THE PINCHAS SAPIR CENTER
FOR DEVELOPMENT
Tel Aviv University

המרכז לפיתוח ע"ש פנחס ספיר
ליד אוניברסיטת תל אביב
"עמותה רשומה"
580010221

THE PINHAS SAPIR CENTER FOR DEVELOPMENT
TEL AVIV UNIVERSITY

The Effect of Privacy on Market Structure and Prices

Daniel Bird and Zvika Neeman

Discussion Paper No. 3-2023

אוניברסיטת תל-אביב, תל אביב יפו 6997801 | טל": 03-6409202 |
Tel Aviv University, Tel Aviv Yafo 6997801 Israel | Tel. 972-3-6409202 | email: sapircen@tauex.tau.ac.il

The Effect of Privacy on Market Structure and Prices

Daniel Bird and Zvika Neeman*

May 7, 2023

Abstract

Protection of consumers' privacy is often motivated by the fear that, without it, consumers may be exploited via personalized pricing. However, personalized pricing seems to be rare in practice. We explain how privacy nevertheless affects prices in search markets through its effect on market structure. If privacy is not protected, then in addition to consumer search, firms may engage in targeted advertising. We show that privacy protection reduces consumer surplus if firms price discriminate between the search and advertising markets. Absent such discrimination, privacy protection may either increase or decrease consumer surplus. We relate our results to the "privacy paradox."

JEL CODES: D40, D83, L10.

KEYWORDS: privacy, price discrimination, search, targeted advertising, interest-based advertising, privacy paradox.

*Eitan Berglas School of Economics, Tel Aviv University (e-mails: `dbird@tauex.tau.ac.il`, `zvika@tauex.tau.ac.il`). Bird and Neeman acknowledge financial support from the Sapir Center for Economic Development..

1 Introduction

The ongoing debate about the need to protect consumers' privacy in retail markets is, to a large extent, driven by the fear that sellers might use consumer data to exploit consumers via personalized pricing. However, empirical evidence suggests that the use of personalized pricing is rather limited in practice. For example, a recent Ipsos, London Economics and Deloitte (2018) field survey conducted on behalf of the European Union found that personalized pricing "was observed in only 6% of matched identical product pairs. Even when price differences were observed, the differences were small, with the median difference being less than 1.6%" (p. 260).[1] Furthermore, Amazon – the largest online retailer in the world – publicly committed to not use personalized pricing.[2]

In this paper, we suggest an alternative channel through which the regulation of privacy may affect consumer welfare. We explain how privacy protection, or the lack thereof, may affect the distribution of market prices through its effect on market structure. Specifically, in many retail markets consumers or firms (or both) must search for one another before they can transact. Protection of consumer privacy effectively shuts down the possibility for firms to search for consumers via targeted advertising. Hence, if consumer privacy is protected, then only a *search market*, in which (only) consumers search for the firms that sell the goods they are interested in, may exist. If, on the other hand, consumer privacy is not protected, then firms may acquire relevant information about consumers, which facilitates targeted advertising. This implies that alongside search markets, there may also exist *advertising markets*, in which firms make direct offers to consumers who are interested in the goods they sell. Equilibrium prices in the search market obviously depend on whether firms may also reach consumers through targeted advertising, and so privacy, or the lack thereof, affects both market structure and pricing.

For simplicity, we focus our attention on the empirically important case in which targeted advertising takes the form of "interest-based advertising."[3] Interest-based advertising is used by a firm that knows nothing about consumers beyond the fact that they

[1] This finding is consistent with previous findings. See the references in the aforementioned report.

[2] In September 2000 Amazon famously outraged consumers by charging different prices to different consumers for the same DVDs. In response to the scandal that erupted, Amazon's CEO Jeff Bezos committed to charge a single price for each good on Amazon (see, e.g., "Amazon apologizes for price-testing program that angered customers," by Todd R. Weiss for Computerworld (9/28/2000) and "Web sites change prices based on customers' habits," by Anita Ramasastry for CNN.com (6/24/2005)).

[3] The Interactive Advertising Bureau estimates interest-based advertising revenues in the United States alone to be equal to \$107.5 billion for 2018. These revenues are expected to continue to grow rapidly.

are possibly interested in the general category of goods that it produces. For example, if a person searches for the term "London" online, then a firm that tracks this person may realize that he may be interested in flying to England in the near future, but it has no idea about the specific dates on which he would like to travel, the exact destination he would like to visit, or the maximal amount he would be willing to pay for airfare.[4,5] Needless to say, interest-based advertising does not allow for personalized pricing.

In this paper we study the effect of consumer privacy on consumer surplus, firms' profits, and social welfare when trade can potentially occur via either a competitive search market or interest-based advertising. To study these questions, we consider a model with a large number of consumers and firms. A small fraction of the consumers in the population are interested in a certain good and the willingness of these consumers to pay for the good is fixed and commonly known by firms. These consumers have access to a search market in which they engage in a "noisy search" for the firms that produce the good. This means that each consumer's search effort yields a random number of new relevant price quotes, which is possibly larger than one. If consumer privacy is not protected, then firms can also reach interested consumers through interest-based advertising. As with search markets, we assume that the number of relevant interest-based ads that reach an interested consumer is random. Importantly, we allow the distribution of the number of relevant price quotes that a consumer obtains through search to be different from the distribution of the number of price quotes that are observed through ads. Finally, we assume that if privacy is not protected, then consumers are first exposed to a random number of relevant ads before deciding whether or not to search for better price quotes.

We show that the effect of privacy protection crucially depends on whether firms price

[4] The website youradchoices.com (see https://youradchoices.com/choices-faq#jr02) that describes interest-based advertising explains that "a typical set of information associated with a user's web browser might include:

Gender: Male

Age range: 25–34

Geography: Washington DC metro area

Interested in baseball

Interested in travel to Europe

Car shopper."

[5] The prevalence of interest-based advertising may be explained by limits on firms' ability to process data in real time: even with all the information that firms are able to glean about consumers, possible interest in the general category of goods they sell is often all the information that firms may be able to obtain, or process, about consumers.

discriminate between the search and advertising markets (henceforth, just "price discriminate"), that is, whether firms quote one price to consumers that they reach through interest-based advertising and another price to unsolicited consumers who reach them through an active search. Such price discrimination is certainly very easy to perform with available technology, for example, by including coupon codes in firms' ads or by using different landing pages for search engines and ads.

We obtain the following main results. First, if firms price discriminate, then protection of consumer privacy hurts consumers. Intuitively, this is because if firms price discriminate, then equilibrium prices in the search market are unaffected by the opening of the advertising market, and so privacy protection denies consumers the option of buying goods through the advertising market. The same intuition implies that privacy protection hurts firms if and only if the advertising market is less competitive than the search market, where the competitiveness of a market is determined by the distribution of the number of price quotes in that market (formal definition below). Since consumers probably pay less attention to the advertising that they are exposed to compared to what they discover through their own search, it seems plausible that the number of price quotes consumers obtain by search would be larger than the number of relevant price quotes obtained from interest-based ads.[6] It follows that, in practice, relaxation of consumer privacy protection is likely to lead to a Pareto improvement. This observation sheds light on the so-called "privacy paradox" by showing how consumers who express concerns about maintaining their privacy in surveys nevertheless willingly share their private information with firms, and, moreover, benefit from doing so.[7]

Second, if firms do not price discriminate, then if we hold constant the degree of competitiveness of the "merged search and advertising market" (merged because each firm charges the same price in both markets), privacy protection helps consumers if and only if the advertising market is sufficiently "large," where the size of the advertising market is measured by the mass of consumers that are exposed to at least one relevant ad. Again, privacy protection hurts firms if and only if the advertising market is less competitive

[6] For example, Acquisti, Taylor and Wagman (2016) write that "despite the large sums of money spent on targeted advertising [...] its effectiveness is unclear" (p. 464) and Lewis and Rao (2015) write that "given the total volume of advertising [that] a typical consumer sees across all media, even an intense campaign only captures about 2% of a user's advertising 'attention' "(p. 1948).

[7] See Acquisti, Taylor and Wagman (2016) and the references therein. In recent work, Madarasz and Pycia (2021) provide an alternative explanation for the paradox by considering the equilibrium of a model in which firms invest in collecting information about consumers' preferences and consumers counter-invest to prevent them from doing so.

than the search market. As before, we find that relaxation of consumer privacy protection may yield a Pareto improvement. However, even when privacy protection increases consumer surplus, each individual consumer benefits from sharing his own private information with firms. This insight provides another possible explanation for the privacy paradox: consumers understand that they benefit collectively from privacy protection, but each consumer personally prefers to share his information with firms.

Finally, we show that price discrimination increases consumer surplus if and only if it reduces firms' aggregate profits. The intuition for this result is that price discrimination determines the distribution of surplus between consumers and firms, but the volume of trade and the number of searches that are performed until a consumer finds an acceptable price offer are identical. This result suggests that if price discrimination benefits firms, it should be constrained by regulators who are concerned about consumer surplus. On the other hand, if price discrimination hurts firms, then they would like to commit to not engage in price discrimination.

Related Literature

Privacy. For an excellent recent survey of the economics of privacy, see Acquisti, Taylor and Wagman (2016). They observe that privacy protection may both benefit consumers by limiting firms' ability to extract their consumer surplus through price discrimination and hurt consumers by increasing their search costs and denying firms information that would allow them to better cater to consumers' tastes (Bergemann and Bonatti, 2011). Thus, privacy protection may either benefit or hurt consumers, depending on which of these two effects is stronger. In some environments, sellers may benefit from committing to full privacy (Calzolari and Pavan, 2006), and in others they may benefit from committing not to engage in price discrimination (Ichihashi, 2020).[8] In yet other cases, it may be possible to design consumer privacy to maximize consumer surplus (Bird and Neeman, 2022).[9] Elliott et al. (2022) study how to design firms' information about consumers in a way that weakens competition between firms and enables firms to extract the full surplus from trade without collusion or any long-term incentives. Mauring (2022) analyzes the effect of privacy when firms can obtain information about consumers' search costs rather

[8] Relatedly, Ali, Lewis and Vasserman (2020) and Pram (2021) show that in order to improve his terms of trade, an informed buyer may wish to give up his privacy by disclosing hard information about himself.

[9] Bergemann, Brooks and Morris (2015) and Haghpanah and Siegel (2020) characterize the distribution of surplus that can be attained via different market segmentation in a canonical buyer–seller setting.

than about their willingness to pay for a good. She finds that, in such settings, privacy protection generally benefits consumers. Finally, Fainmesser, Galeotti and Momot (to appear) study how revenue models of businesses that hold consumers' private information affects how much of it they sell to third parties.

Search and Price Dispersion. Our model of the search market is based on Burdett and Judd's (1983) model of "noisy search." Baye et al. (2006) survey the extensive literature on consumer search and price dispersion. They divide the theoretical part of the literature into two strands. One strand shows that price dispersion can arise in sequential search models in which consumers obtain a deterministic number of price quotes, at a cost, in every period. The other strand employs information clearinghouse models (e.g., Baye and Morgan, 2001) in which some consumers have access to the entire set of firms that sell the good they want, for a small cost. Notice that Burdett and Judd's (1983) model of noisy search belongs to neither of these strands of the literature. Baye et al. (2006) also report on the large empirical literature that shows that price dispersion is extremely common in practice.

Targeted Advertising and Search Markets. Butters (1977) is probably the first to have considered a model with price dispersion in which a search market and an advertising market for a single good operate side by side. Robert and Stahl (1993) consider a model with homogeneous products and study the implications of price advertising when shopping trips are costly to consumers. They do not consider privacy as such, but it is of course possible to interpret lower advertising costs in their model as weaker privacy protection. They showed that as advertising costs vanish, then prices become competitive. By contrast, in our setting, less privacy can hurt consumers in some cases, which suggests that lowering advertising costs might harm consumers.

More recently, Braghieri (2019) has analyzed firms' pricing decisions and consumers' adoption of anonymizing technologies in markets where advertising slots are sold by a two-sided intermediary, but consumers' willingness to pay (for each good) varies. In equilibrium, firms engage in price discrimination, there is no price dispersion, and the introduction of tracking technologies makes all consumers better off. However, unlike us, Braghieri assumes that the quality of the match between consumers and firms is lower in the search market than in the advertising market, which implies that prices in the former are lower than those in the latter. Also, unlike in our model, the search market in his model would collapse if firms were not allowed to price discriminate. Another paper in which targeted advertising intensifies price competition when combined with consumer search is that of De Cornière (2016). He considers a setting with heterogeneity in consumers' willingness to pay and a standard sequential search technology. In his model, advertisers target consumers based on their search queries, after they complete their searches unsuccessfully. He shows that such targeting reduces consumers' search costs, improves matches, and intensifies price competition. Competition among search engines in his model can either increase or decrease welfare, depending on the extent of multi-homing by advertisers.

Price Discrimination. Intuitively, price discrimination benefits firms at the expense of consumers. In his survey of the relevant literature, Armstrong (2006) notes that an increased ability to engage in price discrimination generally boosts the profit of a monopolistic firm, unless the firm cannot commit to its pricing policy. In competitive markets, the effects of price discrimination on consumer surplus, firms' profits, and social welfare depend on the kinds of information and/or tariff instruments available to firms. If firms agree about whether specific information about consumers implies that they should set higher or lower prices, as is the case in our setting, then price discrimination typically increases firms' profits. But this is not so if firms disagree.

* * *

The rest of the paper proceeds as follows. In Section 2 we present the model. In Section 3, we analyze equilibria both in the case where consumer privacy is protected, i.e., where only the search market exists, and in the case where consumer privacy is not protected, i.e., where both the search and advertising markets exist. In the latter case, we distinguish between the subcases in which firms engage and do not engage in price discrimination. In Section 4 we compare equilibria in terms of their induced consumer surplus, firms' profits, and social welfare. In Section 5 we offer concluding remarks.

2 Model

We consider a market for a good that is produced by a large number of firms at a marginal cost of zero. There exists a large population of consumers of which only a small fraction are interested in the good. These interested consumers each have a willingness to pay of $v > 0$ for (one unit of) the good. We normalize the mass of interested consumers to one.

Each firm quotes a search market price for the good. Consumers can search for firms by using a noisy sequential search technology (as in Burdett and Judd, 1983). In each round of search, each consumer pays a cost of $c > 0$ to obtain a random number of new price quotes. After observing the realized price quotes, each consumer decides either to buy the good at the best price obtained up to then, to quit the market, or to pay the search cost c again and obtain yet more new price quotes. For simplicity, we assume that the number of price quotes obtained in each round of the search is at most two. We denote the probability of obtaining $n \in \{0, 1, 2\}$ price quotes in each round of the search by q_n^S. We further assume that[10] $0 < q_1^S, q_2^S < 1$.

The main objective of this paper is to understand the effect that privacy protection has on prices through its effect on market structure. In particular, we say that consumer privacy is protected if firms do not have access to data that enables them to identify the consumers that are interested in the good. That is, privacy protection prevents firms from using interest-based advertising to contact interested consumers. Accordingly, our model can also be used to analyze the impact of other policies that affect the effectiveness of advertising, such as "no solicitation" laws, which prohibit solicitation and the distribution of printed material on designated property.

If consumer privacy is not protected, then firms can contact the consumers who are interested in the good that they produce through costless interest-based advertising. Nevertheless, due to the limited attention consumers pay to firms' ads (and their limited screen size), the fact that firms can contact consumers does not necessarily imply that consumers will notice that they have done so. Hence, as in the case of consumer search, we assume that each consumer is exposed to a random number of relevant ads, and, for symmetry with the search market, we also assume that the number of ads that a consumer observes is at most two. We denote the probability that each consumer receives $n \in \{0, 1, 2\}$ relevant ads by q_n^A, and, as in the case of consumer search, assume that $0 < q_1^A, q_2^A < 1$. Finally, we assume that firms quote an ad price that may be different from their search market price, and that consumers receive interest-based ads once and that they receive them *before* deciding whether or not to search.

We analyze three distinct market regimes. The first regime is where consumer privacy is protected. Under this regime, consumers may search firms that produce the good, but there is no advertising. We denote this regime by $\mathcal{P}$ for "privacy." Under the second

[10] We discuss the assumption that consumers obtain no more than two price quotes in Section 2.2 and explain why this assumption does not impact our qualitative results in Section 5. The other assumption allows us to abstract away from corner cases that complicate the exposition without adding content.

and third regimes, consumer privacy is not protected and firms have access to consumer data, which enables interest-based advertising. Under the second regime, firms may price discriminate between the search and advertising markets by offering a different price in each of these two markets. We denote this regime by $\mathcal{D}$ for "discrimination." The third regime is identical to the second, except that firms are required to charge identical prices in the search and advertising markets. We denote this regime by $\mathcal{ND}$ for "no discrimination." In Appendix B, we also consider a variation of the model, in which the level of privacy protection is a continuous variable that is measured by the probability that consumers receive relevant ads. The qualitative results obtained in that variant are identical to those in the baseline model.

Fix a regime. The timing of the model is as follows. First, firms set prices in the relevant markets. Next, consumers receive interest-based ads (unless the regime is $\mathcal{P}$). Finally, consumers decide either to buy the good through an ad they received (if they received one), to search for the good themselves, or to quit the market. Importantly, we assume that even though consumers receive ads only once, they can search as many times as they want. Moreover, we assume that the search is with perfect recall. That is, after each round of the search consumers can purchase from every price quote they obtained in the past (from ads or searches).

2.1 Strategies and Equilibrium

Consumers

By standard arguments, under all three regimes, consumers' strategies can be described by a single reservation price $\tilde{p}$ such that a consumer who has obtained a price quote (through ads or searches) that is less than or equal to $\tilde{p}$ buys the good immediately at the lowest such price.[11] The consumers' reservation price $\tilde{p}$ is determined such that consumers are indifferent between purchasing the good at price $\tilde{p}$ and the better option of quitting the market and searching one more time for additional price quotes, and then purchasing the good at the lowest price observed up to then.[12] This implies that the

[11] Because a consumer will compare the best available offer with the distribution of offers from search, the same reservation price is maintained in both the advertising and search markets.

[12] At the reservation price $\tilde{p}$ a consumer is exactly indifferent between searching and not searching. Thus, it may be assumed that if the next round of the search does not generate a price that is lower than $\tilde{p}$, the consumer will not search again.

reservation price $\tilde{p}$ satisfies the following condition:

$$v - \tilde{p} = \max\left\{0, v - c - \mathbb{E}\left(\min\{\tilde{p}, p_{min}\}\right)\right\}, \tag{1}$$

where p_{min} is the lowest price quote that is obtained in one additional round of search (if no quotes are obtained in the next round of search, then we set $p_{min} = \infty$).

Firms

Firms' profits in each market (under any regime) depend on the distribution of the number of price quotes observed by consumers before purchasing (Burdett and Judd, 1983). We denote this endogenously generated equilibrium distribution by β. We distinguish between three different distributions of price quotes: a distribution of the number of price quotes in the search market (under regimes $\mathcal{P}$ and $\mathcal{D}$), a distribution of the number of price quotes in the advertising market (under regime $\mathcal{D}$), and a distribution of the number of price quotes in the "merged market" (under regime $\mathcal{ND}$). We refer to the latter as the merged market since in the case where firms advertise and consumers search but firms quote the same price in both the search and advertising markets, it is possible to interpret the general equilibrium as involving just one, merged, market. The formal description of these distributions is given in the relevant sections below.

In equilibrium, firms will offer prices that are no higher than $\tilde{p}$. Thus, a consumer will not search if he has received an ad, or will search until the first time his search yields at least one price quote. Therefore, without loss of generality, for the rest of the paper we assume that the number of price quotes obtained by a consumer before purchasing is no more than two. It follows that in market $z \in \{S, A, M\}$ (where $z = S, A, M$ represents the search, advertising, and merged markets, respectively) the expected profit of a firm that quotes a price p, when the consumers' reservation price is $\tilde{p}$, is given by

$$\Pi^z(p) = \begin{cases} p\Omega^z \sum_{n=1}^{2} \beta_n^z n(1 - F^z(p))^{n-1} & \text{if } p \leq \tilde{p} \\ 0 & \text{if } p > \tilde{p} \end{cases}, \tag{2}$$

where β_n^z is the equilibrium probability that a consumer observes n price quotes before purchasing, Ω^z is the mass of consumers that buy in the market, and $F^z(\cdot)$ is the distribution of prices in market[13] z.

[13]Observe that consumers who obtain two price quotes are doubly represented in the population of consumer–price quote pairs.

Equilibrium

Our model admits the existence of a general equilibrium with a degenerate search market in which consumers do not search because firms quote high prices, and firms quote high prices because consumers do not search.[14] To abstract away from such degenerate general equilibrium, we focus only on general equilibria in which consumers do actively search for the good in the search market. That is, we focus our attention on general equilibria in which a consumer who has observed no price quotes would rather search for price quotes than quit the search market.

In a general equilibrium with active consumer search, it holds that

$$v - \tilde{p} = v - c - \mathbb{E}\left(\min\{\tilde{p}, p_{min}\}\right),$$

and so the consumers' reservation price solves the following simplified version of Equation (1):

$$\tilde{p} = c + \mathbb{E}\left(\min\{\tilde{p}, p_{min}\}\right). \tag{3}$$

In equilibrium, firms take the consumers' reservation price $\tilde{p}$ as given, and set prices optimally in each of the markets that exist under the prevailing regime. Under regime $\mathcal{P}$, only the search market exists; under regime $\mathcal{D}$, both an advertising market and a search market exist; and under regime $\mathcal{ND}$, only a merged advertising and search market exists. We denote the set of markets that exist under regime R by $Z(R)$.

Definition (Equilibrium with Active Consumer Search)**.** *A tuple* $\langle \tilde{p}, \{F^z(\cdot)\}_{z \in Z(R)} \rangle$ *is a general equilibrium with consumer search under regime* R *if:*

1. *The consumers' reservation price* $\tilde{p}$ *solves Equation* (3) *given the distribution of firms' price quotes in the search market.*[15]

2. *Every price* p *that is quoted by firms under regime* R *maximizes firms' profits in the market in which it is quoted.*

3. *A search yields a nonnegative expected payoff for consumers who have received no previous price quotes.*

[14] These equilibria of the search market are similar to those described by Diamond (1971) in which firms quote the monopolistic price and consumers refrain from incurring the cost of search.

[15] Under regime $\mathcal{ND}$, these are the price quotes in the merged market.

Note that an equilibrium with an active search market need not exist for all parameters. In the following section we derive conditions that determine when an equilibrium with active consumer search exists under each of the three regimes that we consider. Henceforth, we refer to equilibria with active consumer search under regime R as equilibria.

The price quoted by a firm in a given market depends on the number of additional price quotes that a consumer who has been exposed to the firm's own price quote is expected to observe. In the degenerate case in which a firm expects to be the only firm that a consumer is exposed to, the firm will act as a monopolist and quote the highest price that consumers are willing to pay, i.e., $\tilde{p}$. In the other extreme case, in which a firm expects a consumer who is exposed to its price quotes to observe (at least) one other price quote, the firm will act as it would in a competitive market and quote a price that is equal to its marginal cost, i.e., zero. In our model, firms are uncertain about the number of additional price quotes consumers are exposed to. As shown by Burdett and Judd (1983), in this case firms adopt a mixed strategy in equilibrium. In this equilibrium, both the price distribution and the consumers' reservation price are jointly determined by the distribution of the number of price quotes obtained by the consumer before purchasing the good.

As the probability that consumers obtain multiple price quotes increases, the market becomes more competitive and market prices decrease. To gain more insight into the roles played by the probabilities $\{q_0^S, q_1^S, q_2^S\}$ and $\{q_0^A, q_1^A, q_2^A\}$, note that the probability that consumers obtain no price quotes per search, q_0^S, is essentially a multiplier of the consumers' search costs because if consumers do search, then they will do so until they obtain at least one price quote. Thus, the consumers' "effective search costs" are given by $\frac{c}{1-q_0^S}$ and the "effective probabilities" of observing one or two price quotes per consumer search are given by $\frac{q_1^S}{q_1^S+q_2^S}$ and $\frac{q_2^S}{q_1^S+q_2^S}$, respectively. Similarly, the probability that consumers receive no ads, q_0^A, is also a parameter that is not directly related to the competitiveness of the advertising market. However, unlike q_0^S, it is possible to interpret q_0^A as an inverse measure of the "size of the advertising market" because the measure of consumers who receive at least one ad and, in equilibrium, go on to purchase the good without searching for it is $1 - q_0^A$. As in the search market, the "effective probabilities" of observing one or two price quotes through interest-based ads are $\frac{q_1^A}{q_1^A+q_2^A}$ and $\frac{q_2^A}{q_1^A+q_2^A}$, respectively.

The above discussion suggests that once the consumers' effective search costs and the size of the advertising market are fixed, we may use $\frac{q_2^S}{q_1^S+q_2^S}$ and $\frac{q_2^A}{q_1^A+q_2^A}$ as measures of the competitiveness of the search and advertising markets, respectively. Accordingly, for the rest of this paper we use these ratios as measures of the competitiveness of the search and advertising markets, respectively.

2.2 Discussion of Assumptions

At Most Two Price Quotes. Our assumption that a consumer receives at most two price quotes in each round is a simplifying assumption that enables us to obtain closed-form solutions for equilibrium outcomes. In Section 5 we explain why our main qualitative results do not depend on this assumption. That being said, we note that previous work suggests that the probability with which a consumer obtains exactly one price quote is the most important statistic of the distribution of the number of price quotes. Burdett and Judd (1983) show that this probability is what determines whether a firm will make a profit, and, if the reservation price is held constant, what this profit will be. More recently, Bergemann, Brooks and Morris (2021) have shown that the probability that the consumer observes one price quote forms an upper bound on firms' profits that is robust to the firms' information structure.

Sequence of Ads and Searches. Our assumption that consumers receive ads before they search aims to capture the following sequence of events. First, an event that creates an interest in some specific good occurs. For example, a consumer realizes that he will need to purchase a good by some future date. At the time when the interest arises, the consumer plans to search (and purchase) the good at some later point in time. In the interim, until the time when the consumer plans to actively search for the good, the consumer may engage with (relevant) ads that he receives. Our model abstracts away from the choice of when to purchase the good, but implicitly assumes that there is some fixed time by which the good needs to be purchased. The existence of such a deadline prevents the consumer from obtaining an infinite number of ads before purchasing, and is consistent with our assumption that the consumer can choose to search multiple times even though he receives ads only once. A noteworthy consequence of this assumption is that, in our model, firms cannot learn about a consumer's preferences through his search activities (such learning is the main focus of De Cornière, 2016).

Privacy Protection is Not a Consumer's Choice. In our model the decision of whether or not to protect consumer privacy is a social choice. It is not a decision that each consumer makes individually, for himself. In fact, in our model each consumer would benefit (individually) from sharing his data because this would not affect prices in a general equilibrium, and would provide the consumer with more opportunities to purchase the good. Thus, our notion of privacy should be interpreted as allowing a regulator to control the manner in which firms use consumer data, rather than as allowing each individual consumer to control how his data is used by firms. Hence, our notion of privacy is distinct from the way privacy is treated in many of the recent laws that protect consumer privacy. For example, the EU's General Date Protection Regulation—in particular, Chapter 3 thereof—establishes the right of the "data subject" to control how his data is used. We discuss this further in our concluding remarks (Section 5).

3 Equilibrium Analysis

In this section we characterize the general equilibrium of our model under all three regimes.

3.1 Equilibrium Analysis under Regime $\mathcal{P}$

When consumer privacy is protected, only the search market exists, and so the analysis is almost identical to that performed by Burdett and Judd (1983). In fact, a minor modification of their Theorem 4 establishes the existence of a unique equilibrium with active consumer search. Our assumption that the number of price quotes obtained in each round of consumer search is at most two enables us to derive a simple condition that characterizes when it is profitable for consumers to engage in a search and, moreover, to derive closed-form expressions of firms' profits and consumer surplus in this equilibrium.

Proposition 1. *Consider regime $\mathcal{P}$. There exists an equilibrium with active consumer search if and only if*

$$v \cdot q_2^S \geq c. \tag{4}$$

Moreover, if Condition (4) *holds, then there exists a unique equilibrium with active consumer search in which the consumer surplus is*

$$CS^{\mathcal{P}} = v - \frac{c}{q_2^S},$$

and the firms' profits are

$$\pi^{\mathcal{P}} = \frac{q_1^S}{1 - q_0^S} \frac{c}{q_2^S}.$$

Condition (4) has a simple interpretation: consumers engage in an active search if and only if the probability of obtaining two price quotes is high enough. In other words, the consumers engage in an active search if and only if the search market is competitive enough to avoid unraveling as in the Diamond paradox (Diamond, 1971), where firms quote the monopolistic price and consumers refrain from incurring the cost of search.

Proposition 1 shows that if consumers engage in an active search, then the total surplus in the search market is given by

$$CS^{\mathcal{P}} + \pi^{\mathcal{P}} = v - \frac{c}{1 - q_0^S}.$$

Because in equilibrium all the consumers purchase the good and receive a payoff of v, social welfare is equal to v minus the consumers' expected search costs until obtaining at least one price quote, i.e., $\frac{c}{1-q_0^S}$. It follows that if the value of q_0^S is held constant, then increasing the competitiveness of the market by increasing the value of q_2^S at the expense of q_1^S transfers surplus from firms to consumers.

3.2 Equilibrium Analysis under Regime $\mathcal{D}$

To characterize the general equilibrium under this regime (in which both the search and advertising markets coexist), note that the consumers' reservation price is determined by the equilibrium distribution of prices in the search market. Moreover, the search market under this regime is identical to the search market under regime $\mathcal{P}$, except that the mass of consumers in the search market is q_0^A rather than one.

Because the size of the search market (the fraction of consumers who engage in an active search) does not affect the firms' pricing decisions, the consumers' reservation price and the firms' pricing decisions in the search market are identical under regimes $\mathcal{P}$ and $\mathcal{D}$. Moreover, the same calculations that were used to establish Proposition 1 can be used both to determine when a general equilibrium with active consumer search exists, to characterize the equilibrium in the advertising market, and to derive the consumer surplus and firms' profits.

Proposition 2. *Consider regime $\mathcal{D}$. There exists a general equilibrium with active consumer search if and only if Condition* (4) *holds. If Condition* (4) *holds, then there exists a unique equilibrium with active consumer search in which the consumer surplus is*

$$CS^{\mathcal{D}} = v - \frac{c(1 - q_2^A)}{q_2^S},$$

and the firms' profits are

$$\pi^{\mathcal{D}} = \left(q_1^A + q_0^A \frac{q_1^S}{1 - q_0^S} \right) \frac{c}{q_2^S}.$$

Note, first, that social welfare is given by

$$CS^{\mathcal{D}} + \pi^{\mathcal{D}} = v - \frac{q_0^A}{1 - q_0^S} c.$$

Thus, social welfare is increasing in the size of the advertising market $1 - q_0^A$. This result is due to the fact that in equilibrium consumers purchase the good when they receive their first price quote. Hence, those consumers who receive an ad avoid the search cost, while those who do not receive an ad incur an expected search cost of $\frac{c}{1-q_0^S}$.

Second, as was the case under regime $\mathcal{P}$, if the consumers' effective search costs and the size of the advertising market are held constant, increasing the competitiveness of either market by increasing the value of q_2^S at the expense of q_1^S or by increasing the value of q_2^A at the expense of q_1^A transfers surplus from firms to consumers.

3.3 Equilibrium Analysis under Regime $\mathcal{ND}$

The characterization of the equilibrium in the merged market is more complicated than under the previous two regimes. The difficulty is due to the fact that the distribution of the number of price quotes that a consumer observes before buying the good (which is the object that determines the firms' pricing decisions) is different from the distribution of the number of price quotes generated by an additional round of search (which is the object that determines the consumers' reservation price).

To derive the equilibrium, we first characterize the expected price that a consumer pays for the good if he decides to search for it. That is, we derive the expectation of the lowest price quote that the consumer receives, conditional on receiving at least one price quote. Let q^S denote the vector $\langle q_0^S, q_1^S, q_2^S \rangle$ and recall that β_1 denotes the fraction of consumers that observe one price quote before buying the good.

Lemma 1. *In equilibrium, the ratio between the expected price that is paid by a consumer who searches for price quotes and his reservation price is*

$$\kappa(q^S, \beta_1) = \frac{\beta_1 \left(2q_2^S \left(\beta_1 \left(1 + \tanh^{-1}(1 - \beta_1) \right) - 1 \right) + (1 - \beta_1) q_1^S \log \left(\frac{\beta_1}{2 - \beta_1} \right) \right)}{2(1 - \beta_1)^2 (q_0^S - 1)}$$

and $\tanh^{-1}(\cdot)$ *denotes the inverse of the hyperbolic tangent function.*

Lemma 1 implies that $\mathbb{E}[p_{min}|\text{at least one price quote}] = \tilde{p} \cdot \kappa(q^S, \beta_1)$. It allows us to characterize when it is profitable for consumers to engage in an active search, and to derive the consumer surplus and the firms' profits. If a consumer engages in an active search then he obtains exactly one price quote before he purchases the good in two cases: when he receives one ad, and when he receives no ads and the first round of the search that generates a price quote yields exactly one price quote. It follows that, when the search market exists, $\beta_1^{\mathcal{ND}} = q_1^A + q_0^A \frac{q_1^S}{1-q_0^S}$.

Proposition 3. *Consider regime $\mathcal{ND}$. There exists an equilibrium with active consumer search if and only if*

$$v \geq \frac{c}{(1-q_0^S)\left(1-\kappa(q^S, \beta_1^{\mathcal{ND}})\right)}. \tag{5}$$

If Condition (5) *holds, then there exists a unique equilibrium with active consumer search in which the consumer surplus is*

$$CS^{\mathcal{ND}} = v - \frac{c}{1-q_0^S}\left(\frac{\kappa(q^S, \beta_1^{\mathcal{ND}})}{1-\kappa(q^S, \beta_1^{\mathcal{ND}})} + q_0^A\right),$$

and the firms' profits are

$$\pi^{\mathcal{ND}} = \frac{c}{1-q_0^S}\frac{\kappa(q^S, \beta_1^{\mathcal{ND}})}{1-\kappa(q^S, \beta_1^{\mathcal{ND}})}.$$

Note that when consumers engage in an active search, social welfare is the same as under regime $\mathcal{D}$, and it is given by $v - \frac{c}{1-q_0^S} q_0^A$. This is because under both regimes all the consumers purchase the good and only those consumers who do not receive an ad must incur the search cost. Moreover, Lemma A.1 in Appendix A shows that $\kappa(q^S, \beta)$ is increasing in β. It follows that, as in the previous two regimes, if the consumers' effective search costs and the size of the advertising market are held constant, increasing the competitiveness of either the advertising market or the search market transfers surplus from firms to consumers.

4 Welfare Analysis

In this section we analyze how the removal of consumer privacy protection affects consumer welfare and firms' profits. In particular, we compare the equilibrium payoffs under regime $\mathcal{P}$ with those of regimes $\mathcal{D}$ and $\mathcal{ND}$. In the remainder of the paper we assume that Conditions (4) and (5) hold, i.e., that there exists a unique equilibrium with active consumer search under all three regimes.

The Value of Privacy to Consumers

First, we evaluate the benefit of privacy to consumers when firms price discriminate between the search and advertising markets. As explained in Section 3, the equilibrium in the search market is identical under regimes $\mathcal{P}$ and $\mathcal{D}$ and, in particular, the consumers' reservation price is the same under both regimes. It follows that the removal of privacy protection is beneficial to consumers because it simply provides them with more opportunities to buy the good, without affecting prices in the search market.

Proposition 4. *Suppose that Condition* (4) *holds. If firms price discriminate between the search and advertising markets, then privacy protection hurts consumers.*

While under price discrimination privacy protection has an unambiguous effect on consumer surplus, when firms do not price discriminate (regime $\mathcal{ND}$) the effect of privacy protection on consumer surplus depends on the size and competitiveness of both markets. On the one hand, privacy protection prevents firms from initiating contact with consumers, which forces consumers to search and therefore increases consumers' search costs. Moreover, the size of this loss is increasing in the size of the advertising market. On the other hand, removal of privacy protection changes the underlying "market competitiveness" as measured by the fraction of consumers who observe more than one price quote before they buy the good. If the advertising market is more competitive than the search market, then the removal of privacy protection increases market competitiveness overall and is unambiguously beneficial to consumers. However, if the advertising market is less competitive than the search market, which seems to be the more likely case in practice, then the removal of privacy protection decreases market competitiveness and consumers can be hurt if the implied reduction in their search costs is outweighed by the decrease in market competitiveness.[16]

Proposition 5. *Fix the values of v, c, and q^S and suppose that Conditions* (4) *and* (5) *hold. If firms do not price discriminate, then the removal of privacy protection increases consumer welfare if and only if the advertising market is sufficiently "large and competitive." That is, there exists a threshold $q^*(\beta_1^{\mathcal{ND}})$ such that the removal of privacy protection increases consumer welfare if and only if $q_0^A < q^*(\beta_1^{\mathcal{ND}})$. Moreover, $q^*(\beta_1^{\mathcal{ND}})$ is decreasing in $\beta_1^{\mathcal{ND}}$.*

[16]Observe that market competitiveness is given by $\frac{q_2^S}{q_1^S+q_2^S}$ under regime $\mathcal{P}$, and by $(1-q_0^A)\frac{q_2^A}{q_1^A+q_2^A} + q_0^A\frac{q_2^S}{q_1^S+q_2^S}$ under regime $\mathcal{ND}$.

Proposition 5 is visualized in Figure 1 below. In this figure, we hold constant the values of q^S, c, and v and show the regions in the probability simplex of q^A in which consumers benefit from the removal of privacy protection (in red), consumers suffer from the removal of privacy protection (in blue), and consumers do not actively engage in a search (in white).

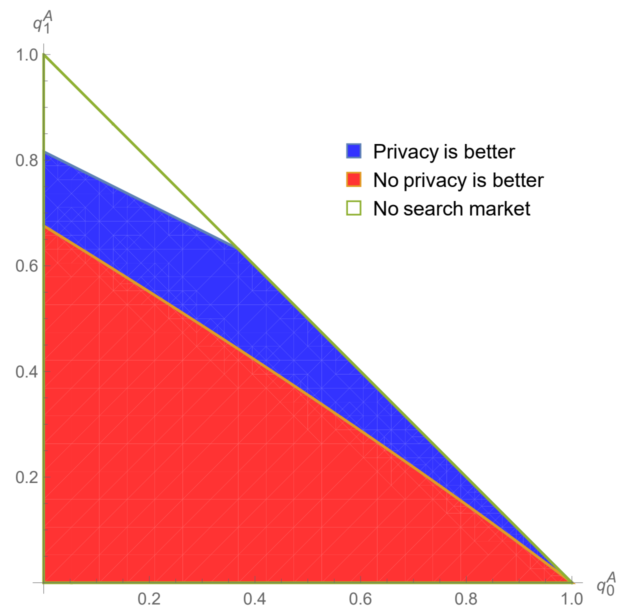

Figure 1: Value of privacy to consumers for $v = 5, c = 1, q_1^S = q_2^S = 1/2$.

Figure 1 provides more details about the characterization provided in Proposition 5. First, note that in the bottom left-hand corner of the simplex, where $q_2^A = 1$, removal of privacy protection benefits consumers. This is clearly a general result because when all consumers receive two ads, then the competition in the merged market generates the lowest possible price for consumers.

Second, note that in the top left-hand corner of the simplex, where $q_1^A = 1$, consumers do not search for the good. This is also a general result because if all consumers receive exactly one ad, which induces them to buy the good with probability one, then firms will set the monopoly price in the merged market. Taken together, these two observations imply that the left-hand boundary of the simplex, where $q_0^A = 0$, must begin (at the bottom) with a segment in which removal of privacy protection benefits consumers and must be followed by another segment in which removal of privacy protection hurts consumers.

Third, observe that along the bottom boundary of the simplex, where $q_1^A = 0$, removal of privacy protection always benefits consumers. This is because along this boundary, removal of privacy protection not only reduces the consumers' search expenditures, but also makes the market more competitive.

Finally, note that the bottom right-hand vertex of the simplex, where $q_0^A = 1$, is on the boundary of both the region in which removal of privacy protection benefits consumers and the region in which removal of privacy protection hurts consumers. As mentioned above, merging the search market with a fully competitive advertising market (i.e., a market where $q_1^A = 0$) is beneficial for consumers regardless of the size of the advertising market. Intuitively, for the removal of privacy protection to hurt consumers, the resulting market must be sufficiently noncompetitive to offset the gain from the reduction in consumers' search expenditures. If the search market is not very competitive, then merging it with a small noncompetitive advertising market (i.e., a market where $q_2^0 = 0$ and q_1^A is small) can hurt consumers (as in the case in Figure 1). However, if the search market is very competitive, then merging it with a small advertising market cannot reduce competition enough to offset the gain from the reduction in cost. Thus, the two regions intersect at the vertex where $q_0^A = 1$, unless the search market is very competitive.[17]

The Value of Privacy for Firms

Firms' profits in the search and advertising markets depend on the competitiveness of these markets: as the market becomes less competitive, i.e., as $\frac{q_1^z}{q_1^z+q_2^z}$, $z \in \{S, A\}$, increases, firms' profits in that market increase as well. The following proposition establishes a stronger result. It shows that firms benefit from advertising if and only if the search market is more competitive than the advertising market. That is, if the search market is more competitive than the putative advertising market, then the removal of privacy protection increases firms' profits regardless of whether or not firms price discriminate between the search and advertising markets, the magnitude of the consumers' search cost, and the size of the advertising market.

Proposition 6. *Suppose that Conditions* (4) *and* (5) *hold. Removal of privacy increases firms' profits if and only if* $\frac{q_1^A}{q_1^A+q_2^A} > \frac{q_1^S}{q_1^S+q_2^S}$.

To see why Proposition 6 holds true, note that if the search and advertising markets are distinct (as in regime $\mathcal{D}$), then the consumers' reservation price does not change if consumer privacy is not protected. Thus, opening up the advertising market (by removing privacy protection) benefits firms if and only if the advertising market is less competitive

[17] A formal proof of this claim is available upon request.

than the search market. This is the case because the consumers' reservation price and the prices in the search market are unaffected by the opening up of the advertising market, and some of the transactions performed in the search market move to the advertising market, where prices are higher on average.

If firms do not price discriminate and the search and advertising markets are merged (regime $\mathcal{ND}$), then the explanation becomes more involved because if firms do not price discriminate, then the removal of privacy protection can affect the consumers' reservation price and hence the distribution of prices in the search market. However, if the advertising market is less competitive than the search market, then removal of privacy protection leads to an increase in market prices, which, in turn, reduces the value of each round of search, increases the consumers' reservation price, and leads to an additional increase in prices. Similarly, if the advertising market is more competitive than the search market, then removal of privacy protection leads to a decrease in the consumers' reservation price, which further reduces firms' profits.

The Social Value of Privacy

Even without consumer privacy protection, the information that firms hold about consumers is extremely general and vague (see the discussion in the Introduction and in particular footnote 4), and so it is plausible that it should be easier for consumers to find firms that produce the goods that they are interested in than for firms to find the consumers who are interested in the specific goods they produce through interest-based advertising. Therefore, it stands to reason that, in many settings, the search market would be more competitive than the advertising market. That is, in many settings, the if and only if condition in Proposition 6 is likely to be satisfied. In such cases, combining the results of Propositions 4 and 6 implies that if firms engage in price discrimination between the search and advertising markets, then the protection of consumer privacy is Pareto inferior to non-protection.

Corollary 1. *Assume that Condition* (4) *holds and that* $\frac{q_1^S}{q_1^S+q_2^S} < \frac{q_1^A}{q_1^A+q_2^A}$. *Equilibrium outcomes under regime* $\mathcal{D}$ *Pareto dominate equilibrium outcomes under regime* $\mathcal{P}$.

The intuition for this result is based on three simple observations. First, removal of consumer privacy protection increases welfare because it decreases consumers' search expenditures. Second, if firms price discriminate, then the removal of privacy protection cannot hurt consumers. Third, if $\frac{q_1^S}{q_1^S+q_2^S} < \frac{q_1^A}{q_1^A+q_2^A}$, then, for any fixed reservation price, firms' profits are greater in the advertising market than in the search market.

It is worth emphasizing that even if the advertising market is almost monopolistic, removing consumer privacy protection leads to a Pareto improvement. For example, if $q_1^A = 9/10, q_0^A = q_2^A = 1/20, v = 5, c = 1$, and $q_1^S = q_2^S = 1/2$, then removing privacy protection raises consumer surplus from 3 to 3.1, and raises firm's profits from 1 to 1.85.

Even if firms do not engage in price discrimination, removal of privacy protection can Pareto dominate the protection of privacy. We illustrate this claim graphically in Figure 2 below. In the figure, the region in the simplex of q^A in which equilibrium outcomes in regime $\mathcal{ND}$ Pareto dominate equilibrium outcomes in regime $\mathcal{P}$ appears in green.

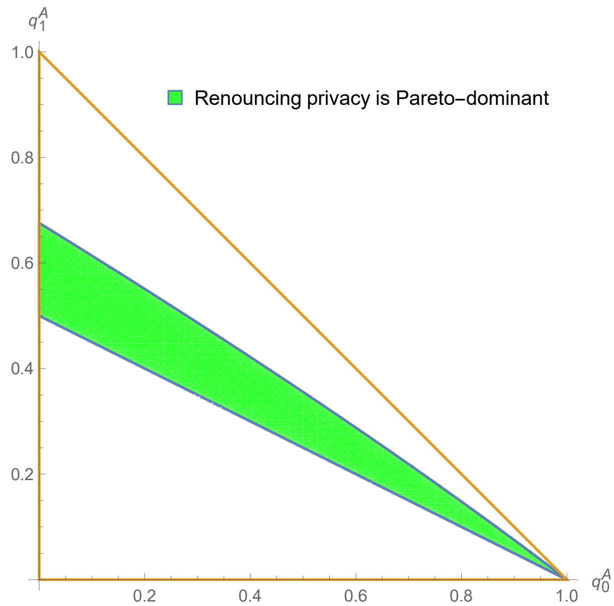

Figure 2: Social value of no privacy protection when $v = 5, c = 1, q_1^S = q_2^S = 1/2$.

In the region of the simplex above the lower boundary of the green triangle, firms prefer regime $\mathcal{ND}$ to regime $\mathcal{P}$, whereas in the region below the upper boundary of the green triangle, consumers prefer regime $\mathcal{ND}$ to regime $\mathcal{P}$. To see why these two regions must intersect—and create a nonempty green region—observe that if $\beta_1^{\mathcal{ND}} = \frac{q_1^S}{q_1^S+q_2^S}$, then market competitiveness is unaffected by the protection of consumer privacy. It therefore follows that the distribution of prices is the same when privacy is protected and when it is not. This, in turn, implies that for this level of β_1, firms' profits are identical under regimes $\mathcal{P}$ and $\mathcal{ND}$. However, at this critical level consumers strictly prefer regime $\mathcal{ND}$ to regime $\mathcal{P}$ because their search expenditures are lower under the former regime than under the latter regime. It follows that if $\beta_1^{\mathcal{ND}}$ is slightly above this critical level, then removal of privacy protection Pareto dominates the protection of privacy regardless of whether or not firms price discriminate.

So far we have focused on the question of whether and under what market conditions the protection of privacy benefits consumers. In some cases, it may be impossible to protect consumer privacy but possible to prevent price discrimination. Whether doing so is beneficial to consumers depends on the comparison between regimes $\mathcal{D}$ and $\mathcal{ND}$.

Proposition 7. *Suppose that Conditions* (4) *and* (5) *hold. Consumer surplus is higher under regime $\mathcal{D}$ than under regime $\mathcal{ND}$ if and only if firms' profits are lower under regime $\mathcal{D}$ than under regime $\mathcal{ND}$.*

Proposition 7 shows that if price discrimination benefits firms, then it harms consumers suffer from it. Thus, in this case, regulators who are concerned about consumer surplus should limit firms' ability to price discriminate. On the other hand, if price discrimination hurts firms, then they would like to commit to not engage in price discrimination.

Importantly, the last observation relies on the assumption that firms collude in their refusal to engage in price discrimination: if the regime is $\mathcal{ND}$, then in the generic case where the search and advertising markets differ in their level of competitiveness, it is profitable for each individual firm to price discriminate. However, because it seems that in many markets price discrimination is frowned upon by consumers, it may not be possible for an individual firm to engage in price discrimination in markets in which this is not an accepted practice.[18]

5 Concluding Remarks

More Than Two Price Quotes. We have assumed that consumers cannot receive more than two ads, or observe more than two price quotes per search. In no case does the intuition for our main qualitative results rely on this assumption. This suggests that this assumption, which allowed us to obtain an explicit solution to the consumers' reservation price, is not essential for our results to hold.

[18] Alternatively, such an assumption can be motivated by a repeated interaction or by a "market leader" (e.g., Amazon) setting an example.

Indeed, note that the result that the combination of price discrimination and no privacy protection is beneficial for consumers (Proposition 4) relies on the fact that interest-based advertising expands the consumers' choice set without impacting equilibrium outcomes in the search market. Hence, this result clearly holds in more general environments as well. Moreover, the result that if the size of the advertising market is large enough, then consumers benefit from no privacy protection even if firms do price discriminate should also generalize. To see this, note that regardless of the number of possible price quotes received by consumers, a firm's equilibrium profit is equal to the product of the probability that a consumer observes one price quote and his reservation price. Thus, if $\frac{q_1^A}{1-q_0^A} = \frac{q_1^S}{1-q_0^S}$, consumers benefit from removing privacy protection regardless of the size of the market. Moreover, since consumer surplus is increasing in the size of the advertising market and decreasing in $\frac{q_1^A}{1-q_0^A}$, it follows that for lower levels of competitiveness in the merged market, there exists a critical size of the advertising market that balances the two forces.

Second, the result that firms benefit from the removal of privacy protection if and only if the probability that consumers observe one price quote is higher in the advertising market than in the search market (Proposition 6) relies on the fact that a firm's equilibrium profit is equal to the product of the probability that a consumer observes one price quote and his reservation price, which, as mentioned above, holds generally. This, in turn, suggests that the results that show that the removal of consumer privacy protection is a Pareto improvement (Corollary 1 and the following claim) should also hold more generally.

Costly Advertising. In this paper we abstracted away from the interaction between firms and the advertisers who create the ads that firms use to contact consumers. In particular, we did not consider how the cost of advertising impacts our welfare results.[19] The value of a single ad to a firm is the expected number of consumers who will observe the ad multiplied by the minimum price charged in the advertising/merged market. Hence, if the marginal cost of creating each ad is below this value, our characterization of the equilibrium in the interaction between firms and consumers remains unchanged. Costly advertising clearly decreases the firms' profits from removing privacy protection, and so it may impact our welfare analysis. However, if the marginal cost of advertising is small, our qualitative results remain unchanged.

The Social Aspect of Privacy. An important difference between the aspect of consumer

[19]The cost of advertising may also include the cost of processing consumer information in order to target the ads.

privacy that we studied in this paper and the aspect of consumer privacy that is related to standard price discrimination through personalized pricing is that in our setting the individual consumer always benefits from sharing his personal data. Intuitively, in a general equilibrium environment an individual consumer's choice does not affect the pricing decisions of firms, and so sharing data and receiving interest-based ads has no downside for an individual consumer. However, as we have shown, if all consumers share their data then this affects market structure and prices in a way that may hurt consumers. This insight suggests that privacy laws that give consumers control over their personal information (e.g., the EU's "General Data Protection Regulation") may be insufficient to protect consumers and that more comprehensive regulation that considers the general equilibrium consequences of consumer privacy regulation may be needed.[20]

References

Acquisti, Alessandro, Curtis Taylor, and Liad Wagman. 2016. "The Economics of Privacy." *Journal of Economic Literature*, 54(2): 442–492.

Ali, Nageeb, Greg Lewis, and Susana Vasserman. 2020. "Voluntary Disclosure and Personalized Pricing." *Working Paper*.

Armstrong, Mark. 2006. "Recent Developments in the Economics of Price Discrimination." In *Advances in Economics and Econometrics 3: Theory and Applications*. , ed. R. Blundell, W. Newey and T. Persson, 41–97. Cambridge University Press.

Baye, Michael R., and John Morgan. 2001. "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets." *American Economic Review*, 91(3): 454–474.

Baye, Michael R., John Morgan, Patrick Scholten, et al. 2006. "Information, Search, and Price Dispersion." In *Handbook on Economics and Information Systems*. Vol. 1, , ed. T. Hendershott, 323–375. Elsevier.

[20]The "public good" aspect of privacy has also been established by Garratt and van Oordt (2021). They show that keeping payment histories private can increase consumer surplus by preventing price discrimination through personalized pricing, even though, if the firm's pricing strategies are held constant, each consumer individually benefits from disclosing his payment history.

Bergemann, Dirk, and Alessandro Bonatti. 2011. "Targeting in Advertising Markets: Implications for Offline versus Online Media." *RAND Journal of Economics*, 42(3): 417–443.

Bergemann, Dirk, Benjamin Brooks, and Stephen Morris. 2015. "The Limits of Price Discrimination." *American Economic Review*, 105(3): 921–957.

Bergemann, Dirk, Benjamin Brooks, and Stephen Morris. 2021. "Search, Information, and Prices." *Journal of Political Economy*, 129(8): 2275–2319.

Bird, Daniel, and Zvika Neeman. 2022. "What Should a Firm Know? Protecting Consumers' Privacy Rents." *American Economic Journal: Microeconomics*, 14(4): 257–295.

Braghieri, Luca. 2019. "Targeted Advertising and Price Discrimination in Intermediated Online Markets." *Working Paper*.

Burdett, Kenneth, and Kenneth L. Judd. 1983. "Equilibrium Price Dispersion." *Econometrica*, 51(4): 955–969.

Butters, Gerard R. 1977. "Equilibrium Distributions of Sales and Advertising Prices." *Review of Economic Studies*, 44: 465–491.

Calzolari, Giacomo, and Alessandro Pavan. 2006. "On the Optimality of Privacy in Sequential Contracting." *Journal of Economic Theory*, 130(1): 168–204.

De Cornière, Alexandre. 2016. "Search Advertising." *American Economic Journal: Microeconomics*, 8(3): 156–188.

Diamond, Peter A. 1971. "A Model of Price Adjustment." *Journal of Economic Theory*, 3(2): 156–168.

Elliott, Matthew, Andrea Galeotti, Andrew Koh, and Wenhao Li. 2022. "Market Segmentation through Information." *Cambridge Working Papers in Economics*.

Fainmesser, Itay P., Andrea Galeotti, and Ruslan Momot. to appear. "Digital Privacy." *Management Science*.

Garratt, Rodney J., and Maarten R. C. van Oordt. 2021. "Privacy as a Public Good: A Case for Electronic Cash." *Journal of Political Economy*, 129(7): 2157–2180.

Haghpanah, Nima, and Ron Siegel. 2020. "Pareto Improving Segmentation of Multiproduct Markets." *Journal of Political Economy*, forthcoming.

Ichihashi, Shota. 2020. "Online Privacy and Information Disclosure by Consumers." *American Economic Review*, 110(2): 569–595.

Ipsos, London Economics, and Deloitte. 2018. "Consumer Market Study on Online Market Segmentation through Personalised Pricing/Offers in the European Union."

Lewis, Randall A., and Justin M. Rao. 2015. "The Unfavorable Economics of Measuring the Returns to Advertising." *Quarterly Journal of Economics*, 130(4): 1941–1973.

Madarasz, Kristof, and Marek Pycia. 2021. "Towards a Resolution of the Privacy Paradox." *Working Paper*.

Mauring, Eeva. 2022. "Search and Price Discrimination Online." *Working Paper*.

Pram, Kym. 2021. "Disclosure, Welfare and Adverse Selection." *Journal of Economic Theory*, 197: 105327.

Robert, Jacques, and Dale O. Stahl. 1993. "Informative Price Advertising in a Sequential Search Model." *Econometrica*, 61(3): 657–686.

A Appendix: Proofs

Proof of Proposition 1

If consumers are willing to pay the search cost to obtain a random number of price quotes, they search until they obtain at least one price quote. Hence, the fraction of consumers that observe n price quotes before purchasing is

$$\beta_n^{\mathcal{P}} = \frac{q_n^S}{1 - q_0^S}.$$

Burdett and Judd (1983) establish that since $\beta_1^{\mathcal{P}} \in (0, 1)$, in equilibrium, the price distribution is continuous and that its support has an upper bound of $\tilde{p}^{\mathcal{P}}$, where $\tilde{p}^{\mathcal{P}}$ is the reservation price under regime $\mathcal{P}$. Moreover, by setting this price, the firm will sell only if it is the only quote a consumer obtains. Hence, since the firms must be indifferent between all prices in the support of $F^S(\cdot)$, the expected profit for each firm is $\beta_1^{\mathcal{P}} \tilde{p}^{\mathcal{P}}$. By the definition of a firm's profit given in Equation (2), it follows that the equilibrium price distribution is given by

$$F^S(p) = \frac{(2 - \beta_1^{\mathcal{P}})p - \beta_1^{\mathcal{P}} \tilde{p}^{\mathcal{P}}}{2(1 - \beta_1^{\mathcal{P}})p},$$

where the lower bound of the support of this distribution is $\underline{p}^{\mathcal{P}} = \frac{\tilde{p}^{\mathcal{P}}\beta_1^{\mathcal{P}}}{2-\beta_1^{\mathcal{P}}}$.

Next, we derive the consumers' reservation price under regime $\mathcal{P}$. Denote by $f^S(p)$ the density of $F^S(p)$ and by $j^S(p) = \frac{d(2F^S(p)-(F^S(p))^2)}{dp}$ the density of the minimum of two (independent) price quotes from the distribution $F^S(p)$. The consumer's expected payment for the good after one round of search that generates at least one quote is given by

$$\beta_1^{\mathcal{P}} \int_{\underline{p}}^{\tilde{p}^{\mathcal{P}}} p f^S(p) dp + \beta_2^{\mathcal{P}} \int_{\underline{p}^{\mathcal{P}}}^{\tilde{p}^{\mathcal{P}}} p j^S(p) dp = \tilde{p}^{\mathcal{P}} \beta_1^{\mathcal{P}}, \tag{6}$$

where the equality is obtained by plugging the previously derived expression into the integral and solving it.

Since firms' prices are never higher than $\tilde{p}^{\mathcal{P}}$, the reservation price can be calculated by assuming that if the next search generates no quotes, then the consumer will pay $\tilde{p}^{\mathcal{P}}$ for the good and otherwise he will pay as per one of the new quotes. Since his expected payment will be $\tilde{p}\beta_1^{\mathcal{P}}$, Equation (3) simplifies to

$$v - \tilde{p}^{\mathcal{P}} = q_0^S(v - \tilde{p}^{\mathcal{P}}) + (1 - q_0^S)(v - \tilde{p}^{\mathcal{P}}\beta_1^{\mathcal{P}}) - c,$$

and the reservation price is

$$\tilde{p}^{\mathcal{P}} = \frac{c}{q_2^S}.$$

Since consumers might have to search multiple times to obtain (at least) one quote, their expected search cost is $\frac{c}{1-q_0^S}$, which, in turn, implies that consumer surplus is given by

$$CS^{\mathcal{P}} = v - \frac{c}{1 - q_0^S} - \tilde{p}^{\mathcal{P}}\beta_1^{\mathcal{P}} = v - \frac{c}{q_2^S} \geq 0.$$

Finally, to obtain the firms' equilibrium profits recall that these profits are given by $\beta_1^{\mathcal{P}}\tilde{p}^{\mathcal{P}} = \frac{q_1^S}{1-q_0^S}\frac{c}{q_2^S}$. □

Proof of Proposition 2

First, note that firms will set a price no higher than $\tilde{p}^{\mathcal{D}}$ for consumers that arrive via searches or ads, where $\tilde{p}^D$ is the reservation price under regime $\mathcal{D}$. Note that the search market is the same under this regime as under regime $\mathcal{P}$, except that it has a mass q_0^A of consumers rather than a mass of one. However, this scaling does not affect equilibrium in the search market, which, in turn, implies that $\tilde{p}^{\mathcal{D}} = \tilde{p}^{\mathcal{P}}$. Hence, the consumer surplus that originates from the fraction q_0^A of consumers that do not receive ads is $q_0^A CS^{\mathcal{P}}$.

A fraction $1 - q_0^A$ of consumers receive ads and purchase a good through the cheapest ad they receive. From the same calculation used to solve Equation (6), it follows that the expected payment such consumers make for the good is $\frac{q_1^A}{1-q_0^A}\tilde{p}^{\mathcal{D}}$. Hence, the consumer surplus that originates from the fraction $1-q_0^A$ of consumers that receive ads is $(1-q_0^A)(v-\frac{q_1^A}{1-q_0^A}\tilde{p}^{\mathcal{D}})$. Thus, consumer surplus is given by

$$CS^{\mathcal{D}} = q_0^A CS^{\mathcal{P}} + (1-q_0^A)(v - \frac{q_1^A}{1-q_0^A}\tilde{p}^{\mathcal{D}}) = v - \frac{c(1-q_2^A)}{q_2^S}.$$

Next, we calculate the firms' profits. The derivation in Proposition 1 implies that with a measure one of firms and a measure S of consumers that engage in a noisy search using a reservation price $\tilde{p}$, and in which β of the consumers observe one price quote before purchasing, firms' profits are given by $S\beta\tilde{p}$.

The size of the advertising market is $(1-q_0^A)$ and in this market $\beta_1^A = \frac{q_1^A}{1-q_0^A}$. Hence, the firms' profits in this market are $(1-q_0^A)\beta_1^A\tilde{p}^{\mathcal{D}} = q_1^A\tilde{p}^{\mathcal{D}}$. Similarly, the size of the search market is q_0^A and in this market $\beta_1^S = \frac{q_1^S}{1-q_0^S}$, and so the firms' profits in the search market are $q_0^A\beta_1^S\tilde{p}^{\mathcal{D}} = q_0^A\frac{q_1^S}{1-q_0^S}\tilde{p}^{\mathcal{D}}$. Thus, firms' profits are given by

$$\pi^{\mathcal{D}} = q_1^A\tilde{p}^{\mathcal{D}} + q_0^A\frac{q_1^S}{1-q_0^S}\tilde{p}^{\mathcal{D}} = (q_1^A + q_0^A\frac{q_1^S}{1-q_0^S})\frac{c}{q_2^S}.$$

□

Proof of Lemma 1

To calculate the best expected price obtained in one round of search, conditional on obtaining at least one price quote, we must first calculate the firms' pricing strategy. For any given β_1 and $\tilde{p}$, an analogous argument to the one used in the proof of Proposition 1 shows that the equilibrium price distribution is

$$F(p) = \frac{(2-\beta_1)p - \beta_1\tilde{p}}{2(1-\beta_1)p}.$$

Denote by $\underline{p}$ the lower bound of the support of this distribution (recall that the upper bound is the reservation price $\tilde{p}$), by $f(p)$ the density of $F(p)$, and by $j(p) = \frac{d(2F(p)-(F(p))^2)}{dp}$ the density of the minimum of two (independent) price quotes from the distribution $F(p)$. The consumer's expected payment from buying after one round of search that generated at least one quote is thus given by

$$\frac{q_1^S}{1-q_0^S}\int_{\underline{p}}^{\tilde{p}} pf(p)dp + \frac{q_2^S}{1-q_0^S}\int_{\underline{p}}^{\tilde{p}} pj(p)dp =$$
$$\tilde{p}\times\left(\frac{\beta_1\left(2q_2^S\left(\beta_1\left(1+\tanh^{-1}(1-\beta_1)\right)-1\right)+(1-\beta_1)q_1^S\log\left(\frac{\beta_1}{2-\beta_1}\right)\right)}{2(1-\beta_1)^2(q_0^S-1)}\right).$$

□

Proof of Proposition 3

If the consumers' value from searching one more time is nonnegative, then the reservation price can be calculated by assuming that a consumer that receives a price quote of $\tilde{p}$ (through ads or previous searches) will search one more time and then buy the good. Hence, Equation (3) simplifies to

$$v-\tilde{p}^{\mathcal{ND}} = q_0^S(v-\tilde{p}^{\mathcal{ND}}) + (1-q_0^S)(v-\tilde{p}^{\mathcal{ND}}\kappa(q^S,\beta_1^{\mathcal{ND}})) - c,$$

and so the reservation price is

$$\tilde{p}^{\mathcal{ND}} = \frac{c}{(1-q_0^S)(1-\kappa(q^S,\beta_1^{\mathcal{ND}}))}.$$

The search market exists if the value of the good minus the sum of the expected search cost and the expected price paid is nonnegative. That is,

$$v-\frac{c}{1-q_0^S}-\tilde{p}^{\mathcal{ND}}\kappa(q^S,\beta_1^{\mathcal{ND}})\geq 0.$$

Hence, the search market exists if

$$v\geq\frac{c}{(1-q_0^S)(1-\kappa(q^S,\beta_1^{\mathcal{ND}}))}.$$

If the search market exists, then the consumers' expected search costs are $q_0^A\frac{c}{1-q_0^S}$ and by Lemma 1 their expected payment for the good is $\tilde{p}^{\mathcal{ND}}\kappa(q^S,q_1^A+q_0^A\frac{q_1^S}{1-q_0^S})$. Thus, consumer surplus is given by

$$CS^{\mathcal{ND}} = v-\frac{c}{1-q_0^S}\left(\frac{\kappa(q^S,\beta_1^{\mathcal{ND}})}{1-\kappa(q^S,\beta_1^{\mathcal{ND}})}+q_0^A\right).$$

Since, in this case, all the consumers buy the good, firms' profits are equal to the expected price paid by consumers, i.e.,

$$\pi^{\mathcal{ND}} = \tilde{p}^{\mathcal{ND}}\kappa(q^S,\beta_1^{\mathcal{ND}}) = \frac{c}{1-q_0^S}\frac{\kappa(q^S,\beta_1^{\mathcal{ND}})}{1-\kappa(q^S,\beta_1^{\mathcal{ND}})}.$$

$\square$

Proof of Proposition 4

To establish this result we must show that $CS^{\mathcal{D}} > CS^{\mathcal{P}}$. Plugging in the expressions derived above shows that this is equivalent to

$$v - \frac{c(1-q_2^A)}{q_2^S} > v - \frac{c}{q_2^S} \Leftrightarrow q_2^A > 0.$$

$\square$

Proof of Proposition 5

To determine whether removing privacy protection is beneficial for consumers, we must sign the expression $CS^{\mathcal{ND}} - CS^{\mathcal{P}}$. Plugging in the values derived in the previous section shows that this expression is equal to

$$c\left(\frac{q_0^A}{1-q_0^S} + \frac{1}{1-q_0^S}\frac{\kappa(q^S,\beta_1^{\mathcal{ND}})}{1-\kappa(q^S,\beta_1^{\mathcal{ND}})} - \frac{1}{q_2^S}\right). \tag{7}$$

Consider an iso-curve of $\beta_1^{\mathcal{ND}}$ in the q^A probability simplex, that is, the set of pairs $\langle q_0^A, q_1^A\rangle$ for which $q_1^A + q_0^A\frac{q_1^S}{1-q_0^S}$ is constant. Along such a curve, expression (7) is linear and increasing in q_0^A. Therefore, for every $\beta_1^{\mathcal{ND}}$ there exists $q^*(\beta_1^{\mathcal{ND}})$ such that removing privacy protection increases firms' profits if and only if[21] $q_0^A > q^*(\beta_1^{\mathcal{ND}})$. Moreover, since by Lemma A.1 $\kappa(q^S,\beta_1)$ is increasing in β_1, it follows immediately that $q^*(\beta_1^{\mathcal{ND}})$ is decreasing in $\beta_1^{\mathcal{ND}}$. $\square$

Proof of Proposition 6

First, consider the case in which firms price discriminate. Removing consumer privacy protection increases firms' profits if $\pi^{\mathcal{D}} > \pi^{\mathcal{P}}$. Plugging in these expressions yields

$$(q_1^A + q_0^A\frac{q_1^S}{1-q_0^S})\frac{c}{q_2^S} > \frac{q_1^S}{1-q_0^S}\frac{c}{q_2^S} \Leftrightarrow \frac{q_1^A}{1-q_0^A} > \frac{q_1^S}{1-q_0^S}.$$

Second, consider the case in which firms do not price discriminate. Removing consumer privacy protection increases firms' profits if $\pi^{\mathcal{ND}} > \pi^{\mathcal{P}}$. Plugging in these expressions and simplifying shows that this is equivalent to

$$\frac{\kappa(q^S,\beta_1^{\mathcal{ND}})}{1-\kappa(q^S,\beta_1^{\mathcal{ND}})} > \frac{q_1^S}{q_2^S}.$$

[21] It may be the case that $q^*(\beta_1^{\mathcal{ND}}) < 0$ for some level of $\beta_1^{\mathcal{ND}}$, in which case removing privacy protection is profitable for all advertising technologies that are consistent with $\beta_1^{\mathcal{ND}}$ given q^S. Similarly, if $q^*(\beta_1^{\mathcal{ND}}) > 1$ for some level of $\beta_1^{\mathcal{ND}}$, then removing privacy protection is not profitable for all advertising technologies that are consistent with $\beta_1^{\mathcal{ND}}$ given q^S.

Evaluating $\kappa(q^S, \beta_1^{\mathcal{ND}})$ at $\beta_1^{\mathcal{ND}} = \frac{q_1^S}{1-q_0^S}$ yields $\kappa(q^S, \frac{q_1^S}{1-q_0^S}) = \frac{q_1^S}{q_2^S}$, and so for this value of β_1 the firms' profits under regimes $\mathcal{P}$ and $\mathcal{ND}$ are the same. In Lemma A.1 we showed that κ is increasing in $\beta_1^{\mathcal{ND}}$, and so removing consumer privacy protection increases firms' profits if and only if

$$\frac{q_1^S}{1-q_0^S} < \beta_1^{\mathcal{ND}} = q_1^A + q_0^A \frac{q_1^S}{1-q_0^S} \Leftrightarrow \frac{q_1^A}{q_1^A + q_2^A} > \frac{q_1^S}{q_1^S + q_2^S}.$$

□

Proof of Proposition 7

If Conditions (4) and (5) hold, then Propositions 2 and 3 jointly imply that total welfare is the same under regimes $\mathcal{D}$ and $\mathcal{ND}$. The proposition follows from this observation. □

Lemma A.1. *$\kappa(q^S, \beta_1)$ is strictly increasing in β_1.*

Proof. Differentiating κ with respect to β_1 gives

$$\frac{\partial \kappa(q^S, \beta_1)}{\partial \beta_1} = \frac{(\beta_1 - 1)\left(-4q_0^S + 2(\beta_1 - 3)q_1^S + (\beta_1 - 2)q_1^S \log\left(\frac{\beta_1}{2-\beta_1}\right) + 4\right) + 4(\beta_1 - 2)\beta \tanh^{-1}(1-\beta_1)(q_0^S + q_1^S - 1)}{2(\beta_1 - 2)(\beta_1 - 1)^3(q_0^S - 1)}.$$

Note that this expression is linear in q_1^S, and so to establish that this derivative is positive it suffices to show that it is positive when evaluated at $q_1^S = 0$ and $q_1^S = 1 - q_0^S$.

The derivative of κ evaluated at $q_1^S = 0$ is

$$\frac{-2\beta_1 + 2(\beta_1 - 2)\beta_1 \tanh^{-1}(1-\beta_1) + 2}{(\beta_1 - 2)(\beta_1 - 1)^3},$$

an expression that is positive for all $\beta_1 \in (0, 1)$.

The derivative of κ evaluated at $q_1^S = 1 - q_0^S$ is

$$\frac{2(\beta - 1) + (\beta - 2)\log\left(\frac{\beta}{2-\beta}\right)}{2(2-\beta)(\beta - 1)^2}.$$

The denominator of this expression is clearly positive. The numerator evaluated at $\beta = 1$ is zero, and the first derivative thereof is also zero at that point. Moreover, the numerator is a convex function of β_1. Thus, for any $\beta_1 \in (0, 1)$, Taylor's theorem with the remainder written in the Lagrange form establishes that the numerator is positive. □

B Appendix: Continuous Privacy

In our welfare analysis (and baseline model) we assumed that the level of privacy protection is binary: either privacy is protected or it is not. However, in practice, privacy regulation is often more nuanced and specifies the level of privacy protection in a more continuous manner. In this section, we generalize our analysis to such settings in which the level of privacy protection can be chosen continuously.

We define an ϵ-increase in the level of privacy protection as a change that decreases the mass of consumers who are exposed to interest-based ads by ϵ, without altering the level of competition in the advertising market. Formally, for a given distribution of the number of ads received by each consumer, $\{q_0^A, q_1^A, q_2^A\}$, an ϵ-increase in the level of privacy protection changes that distribution to $\tilde{q}_0^A = q_0^A + \epsilon, \tilde{q}_0^A = q_1^A - \frac{q_1^A}{q_1^A+q_2^A}\epsilon$, and $\tilde{q}_2^A = q_2^A - \frac{q_2^A}{q_1^A+q_2^A}\epsilon$. Note that since, in equilibrium, consumers who receive an ad do not search, an ϵ-increase in the level of privacy protection, in essence, increases (decreases) the size of the search (advertising) market by ϵ.

The Marginal Value of Privacy to Consumers

If firms engage in price discrimination, then removing privacy protection provides consumers with additional options to purchase the good, without affecting their options in the search market. Intuitively, this suggests that an ϵ-increase in the level of privacy protection is harmful to consumers as it makes it less likely that they will receive this option.

Proposition 4'. *Suppose that Condition* (4) *holds. If firms price discriminate between the search and advertising markets, then an ϵ-increase in the level of privacy protection hurts consumers.*

Proof. In Proposition 2 we established that when firms engage in price discrimination consumer welfare is $v - \frac{c(1-q_2^A)}{q_2^S}$. Hence, marginally increasing the level of privacy protection reduces consumer welfare by $\frac{cq_2^A}{(1-q_0^A)q_2^S}$. □

On the other hand, if firms do not price discriminate, privacy protection impacts consumer welfare in two ways: through changes in their expected search costs, and through the change in the competitiveness of the market. As in the baseline model, reducing the level of privacy protection decreases expected search costs, and increases the expected price if and only if the advertising market is less competitive than the search market. Thus, increasing the level of privacy protection is beneficial for consumers if and only if the search market is sufficiently more competitive than the advertising market.

Proposition 5′. *Fix the values of $v, c,$ and q^S and suppose that Condition* (5) *holds. If firms do not price discriminate, then an ϵ-increase in the level of privacy protection benefits consumers if and only if the search market is sufficiently more competitive than the advertising market.*

Proof. In Proposition 3 we established that when firms do not price discriminate consumer welfare is $v - \frac{c}{1-q_0^S}\left(\frac{\kappa(q^S,\beta_1^{\mathcal{ND}})}{1-\kappa(q^S,\beta_1^{\mathcal{ND}})} + q_0^A\right)$. Hence, the marginal value of increasing the level of privacy protection is

$$-\frac{c}{1-q_0^S} - \frac{c}{1-q_0^S}\frac{1}{(1-\kappa(q^S,\beta_1^{\mathcal{ND}}))^2}\frac{\partial\kappa_2(q^S,\beta_1^{\mathcal{ND}})}{\partial\beta_1^{\mathcal{ND}}}\left(\frac{q_2^A}{q_1^A+q_2^A} - \frac{q_2^S}{q_1^S+q_2^S}\right).$$

By Lemma A.1 the derivative of $\kappa(q^S,\beta_1^{\mathcal{ND}})$ with respect to $\beta_1^{\mathcal{ND}}$ is positive, and so the marginal impact of increasing privacy protection is positive if and only if $\frac{q_2^S}{q_1^S+q_2^S}$ is sufficiently large relative to $\frac{q_2^A}{q_1^A+q_2^A}$. □

The Marginal Value of Privacy to Firms

Firms' profits are determined by the level of competitiveness in the market. In particular, since firms make higher profits in less competitive markets, firms benefit from full privacy protection if and only if the advertising market is more competitive than the search market. Clearly, if the advertising market is less competitive than the search market, an ϵ-increase in the level of privacy protection, which decreases the size of the advertising market while increasing the size of the search market, would also be beneficial for firms.

Proposition 6′. *Suppose that Conditions* (4) *and* (5) *hold. An ϵ-increase in the level of privacy increases firms' profits if and only if $\frac{q_2^A}{q_1^A+q_2^A} > \frac{q_2^S}{q_1^S+q_2^S}$.*

Proof. If firms engage in price discrimination, then by Proposition 2 their profits are $\left(q_1^A + q_0^A\frac{q_1^S}{1-q_0^S}\right)\frac{c}{q_2^S}$. Hence, marginally increasing the level of privacy protection increases firms' profits by $\frac{c}{q_2^S}\left(\frac{q_2^A}{q_1^A+q_2^A} - \frac{q_2^S}{q_1^S+q_2^S}\right)$.

If firms do not engage in price discrimination, then by Proposition 3 their profits are $\frac{c}{1-q_0^S}\frac{\kappa(q^S,\beta_1^{\mathcal{ND}})}{1-\kappa(q^S,\beta_1^{\mathcal{ND}})}$. Hence, marginally increasing the level of privacy protection increases firms' profits by $\frac{c}{1-q_0^S}\left(\frac{1}{(1-\kappa(q^S,\beta_1^{\mathcal{ND}}))^2}\frac{\partial\kappa_2(q^S,\beta_1^{\mathcal{ND}})}{\partial\beta_1^{\mathcal{ND}}}\left(\frac{q_2^A}{q_1^A+q_2^A} - \frac{q_2^S}{q_1^S+q_2^S}\right)\right)$. The result follows since $\kappa(q^S,\beta_1^{\mathcal{ND}})$ is increasing in $\beta_1^{\mathcal{ND}}$ (Lemma A.1). □