the conditions for assortative matching under different assumptions. For example, Becker (1973) studies these conditions in the frictionless marriage model, and Morgan (1996), Shimer and Smith (2000), Atakan (2006), and more recently Bonneton and Sandmann (2022) study them in the context of marriage markets in which utility is transferable and/or there is an explicit search cost.

[14] According to a recent survey by Pew Research Center (2020), "Roughly half or more of 18- to 29-year-olds ... say they have ever used a dating site or app."

have affected both the speed of search and the difficulty of learning about potential partners. We now explore the implications of such advances and, in particular, their effect on equilibrium sorting. To study these effects, we characterize the steady-state equilibrium when either the learning friction vanishes or the search friction vanishes.[15]

Proposition 4

1. *As $\lambda \to \infty$, agents are willing to date everyone: $q^\star(x) \underset{\lambda\to\infty}{\longrightarrow} 0$ for every $x \in X$. Hence, as learning frictions vanish, dating becomes non-assortative.*

2. *As $\mu \to \infty$, agents are willing to date only agents of their own pizzazz and higher: $q^\star(x) \underset{\mu\to\infty}{\longrightarrow} q_0(x,x)$ for every $x \in X$. Hence, as search frictions vanish, dating becomes fully assortative, and the average number of partners each agent dates before marrying goes to infinity.*

As $\lambda \to \infty$, the marginal benefit of dating goes to infinity, as long as a couple is not certain that they are incompatible. Since both the direct and indirect costs of dating are finite, as long as an agent believes that s/he is compatible with her/his dating partner with strictly positive probability, in the limit, s/he finds it optimal to continue dating that partner. This observation is independent of the partner's pizzazz.

To gain intuition for the second part of this result, suppose that as $\mu \to \infty$, agent x meets singles instantaneously (note that this is not guaranteed, as the pool of singles is endogenous). In this case, agent x has no reason to date a potential partner for more than an infinitesimal amount of time: s/he can break up with the partner if a click does not occur immediately and instantaneously begin dating a new one with the same pizzazz. Thus, in the limit $\mu \to \infty$, agent x would choose to date only the highest-pizzazz agents that are willing to date her/him, and hence the breakup threshold $q^\star(x)$ converges to $q_0(x,x)$. The main challenge in the proof is to show that the singles pool $u(x)$ does not vanish as $\mu \to \infty$.

In the latter case in which search frictions vanish, agents invest a negligible amount of time in dating each partner that they meet, and hence date an arbitrarily large number of partners before marrying. This result is consistent with the phenomenon often referred to as the "dating apocalypse" (see, e.g., Sales, 2020), whereby the growing ease with which people find dating partners through modern dating apps creates a difficulty in establishing long-term relationships.

[15] In the matching-with-frictions literature, comparative static results typically take the form of either limit results or use a simplifying "cloning" assumption; see Chade, Eeckhout and Smith (2017).

The search and learning technologies also affect the distribution of agents that are married, dating, and single. In particular, we show that in both limiting cases high-pizzazz agents are more likely to be married than low-pizzazz agents, relative to their weight in the population. Let $\eta^u(x) = \int_X q_0(x,y)u(y)dy$ denote the (scaled) probability that agent x is compatible with a random agent drawn from the singles pool. Note that $\eta^u(x)$ is strictly increasing in x for any $u(\cdot)$.

Proposition 5 *In both limiting cases, higher-pizzazz agents are underrepresented among unmarried agents relative to their share in the population:*

1. *As learning frictions vanish,*

$$\frac{u(x)+d(x)}{g(x)} = \frac{1}{1+\frac{\mu}{\delta}\eta^u(x)}.$$

2. *As search frictions vanish,*

$$\frac{u(x)+d(x)}{g(x)} = \frac{1}{1+\frac{\lambda}{\delta}q_0(x,x)}.$$

Proposition 5 also highlights two qualitative implications of dating. First, a strictly positive fraction of agents of every level of pizzazz are unmarried in the steady-state equilibrium even when search frictions vanish. Second, dating causes high-pizzazz agents to be scarce in the limit.

As learning frictions vanish, the amount of time a couple spend dating converges to zero. Since the rate at which meetings occur is finite, this implies that the measure of dating agents also converges to zero. Hence, Proposition 5 implies that higher-pizzazz agents are underrepresented in the singles pool relative to the population. That is, the ratio $u(x)/g(x)$ is decreasing in x.

Although high-pizzazz agents are underrepresented among unmarried agents in both limiting cases, this occurs for distinct reasons. When learning frictions vanish, every compatible couple that meets ends up marrying. Hence, agents that are more likely to be compatible with other agents – namely, high-pizzazz agents – date fewer agents before marrying than low-pizzazz agents. On the other hand, when search frictions vanish, dating is fully assortative. Hence, high-pizzazz agents are always more likely to be compatible with their dating partners than low-pizzazz agents, and therefore click with their partners at a higher rate.

5 Concluding Remarks

We introduced a model of two-sided matching with learning frictions alongside the traditional search frictions. The main innovation is that agents can spend time together to learn about their compatibility before marrying. Our analysis shows that equilibrium dating is inefficient (results in overdating) and illustrates how different social norms that generate asymmetry in dating costs between men and women may mitigate this inefficiency. Moreover, we show that if there is complementarity between agents types, then equilibrium matching satisfies a new notion of positive assortative matching: single-crossing in marriage probabilities, but that positive sorting need not be efficient. Finally, we show that the search and learning frictions have radically different effects on equilibrium dating and marriage patterns and on the manner in which agents sort themselves. On the one hand, advances in search technologies reduce the amount of time agents spend dating each partner, increase the number of partners whom they date, and result in more sorting in equilibrium. On the other hand, advances in learning technologies reduce the number of partners whom agents date and result in less sorting in equilibrium.

Throughout the paper we use marriage-market and dating terminology. However, pre-matching learning is also prominent in other markets in which agents trade bilaterally and engage in time-consuming search. For example, in the context of the labor market it can take the form of job interviews or probationary periods during the hiring process, and in markets in which entrepreneurs and investors match to develop joint ventures it may take the form of a due diligence process. In such markets, additional factors may come into play as buyers and sellers may negotiate different terms of trade. However, as long as utility is not fully transferable, our insights remain relevant. In fact, the results of Section 3 hold even in markets in which utility is fully transferable, as the symmetric model presented in that section is equivalent to a symmetric model in which utility is transferable and the terms of trade are settled via Nash bargaining with equal weights. These results suggest that hiring processes and probationary periods may be inefficiently long, and that investment in assessing the viability of joint ventures may be excessive.

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A Appendix

Proofs

Proof of Proposition 1. The proof consists of three steps: characterizing the equilibrium, establishing existence and uniqueness, and showing that equilibrium dating is excessively long.

Step 1 – Characterization of the equilibrium. Denote the likelihood ratio that a couple is compatible after having dated for $t \geq 0$ units of time by $l_t = \frac{q_t}{1-q_t}$. We first show that the set of steady-state equilibria are represented by the solutions to the equation

$$\frac{(1+l_0)r(\lambda+r)(l^\star(\lambda - c(\delta+r)) - c(\delta+r))}{(1+c)\lambda\left(r(l_0 - l^\star) - \lambda l^\star\left(1 - \left(\frac{l^\star}{l_0}\right)^{r/\lambda}\right)\right)} = \frac{2\mu}{1+\sqrt{1+\frac{4\mu}{\delta\lambda(1+l_0)}\left((\delta+\lambda)(l_0 - l^\star) + \delta\log\left(\frac{l_0}{l^\star}\right)\right)}}, \tag{A.1}$$

where $l^\star \equiv \frac{q^\star}{1-q^\star}$.

In a steady-state equilibrium $rW_s = \mu u V_d$ (see (1)). In this step, we express W_s, V_d, and u as functions of $l^\star$, and show that this condition is equivalent to (A.1).

The learning technology implies that $l_0 e^{-\lambda t} = l_t$. Hence,

$$T^\star = \frac{1}{\lambda}\log\left(\frac{l_0}{l^\star}\right). \tag{A.2}$$

The probability that a couple that dates for T units of time eventually marries is $q_0(1-e^{-\lambda T})$. Hence, from (A.2), it follows that

$$\alpha^\star \equiv q_0(1-e^{-\lambda T^\star}) = \frac{l_0 - l^\star}{1+l_0}. \tag{A.3}$$

From (3), it follows that

$$l^\star = \frac{(\delta + r)(c + rW_s)}{\lambda - c(\delta + r) - rW_s(\delta + \lambda + r)}.$$

Rearranging, we obtain the continuation value of a single agent as a function of $l^\star$,

$$W_s = \frac{\lambda l^\star - c(l^\star + 1)(\delta + r)}{(l^\star + 1)r(\delta + r) + \lambda l^\star r}. \tag{A.4}$$

Next, we use the above expressions to derive the capital gain from dating, which can be written as

$$V_d = \alpha^\star \sigma^\star \frac{1 - rW_s}{\delta + r} - (c + rW_s)\kappa^\star,$$

where $\sigma^\star$ is the expected discounting factor at the time at which a dating couple marries, conditional on the couple marrying, and $\kappa^\star$ is the expected discounted amount of time for which the couple will date. Since

$$\sigma^\star \equiv \int_0^{T^\star} e^{-rt} \frac{\lambda e^{-\lambda t}}{1 - e^{-\lambda T^\star}} dt = \frac{\lambda}{\lambda + r} \cdot \frac{l_0 - l^\star \left(\frac{l_0}{l^\star}\right)^{-\frac{r}{\lambda}}}{l_0 - l^\star}$$

and

$$\begin{aligned} \kappa^\star &\equiv (1 - \alpha^\star)\frac{1 - e^{-rT^\star}}{r} + \alpha^\star \int_0^{T^\star} \frac{(1 - e^{-rt})}{r} \frac{\lambda e^{-\lambda t}}{1 - e^{-\lambda T^\star}} dt \\ &= \frac{\lambda + r(l_0 + 1) - (\lambda + r(l^\star + 1))\left(\frac{l_0}{l^\star}\right)^{-r/\lambda}}{r(\lambda + r)(l_0 + 1)}, \end{aligned}$$

the expected cost of dating is given by

$$(c + rW_s)\kappa^\star = \frac{(1 + c)\lambda l \left((l_0 + 1)r + \lambda - ((l^\star + 1)r + \lambda)\left(\frac{l_0}{l}\right)^{-\frac{r}{\lambda}}\right)}{(1 + l_0)r(\lambda + r)(\lambda l^\star + (l^\star + 1)(\delta + r))}.$$

The above derivations imply that

$$V_d = \frac{(1 + c)\lambda \left(l_0 r + l^\star \left(\lambda \left(\frac{l_0}{l^\star}\right)^{-\frac{r}{\lambda}} - (\lambda + r)\right)\right)}{(l_0 + 1)r(\lambda + r)(\lambda l^\star + (l^\star + 1)(\delta + r))}.$$

Finally, we represent the mass of agents in the singles pool as a function of $l^\star$. The mass of dating agents is given by

$$d = \mu u^2 \int_0^{T^\star} 1 - q_0(1 - e^{-\lambda t})dt = \frac{\mu u^2 \left(\log\left(\frac{l_0}{l^\star}\right) + l_0 - l^\star\right)}{\lambda(1 + l_0)}.$$

Hence, the balanced flow condition (5) becomes

$$1 - u = \frac{\mu u^2}{\lambda(1 + l_0)} \left((1 + \frac{\lambda}{\delta})(l_0 - l^\star) + \delta \log\left(\frac{l_0}{l^\star}\right)\right).$$

This quadratic equation has a single positive root that is given by

$$u^\star = \frac{2}{1 + \sqrt{1 + \frac{4\mu}{\delta\lambda(1+l_0)}\left((\delta + \lambda)(l_0 - l^\star) + \delta \log\left(\frac{l_0}{l^\star}\right)\right)}}. \tag{A.5}$$

Plugging W_s, V_d, and u^* into (1) and rearranging yields (A.1).

Step 2 – Equilibrium existence and uniqueness. We begin by establishing that (A.1) has at least one solution, and that the number of its solutions is odd. Recall that $l^\star \in [0, l_0]$ and observe that (i) the limit of the LHS (resp., RHS) of (A.1) as $l^\star \to l_0$ from below goes to ∞ (resp., converges to a finite number), and (ii) the limit of the LHS (resp., RHS) of (A.1) as $l^\star \to 0$ from above converges to a negative number (resp., to zero). Since both sides of (A.1) are continuous in $l^\star$, the intermediate value theorem implies that (A.1) has a solution. Moreover, the number of solutions must be odd, as each solution is a point at which there is a change in the sign of the difference between the LHS and RHS of (A.1).[16]

Next, we show that the number of solutions to (A.1) is at most two. Plugging $l^\star = l_0 e^{-\lambda T^\star}$ into (A.1) and rearranging yields

$$\begin{aligned} &-(l_0 + 1)r(\lambda + r)\left(\delta + \psi(T^\star)\right)\left(c(\delta + r)\left(l_0 + e^{\lambda T^\star}\right) - \lambda l_0\right) = \\ &\qquad 2(c + 1)\delta\lambda\mu l_0 \left(r\left(e^{\lambda T^\star} - 1\right) + \lambda\left(e^{-rT^\star} - 1\right)\right), \end{aligned} \tag{A.6}$$

where

$$\psi(T^\star) = \sqrt{\frac{\delta\left(\delta\lambda(l_0 + 1) + 4\mu(\delta + \lambda)\left(l_0 - l_0 e^{-\lambda T^\star}\right) + 4\delta\lambda\mu T^\star\right)}{\lambda(l_0 + 1)}}.$$

The RHS of (A.6) is convex in $T^\star$, as its second derivative is

$$2(c + 1)\delta\lambda^2\mu l_0 r e^{-rT^\star}\left(\lambda e^{T^\star(\lambda + r)} + r\right) > 0.$$

[16] Equation (A.1) cannot have a continuum of solutions. To see this, observe that both the LHS and RHS of this equation belong to C^∞. Hence, if the LHS and RHS were equal over any nonempty subinterval of $[0, l_0]$, they would be equal over all of $(0, l_0)$.

On the other hand, the LHS of (A.6) is concave in $T^\star$. To see this note that the second derivative of the LHS is

$$-(l_0+1)r(\lambda+r)\Bigg\{c\lambda^2(\delta+r)e^{\lambda T^\star}\left(\delta+\Psi(T^\star)\right)+\frac{4c\delta\lambda\mu(\delta+r)\left(l_0(\delta+\lambda)+\delta e^{\lambda T^\star}\right)}{(l_0+1)\Psi(T^\star)}$$
$$+\left(\lambda l_0-c(\delta+r)\left(l_0+e^{\lambda T^\star}\right)\right)\left(\frac{4\delta^2\mu^2\left(\delta+l_0(\delta+\lambda)e^{-\lambda T^\star}\right)^2}{(l_0+1)^2\left(\Psi(T^\star)\right)^3}+\frac{2\delta\lambda\mu l_0(\delta+\lambda)e^{-\lambda T^\star}}{(l_0+1)\Psi(T^\star)}\right)\Bigg\}.$$

The term outside the curly brackets is negative, whereas all terms inside the curly brackets are unambiguously positive with the exception of $\left(\lambda l_0-c(\delta+r)\left(l_0+e^{\lambda T^\star}\right)\right)$. However, plugging $e^{\lambda T^\star}=l_0/l^\star$ into this expression and rearranging shows that it is positive if and only if $\lambda q^\star\frac{1}{r+\delta}>c$. The latter inequality states that the marginal value of dating is greater than the direct cost of dating. Hence, in the relevant range of cutoff beliefs, this inequality must hold.

Since a convex function and a concave function can have at most two intersections, it follows that there are at most two solutions to (A.1). As we have shown that a solution exists, and that the number of solutions is odd, this implies that there is a unique solution to (A.1).

Step 3 – Inefficiency of equilibrium dating. Consider an agent who dates all partners for T units of time. Denote by $W_d(T,\tilde{T})$ this agent's continuation utility at the moment s/he meets a potential partner, given that all other couples date for $\tilde{T}$ units of time. This value can be written as

$$W_d(T,\tilde{T})=\overline{v}_d(T)+\sigma(T)\frac{\mu\overline{u}(\tilde{T})}{\mu\overline{u}(\tilde{T})+r}W_d(T,\tilde{T}),$$

where $\overline{v}_d(T)$ denotes the actual expected payoff while the two agents are together (i.e., a flow cost of c while the agents are dating and a flow benefit of 1 while they are married), $\overline{u}(\tilde{T})$ denotes the mass of singles when all other couples date for $\tilde{T}$ units of time, $\sigma(T)$ is the expected discount factor when the agent breaks up (before or after marriage) with her/his current partner, and $\frac{\mu\overline{u}(\tilde{T})}{\mu\overline{u}(\tilde{T})+r}$ is the expected discounting over the amount of time the agent will wait between the time at which s/he breaks up with the current partner and the time s/he will meet the next partner. Rearranging yields

$$W_d(T,\tilde{T})=\frac{r+\mu\overline{u}(\tilde{T})}{r+\mu\overline{u}(\tilde{T})(1-\sigma(T))}\overline{v}_d(T).$$

If the agent were unconstrained by the choices of her/his dating partner, the necessary and sufficient first-order condition for the agent's problem would be[17]

$$\frac{\overline{v}_d'(T)}{1-\frac{\sigma(T)}{\frac{r}{\mu\overline{u}(\tilde{T})}+1}}+\frac{\overline{v}_d(T)\sigma'(T)}{\left(\frac{r}{\mu\overline{u}(\tilde{T})}+1\right)\left(1-\frac{\sigma(T)}{\frac{r}{\mu\overline{u}(\tilde{T})}+1}\right)^2}=0.$$

In a symmetric equilibrium, all couples date for the same amount of time, $T^\star$, which implies that the dating time in the unique equilibrium is given by

$$\frac{\overline{v}_d'(T^\star)}{1-\frac{\sigma(T^\star)}{\frac{r}{\mu\overline{u}(T^\star)}+1}}+\frac{\overline{v}_d(T^\star)\sigma'(T^\star)}{\left(\frac{r}{\mu\overline{u}(T^\star)}+1\right)\left(1-\frac{\sigma(T^\star)}{\frac{r}{\mu\overline{u}(T^\star)}+1}\right)^2}=0.$$

The social planner maximizes a weighted average of the continuation utilities of agents that are single, dating, and married. We first solve the social planner's problem for the case in which s/he assigns a weight of one to the utility of agents that are dating, and then show that the dating times for these weights are longer than for any other weights.[18] For these weights, the social planner's value from instructing all agents to date for $\hat{T}$ units of time is

$$W_d(\hat{T},\hat{T})=\frac{r+\mu\overline{u}(\hat{T})}{r+\mu\overline{u}(\hat{T})(1-\sigma(\hat{T}))}\overline{v}_d(\hat{T}).$$

The necessary and sufficient first-order condition for the social planner's problem is therefore

$$\frac{r\sigma(\hat{T})\overline{v}_d(\hat{T})u'(\hat{T})}{\mu(\overline{u}(\hat{T}))^2\left(\frac{r}{\mu\overline{u}(\hat{T})}+1\right)^2\left(1-\frac{\sigma(\hat{T})}{\frac{r}{\mu\overline{u}(\hat{T})}+1}\right)^2}+\left\{\frac{\overline{v}_d'(\hat{T})}{1-\frac{\sigma(\hat{T})}{\frac{r}{\mu\overline{u}(\hat{T})}+1}}+\frac{\overline{v}_d(\hat{T})\sigma'(\hat{T})}{\left(\frac{r}{\mu\overline{u}(\hat{T})}+1\right)\left(1-\frac{\sigma(\hat{T})}{\frac{r}{\mu\overline{u}(\hat{T})}+1}\right)^2}\right\}=0.$$

Note that the term inside the curly brackets is identical to the agents' FOC, whereas the term outside the curly brackets is negative since $\overline{u}(T)$ is decreasing in T. To see that $\overline{u}(T)$ is decreasing in T, recall that by (A.5) the mass of single agents is increasing in l, and by (A.2) l is decreasing in T. Since the capital gain from dating is concave in the dating time, the term in curly brackets is decreasing in $\hat{T}$. It follows that, for these weights, the social planner would instruct agents to date for less time than they would choose to date in equilibrium.

To conclude the proof, note that the continuation utility of a married agent is given by the

[17]The first-order condition is necessary and sufficient as the value of dating is concave, and, by assumption the direct cost of dating is not prohibitive.

[18]Within the set of dating agents, it is sufficient to focus on the continuation utility of those agents who have just started dating, due to the dynamic consistency in the choice of dating times.

sum of the discounted flow utility s/he will derive while married, and the continuation value of a single agent, discounted by the exogenous time at which her/his marriage will be dissolved. The continuation utility of an agent that is single is $\frac{\mu\overline{u}(T)}{\mu\overline{u}(T)+r}$ times the continuation utility of an agent upon meeting a potential partner. Since $\frac{\mu\overline{u}(T)}{\mu\overline{u}(T)+r}$ is decreasing in T, it follows that if the social planner assigns any positive weights to the welfare of single and married agents, the planner will choose a shorter dating time than $\hat{T}$. Since $\overline{u}(T)$ is decreasing in T, this implies that the singles pool under the efficient outcome is larger than in equilibrium. ■

Proof of Proposition 2. We first consider changes in the search technology. Recall that in the proof of Proposition 1 we showed that the difference between the LHS of (A.1) and the RHS of (A.1) crosses zero exactly once and, moreover, that it does so from below. Hence, in order to establish this result it is sufficient to show that increasing μ: (i) does not reduce the LHS of (A.1), and (ii) increases the RHS of (A.1).

Point (i) is immediate as the LHS of (A.1) is independent of μ. To establish point (ii), we can differentiate the RHS w.r.t. μ and verify that this derivative is positive:

$$\frac{\partial RHS}{\partial \mu} = \frac{1}{\sqrt{\frac{4\mu(\delta+\lambda)(l_0-l^\star)+4\delta\mu\log\left(\frac{l_0}{l^\star}\right)+\delta\lambda(l_0+1)}{\delta\lambda(l_0+1)}}} > 0.$$

Next consider changes in the learning technology. The equilibrium breakup threshold is given by the solution to (A.1), which can also be represented as the root of the function

$$F(l) = \overbrace{(l_0+1)r(\lambda+r)(\lambda l - c(l+1)(\delta+r))}^{I} - \overbrace{\frac{2(c+1)\delta\lambda\mu\left(l_0 r - l\left(\lambda - \lambda\left(\frac{l}{l_0}\right)^{r/\lambda} + r\right)\right)}{\delta + \sqrt{\frac{\delta\left(4\mu(\delta+\lambda)(l_0-l)+4\delta\mu\log\left(\frac{l_0}{l}\right)+\delta\lambda(l_0+1)\right)}{\lambda(l_0+1)}}}}^{II}.$$

From the implicit function theorem,

$$\frac{\partial l^\star}{\partial \lambda} = -\frac{\frac{\partial F}{\partial \lambda}(l^\star)}{\frac{\partial F}{\partial l}(l^\star)}.$$

In the proof of Proposition 1 we showed that $\frac{\partial F}{\partial l}(l^\star) > 0$. Hence, $\frac{\partial l^\star}{\partial \lambda}$ and $\frac{\partial F}{\partial \lambda}(l^\star)$ have the opposite sign. The limit of the derivative of II w.r.t. λ, as $\delta \to 0$, is equal to zero, and the limit of the derivative of I w.r.t. λ, as $\delta \to 0$, is equal to

$$\frac{l_0}{l}(l_0+1)r\Big(l(2\lambda+r) - c(l+1)r\Big).$$

The condition $c \le \lambda q \frac{1}{r}$ is necessary for agents to choose to continue dating their partner. Since $q = \frac{l}{l+1}$, it follows that if δ is small, $\frac{\partial F}{\partial \lambda}(l^\star) > 0$, which in turn implies that $\frac{\partial l^\star}{\partial \lambda} < 0$. ■

Proof of Lemma 1. The discussion preceding the lemma establishes the strict monotonicity of $W_s(\cdot)$ and $q^\star(\cdot)$. We now establish the Lipschitz continuity of both functions.

By (1), we have that

$$W_s(x+\epsilon) - W_s(x) = \frac{\mu}{r} \int_X \left(V_d(x+\epsilon; y) - V_d(x; y)\right) u(y) dy. \tag{A.7}$$

Fix $x, y \in X$ and $\epsilon > 0$, such that $x + \epsilon \in X$. We begin by showing that there exists $K > 0$ independent of x, y, and ϵ, such that $V_d(x+\epsilon; y) - V_d(x; y) < K\epsilon$.

The probability that any couple is compatible is at most $\overline{q} < 1$. Moreover, in optimum, every pair of agents who start dating either marry or break up before their belief about their compatibility reaches q_{min}, where $q_{min}\lambda\frac{1}{r} = c$. By the assumption that dating costs are non-prohibitive, $q_{min} < \overline{q}$. In combination with the fact that $\dot{q}_t = -\lambda q_t(1-q_t)$, it follows that $\epsilon\tilde{K}$, where $\tilde{K} = \frac{1}{\min_{q \in [\frac{cr}{\lambda}, \overline{q}]}\{\lambda q(1-q)\}}$ is a uniform upper bound on the amount of time it takes a couple's belief about their compatibility to drift down by ϵ.

Denote by τ the amount of time a couple $\langle x+\epsilon, y\rangle$ must spend dating so that, in the absence of a click, they believe that they are compatible with probability $q_0(x, y)$ (i.e., τ is defined implicitly by $q_\tau(x+\epsilon, y) = q_0(x, y)$). Note that $\tau < \tilde{K}\overline{d}\epsilon$, where $\overline{d}$ is an upper bound on $\frac{\partial q(x,y)}{\partial x}$, a derivative that is bound by assumption. The marginal gain from dating is bounded from above by λ/r for every agent. Since the marginal cost of dating is non-negative, it follows that the capital gain that agent $x+\epsilon$ derives from the first τ units of time for which s/he dates y is bounded from above by $\tilde{K}\frac{\lambda\overline{d}}{r}\epsilon$.

For any $t > 0$, the probability that $\langle x+\epsilon, y\rangle$ click after having dated for $t+\tau$ units of time is the same as the probability that $\langle x, y\rangle$ click after having dated for t units of time, conditional on neither couple clicking earlier. Moreover, since $W_s(\cdot)$ is increasing, agent $x+\epsilon$ has a higher marginal cost of dating than agent x. Furthermore, the monotonicity of $W_s(\cdot)$ implies that the dating time of the couple $\langle x, y\rangle$ is greater than the amount of time for which the couple $\langle x+\epsilon, y\rangle$ will continue to date after they have already dated for τ units of time. It follows that the additional capital gain that agent $x+\epsilon$ derives from dating y after they have already dated for τ units of time is smaller than agent x's capital gain from dating y. We have therefore shown that (i) the capital gain agent $x+\epsilon$ derives from dating y for τ units of time is less than $\tilde{K}\frac{\lambda\overline{d}}{r}\epsilon$, and (ii) the capital gain agent x derives from dating agent y is greater than the additional capital gain that agent $x+\epsilon$ derives from dating agent y after

τ units of time passed without a click occurring. Hence, $V_d(x+\epsilon;y) - V_d(x;y) < \tilde{K}\frac{\bar{d}\lambda}{r}\epsilon$.

Since $\int_X u(y)dy \le 1$, from (A.7) it follows that $W_s(x+\epsilon) - W(x) \le \overline{K}\epsilon$, where $\overline{K} = \frac{\mu}{r}\frac{\lambda\bar{d}}{r}\tilde{K}$. Since $W_s(\cdot)$ is increasing, this implies that $W_s(\cdot)$ is Lipschitz continuous with modulus $\overline{K}$. The Lipschitz continuity of $q^\star(\cdot)$ then follows immediately from condition (3) and the observation that $rW_s(\cdot) \le r\frac{\mu}{r+\mu}\frac{1}{r} < 1$, where the first inequality follows from the fact that $rW_s(\cdot)$ is bounded from above by r times the expected discounted time at which the next partner is met, $\frac{\mu}{r+\mu}$, and the payoff from a marriage that lasts forever, $1/r$. ■

Proof of Theorem 1. To establish the existence of a steady-state equilibrium, we show that (1) value functions have a continuous impact on the conversion rate of any two agents, (2) conversion rates have a continuous impact on the distribution of agents in the singles pool, and (3) the value functions are given by a fixed point of a continuous operator. We then invoke Schauder's fixed point theorem to establish that a fixed point exists.

The continuation value of any single agent is bounded from above by her/his expected value from meeting compatible partners at rate μ while single, marrying them immediately, and returning to the market once the marriage is hit by a dissolution shock. This upper bound is given by $\overline{W} \equiv \frac{\mu}{(r+\mu+\delta)r}$. Define the family $\mathcal{F}$ of functions from X to $[0,\overline{W}]$ that are weakly increasing and Lipschitz continuous with modulus $K^\star > 0$. $\mathcal{F}$ is a subset of $C[0,\overline{W}]$ that is nonempty, bounded, closed, and convex. We endow this family of functions with the sup norm $||W_s|| = sup_{x\in X}|W_s(x)|$.

If a dating couple $\langle x,y\rangle$ breaks up when they believe that they are compatible with probability q, the maximal length of time for which they will date, $T(x,y,q)$, is given implicitly by $e^{-\lambda T(x,y,q)}\frac{q_0(x,y)}{1-q_0(x,y)} = \frac{q}{1-q}$. Hence, the probability that they will eventually marry is

$$\alpha(x,y,q) = \begin{cases} q_0(x,y)(1-e^{-\lambda T(x,y,q)}) = \frac{q_0(x,y)-q}{1-q} & \text{, if } q < q_0(x,y) \\ 0 & \text{, otherwise} \end{cases}. \tag{A.8}$$

As explained in the proof of Lemma 1, every agent will either marry or break up with her/his partner by the time the belief about the couple's compatibility reaches $q_{\min} = rc/\lambda$. Therefore, for every $x,y \in X$, it holds that $\alpha(x,y) \in [0, \frac{q_0(x,y)-q_{min}}{1-q_{min}}]$.

We denote by $q^{W_s}(x)$ the minimum between (i) agent x's breakup threshold given by the optimality condition (3) when her/his continuation value while single is $W_s(x)$, and (ii) the maximal probability that a couple is compatible. That is, $q^{W_s}(x) \equiv \min\{\frac{rW_s(x)+c}{1-rW_s(x)}\frac{r+\delta}{\lambda}, \overline{q}\}$.[19]

[19]In equilibrium, all agents must date someone, and so for any $W_s(x)$ that is part of an equilibrium, $q^{W_s}(x)$ is equal to the breakup threshold given by (3). The need to define $q^{W_s}(\cdot)$ in this way is due to the fact that

We denote by $\alpha^{W_s}: X^2 \to [0, \frac{q_0(x,y)-q_{min}}{1-q_{min}}]$ a mapping that specifies the conversion rate for any pair of agents, when they behave according to the breakup thresholds given by $q^{W_s}(\cdot)$. We endow this family with the sup norm $||\alpha^{W_s}|| = sup_{x,y\in X^2}|\alpha^{W_s}(x,y)|$.

Lemma A.1 *$\alpha^{W_s}(\cdot,\cdot)$ is continuous in W_s.*

Proof of Lemma A.1. From (A.8), it follows that

$$\alpha^{W_s}(x,y) = \begin{cases} \frac{q_0(x,y)-\max\{q^{W_s}(x),q^{W_s}(y)\}}{1-\max\{q^{W_s}(x),q^{W_s}(y)\}} & \text{, if } \max\{q^{W_s}(x),q^{W_s}(y)\} \le q_0(x,y) \\ 0 & \text{, otherwise} \end{cases}.$$

Since $W_s \in [0,\overline{W}]$, it holds that $rW_s(\cdot)$ is bounded away from 1. Thus, from (3), it follows that $\frac{dq^{W_s}(x)}{dW_s(x)}$ is uniformly bounded from above. The derivative of the conversion rate of the couple $\langle x,y\rangle$ with respect to $\max\{q^{W_s}(x),q^{W_s}(y)\}$ is $\frac{q_0(x,y)-1}{(1-\max\{q^{W_s}(x),q^{W_s}(y)\})^2}$. Since $q^{W_s}(\cdot)$ is bounded from above by $\overline{q}<1$, the absolute value of this derivative is also uniformly bounded. It follows that $\alpha^{W_s}(\cdot,\cdot)$ is continuous in $W_s(\cdot)$ in the sup norm. ■

Let $u_\alpha(\cdot)$ and $d_\alpha(\cdot)$ denote, respectively, the steady-state measure of agents in the singles pool and the measure of agents who are dating, as functions of the conversation rate $\alpha(\cdot,\cdot)$. We endow both measures with the sup norm. The next lemma establishes that for any viable $\alpha(\cdot,\cdot)$ there is a unique $u_\alpha(\cdot)$ for which the balanced flow condition (5) holds, and that this mapping is continuous. The proof of this lemma is analogous to the proof of step 1 of Lemma 4 in Shimer and Smith (2000).

Lemma A.2 *$u_\alpha(\cdot)$ and $d_\alpha(\cdot)$ are well defined and continuous.*

Proof of Lemma A.2. First, we show that for any viable $\alpha(\cdot,\cdot)$ there is a unique $u_\alpha(\cdot)$ for which the balanced flow condition (5) holds.

Fix $\alpha(\cdot,\cdot)$. The couple $\langle x,y\rangle$ date for at most

$$T^\star_\alpha(x,y) = -\frac{1}{\lambda}log\left(1-\frac{\alpha(x,y)}{q_0(x,y)}\right)$$

units of time, and so the measure of couples with pizzazz x and y that are dating is

$$d_\alpha(x,y) = \mu u_\alpha(x)u_\alpha(y)\int_0^{T^\star_\alpha(x,y)}(1-q_0(x,y)(1-e^{-\lambda t}))dt.$$

$\overline{W}$ is a loose bound on the value function.

Integrating yields

$$d_\alpha(x,y) = \mu u_\alpha(x) u_\alpha(y) \frac{\alpha(x,y) - (1-q_0(x,y))\log(1-\frac{\alpha(x,y)}{q_0(x,y)})}{\lambda}. \tag{A.9}$$

Plugging (A.9) into the balanced flow condition (5) and rearranging yields

$$u_\alpha(x) = \frac{g(x)}{1+\frac{\mu}{\lambda}\int_X \left\{\frac{\lambda}{\delta}\alpha(x,y) - (1-q_0(x,y))\log\left(\frac{q_0(x,y)-\alpha(x,y)}{q_0(x,y)}\right)\right\} u_\alpha(y)dy}. \tag{A.10}$$

Define Ω to be the space of measurable functions from X to $[\log\{\underline{l}\} - \log\{1+\frac{\mu}{\lambda}\bar{l}\}, \log\{\bar{l}\}]$, where $\underline{l} = \underline{g}$ and $\bar{l} = \bar{g} \cdot \max\{1, \int_X \left\{\frac{\lambda}{\delta}\frac{q_0(x,y)-q_{min}}{1-q_{\min}} - (1-q_0(x,y))\log\left(\frac{q_{\min}(1-q_0(x,y))}{q_0(x,y)(1-q_{\min}}\right)\right\} dy\}$. For all $x \in X$ and $\nu \in \Omega$, define

$$\Psi_\alpha \nu(x) = \log\{\frac{g(x)}{1+\frac{\mu}{\lambda}\int_X \left\{\frac{\lambda}{\delta}\alpha(x,y) - (1-q_0(x,y))\log\left(\frac{q_0(x,y)-\alpha(x,y)}{q_0(x,y)}\right)\right\} e^{\nu(y)}dy}\},$$

where $u \equiv e^\nu$. Note that $u_\alpha(\cdot)$ solves the balanced flow condition (5) if and only if $\nu = \Psi\nu$. Next, we show that Ψ is a contraction mapping, and so it has a unique fixed point.

It is straightforward to verify that Ψ_α is a map from Ω to Ω. For any $x \in X$ and $\nu^1, \nu^2 \in \Omega$, it holds that

$$\Psi_\alpha \nu^2(x) - \Psi_\alpha \nu^2(x) = \log\left\{\frac{1+\frac{\mu}{\lambda}\int_X \left\{\frac{\lambda}{\delta}\alpha(x,y) - (1-q_0(x,y))\log\left(\frac{q_0(x,y)-\alpha(x,y)}{q_0(x,y)}\right)\right\} e^{\nu^1(y)}dy}{1+\frac{\mu}{\lambda}\int_X \left\{\frac{\lambda}{\delta}\alpha(x,y) - (1-q_0(x,y))\log\left(\frac{q_0(x,y)-\alpha(x,y)}{q_0(x,y)}\right)\right\} e^{\nu^2(y)}dy}\right\}$$

$$\leq \log\left\{\frac{1+\frac{\mu}{\lambda}e^{||\nu^1-\nu^2||}\int_X \left\{\frac{\lambda}{\delta}\alpha(x,y) - (1-q_0(x,y))\log\left(\frac{q_0(x,y)-\alpha(x,y)}{q_0(x,y)}\right)\right\} e^{\nu^2(y)}dy}{1+\frac{\mu}{\lambda}\int_X \left\{\frac{\lambda}{\delta}\alpha(x,y) - (1-q_0(x,y))\log\left(\frac{q_0(x,y)-\alpha(x,y)}{q_0(x,y)}\right)\right\} e^{\nu^2(y)}dy}\right\}. \tag{A.11}$$

The first inequality uses the fact that the integrand is positive, and that $e^{\nu^1(y)} \leq e^{\nu^2(y)}e^{||\nu^1-\nu^2||}$ for all $y \in X$ under the sup norm. The term in curly brackets is increasing in $\alpha(\cdot,\cdot)$ and $u_\alpha(\cdot) \leq \bar{g}$. Since $\alpha \leq \frac{q_0(x,y)-q_{min}}{1-q_{\min}}$, it follows that the integral is bounded from above by $\bar{l}$. Since $e^{||\nu^2-\nu^1||} > 1$, the fraction in (A.11) is increasing in the integral, and so

$$\Psi_\alpha \nu^2(x) - \Psi_\alpha \nu^2(x) \leq \log\left\{\frac{1+\frac{\mu}{\lambda}e^{||\nu^1-\nu^2||}\bar{l}}{1+\frac{\mu}{\lambda}\bar{l}}\right\}.$$

Finally, observe that

$$\frac{\log\{1+e^{||\nu^1-\nu^2||}\frac{\mu}{\lambda}\bar{l}-\log\{1+\frac{\mu}{\lambda}\bar{l}\}}{||\nu^1-\nu^2||} \leq \frac{\log\{\underline{l}+\frac{\mu}{\lambda}\bar{l}^2(1+\frac{\mu}{\lambda}\bar{l})\}-\log\{\underline{l}(1+\frac{\mu}{\lambda}\bar{l})\}}{\log\{\bar{l}\}-\log\{\underline{l}\}+\log\{1+\frac{\mu}{\lambda}\bar{l}\}} \equiv \chi \in (0,1).$$

It follows that $|\psi_\alpha\nu^2(x)-\psi_\alpha\nu^1(x)| \leq \chi||\nu^1-\nu 2||$. Thus, Ψ_α is a contraction mapping, and there is a unique steady-state mass of singles that is consistent with any viable $\alpha(\cdot,\cdot)$. This, in turn, implies that $d_\alpha(x,y)$ is well defined for any $x,y\in X$ (Equation (A.10)), and hence $d_\alpha(\cdot)$ is well defined.

We now establish the continuity of $u_\alpha(\cdot)$. Rearranging (A.10) yields

$$\int_X u_\alpha(y)\left\{\alpha(x,y)(\frac{\mu}{\delta}+\frac{\mu}{\lambda})-\frac{\mu}{\lambda}(1-q_0(x,y))\log\left(\frac{q_0(x,y)-\alpha(x,y)}{q_0(x,y)}\right)\right\}dy=\frac{g(x)}{u_\alpha(x)}-1. \tag{A.12}$$

Consider how changing the value of $\alpha(\cdot,\cdot)$ by at most ϵ (for any element in its domain) impacts the term in curly brackets in (A.12). The change in the first term inside the curly brackets is at most

$$\epsilon(\frac{\mu}{\delta}+\frac{\mu}{\lambda}).$$

Since $\alpha(x,y) \leq \frac{q_0(x,y)-q_{min}}{1-q_{min}}$, it follows that $q_0(x,y)-\alpha(x,y) \geq q_{min}\frac{1-q_0(x,y)}{1-q_{min}}$, and so the absolute value of the change in the second term inside the curly brackets is at most

$$\frac{\mu(1-q_{min})}{\lambda q_{min}}\epsilon.$$

Moreover, note that since $u_\alpha(x)\in[0,\overline{g}]$ for any $\alpha(\cdot,\cdot)$, the absolute value of the change in $u_\alpha(x)$ due to any change in $\alpha(\cdot,\cdot)$ is at most $\overline{g}$. It follows that such a change in $\alpha(\cdot,\cdot)$ can change the absolute value of the LHS of (A.12) by at most

$$\overline{g}\left(\frac{\mu}{\delta}+\frac{\mu}{\lambda}+\frac{\mu(1-q_{min})}{\lambda q_{min}}\right)\epsilon.$$

Therefore, $u_\alpha(\cdot)$ is continuous in $\alpha(\cdot,\cdot)$ in the sup norm. ∎

Next, we construct the operator whose fixed points represent the set of equilibria in our model. By (1), an equilibrium value function must satisfy

$$rW_s(x)=\mu\int_X(W_d^{W_s}(x;y)-W_s(x))u^{W_s}(y)dy,$$

where $W_d^{W_s}(x;y)$ is agent x's continuation utility upon meeting agent y given $W_s(\cdot)$ (note that $W_d^{W_S}(x;y)$ is not the capital gain from dating), and $u^{W_s}(\cdot)$ are the densities in the singles pool that are consistent with value functions $W_s(\cdot)$. Adding the expectation of $W_s(x)$ to both sides and rearranging, we define the operator Γ by

$$\Gamma W_s(x) = \frac{\mu}{r + \mu \overline{u}^{W_s}} \int_X W_d^{W_s}(x;y) u^{W_s}(y) dy, \tag{A.13}$$

where $\overline{u}^{W_s} = \int_X u^{W_s}(y)dy$.

Lemma A.3 *If $K^\star$ is sufficiently large, then $W_s \in \mathcal{F}$ implies that $\Gamma(W_s) \in \mathcal{F}$.*

Poof of Lemma A.3. First, we show that $\Gamma W_s(x)$ lies in $[0, \overline{W}]$. Since $W_d^{W_s}$ is nonnegative, it is immediate that $\Gamma W_s(x) \geq 0$. An upper bound on agent x's value upon meeting agent y is given by her/his value from marrying a compatible partner immediately, separating when the dissolution shock occurs, and then being matched again to agent y according to a Poisson process with arrival rate μ. Thus,

$$W_d^{W_s}(x;y) \leq \frac{1}{r+\delta} + \frac{\delta}{r+\delta}\frac{\mu}{r+\mu} W_d^{W_s}(x;y),$$

which implies that

$$W_d^{W_s}(x;y) \leq \frac{\mu + r}{r(\delta + \mu + r)}.$$

It follows that

$$\Gamma W_s(x) \leq \frac{\mu \overline{u}^{W_s}}{r + \mu \overline{u}^{W_s}} \frac{\mu + r}{r(\delta + \mu + r)} \leq \overline{W}.$$

Next, note that $\Gamma W_s(x)$ is weakly increasing in x. This follows from $W_s(\cdot)$ being increasing, as it is an element of $\mathcal{F}$, which implies that an agent with pizzazz x can perfectly duplicate the dating time of an agent with pizzazz $x' < x$ with any potential partner.

Finally, we show that for sufficiently large $K^\star$, $\Gamma W_s(x)$ is Lipschitz continuous with modulus $K^\star$. Since ΓW_s is increasing in x, it suffices to show that $\Gamma W_s(x+\epsilon) - \Gamma W_s(x) \leq \epsilon K^\star$. Note that

$$W_d^{W_s}(x;y) = V_d^{W_s}(x;y) + W_s(x),$$

where $V_d^{W_s}(x;y)$ is agent x's capital gain from meeting agent y, given value functions W_s.

Using this representation, it follows that

$$\Gamma W_s(x+\epsilon) - \Gamma W_s(x) = \frac{\mu}{r+\mu\overline{u}^{W_s}} \int_X (V_d^{W_s}(x+\epsilon, y) - V_d^{W_s}(x;y))u^{W_s}(y)dy \\ + \frac{\mu}{r+\mu\overline{u}^{W_s}} \int_X (W_s(x+\epsilon) - W_s(x))u^{W_s}(y)dy.$$

Since $W_s(\cdot)$ is increasing and continuous (as it is an element of $\mathcal{F}$), the same arguments used in the proof of Lemma 1 show that $\int_X V_d^{W_s}(x;y)u^{W_s}(y)dy$ is Lipschitz continuous in x with modulus $\frac{r}{\mu}\overline{K}$. Thus,

$$\Gamma W_s(x+\epsilon) - \Gamma W_s(x) \leq \frac{\mu}{r+\mu\overline{u}^{W_s}} \int_X (\frac{r}{\mu}\overline{K}\epsilon + K^\star\epsilon)u^{W_s}(y)dy \leq (\frac{r}{r+\mu}\overline{K} + \frac{\mu}{r+\mu}K^\star)\epsilon.$$

Hence, if $K^\star$ is sufficiently large, then ΓW_s is Lipschitz continuous with modulus $K^\star$, and thus $\Gamma : \mathcal{F} \to \mathcal{F}$. ■

Lemma A.4 *The operator* Γ *is continuous.*

Proof of Lemma A.4. If $W_s^1(\cdot)$ and $W_s^2(\cdot)$ are close under the sup norm, then the dating times of any couple are close under these two value functions. Since small changes in $W_s(\cdot)$ also induce small changes in $u^{W_s}(\cdot)$ (by Lemmata A.1 and A.2), it follows that small changes in $W_s(\cdot)$ have a small impact on ΓW_s. ■

We have therefore shown that $\mathcal{F}$ is closed, bounded, convex, and nonempty. Moreover, since $\mathcal{F}$ is family of Lipschitz continuous functions with the same modulus, it is equicontinuous. We have also shown that Γ is a continuous mapping from $\mathcal{F}$ to $\mathcal{F}$. Thus Schauder's fixed point theorem (Theorem 17.4 in Stokey and Lucas, 1989) establishes that Γ has a fixed point, which proves Theorem 1. ■

Proof of Proposition 3. Assume by way of contradiction that, in equilibrium, X is partitioned into $n \geq 2$ blocks such that agents in a given block are (i) willing to date all agents in their own block, and (ii) unwilling to date agents from lower blocks. Let X_1 denote the highest block in the partition of X, and denote $x^\dagger = \inf X_1 > 0$. Since $q^\star(\cdot)$ is strictly increasing, for the strategy of an agent $y \in X_1$ to be consistent with (i), it must be the case that $q^\star(y) \leq q_0(y, x^\dagger + \epsilon)$ for all $\epsilon > 0$. Similarly, for the strategy to be consistent with (ii), it must be the case that $q^\star(y) > q_0(y, x^\dagger - \epsilon)$ for all $\epsilon > 0$. Hence, $q^\star(y) = q_0(y, x^\dagger)$ for all $y \in X_1$. It follows that $\lim_{x \to x^\dagger} \alpha(x, y) = 0$ for all $y \in X_1$, and so as $x \to x^\dagger$ from above, $W_s(x) \to 0$. Since $W_s(x) \geq 0$ for all x, this contradicts the strict monotonicity of $W_s(\cdot)$. ■

Proof of Theorem 2. Fix $x'' > x'$. First, consider potential partners with pizzazz $y \geq x''$. Since $q^\star(\cdot)$ is increasing, in both the couple $\langle x'', y\rangle$ and the couple $\langle x', y\rangle$, agent y is the one that breaks up with her/his partner. Moreover, s/he does so when the belief about the couple's compatibility drops to $q^\star(y)$. Since $q_0(\cdot,\cdot)$ is increasing, it follows that for all $y \in A_\alpha(x', x'')$ such that $y \geq x''$, agent y dates x'' for strictly longer than s/he dates x'. Since the couple $\langle x'', y\rangle$ is also ex-ante more likely to be compatible than $\langle x', y\rangle$, it follows that $\alpha(x', y) < \alpha(x'', y)$ for all $y > x''$.

Next, consider potential partners with pizzazz $y \leq x''$. For such potential partners, x'' is the one that breaks up with y. Moreover, x'' will date such partners with positive probability if and only if $q_0(x'', y) > q^\star(x'')$. Since $q_0(\cdot,\cdot)$ is increasing, the set of such agents that x'' marries is an interval. Thus, if either (i) the sets of potential partners with $y \leq x''$ whom x'' and x' marry do not intersect, or (ii) $\alpha(x'', y) > \alpha(x', y)$ for all $y \in A_\alpha(x', x'')$ such that $y \leq x''$, then the proposition is established.

In the remainder of the proof we therefore assume that there exists $y < x''$ such that $\alpha(x', y) > \alpha(x'', y) > 0$. Since $q^\star(\cdot)$ is continuous, it follows that the probability that a couple marries is also continuous in the pizzazz of both agents (see (A.8)). Since $\alpha(x', x'') < \alpha(x'', x'')$, the intermediate value theorem implies that there exists $y^\star < x''$ for which $\alpha(x', y^\star) = \alpha(x'', y^\star)$.

To conclude the proof, we show that $\alpha(x', y) > \alpha(x'', y)$ for every potential partner $y \in A_\alpha(x', x'')$ with pizzazz $y < y^\star$. For such y, the couple $\langle x'', y\rangle$ breaks up when $q_t(x, y) = q^\star(x'')$. Thus, by (A.8), if a couple $\langle x'', y\rangle$ date, they marry with probability

$$\alpha(x'', y) = \frac{q_0(x'', y) - q^\star(x'')}{1 - q^\star(x'')}.$$

Hence, for any $y < y^\star$ that x'' marries with a strictly positive probability, it holds that

$$\frac{d\alpha(x'', y)}{dy} = \frac{\frac{dq_0}{dy}(x'', y)}{1 - q^\star(x'')}.$$

Similarly, the probability that a dating couple $\langle x', y\rangle$ will marry is

$$\alpha(x', y) = \frac{q_0(x', y) - q^\star(\max\{x', y\})}{1 - q^\star(\max\{x', y\})}.$$

Since $q^\star(\cdot)$ is monotone, it is differentiable almost everywhere. Hence, for almost all $y < y^\star$,

$$\frac{d\alpha(x',y)}{dy} = \frac{\frac{dq_0}{dy}(x',y)}{1-q^\star(\max\{x',y\})} - \frac{1-q_0(x',y)}{(1-q^\star(\max\{x',y\}))^2}\frac{dq^\star(\max\{x',y\})}{dy}.$$

Since $x'' > \max\{x',y\}$, $q^\star(\cdot)$ is increasing, and $q_0(\cdot,\cdot)$ is supermodular, it follows that $\frac{d\alpha(x'',y)}{dy} > \frac{d\alpha(x',y)}{dy}$ for all $y < y^\star$ at which $q^\star(\cdot)$ is differentiable. By Lemma 1, $q^\star(\cdot)$ is Lipschitz continuous and hence absolutely continuous, which in turn implies that $\alpha(\cdot,\cdot)$ is also absolutely continuous. Since, by definition, $\alpha(y^\star,x'') = \alpha(y^\star,x')$, the proposition follows from the fundamental theorem of calculus. ■

Proof of Proposition 4. First, consider the limiting case in which $\lambda \to \infty$. Since an agent meets potential partners at a rate of at most $\mu < \infty$, the expected discounted amount of time during which an agent is single is bounded away from zero uniformly over λ and x. Therefore, $rW_s(x)$ is bounded away from 1 for every $x \in X$, and the first part of the proposition follows directly from (3).

Next, consider the limiting case in which $\mu \to \infty$. Equation (A.9) determines the connection between $d(\cdot,\cdot)$ and $\alpha(\cdot,\cdot)$. Integrating that equation over y yields

$$\begin{aligned} d(x) &= \int_X \mu u(x)d(x,y)u(y)dy \\ &= u(x)\int_X \frac{\mu}{\lambda}\left(\alpha(x,y) + (1-q_0(x,y))\log\left(\frac{q_0(x,y)}{q_0(x,y)-\alpha(x,y)}\right)\right)u(y)dy. \end{aligned}$$

Rearranging the balanced flow condition (5) yields

$$u(x)\left(1+\frac{\mu}{\delta}u(x)\int_X \alpha(x,y)u(y)dy\right) = g(x) - d(x).$$

Plugging in the expression for $d(x)$ derived above and rearranging yields

$$u(x) = \frac{g(x)}{1+\mu\int_X\left(\frac{\lambda+\delta}{\lambda\delta}\alpha(x,y) + \frac{1-q_0(x,y)}{\lambda}\log\left(\frac{q_0(x,y)}{q_0(x,y)-\alpha(x,y)}\right)\right)u(y)dy}. \tag{A.14}$$

Next, assume by way of contradiction that $\liminf_{\mu\to\infty} u(x) = 0$ for some $x \in X$. From (A.14), it follows that there exists $Y \subset X$ with strictly positive measure such that for every $y \in Y$, (i) $\alpha(x,y) > 0$, and (ii) $\liminf_{\mu\to\infty} \mu u(y) = \infty$. By construction, every agent with pizzazz $y \in Y$ is willing to date agent x for a strictly positive amount of time. Since agent x can start dating any $y \in Y$ after searching for an arbitrarily small amount of time, it follows that, in the limit, for almost all $y \in Y$ it must be the case that $\alpha(x, y) = 0$. Hence, any $Y \subset X$ that satisfies properties (i) and (ii) must be of measure zero, contradicting the assumption that Y has a positive measure. Therefore, $\lim_{\mu \to \infty} \mu u(x) = \infty$ for all $x \in X$.

Since each agent must wait an arbitrarily small amount of time before meeting a potential partner with every possible level of pizzazz, each agent will date only agents with the highest pizzazz among those that are willing to date her/him. Hence, $\lim_{\mu \to \infty} q^\star(x) = q_0(x, x)$. ■

Proof of Proposition 5. First, consider the limiting case in which $\lambda \to \infty$. By Proposition 4, $\lim_{\lambda \to \infty} q^\star(x) = 0$ for all $x \in X$. Hence, for all $x, y \in X$, the conversion rate is $\alpha(x, y) = q_0(x, y)$. Moreover, in this limiting case, dating is essentially instantaneous, and so $d(x, y) = 0$ for all $x, y \in X$. Plugging these two expressions into the balanced flow condition (5) yields

$$\mu u(x) \eta^u(x) = \delta(g(x) - u(x)).$$

Rearranging this expression yields the desired result.

Next, consider the limiting case in which $\mu \to \infty$. In this case, by Proposition 4, single agents with pizzazz x date only other agents with pizzazz x. Moreover, they date each potential partner for an infinitesimal amount of time. Hence, an unmarried agent with pizzazz x transitions from singlehood into marriage at a constant rate of $\lambda q_0(x, x)$, and so the expected time for which such an agent remains unmarried is $\frac{1}{\lambda q_0(x,x)}$. Similarly, a married agent with pizzazz x transitions from marriage into singlehood at a constant rate of δ, and so the expected time for which a marriage lasts is $1/\delta$. It follows that

$$\frac{\frac{1}{\lambda q_0(x,x)}}{\frac{1}{\delta}} = \frac{u(x) + d(x)}{g(x) - (u(x) + d(x))},$$

which in turn implies that

$$\frac{g(x)}{u(x) + d(x)} = \frac{\lambda}{\delta} q_0(x, x) + 1.$$

■

Derivation of the Capital Gain from Dating

Agent x's capital gain from meeting a potential partner y and then dating her/him for (at most) T units of time is

$$q_0(x,y)\int_0^T \lambda e^{-\lambda t}\left(e^{-rt}\frac{1-rW_s(x)}{r+\delta}-\frac{1-e^{-rt}}{r}(c+rW_s(x))\right)dt \\ -\left(1-q_0(x,y)(1-e^{-\lambda T})\right)\frac{1-e^{-rT}}{r}(c+rW_s(x)). \quad \text{(A.15)}$$

The first term in this expression is agent x's expected gain in case s/he clicks with agent y while dating, and the second term represents the expected cost agent x incurs when they do not click and eventually separate without marrying. Antler, Bird and Oliveros (2023) show formally that the value of learning in such problems is concave. Hence, agent x's preferred dating time is either zero, or is given by equating the derivative of (A.15) with respect to T with zero. This derivative is given by (2), and equating it with zero yields (3).

Next, we connect this breakup threshold to the dating times (i.e., derive (4)). Integrating $\dot{q}_t = -\lambda q_t(1-q_t)$ implies that

$$\frac{q_t(x,y)}{1-q_t(x,y)} = e^{-\lambda t}\frac{q_0(x,y)}{1-q_0(x,y)}. \quad \text{(A.16)}$$

For a partner y whom agent x does not date, i.e., $q_0(x,y) < q^\star(x)$, the dating time is zero. For a partner y whom agent x does date, plugging in $q_{T_x^\star(y)}(x,y) = q^\star(x)$ and rearranging yields (4). Since dating requires mutual consent, $V_d(x;y)$ is obtained by evaluating (A.15) at $T = \min\{T_x^\star(y), T_y^\star(x)\}$.