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Efficient Resolution of Partnership Disputes*

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Abstract

We study efficient resolution of partnership disputes where, in a departure from the literature on partnership dissolution, dissolution need not be the efficient outcome and hence is treated as endogenous. The endogeneity of efficient dissolution implies that a dispute in a more effective partnership may be less costly to resolve efficiently, in contrast with environments in which the efficiency of dissolution is certain. We characterize which disputes can be resolved efficiently without running a budget deficit, and show that a partnership's effectiveness has a non-monotonic effect on the cost of efficiently resolving the dispute. The latter implies unless a partnership is sufficiently ineffective, efficient resolution is impossible.

JEL classification: D47; D82; L14

Keywords: Partnership dissolution, arbitration, dispute resolution.

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1 Introduction

Disputes between partners are often resolved using the services of a third party without resorting to dissolution. Indeed, over the past few decades, Alternative Dispute Resolution (ADR) – a variety of dispute resolution methods providing an alternative to traditional litigation – has become an integral part of the legal system in many countries. Such methods are perceived as having an advantage in preserving business relationships, and have become increasingly popular.[1] The American Arbitration Association (AAA), for example – a not-for-profit organization that provides such ADR services – assures disputing parties that its services "enhance the likelihood of continuing the business relationship." The services of the International Centre for Dispute Resolution (ICDR), focusing on international arbitration, "assists in minimizing the impact of disputes by resolving them early and/or with less disruption to business."[2]

Emanating from the seminal paper of Cramton, Gibbons, and Klemperer (1987), a large literature on partnership dissolution has studied the possibility or impossibility of efficient dissolution of a partnership; that is, the existence of an individually rational mechanism that implements efficient allocation of a jointly owned asset without running a deficit. In studying this question, the literature has typically focused on disputes in which efficiency implies dissolution of the partnership and allocation of full ownership to one of the parties. Dissolution is therefore taken as given. This paper studies efficient resolution of partnership disputes where, departing from the previous literature, dissolution need not be efficient, and hence is viewed as endogenous. This endogeneity introduces new tradeoffs, with implications for the design of arbitration.

We consider a partnership between two agents who jointly own an indivisible asset. Each of the agents has private information representing their value from sole ownership of the asset. The value of the partnership (i.e., joint ownership of the asset) is a function of both parties' private information.[3] Therefore, whether or not it is desirable to dissolve the partnership, from the perspective of each party or social welfare, depends on the realization of each of the agents' private information. In particular, dissolving the partnership and awarding the asset to one of the agents is efficient if that agent's value for the asset exceeds both the other's and the value of the asset under joint ownership. However, if the value of joint ownership exceeds both agents' values, dissolution is inefficient. For concreteness, it is convenient to

[1] See, for example, Allison (1990), Lipsky and Seeber (1998), and Shontz, Kipperman, and Soma (2011).

[2] See https://www.adr.org and https://www.icdr.org.

[3] Our use of an interdependent values setting to study partnership dissolution builds on Fieseler, Kittsteiner, and Moldovanu (2003). As discussed below, however, our frameworks departs from previous literature in allowing for the possibility that dissolution is not efficient (and is therefore not taken as given).

view the dispute between the partners as arising due to an unforeseen event which gives rise to (or enhances) the uncertainty the agents face about the desirability of continuing the partnership. Such an event may take the form of a falling out between the partners, changes in the circumstances in which the partnership operates (e.g., the arrival of a competitor), or death of one of the partners, triggering the deceased partner's potential replacement by a legal heir. We abstract from the nature of the event leading to the dispute, and take as a starting point the resulting uncertainty about the partnership's desirability.

For example, the asset may correspond to a jointly owned business, to which both partners contribute their expertise.[4] Each partner may be in charge of different parts of the business, and hence the value of the partnership as a whole is a function of both partners' information. We imagine here partnerships which involve joint control over an indivisible asset which is not easily replaceable, rather than a mere profit-sharing arrangement. A restaurant may enjoy a favorable reputation established over many years, a retail company may benefit from a recognizable brand, a tech company may have rights to a technology which may not be replicated (e.g., for legal reasons), and a service provider may enjoy exclusivity in a certain area. In such partnerships, the parties cannot decide to simply break-up their partnership and operate independently. In case of dissolution, only the party awarded the asset benefits from it, and the other must be compensated.

We characterize which disputes can be resolved efficiently. That is, we characterize the existence of an individually rational mechanism that implements the ex-post efficient allocation – which in our setting may or may not involve dissolution – without running a deficit. Whether or not a dispute can be resolved efficiently crucially depends on a partnership's *effectiveness*, or equivalently, the *level of disagreement* between the partners. We say that a partnership is more effective (equivalently, the level of disagreement is lower) if the value of joint ownership is greater for any realization of the agents' values.[5] If the partnership is not sufficiently ineffective – alternatively, the devaluation as a result of the dispute is not sufficiently large – the dispute cannot be resolved efficiently. Furthermore, the budget deficit

[4]Consider the following example from *Harvard Business Review* (Thurston (1986)): "Two partners owned a company that assembled and marketed an electronic product. One managed the design, marketing, and sales activities. The other handled the procurement, assembly, and finances. At first they disagreed about their customers' needs and their product's features. 'You have overdesigned the product! We can't compete in that market!' one said. 'The market demands an upgraded product! The trouble lies with your failure to assemble to specs!' the other replied... The partners' pressure on each other mounted when the company began to lose money. Antagonism and mutual recrimination sapped energy that might have helped to resolve their market-position and quality concerns."

[5]Formally, denoting by θ_1 and θ_2 the partners' values for sole ownership, say that one partnership is more effective than another if the value of joint ownership under the former, $W(\theta_1, \theta_2)$, is greater than the value of joint ownership under the latter, $V(\theta_1, \theta_2)$. That is, $W(\theta_1, \theta_2) \geq V(\theta_1, \theta_2)$ for all θ_1, θ_2. We also consider a different measure of a partnership's effectiveness, based on the distribution of agents' private information.

(or surplus) is non-monotonic in the size of this devaluation.

More generally, these results are a consequence of the fact that a dispute in a more effective partnership may be less costly to resolve efficiently. That is, the budget deficit resulting from efficient resolution of the dispute is smaller (alternatively, the resulting surplus is greater). This observation is a consequence of a new tradeoff that results from the fact that the efficiency of dissolution is endogenous. To illustrate this point, consider an environment in which the efficiency of dissolution is certain. As we argue in Section 3, when this is the case, a more effective partnership is always more difficult to dissolve efficiently. This is because a more effective partnership tightens the partners' participation constraints relative to a less effective one by making the "status quo" of joint ownership more desirable, but does not alter the efficient allocation rule. When efficient dissolution is endogenous, however, the region in which dissolution is efficient also changes as a function of the effectiveness of the partnership. This leads to an additional change in the agents' incentive constraints, which may outweigh the tightening of the participation constraints resulting from the improved status quo.

As we show in Section 3, determining which of these forces is more salient requires understanding the properties of two components under efficient, incentive compatible mechanisms: (i) the expected subsidy $\mathcal{S}$ that must be provided by the arbitrator, and (ii) the sum of the net expected utilities of the partners' (endogenously determined) "worst-off types" from participation, $\mathcal{L}$. Both $\mathcal{S}$ and $\mathcal{L}$ change as a function of the effectiveness of a partnership. When the efficiency of dissolution is endogenous, the relaxation of participation constraints resulting from a lower status quo, measured by the change in $\mathcal{L}$, may be outweighed by the change in the efficient allocation rule, which determines the change in the expected subsidy $\mathcal{S}$. As these "worst-off" types are typically interior and need not be expressible in closed-form, for illustrative purposes we begin our analysis by specializing our environment to one in which the value of joint ownership is given by a simple additive function of agents' values, which allows an explicit characterization of these types. We then extend the analysis to general partnership functions in Subsection 3.4, by identifying certain general properties of these worst-off types.

In a seminal paper, Cramton, Gibbons, and Klemperer (1987) establish that when ownership is sufficiently close to being symmetric, ex-post efficient dissolution of a partnership (i.e., allocating an asset to the agent with the highest valuation) is possible, but sufficiently asymmetric ownership precludes the possibility of efficient dissolution.[6] In particular, this

[6] In the context of a public-goods problem with private values, Neeman (1999) shows that efficiency can be obtained only for intermediate property-rights allocations.

result contrasts with the impossibility result in Myerson and Satterthwaite (1983), and shows that, with private values, asymmetric ownership rather than asymmetric information is the key factor hindering efficiency. Schweizer (2006) shows that even if agents' types are not identically distributed, there always exists an initial distribution of shares that permits ex-post efficient dissolution of the partnership.

Fieseler, Kittsteiner, and Moldovanu (2003) study an environment with interdependent values, and show that if the interdependence is positive and sufficiently strong, ex-post efficient dissolution may be impossible given any initial ownership structure. Subsequent work studying partnership dissolution with interdependent values includes Kittsteiner (2003), Jehiel and Pauzner (2006), Ornelas and Turner (2007), Segal and Whinston (2011), Turner (2013) and Loertscher and Wasser (2018). In recent work, Loertscher and Wasser (2018) provide a comprehensive study of partnership dissolution with interdependent values. Allowing for general type distributions, they derive optimal dissolution mechanisms that maximize convex combinations of revenue and social surplus. The key departure from the literature in the present paper is that efficiency need not imply dissolution.

While this paper adds to the mechanism design literature on partnership dissolution, a related strand of literature has studied specific, widely used mechanisms for partnership dissolution in different environments. McAfee (1992) compares several simple mechanisms for dissolving equal-share partnerships in an independent private values environment. Versions of the $k+1$-auctions in Cramton, Gibbons, and Klemperer (1987) are studied in de Frutos (2000), Kittsteiner (2003) and Wasser (2013) (see also references therein) in different settings. De Frutos and Kittsteiner (2008), Brooks, Landeo, and Spier (2010) and Landeo and Spier (2013) study versions of the popular Texas Shootout mechanism.[7] Kittsteiner, Ockenfels, and Trhal (2012) and Brown and Velez (2016) experimentally compare different partnership dissolution mechanisms under incomplete and complete information, respectively. In recent work, Van Essen and Wooders (2016) introduce and study a dynamic auction for efficiently dissolving a partnership.

The paper is organized as follows. Section 2 introduces the model. Section 3 contains the main results of the paper concerning efficient resolution of partnership disputes, and Section 4 concludes. Appendix A contains all proofs omitted from the main text, and Appendix B contains the results extending the analysis to general partnership functions.

[7] Consistent with the motivation for this work, Brooks, Landeo, and Spier (2010) note in their conclusion that "certain deadlock situations might be resolved without an actual dissolution of the partnership".

2 Model

A partnership consists of two risk neutral agents $i = 1, 2$ who jointly own an asset. Agent i owns share $0 < r_i < 1$ of the asset, with $r_1 + r_2 = 1$. Each partner has private information θ_i which represents her value for sole ownership of the asset in the absence of the other partner and is drawn independently from a commonly known, continuously differentiable distribution function F on $[0, 1]$, with strictly positive density f. Partner i's payoff from joint ownership of the asset is equal to $r_i V(\theta_1, \theta_2)$, where $V(\theta_1, \theta_2) \in \mathbb{R}$ is the value of the asset when it is jointly owned by the two partners, and hence is a function of both partners' information.

As agents posses private information, whether or not it is desirable to dissolve the partnership, from the perspective of each party or from the perspective of welfare maximization, depends on the realization of the agents' types θ_i. In particular, dissolving the partnership and awarding the asset to agent 1 is efficient if $\theta_1 > \max\{\theta_2, V(\theta_1, \theta_2)\}$ (analogously for agent 2), but if $V(\theta_1, \theta_2) > \max\{\theta_1, \theta_2\}$, it is efficient to keep the partnership intact. We view the dispute between the partners, characterized by (r_1, r_2, F, V), as arising due to an unforeseen event (e.g., a falling out between the partners, an exogenous change in the circumstances in which the partnership operates, or death of one of the partners, triggering the deceased partner's potential replacement by a legal heir) which creates uncertainty about the desirability of continuing the partnership.[8] We take as a starting point this resulting uncertainty, rather than explicitly modeling the event leading to it.

For concreteness, suppose the asset corresponds to a jointly owned business (e.g., a restaurant) to which each of the partners contributes. Each partner is responsible for different parts of the business, and hence the value of the partnership as a whole is a function of both partners' information. The business may enjoy a favorable reputation established over many years, an invaluable location, a recognizable brand, exclusive rights to a technology, or exclusive rights to operate in a certain area. Thus, in case of dissolution, only the party awarded the asset benefits from it, and the other must be compensated (in particular, the parties cannot simply decide to end the relationship and operate independently).[9]

To illustrate the new tradeoffs that arise from the endogeneity of efficient dissolution in the simplest way possible, we first specialize our analysis to a simple additive partnership function:

$$V(\theta_1, \theta_2) = \theta_1 + \theta_2 - k, \tag{1}$$

[8]We assume partners cannot simply infer the other's type through observed profits. Indeed we are concerned with circumstances in which arbitration is necessary in order to resolve such uncertainty. For example, profits may be noisy, or may be revealed only at a future date.

[9]In contrast, in some profit-sharing arrangement such as that of two physicians sharing a practice, the parties may decide to part ways and open independent practices.

with $k \in [0,1]$ capturing the *level of disagreement* between the partners.[10] In Section 3.4 and in Appendix B we show that the results we obtain for this case hold for general partnership functions V which are increasing, concave, and satisfy an appropriate single-crossing property. The advantage of presenting the results in the context of this case are twofold. First, for computational reasons: it allows an explicit characterization of the "worst-off" participating types (see the discussion in the next section), which are otherwise complex functions of the model's parameters and V. Second, it is convenient to illustrate some of the results using a single parameter k capturing the devaluation of the partnership resulting from the dispute, with $k = 0$ capturing the case in which dissolution is never efficient, and $k = 1$ the case in which dissolution is always efficient.

A *partnership dispute* can be summarized by (r_1, F, k). Depending on each of the agents' valuations for the asset and the devaluation k, (ex-post) efficient allocation of the asset may involve either (a) retaining joint ownership of the asset if $\theta_1, \theta_2 \geq k$, or (b) dissolving the partnership and allocating the asset to agent i if $\theta_i > \theta_{-i}$, and $\theta_{-i} < k$. Denote $X = \{d_1, d_2, 0\}$, where d_i denotes the decision to dissolve the partnership and allocate it to i, and 0 the decision to keep the partnership intact.

We study the design of mechanisms with the goal of implementing the efficient allocation rule without the use of external subsidy. By the revelation principle, it is without loss of generality to restrict attention to direct truthful mechanisms.[11] We therefore consider mechanisms of the form (q, t), where $q : [0,1]^2 \to X$ and $t : [0,1]^2 \to \mathbb{R}^2$ denote an allocation and payment rule, respectively. Given a mechanism (q, t) and reported types (θ_1, θ_2), partner i's ex-post net utility is given by $u_i(\theta_1, \theta_2; q, r_i, k) + t_i(\theta_1, \theta_2) - r_i(\theta_1 + \theta_2 - k)$, where

$$u_i(\theta_1, \theta_2; q, r_i, k) = \begin{cases} \theta_i & \text{if } q(\theta_1, \theta_2) = d_i, \\ 0 & \text{if } q(\theta_1, \theta_2) = d_{-i}, \\ r_i(\theta_1 + \theta_2 - k) & \text{if } q(\theta_1, \theta_2) = 0. \end{cases}$$

If agent i has a true type θ_i, reports θ_i', and believes the other agent reports truthfully, her interim net expected utility is given by

$$U_i(\theta_i', \theta_i; r_i, k) = \int_0^1 u_i\left(\theta_1, \theta_2; q(\theta_i', \theta_{-i}), r_i, k\right) + t_i(\theta_i', \theta_{-i}) - r_i(\theta_i + \theta_{-i} - k)\, \mathrm{d}F(\theta_{-i}). \quad (2)$$

[10] Natural generalizations of (1) include allowing for more general functions of the types: $V(\theta_1, \theta_2) = g_1(\theta_1) + g_2(\theta_2) - k$ or allowing the function V to differ across the agents. The restriction to (1) does not distort the main messages of the paper, and is merely meant to simplify the characterizations.

[11] Implicit in our specification is the assumption that the arbitrator charges the agents before they observe their payoffs (see, e.g., Mezzetti (2004)).

Denote $U_i(\theta_i; r_i, k) = U_i(\theta_i, \theta_i; r_i, k)$.

A mechanism is *(interim) incentive compatible* (IC) if $U_i(\theta_i; r_i, k) \geq U_i(\theta_i', \theta_i; r_i, k)$ for all $\theta_i, \theta_i' \in [0,1]$, *(interim) individually rational* (IR) if $U_i(\theta_i; r_i, k) \geq 0$ for all $\theta_i \in [0,1]$, and *(ex-ante) budget balanced* (BB) if the arbitrator does not expect to incur positive subsidy payments to the partners: $\mathbb{E}_{\theta_1,\theta_2}\big(t_1(\theta_1,\theta_2) + t_2(\theta_1,\theta_2)\big) \leq 0$. A mechanism is *(ex-post) efficient* if the allocation rule is the efficient one,

$$q^*(\theta_1,\theta_2) = \begin{cases} 0 & \text{if } \theta_1 > k,\ \theta_2 > k, \\ d_1 & \text{if } \theta_1 > \theta_2,\ \theta_2 < k, \\ d_2 & \text{if } \theta_1 < k,\ \theta_2 > \theta_1, \end{cases} \tag{3}$$

illustrated in Figure 1. We say a partnership dispute (r_1, F, k) can be *resolved efficiently* if there exists a mechanism that is efficient, IC, IR and BB.

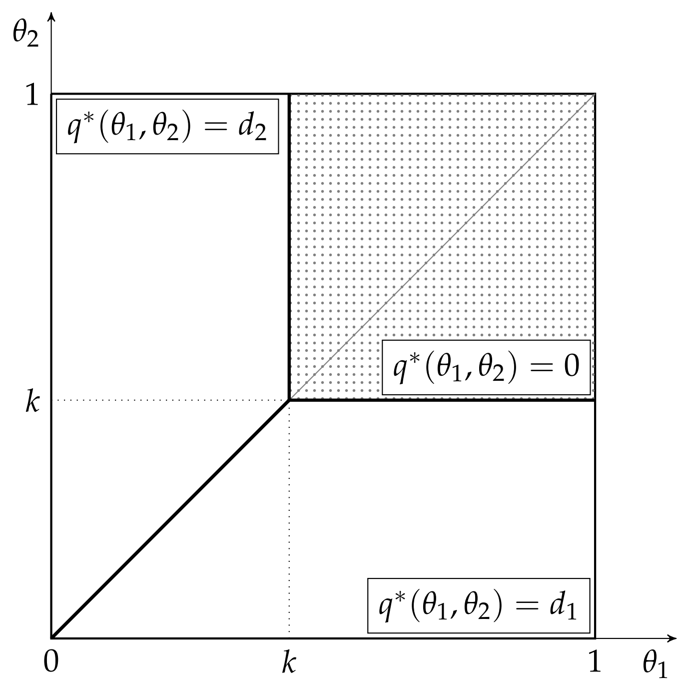

Figure 1: The efficient allocation rule

3 Efficient resolution of disputes

3.1 Preliminaries

As a first step in characterizing which disputes can be resolved efficiently, we consider the following transfer scheme: For each $i = 1, 2$,

$$t_i^*(\theta_1, \theta_2) = \begin{cases} 0 & \text{if } \theta_i > k,\ \theta_{-i} > k \\ r_i \theta_{-i} & \text{if } \theta_i < k,\ \theta_{-i} > \theta_i \\ -(1 - r_i)\theta_{-i} & \text{if } \theta_i > \theta_{-i},\ \theta_{-i} < k \end{cases} . \tag{4}$$

Under the mechanism $\Gamma^* = (q^*, t^*)$, if the partnership remains intact $(\theta_i, \theta_{-i} > k)$, the partners do not make transfers. If the asset is allocated to partner $-i$, partner i receives a transfer equal to his share in the (higher) value generated by $-i$'s sole ownership. In this case, partner $-i$ pays an amount equal to i's share in the (lower) value generated by i's sole ownership.

The following Lemma will be useful in characterizing which disputes can be resolved efficiently.

Lemma 1 *The mechanism $\Gamma^* = (q^*, t^*)$ is efficient, IC and IR; it is not BB.*

The proof of Lemma 1 is straightforward and is therefore omitted. The payments t^* guarantee that participation and truthful reporting constitute an ex-post equilibrium. Importantly, however, note that under Γ^*, for any pair of realizations (θ_1, θ_2) such that the partnership is dissolved, the party receiving the asset pays an amount smaller than the amount the other party receives. An implication of this result is that unless both partners are either certain the partnership will be dissolved or certain it will not be dissolved, an efficient mechanism that requires that no payments be made whenever the partnership remains intact cannot be budget balanced, as dissolution necessarily involves a deficit. In other words, the agents must pay in order to resolve uncertainty about their partnership.

The mechanism Γ^* can nevertheless be used to characterize when efficient resolution is possible. To this end, we introduce the following definitions. Given Γ^* and a partnership dispute (r_1, F, k), let $\mathcal{S}(k)$ denote the *expected subsidy* the arbitrator must incur under Γ^*. Under Γ^*, no payments are made if the partnership is not dissolved, and if it is dissolved, the party i awarded the asset pays $r_{-i}\theta_{-i}$ while the other party $-i$ receives $r_{-i}\theta_i$. The arbitrator's expected subsidy is

$$\mathcal{S}(r_1, r_2, k) = \sum_{i=1,2} \int_0^1 \int_0^{\min\{\theta_i, k\}} r_{-i}(\theta_i - \theta_{-i}) \,\mathrm{d}F(\theta_{-i}) \,\mathrm{d}F(\theta_i). \tag{5}$$

Next, define the *worst-off type* of each agent i as $\theta_i^*(r_i, k) \in \mathbf{argmin}_{\theta_i \in [0,1]} U_i(\theta_i; r_i, k)$, and let

$$\mathcal{L}(r_1, r_2, k) = U_1(\theta_1^*(r_1, k); r_1, k) + U_2(\theta_2^*(r_2, k); r_2, k) \tag{6}$$

denote the largest *lump-sum fee* that can be charged from the agents under Γ^* without violating their participation constraints; i.e., the sum of the maximal *participation fees* $U_i(\theta_i^*(r_i, k); r_i, k)$ that the agents are willing to pay under Γ^*. We refer to

$$\mathcal{B}(r_1, r_2, k) = \mathcal{L}(r_1, r_2, k) - \mathcal{S}(k)$$

as the *budget surplus* when $\mathcal{B} \geq 0$, and as the *budget deficit* when $\mathcal{B} < 0$.

Lemma 2 *$\mathcal{S}(r_1, r_2, k)$ is independent of (r_1, r_2) and satisfies*

$$\begin{aligned}\mathcal{S}(r_1, r_2, k) &= \mathbb{E}_{\theta_1, \theta_2}((\theta_1 - \theta_2) \mathbb{1}_{\{\theta_2 < min\{\theta_1, k\})\}}) \\ &= F(k) \int_k^1 1 - F(\theta) \,\mathrm{d}\theta + \int_0^k F(\theta)\big(1 - F(\theta)\big) \,\mathrm{d}\theta. \end{aligned} \tag{7}$$

The lump sum fee $\mathcal{L}(r_1, r_2, k)$ satisfies

$$\mathcal{L}(r_1, r_2, k) = k - \int_0^{\theta_1^*(r_1, k)} r_1 - F(\theta_2) \,\mathrm{d}\theta_2 - \int_0^{\theta_2^*(r_2, k)} r_2 - F(\theta_1) \,\mathrm{d}\theta_1. \tag{8}$$

The next result shows that in order to study the possibility of budget balance, it is sufficient to compare the lump-sum fee with the expected subsidy under Γ^*.

Lemma 3 *Under any efficient, IC and IR mechanism, the worst-off types are equal to*

$$\theta_i^*(r_i, k) = min\{F^{-1}(r_i), k\}.$$

Moreover, the dispute (r_1, F, k) can be resolved efficiently if and only if $\mathcal{B}(r_1, r_2, k) \geq 0$.

The proof of Lemma 3, in Appendix A, follows arguments similar to those in Fieseler, Kittsteiner, and Moldovanu (2003) and Kos and Messner (2013). Whenever the partnership is dissolved under Γ^*, there must be a deficit ($\mathcal{B} < 0$). However, the participation constraints of the agents typically do not bind, which permits the use of "participation fees" to extract additional surplus from the parties. The highest participation fees that can be charged are those that make the participation constraints of the agents' worst-off types bind. Therefore, a comparison of these highest total participation fees with the expected subsidy determines whether the efficient allocation rule q^* can be implemented without a budget deficit. Finally, a revenue equivalence argument establishes the result.

Note that, as in Cramton, Gibbons, and Klemperer (1987), agents' worst-off types are interior. Intuitively, if a partner's type is high, she not only values the asset more, but it is also more likely that she will receive it if the partnership is dissolved. Similarly, the lower a partner's value for the asset the more likely it is that the other partner will be awarded the asset, and the higher the compensation she expects to receive. Hence, the participation constraints of extreme types are more relaxed, resulting in lower information rents for these types.[12]

3.2 The endogeneity of efficient dissolution

The main new feature this paper introduces into the partnership dissolution problem is the possibility that dissolution is not the efficient outcome. Before presenting our main results, we briefly discuss the key new tradeoffs resulting from the endogeneity of efficient dissolution.

Consider first the case in which dissolution is efficient with certainty. In the simple setting in which V is given by (1), this case can be captured by setting $k = 1$. Suppose then that k is increased above 1.[13] In this case, expression (7) for the expected subsidy must be adjusted to allow for $k > 1$ by replacing k with $\min\{k, 1\}$. For $k \geq 1$, we have:

$$\mathcal{S}(k) = \sum_{i=1,2} \int_0^1 \int_0^{\theta_i} r_{-i}(\theta_i - \theta_{-i}) \, \mathrm{d}F(\theta_{-i}) \, \mathrm{d}F(\theta_i).$$

The expressions for the agents' net expected utilities, (2), as well as the lump-sum fee $\mathcal{L}$, (8), remain unchanged. Note that when $k \geq 1$, the devaluation k enters the agents' interim net expected utility (2), and hence the lump-sum fee $\mathcal{L}$, only through $-r_i(\theta_1 + \theta_2 - k)$. Furthermore, in this case the expected subsidy $\mathcal{S}$ is independent of k. By Lemma 3, an increase in k when $k \geq 1$ must increase $\mathcal{B}$.

[12] A similar intuition underlies the literature on countervailing incentives (see, for example, Lewis and Sappington (1989)).

[13] Note that, to ease the exposition, we have restricted k to be in $[0, 1]$; we consider here an increase in k above 1 only for illustrative purposes.

If dissolution need not be efficient (i.e., $k < 1$), this is no longer the case. A change in k alters the region in which dissolution is efficient, and in turn may alter also the worst-off types, their net expected utility, and the expected subsidy. More specifically, a greater devaluation k typically increases both the expected subsidy and the lump-sum fee. Therefore, given Lemma 3, the relationship between the rate of these two changes determines the overall change in $\mathcal{B}$ resulting from efficient resolution. The relaxation of participation constraints, measured by the change in the lump-sum fee $\mathcal{L}$, may be outweighed by the change in the allocation rule, which determines the change in the expected subsidy $\mathcal{S}$. Hence, in contrast with the case in which dissolution is known to be efficient with certainty, a dispute in a more effective partnership may be *less* costly to resolve. This intuition plays a key role in the analysis that follows. Furthermore, as we show in subsection 3.4, this intuition holds generally, beyond the simple additive case.

3.3 Efficient resolution and a partnership's effectiveness

The following example illustrates the budget surplus $\mathcal{B}$ (or deficit $-\mathcal{B}$) as a function of k for the simple case of a symmetric partnership with uniformly distributed types. As can be gleaned from this example, the surplus from efficient resolution of disputes is negative and decreasing below a certain threshold, indicating that efficient resolution of disputes within this region is impossible. Furthermore, $\mathcal{B}$ is non-monotonic. Note that the parameter region for which disputes cannot be resolved efficiently is far from negligible.[14]

Example 1 *Suppose* $r_1 = r_2 = \frac{1}{2}$ *and* $\theta_i \sim U[0,1]$. *In this case, it can easily be verified that the worst-off types are equal to* $\theta_i^*(1/2, k) = min\{k, \frac{1}{2}\}$, *the expected subsidy is equal to* $\mathcal{S}(k) = \frac{k}{2} - \frac{k^2}{2} + \frac{k^3}{6}$, *and* $\mathcal{L}\left(\frac{1}{2}, \frac{1}{2}, k\right)$ *is equal to* k^2 *when* $k \leq 1/2$ *and equal to* $k - 1/4$ *when* $k > 1/2$. *These functions are plotted in Figure 2.*

The derivative of the budget surplus $\mathcal{B}$ *is given by*

$$\frac{\partial}{\partial k}\left(\mathcal{L}\left(\frac{1}{2}, \frac{1}{2}, k\right) - \mathcal{S}(k)\right) = -\frac{1}{2}(k-1)^2 + \begin{cases} 2k & \text{if } k < 1/2, \\ 1 & \text{if } k > 1/2. \end{cases}$$

Therefore, the budget deficit from efficient resolution increases for $k < 3 - 2\sqrt{2} \approx 0.172$, *and otherwise decreases. This is the region to the left of the first vertical grid line in Figure 2. Using Lemma 3, it can be shown that the partnership dispute can be resolved efficiently if and only if* $k \geq k^* \approx 0.347$ *(represented by the second vertical grid line) or* $k = 0$.

[14] The average deficit from efficient resolution is of course a function of the distribution of k in the market.

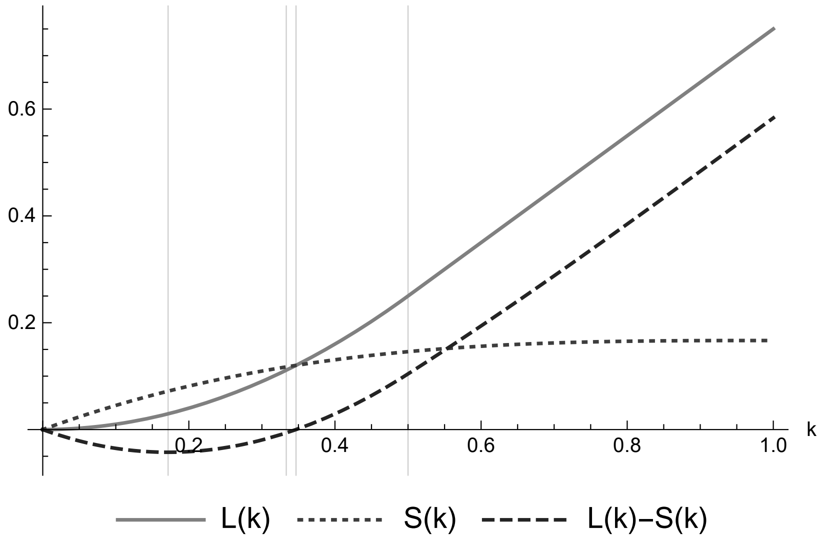

Figure 2: Lump-sum fee, expected subsidy, and budget surplus in Example 1

We now establish the key features described above, for any distribution F, and any initial share allocation (r_1, r_2).

Proposition 1 *For every share allocation* (r_1, r_2) *and distribution* F*, the budget surplus* $\mathcal{B}(r_1, r_2, k)$ *is non-monotonic in* k*. In particular, there exist* $0 < \underline{k} < \overline{k} < 1$ *such that the budget surplus is decreasing on* $(0, \underline{k})$ *and increasing on* $(\overline{k}, 1)$*.*

The proof is in Appendix A. The budget surplus is non-monotonic in the partnership's effectiveness, as measured by k. It initially decreases, and eventually increases.

To gain some intuition, consider a small increase in k, from k_1 to $k_2 > k_1$. Suppose first that k_1 is close to 1. The increase to k_2 raises the expected subsidy $\mathcal{S}$ as dissolution becomes efficient for type profiles just above k_1, creating additional deficit. However, when k_1 is close to 1, these additional pairs of types become increasingly similar and therefore have a lower impact on the deficit. Hence, the effect of a small increase in k on the expected subsidy vanishes as k approaches 1. Furthermore, for values of k close enough to 1, an increase in k does not change the worst-off types, but increases their net expected utility by worsening the status quo. Since this effect does not vanish as k approaches 1, the increase in the lump-sum fee $\mathcal{L}$ eventually dominates the increase in the expected subsidy $\mathcal{S}$.

Now suppose k is low. An increase in k results in an increase in the expected subsidy, which does not vanish as k approaches 0. This follows from the fact that the mass of additional type pairs for which dissolution becomes efficient is bounded away from 0, and the expected deficit created by these pairs increases as k approaches 0. On the other hand, as opposed to the case of high k, the change in the lump-sum fee for low values of k consists of a change in both the worst-off types and the status quo utilities. If k is small enough, partner i's worst-off type is equal to k. It can easily be checked that in this case the net expected utility of i's worst-off type is equal to the (unconditional) expected difference in valuations, $\theta_i - \theta_{-i}$, over the region of θ_{-i} values in which i is awarded the asset. An increase in k increases this difference uniformly for every type of the other partner (below k), therefore the increase in partner i's net expected utility is be proportional to the probability of i being awarded the asset, $F(k)$. As a result, as k approaches 0, this effect vanishes for both partners, and the change in the lump-sum fee is dominated by the change in the expected subsidy.

Since the budget is balanced ($\mathcal{B} = 0$) when $k = 0$, an immediate consequence of the previous Proposition is the following.

Proposition 2 *For every share allocation (r_1, r_2) and distribution F, there exists $\hat{k} \in (0, 1)$ such that any dispute with $k \in (0, \hat{k})$ cannot be resolved efficiently.*

That is, given any initial share allocation, disputes cannot be resolved efficiently unless they render the partnership sufficiently ineffective.

An alternative measure of a partnership's effectiveness is the distribution of types F. On one hand, the effect of a first-order stochastic improvement in F can be shown to create a shift downwards in the net expected utility curves, resulting in lower participation fees. On the other hand, an improved type distribution has an ambiguous effect on the expected subsidy $\mathcal{S}$. The following result constructs an example that shows that a dispute in more effective partnership – as measured by a first-order stochastically dominating distribution of types – may be *less* costly to resolve efficiently. In the special case in which the probability of dissolution remains the same, the dispute in the more effective partnership is more costly to resolve (i.e., yields a lower budget surplus, or a greater deficit).

Proposition 3 *For any two partnership disputes (r_1, F, k) and (r_1, G, k) such that G first-order stochastically dominates F:*

1. *The dispute (r_1, G, k) may be less costly to resolve than (r_1, F, k).*
2. *If the probability with which dissolution is efficient is the same under F and G, (r_1, G, k) is more costly to resolve.*

The proof is in Appendix A. The result is consistent with the intuition discussed in Subsection 3.2. In the example of part 1, a mass of types is reallocated from slightly below the dissolution threshold to slightly above it. The participation fee of a partner whose worst-off type is k is equal to the (unconditional) expected difference in valuations, $\theta_i - \theta_{-i}$, over the region in which i is awarded the asset, at $\theta_i = k$, and the total expected subsidy is the expected value of this unconditional expected difference with respect to θ_i. Since the change is near the threshold, the effect of such reallocation of mass on the unconditional expected difference in valuations is small for θ_i types below the threshold, but continuously increasing for θ_i types above the threshold. In the example, its effect on the lump-sum fee is smaller than the corresponding effect on the expected subsidy.

3.4 General partnership disputes

The analysis above extends to a more general environment. Assume the value of the partnership $V\colon [0,1]^2 \to \mathbb{R}$ is a twice differentiable, concave function with $\frac{\partial V(\theta_1,\theta_2)}{\partial \theta_i} \in (0,1)$, $i = 1,2$. The assumptions guarantee that the value of the partnership is increasing in both partners' types, but the marginal value is decreasing. Furthermore, the difference between a partner's private value θ_i and the value of the partnership $V(\theta_1,\theta_2)$ is strictly increasing in θ_i for all θ_{-i}, which ensures that if it is efficient to assign the asset to partner i then this is also the case for higher types θ_i, holding θ_{-i} fixed. Let the agents' types be drawn from independent, continuously differentiable distribution functions F_1 and F_2, with positive densities f_1 and f_2.

Given the assumptions on V, there exist threshold types $\underline{\theta}_i(\theta_{-i})$ and $\overline{\theta}_i(\theta_{-i})$ such that: (a) when $\theta_i < \underline{\theta}_i(\theta_{-i})$, or equivalently $\theta_{-i} > \overline{\theta}_{-i}(\theta_i)$, social surplus is maximized when the partnership is dissolved and the asset is allocated to partner $-i$; (b) when $\theta_i \in \big(\underline{\theta}_i(\theta_{-i}), \overline{\theta}_i(\theta_{-i})\big)$, it is efficient to keep the partnership intact. Formally, for any $\theta_{-i} \in [0,1]$, the thresholds are defined by:

$$\underline{\theta}_i(\theta_{-i}) = \min\left(\{\theta_i \in [0,\theta_{-i}]\colon V(\theta_1,\theta_2) \geq \theta_{-i}\} \cup \{\theta_{-i}\}\right),$$
$$\overline{\theta}_i(\theta_{-i}) = \min\left(\{\theta_i \in [\theta_{-i},1]\colon V(\theta_1,\theta_2) \leq \theta_i\} \cup \{1\}\right).$$

Given the thresholds $\underline{\theta}_i(\theta_{-i})$ and $\overline{\theta}_i(\theta_{-i})$, the efficient allocation rule, illustrated in Figure 3, is given by

$$q^*(\theta_1,\theta_2) = \begin{cases} d_{-i} & \text{if } \theta_i < \underline{\theta}_i(\theta_{-i}) \\ 0 & \text{if } \theta_i \in \big(\underline{\theta}_i(\theta_{-i}), \overline{\theta}_i(\theta_{-i})\big) \\ d_i & \text{if } \theta_i > \overline{\theta}_i(\theta_{-i}) \end{cases}. \tag{9}$$

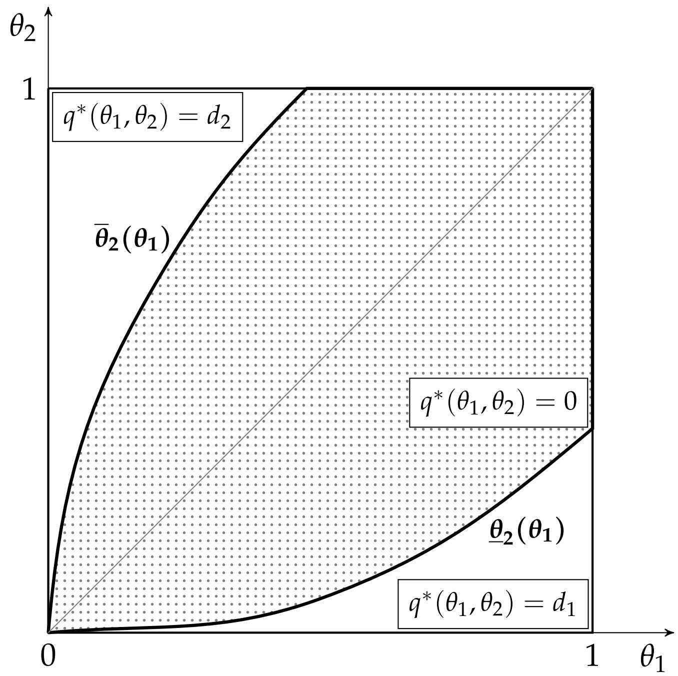

Figure 3: Efficient allocation rule: general V

Denote $\Gamma^* = (q^*, t^*)$, with the transfer rule t^* defined by

$$t_i^*(\theta_1, \theta_2) = \begin{cases} r_i \theta_{-i} & \text{if } \theta_{-i} > \overline{\theta}_{-i}(\theta_i) \\ 0 & \text{if } \theta_{-i} \in (\underline{\theta}_{-i}(\theta_i), \overline{\theta}_{-i}(\theta_i)) \\ -r_{-i}\overline{\theta}_i(\theta_{-i}) & \text{if } \theta_{-i} < \underline{\theta}_{-i}(\theta_i) \end{cases}. \tag{10}$$

Under Γ^*, if the partnership remains intact, the partners do not make transfers. If the asset is allocated to partner $-i$, partner i receives a transfer equal to his share in the (higher) value generated by $-i$'s sole ownership. Partner $-i$ makes a payment equal to the utility that agent i could have derived either in the best possible effective partnership with $-i$ or from sole ownership of the asset, whichever is higher.

In Appendix B, we show that the results above continue to hold for general partnership disputes (r_1, F_1, F_2, V). While the generality of the environment no longer permits closed form expressions for the worst-off types of the agents, it is nevertheless possible to derive certain general properties of these worst-off types and their net expected utilities (Lemma 4 in Appendix B), which permit to characterize how different properties of the dispute affect $\mathcal{B}$, and in particular the possibility of efficient resolution.

We say that partnership W is more effective than partnership V if $W(\theta_1, \theta_2) \geq V(\theta_1, \theta_2)$

for all $\theta_1, \theta_2 \in [0,1]$. A less effective partnership implies both a higher expected subsidy and a higher lump-sum fee (Lemma 6 in Appendix B). The relationship between these two changes determines the overall change in $\mathcal{B}$, as in Lemma 3. The expected subsidy depends only on the thresholds of dissolution, whereas the net expected utility, and hence the lump-sum fee, depend on other features of the partnership's value. In the particular case in which the region of efficient dissolution is the same for both partnerships, $\mathcal{B}$ is greater for the less effective partnership. However, if the region of dissolution differs, the change in the allocation rule, which determines the change in the expected subsidy, may outweigh the relaxation of participation constraints resulting from a lower status quo. Hence, a more effective partnership may be yield a higher $\mathcal{B}$ (Proposition 4 in Appendix B). In Appendix B we also obtain a version of Proposition 1 by showing that sufficiently small disputes, for which the relative gain from making the efficient allocation decision is smaller, cannot be resolved without budget deficit (Proposition 5, Appendix B).

4 Concluding remarks

The literature on partnership dissolution has accumulated a rich set of results on the possibility of efficient dissolution of a partnership, and mechanisms that can be used for such dissolution. This literature, however, implicitly takes as given the inefficiency of sustaining the partnership. This paper adds to such problems the possibility that, for some realizations of the underlying uncertainty, dissolving the partnership may be inefficient. Given this possibility, we ask under what conditions a partnership dispute can be resolved efficiently. The (im)possibility of efficient resolution hinges subtly on the level of disagreement between the agents. When the efficiency of dissolution is certain, more effective partnerships make dissolution more costly. When dissolution need not be efficient, this relationship becomes non-monotonic due to the change in incentive constraints resulting from changes in the efficient allocation rule. An important implication of this is that disputes cannot be resolved efficiently unless they render the partnership sufficiently ineffective.

As a first step in studying the implications of the potential inefficiency of dissolution, we have focused – in line with much of the partnership dissolution literature – on environments in which the value of a partnership is not a function of the ownership structure. Relaxing this assumption would open the door to different interesting questions. For instance, how might the partners be incentivized to trade shares of an asset with the goal of reaching an optimal ownership structure, given their different valuations for the asset? Furthermore, a related literature pioneered by Grossman and Hart (1986) and Hart and Moore (1990) studies how the allocation of property rights shapes incentives to invest in an asset's improvement. Studying such incentives in the current context seems a particularly interesting direction for future research.

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A Proofs

Proof of Lemma 2. The derivation of (7) follows from the fact that

$$\begin{aligned}
&\int_0^1 \int_0^{\min\{\theta_i,k\}} \theta_i - \theta_{-i}\,\mathrm{d}F(\theta_{-i})\,\mathrm{d}F(\theta_i)\\
&= \int_k^1 \int_0^k \theta_i - \theta_{-i}\,\mathrm{d}F(\theta_{-i})\,\mathrm{d}F(\theta_i) + \int_0^k \int_0^{\theta_i} \theta_i - \theta_{-i}\,\mathrm{d}F(\theta_{-i})\,\mathrm{d}F(\theta_i)\\
&= \int_k^1 F(k)(\theta_i - k) + \int_0^k F(\theta_{-i})\,\mathrm{d}\theta_{-i}\,\mathrm{d}F(\theta_i) + \int_0^k \int_0^{\theta_i} F(\theta_{-i})\,\mathrm{d}\theta_{-i}\,\mathrm{d}F(\theta_i)\\
&= F(k)\int_k^1 1 - F(\theta)\mathrm{d}\theta + (1-F(k))\int_0^k F(\theta)\,\mathrm{d}\theta + F(k)\int_0^k F(\theta)\,\mathrm{d}\theta - \int_0^k F(\theta)^2\,\mathrm{d}\theta\\
&= F(k)\int_k^1 1 - F(\theta)\mathrm{d}\theta + \int_0^k F(\theta)\,(1-F(\theta))\,\mathrm{d}\theta,
\end{aligned}$$

where the second and third equalities follows from integration by parts.

Next, it can easily be verified that under the mechanism Γ^*,

$$\begin{aligned}
U_i(\theta_i^*(r_i;k);r_i,k) &= \int_0^{\theta_i^*(r_i;k)} \theta_i^*(r_i;k) - \theta_{-i}\,\mathrm{d}F(\theta_{-i}) - r_i(\theta_i^*(r_i;k) - k)\\
&= \theta_i^*(r_i;k)F(\theta_i^*(r_i;k)) - \int_0^{\theta_i^*(r_i;k)} \theta_{-i}\,\mathrm{d}F(\theta_{-i}) - r_i(\theta_i^*(r_i;k) - k)\\
&= \int_0^{\theta_i^*(r_i;k)} F(\theta_{-i})\,\mathrm{d}\theta_{-i} - r_i(\theta_i^*(r_i;k) - k),
\end{aligned}$$

where the third equality follows from integration by parts. Formula (8) follows immediately since $\mathcal{L}(r_1,r_2,k) = U_1(\theta_1^*(r_1,k);r_1,k) + U_2(\theta_2^*(r_2,k);r_2,k)$. ■

Proof of Lemma 3. The result in Lemma 3 follows from arguments similar to those in Williams (1999), Fieseler, Kittsteiner, and Moldovanu (2003) and Kos and Messner (2013). In particular, revenue equivalence implies that any two efficient and IC mechanisms induce the same interim expected transfer, up to a constant. For any dispute (r_1, F, k), therefore, an efficient, IR, IC and BB mechanism exists if and only if for any efficient IC mechanism Γ it holds that

$$U_1^\Gamma(\theta_1^{*\Gamma}(r_1,k);r_1,k) + U_2^\Gamma(\theta_2^{*\Gamma}(r_2,k);r_2,k) \geq \mathbb{E}_\theta(t_1^\Gamma(\theta_1,\theta_2) + t_2^\Gamma(\theta_1,\theta_2)),$$

where $\theta_i^{*\Gamma}, U_i^\Gamma, t_i^\Gamma$ denote i's worst-off type, net expected utility and payments under Γ.

From Lemma 1, Γ^* is efficient and IC, which yields the necessary and sufficient condition $\mathcal{L}(r_1, r_2, k) \geq \mathcal{S}(k)$.

Finally, to see that $\theta_i^*(r_i, k) = \min\{F^{-1}(r_i), k\}$, first note that

$$U_i^\Gamma(\theta_i; r_i, k) = \begin{cases} \int_0^{\theta_i} \theta_i - \theta_{-i} \, \mathrm{d}F(\theta_{-i}) - r_i(\theta_i - k) & \text{if } \theta_i < k, \\ \int_0^{k} \theta_i - \theta_{-i} \, \mathrm{d}F(\theta_{-i}) - F(k) r_i(\theta_i - k) & \text{if } \theta_i > k. \end{cases}$$

Therefore,

$$\frac{\partial U_i^\Gamma(\theta_i; r_i, k)}{\partial \theta_i} = \begin{cases} F(\theta_i) - r_i & \text{if } \theta_i < k, \\ (1 - r_i) F(k) & \text{if } \theta_i > k, \end{cases} \quad \text{and} \quad \frac{\partial^2 U_i^\Gamma(\theta_i; r_i, k)}{\partial \theta_i^2} = \begin{cases} f(\theta_i) & \text{if } \theta_i < k, \\ 0 & \text{if } \theta_i > k. \end{cases}$$

Note that U_i^Γ is strictly convex on $[0, k]$, strictly decreasing at $\theta_i = 0$, and linear and strictly increasing on $[k, 1]$. Therefore, it has a unique minimum at either $F^{-1}(r_i)$ or k. ■

Proof of Proposition 1. The derivative of the expected subsidy can be written as

$$\frac{\mathrm{d}\mathcal{S}(k)}{\mathrm{d}k} = f(k) \int_k^1 1 - F(\theta) \, \mathrm{d}\theta.$$

For sufficiently small k, $\theta_1^*(r_1, k) = \theta_2^*(r_2, k) = k$; hence, from (8), $\frac{\partial \mathcal{L}(r_1, r_2, k)}{\partial k} = 2F(k)$. Since $F(k) \to 0$ as $k \to 0$, the budget surplus $\mathcal{B}(r_1, r_2, k) = \mathcal{L}(r_1, r_2, k) - \mathcal{S}(k)$ is decreasing on $(0, \underline{k})$, for some $\underline{k} \in (0, 1)$. Similarly, for k close enough to 1, $\theta_i^*(r_i, k) = F^{-1}(r_i)$, and from (8), $\frac{\partial \mathcal{L}(r_1, r_2, k)}{\partial k} = 1$. Furthermore, $\frac{\mathrm{d}\mathcal{S}}{\mathrm{d}k} \to 0$ as $k \to 1$, but $\frac{\partial \mathcal{L}}{\partial k}$ remains bounded away from 0 as $k \to 1$. Hence, there exists $\bar{k} \in (\underline{k}, 1)$ such that the surplus is increasing on $(\bar{k}, 1)$. ■

Proof of Proposition 2. The proof follows immediately from Proposition 1 and the fact that $\mathcal{B} = 0$ when $k = 0$. ■

Proof of Proposition 3. *Part 1.* To show that the budget surplus can be larger under a first-order stochastically dominating distribution G, consider a sequence of disputes involving symmetric shares and the following sequences of distributions, defined by their pdf's (where $n > 5$):

$$f_n(\theta) = \begin{cases} 0.5 & \text{if } \theta \in [0, 0.2 - \frac{1}{n}) \\ 0.5 + 0.1n & \text{if } \theta \in [0.2 - \frac{1}{n}, 0.2) \\ 1 & \text{if } \theta \in [0.2, 1] \end{cases}, \quad g_n(\theta) = \begin{cases} 0.5 & \text{if } \theta \in [0, 0.2 - \frac{1}{n}) \\ 1 & \text{if } \theta \in [0.2 - \frac{1}{n}, 0.2) \cup [0.2 + \frac{1}{n}, 1] \\ 0.5 + 0.1n & \text{if } \theta \in [0.2, 0.2 + \frac{1}{n}) \end{cases}.$$

Fix $k = 0.2$. The median of both distributions is 0.5; therefore, k is the worst-off type of both partners under F_n and G_n. It is easy to see that for every n, the respective cdf's, F_n and G_n, are integrable and G_n first-order stochastically dominates F_n. Using formulas (7) and (8) for the expected subsidy and the lump-sum payment, it is straightforward to show that the difference in the budget surplus under G_n and F_n is equal to

$$(F_n(0.2) - G_n(0.2))(1-0.2) + \int_0^{0.2} F_n(\theta) - G_n(\theta)\,\mathrm{d}\theta - \left(F_n(0.2)\int_{0.2}^1 F_n(\theta)\,\mathrm{d}\theta - G_n(0.2)\int_{0.2}^1 G_n(\theta)\,\mathrm{d}\theta\right)$$
$$- \int_0^{0.2} F_n^2(\theta) - G_n^2(\theta)\,\mathrm{d}\theta - 2\int_0^{0.2} F_n(\theta) - G_n(\theta)\,\mathrm{d}\theta. \tag{11}$$

Since F_n and G_n are identical on $[0, 0.2 - \frac{1}{n}]$ and $[0.2 + \frac{1}{n}, 1]$, they pointwise converge on the set $[0, 0.2) \bigcup (0.2, 1]$ to the same (integrable) function H as $n \to \infty$. Therefore, by the dominated convergence theorem, the change in the budget surplus (11) converges to

$$\lim_{n\to\infty}(F_n(0.2) - G_n(0.2))\left(1 - 0.2 - \int_{0.2}^1 H(\theta)\,\mathrm{d}\theta\right) = 0.032 > 0,$$

where H is equal to the cdf of the uniform distribution on $(0.2, 1]$.

Part 2. Assume without loss of generality that $r_1 \geq 1/2 \geq r_2$. Using formulas (7) and (8) for the expected subsidy and the lump-sum payment, it is easy to show that the difference in the budget surplus under G and F is equal to[15]

$$\Delta\mathcal{B} = (F(k) - G(k))(1-k) + \int_0^k F(\theta) - G(\theta)\,\mathrm{d}\theta - \left(F(k)\int_k^1 F(\theta)\,\mathrm{d}\theta - G(k)\int_k^1 G(\theta)\,\mathrm{d}\theta\right)$$
$$- \int_0^k F^2(\theta) - G^2(\theta)\,\mathrm{d}\theta - \int_{\theta_1^{*F}}^{\theta_1^{*G}} r_1 - G(\theta)\,\mathrm{d}\theta - \int_{\theta_2^{*F}}^{\theta_2^{*G}} r_2 - G(\theta)\,\mathrm{d}\theta$$
$$- \int_0^{\theta_1^{*F}} F(\theta) - G(\theta)\,\mathrm{d}\theta - \int_0^{\theta_2^{*F}} F(\theta) - G(\theta)\,\mathrm{d}\theta, \tag{12}$$

where θ_i^{*F} and θ_i^{*G} denote the worst-off types of partner i under F and G.

Assume G first-order stochastically dominates F. Then $\theta_i^{*F} \leq \theta_i^{*G}$, and $G(\theta) \leq r_i$ for all $\theta \leq \theta_i^{*G} = \min\{k, G^{-1}(r_i)\}$. Therefore, the first two terms are non-negative while all other terms are non-positive in this expression. Using $F(k) = G(k)$ and dropping non-positive terms we have

$$\Delta\mathcal{B} \leq \int_0^k F(\theta) - G(\theta)\,\mathrm{d}\theta - \int_0^k F^2(\theta) - G^2(\theta)\,\mathrm{d}\theta - \int_{\theta_1^{*F}}^{\theta_1^{*G}} r_1 - G(\theta)\,\mathrm{d}\theta - \int_0^{\theta_1^{*F}} F(\theta) - G(\theta)\,\mathrm{d}\theta$$
$$= \int_{\theta_1^{*F}}^k F(\theta) - G(\theta)\,\mathrm{d}\theta - \int_0^k F^2(\theta) - G^2(\theta)\,\mathrm{d}\theta - \int_{\theta_1^{*F}}^{\theta_1^{*G}} r_1 - G(\theta)\,\mathrm{d}\theta. \tag{13}$$

[15] In this proof, we drop the arguments r_1, r_2 and k of the functions $\mathcal{L}$, $\mathcal{S}$, θ_i^{*F}, and θ_i^{*G} to ease the notation.

In the last line, the first term is again non-negative while the other two terms are non-positive. If $\theta_1^{*F} = k$, the first term disappears and we get the desired result. In the $\theta_1^{*F} < k$ case, the following algebra shows the nonnegativity of the difference in budget surplus:

$$\begin{aligned}
\Delta\mathcal{B} &\le \int_{\theta_1^{*F}}^{k} F(\theta) - G(\theta)\,\mathrm{d}\theta - \int_0^k F^2(\theta) - G^2(\theta)\,\mathrm{d}\theta - \int_{\theta_1^{*F}}^{\theta_1^{*G}} r_1 - G(\theta)\,\mathrm{d}\theta \\
&= \int_{\theta_1^{*G}}^{k} (F(\theta) - G(\theta))(1 - (F(\theta) + G(\theta)))\mathrm{d}\theta \\
&\qquad + \int_{\theta_1^{*F}}^{\theta_1^{*G}} F(\theta) - G(\theta)\,\mathrm{d}\theta - \int_0^{\theta_1^{*G}} F^2(\theta) - G^2(\theta)\,\mathrm{d}\theta - \int_{\theta_1^{*F}}^{\theta_1^{*G}} r_1 - G(\theta)\,\mathrm{d}\theta \\
&\le \int_{\theta_1^{*F}}^{\theta_1^{*G}} F(\theta) - G(\theta)\,\mathrm{d}\theta - \int_0^{\theta_1^{*G}} F^2(\theta) - G^2(\theta)\,\mathrm{d}\theta - \int_{\theta_1^{*F}}^{\theta_1^{*G}} r_1 - G(\theta)\,\mathrm{d}\theta \\
&= \int_{\theta_1^{*F}}^{\theta_1^{*G}} F(\theta)(1 - F(\theta)) + G^2(\theta) - r_1\,\mathrm{d}\theta - \int_0^{\theta_1^{*F}} F^2(\theta) - G^2(\theta)\,\mathrm{d}\theta \\
&\le \int_{\theta_1^{*F}}^{\theta_1^{*G}} F(\theta)(1 - F(\theta)) + G^2(\theta) - r_1\,\mathrm{d}\theta \\
&\le \int_{\theta_1^{*F}}^{\theta_1^{*G}} F(\theta)(1 - F(\theta)) - r_1(1 - r_1)\,\mathrm{d}\theta \le 0. \qquad (14)
\end{aligned}$$

The first inequality is simply a repetition of the previous formula. The validity of the second inequality can be seen as follows. If $\theta_1^{*G} = k$, the first integral is zero. If $\theta_1^{*G} < k$, then $\theta_1^{*G} = G^{-1}(r_1)$ must hold. Therefore, $F(\theta) \ge G(\theta) \ge r_1 \ge 1/2$ is true for every $\theta_1 \ge \theta_1^{*G}$, guaranteeing the non-positivity of the integrand in the first integral. The third inequality drops a non-positive term, and the forth inequality follows from the fact that $G(\theta) \le r_1$ for $\theta \le \theta_1^{*G} \le G^{-1}(r_1)$. Finally, the last inequality holds since (i) $F(\theta) \ge r_1 \ge 1/2$ for $\theta \ge \theta_1^{*F} = F^{-1}(r_1)$ and (ii) the function $x \mapsto x(1-x)$ being decreasing for $x \ge 1/2$ together guarantee the non-positivity of the integrand.

To show that the decrease in the budget surplus is strict, first notice that the continuity of the cdf's and stochastic dominance implies that the set $\{\theta \in [0,1] : F(\theta) > G(\theta)\}$ must be of positive measure. If F and G are different for some $\theta \in [k,1]$, then the third term in (12) is negative, and dropping it makes the inequality in (13) strict. If F and G are different for some $\theta \in [0,k]$, then either $\theta_1^{*F} = k$, or $\theta_1^{*F} < \theta_1^{*G} \le k$, or $\theta_1^{*F} = \theta_1^{*G} < k$. In the first case, the right-hand side of the first inequality in (14) is already negative since the first and third terms are 0, and the second term is negative. In the second case, the last inequality in (14) must be strict since the integrand is negative for $\theta \in (\theta_1^{*F}, \theta_1^{*G}]$, and this set is of positive measure. Finally, consider the third case. If F and G are different on $[\theta_1^{*G}, k]$, then the second inequality in (14) is strict since the first integrand on the left-hand side is negative on a set of positive measure. Otherwise, the reasoning is the same as in the case $\theta_1^{*F} = k$. ■

B General partnership functions

This section contains the analysis for Subsection 3.4, which studies general partnership functions.

First note that given the assumptions on V, the threshold functions are increasing and twice differentiable almost everywhere. Furthermore, $\underline{\theta}_i$ is convex and $\overline{\theta}_i$ concave whenever they are not equal to 0, 1 and do not intersect with the 45-degree line. For each $i = 1, 2$, denote partner i's net expected utility by $U_i(\theta_i; r_i, V)$, where

$$U_i(\theta_i; r_i, V) = \int_0^{\underline{\theta}_{-i}(\theta_i)} \theta_i - \overline{\theta}_i(\theta_{-i}) - r_i\big(V(\theta_1, \theta_2) - \overline{\theta}_i(\theta_{-i})\big)\,\mathrm{d}F_{-i}(\theta_{-i}) + \\ + \int_{\overline{\theta}_{-i}(\theta_i)}^1 r_i\,(\theta_{-i} - V(\theta_1, \theta_2))\,\mathrm{d}F_{-i}(\theta_{-i}). \tag{15}$$

Given the mechanism Γ^* defined by the allocation rule (9) and the payment rule (10), denote the expected subsidy the arbitrator must incur by

$$\mathcal{S}(r_1, r_2, V) = r_1 \int_0^1 \int_{\overline{\theta}_2(\theta_1)}^1 \theta_2 - \overline{\theta}_2(\theta_1)\,\mathrm{d}F_2(\theta_2)\,\mathrm{d}F_1(\theta_1) \\ + r_2 \int_0^1 \int_0^{\underline{\theta}_2(\theta_1)} \theta_1 - \overline{\theta}_1(\theta_2)\,\mathrm{d}F_2(\theta_2)\,\mathrm{d}F_1(\theta_1). \tag{16}$$

Define the worst-off type of each agent i as $\theta_i^*(r_i, V) \in \mathbf{argmin}_{\theta_i \in [0,1]} U_i(\theta_i; r_i, V)$, and let $\mathcal{L}(r_1, r_2, V) = U_1(\theta_1^*(r_1, V); r_1, V) + U_2(\theta_2^*(r_2, V); r_2, V)$ denote the largest lump-sum fee that can be charged from the agents without violating their participation constraints. (i.e., the sum of the maximal participation fees the agents are willing to pay).

The following lemma summarizes several useful properties of the net expected utility and the worst-off type.

Lemma 4 *Given the mechanism Γ^*, the following properties hold:*

1. $\frac{\partial U_i(\theta_i; r_i, V)}{\partial \theta_i}$ *is non-increasing in r_i;*
2. *For all $r_i \in [0, 1]$, the function $U_i(\theta_i; r_i, V)$ is convex in θ_i;*
3. *For all $r_i \in [0, 1]$, the function $\theta_i^*(r_i, V)$ is non-decreasing in r_i;*
4. *$U_i(\theta_i^*(r_i, V); r_i, V)$ is a concave function of r_i.*

PROOF. Using the Leibniz rule to compute the first derivative of $U_i(\theta_i; r_i, V)$ with respect to θ_i,

$$\frac{\partial U_i(\theta_i; r_i, V)}{\partial \theta_i} = \int_0^{\underline{\theta}_{-i}(\theta_i)} 1 - r_i \frac{\partial V(\theta_1, \theta_2)}{\partial \theta_i} \, \mathrm{d}F_{-i}(\theta_{-i}) - \int_{\overline{\theta}_{-i}(\theta_i)}^1 r_i \frac{\partial V(\theta_1, \theta_2)}{\partial \theta_i} \, \mathrm{d}F_{-i}(\theta_{-i}).$$

Differentiating with respect to r_i, part 1 is immediate. For part 2, computing the second derivative,

$$\begin{aligned}\frac{\partial^2 U_i(\theta_i; r_i, V)}{\partial \theta_i^2} = &- \int_0^{\underline{\theta}_{-i}(\theta_i)} r_i \frac{\partial^2 V(\theta_1, \theta_2)}{\partial \theta_i^2} \, \mathrm{d}F_{-i}(\theta_{-i}) + \left(1 - r_i \frac{\partial V(\theta_i, \underline{\theta}_{-i}(\theta_i))}{\partial \theta_i}\right) f_{-i}(\underline{\theta}_{-i}(\theta_i)) \frac{\mathrm{d}\underline{\theta}_{-i}(\theta_i)}{\mathrm{d}\theta_i} \\ &- \int_{\overline{\theta}_{-i}(\theta_i)}^1 r_i \frac{\partial^2 V(\theta_1, \theta_2)}{\partial \theta_i^2} \, \mathrm{d}F_{-i}(\theta_{-i}) + r_i \frac{\partial V(\theta_i, \overline{\theta}_{-i}(\theta_i))}{\partial \theta_i} f_{-i}(\overline{\theta}_{-i}(\theta_i)) \frac{\mathrm{d}\overline{\theta}_{-i}(\theta_i)}{\mathrm{d}\theta_i}.\end{aligned}$$

The concavity of V, the assumption that $\frac{\partial V(\theta_1, \theta_2)}{\partial \theta_i} \in (0, 1)$ for all θ_i and θ_{-i}, and the observation that $\frac{\mathrm{d}\overline{\theta}_{-i}(\theta_i)}{\mathrm{d}\theta_i}, \frac{\mathrm{d}\underline{\theta}_{-i}(\theta_i)}{\mathrm{d}\theta_i} \geq 0$ for almost every θ_i guarantee that this expression is non-negative, proving part 2. Part 3 is now straightforward from parts 1 and 2. For part 4, let $r_1, r_1' \in [0, 1]$ and $\lambda \in [0, 1]$. Then

$$\begin{aligned}&U_i\Big(\theta_i^*(\lambda r_i + (1 - \lambda) r_i', V); \lambda r_i + (1 - \lambda) r_i', V\Big) \\ &= \lambda U_i\Big(\theta_i^*(\lambda r_i + (1 - \lambda) r_i', V); r_i, V\Big) + (1 - \lambda) U_i\Big(\theta_i^*(\lambda r_i + (1 - \lambda) r_i', V); r_i', V)\Big) \\ &\geq \lambda U_i(\theta_i^*(r_i, V); r_i, V) + (1 - \lambda) U_i(\theta_i^*(r_i', V); r_i', V),\end{aligned}$$

using the linearity of the net expected utility in r and the definition of θ^*. ■

Whether or not a dispute can be resolved efficiently hinges on the relationship between the net expected utilities of the worst-off types and the expected subsidy under Γ^*.

Lemma 5 *Under any efficient, IC and IR mechanism, the partnership dispute* (r_1, F_1, F_2, V) *can be resolved efficiently if and only if* $\mathcal{B}(r_1, r_2, V) = \mathcal{L}(r_1, r_2, V) - \mathcal{S}(r_1, r_2, V) \geq 0$.

The proof follows the same arguments as the one for Lemma 3. It is useful to rearrange the net expected utility as follows:

$$\begin{aligned}U_i(\theta_i; r_i, V) &= r_i \int_0^1 \Big(\max\{\theta_i, \theta_{-i}, V(\theta_1, \theta_2)\} - V(\theta_1, \theta_2)\Big) \mathrm{d}F_{-i}(\theta_{-i}) \\ &\quad + r_{-i} \int_0^1 \max\{0, \theta_i - \overline{\theta}_i(\theta_{-i})\} \, \mathrm{d}F_{-i}(\theta_{-i}) \\ &= r_i \mathbb{E}_{\theta_{-i}} \operatorname{Surp}(\theta_1, \theta_2; V) + r_{-i} \mathbb{E}_{\theta_i} \operatorname{Impr_i}(\theta_1, \theta_2, V),\end{aligned} \tag{17}$$

where $\text{Surp}(\theta_1, \theta_2; V) = \max\{\theta_1, \theta_2, V(\theta_1, \theta_2)\} - V(\theta_1, \theta_2)$ is the ex-post surplus from resolving the partnership dispute, and $\text{Impr}_\text{i}(\theta_1, \theta_2, V) = \max\{0, \theta_i - \overline{\theta}_i(\theta_{-i})\}$ is equal to the ex-post improvement generated by partner i's sole ownership of the asset relative to the value of the best alternative (the best effective partnership or $-i$'s sole ownership), when this improvement is positive, and 0 otherwise.

We can now examine how a partnership's effectiveness affects the possibility of its efficient resolution.

Definition 1 *Partnership W is more effective than partnership V if $W(\theta_1, \theta_2) \geq V(\theta_1, \theta_2)$ for all $\theta_1, \theta_2 \in [0, 1]$.*

Lemma 6 *Assume that the dispute in partnership W is more effective than V.*

1. *The region of efficient dissolution for partnership W is a subset of the efficient region for partnership V. Denoting the analogous thresholds for W by $\underline{\omega}_i$ and $\overline{\omega}_i$, it holds that $\underline{\omega}_i \leq \underline{\theta}_i$ and $\overline{\theta}_i \leq \overline{\omega}_i$, for each $i = 1, 2$.*

2. *For all θ_i and i, the net expected utility is greater under V: $U_i(\theta_i; r_i, V) \geq U_i(\theta_i; r_i, W)$. Consequently, the largest lump-sum fee that can be charged is smaller for a more effective partnership.*

3. *The expected subsidy is greater under V: $\mathcal{S}(r_1, r_2, V) \geq \mathcal{S}(r_1, r_2, W)$.*

PROOF. Part 1 holds by definition of the threshold functions. Part 2 follows immediately from the first point and equation (17), since both integrands are lower for partnership W. Part 3 is a consequence of equation (16); smaller functions are integrated over smaller sets in the case of partnership W. ■

The partnership's effectiveness shapes both its value when kept intact and the region of efficient dissolution. As the following proposition shows, efficient resolution of the dispute crucially depends on the interactions between these two effects.

Proposition 4 *Assume partnership W is more effective than partnership V.*

1. *If the region of efficient dissolution is the same for partnerships V and W (i.e., $\overline{\theta} = \overline{\omega}$ and $\underline{\theta} = \underline{\omega}$), then a partnership dispute is less costly to resolve for partnership V.*

2. *Otherwise, a more effective partnership may be less costly to resolve efficiently.*

PROOF. *Part 1.* Note first that the expected subsidy requires information only about the threshold functions, but not about the other characteristics of the value function. Since the dissolution thresholds are identical for the two partnerships, the expected subsidy paid is the same for V and W. The lump-sum fee, however, is weakly greater for the less effective partnership according to part 2 of Lemma 6. Therefore, the partnership dispute under V is less costly to resolve than the partnership dispute under W.

Part 2. Consider the extreme case in which partner 2 has full ownership (i.e., $r_2 = 1$). Rewriting equation (15) for this case gives:

$$U_1(\theta_1; 0, V) = \int_0^{\underline{\theta}_2(\theta_1)} \theta_1 - \overline{\theta}_1(\theta_2)\, \mathrm{d}F_2(\theta_2),$$
$$U_2(\theta_2; 1, V) = \int_0^{\underline{\theta}_1(\theta_2)} \theta_2 - V(\theta_1, \theta_2)\, \mathrm{d}F_1(\theta_1) + \int_{\overline{\theta}_1(\theta_2)}^1 \theta_1 - V(\theta_1, \theta_2)\, \mathrm{d}F_1(\theta_1).$$

Note that $\theta_1 = 0$ is a worst-off type of player 1. First, the net expected utility of player 1 is non-negative by the individual rationality of the implementing mechanism. Second, by assumption, $0 \leq \underline{\theta}_2(\theta_1) \leq \theta_1$; therefore, it must be that $\underline{\theta}_2(0) = 0$. Consequently, $U_1(0; 0, V) = 0$; a partner without a share is never willing to pay any positive lump-sum fee ex ante.

Fix two value functions ($0 < \varepsilon < 1$):

$$W(\theta_1, \theta_2) = \min\left\{(1-\varepsilon)\theta_1 + \frac{\varepsilon}{2}(1+\theta_2), \frac{1+\theta_2}{2}\right\},$$
$$V(\theta_1, \theta_2) = \min\left\{(1-\varepsilon)\theta_1 + \frac{\varepsilon}{2}(1+\theta_2), \frac{1+\theta_2}{2}\right\} - \frac{\varepsilon}{4}(1-\theta_2).$$

Note that both V and W satisfy the assumptions of the model: they are both piecewise linear, concave functions, increasing in both types.[16] Partnership W is more effective than V for any $\varepsilon > 0$. For a given type θ_2, the threshold values are

$$\underline{\omega}_1(\theta_2) = \max\left\{\frac{1-\varepsilon/2}{1-\varepsilon}\theta_2 - \frac{\varepsilon/2}{1-\varepsilon}, 0\right\}, \quad \overline{\omega}_1(\theta_2) = \frac{1+\theta_2}{2},$$
$$\underline{\theta}_1(\theta_2) = \max\left\{\frac{1-3\varepsilon/4}{1-\varepsilon}\theta_2 - \frac{\varepsilon/4}{1-\varepsilon}, 0\right\}, \quad \overline{\theta}_1(\theta_2) = \frac{1+3\theta_2}{4}.$$

These threshold functions are illustrated in Figure 4. The two shaded regions represent the two different linear segments in the definition of the function W.

[16]Note that these functions are not strictly increasing in θ_1. The example can modified to accommodate such strict monotonicity. The assumption that $r_1 = 1$ is also merely a simplification. To avoid additional complexity, we focus on the simple example above.

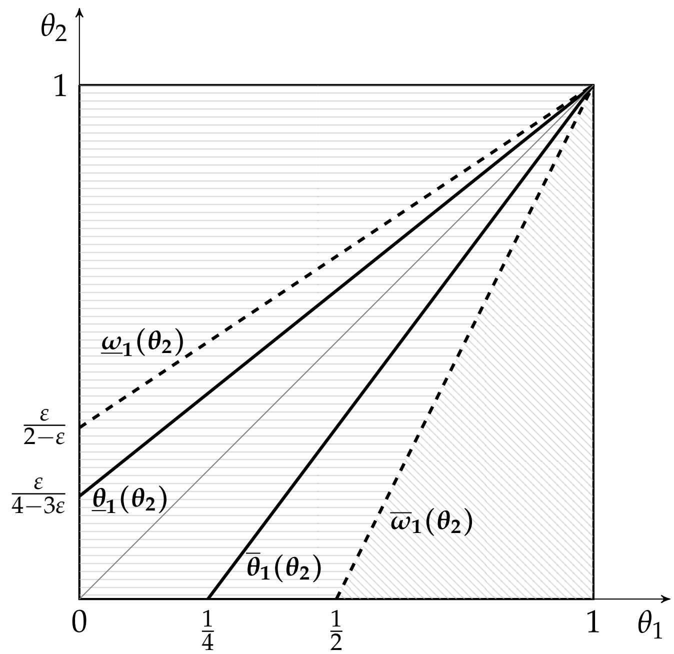

Figure 4: Threshold functions used in the proof of Proposition 4

Computing the difference in net expected utility for partner 2 under V and W:

$$\begin{aligned}
U_2(\theta_2;1,V) - U_2(\theta_2;1,W) &= \int_0^{\underline{\omega}_1(\theta_2)} W(\theta_1,\theta_2) - V(\theta_1,\theta_2)\,\mathrm{d}F_1(\theta_1) + \int\limits_{\underline{\omega}_1(\theta_2)}^{\underline{\theta}_1(\theta_2)} \theta_2 - V(\theta_1,\theta_2)\,\mathrm{d}F_1(\theta_1) \\
&+ \int_{\overline{\theta}_1(\theta_2)}^{\overline{\omega}_1(\theta_2)} \theta_1 - V(\theta_1,\theta_2)\,\mathrm{d}F_1(\theta_1) + \int_{\overline{\omega}_1(\theta_2)}^{1} W(\theta_1,\theta_2) - V(\theta_1,\theta_2)\,\mathrm{d}F_1(\theta_1) \\
&= \int_0^{\underline{\omega}_1(\theta_2)} \underbrace{\frac{\varepsilon}{4}(1-\theta_2)}_{\leq \frac{\varepsilon}{4}}\,\mathrm{d}F_1(\theta_1) \\
&+ \int_{\underline{\omega}_1(\theta_2)}^{\underline{\theta}_1(\theta_2)} \underbrace{-\frac{\varepsilon}{4} - (1-\varepsilon)\theta_1 + \left(1 - \frac{3\varepsilon}{4}\right)\theta_2}_{\leq \frac{\varepsilon}{4}(1-\theta_2) \leq \frac{\varepsilon}{4},\ \text{using } \theta_1 \geq \underline{\omega}_1(\theta_2)}\,\mathrm{d}F_1(\theta_1) \\
&+ \int_{\overline{\theta}_1(\theta_2)}^{\overline{\omega}_1(\theta_2)} \underbrace{-\frac{\varepsilon}{4} + \varepsilon\theta_1 - \frac{3\varepsilon}{4}\theta_2}_{\leq \frac{\varepsilon}{4}(1-\theta_2) \leq \frac{\varepsilon}{4},\ \text{using } \theta_1 \leq \overline{\omega}_1(\theta_2)}\,\mathrm{d}F_1(\theta_1) + \int_{\overline{\omega}_1(\theta_2)}^{1} \underbrace{\frac{\varepsilon}{4}(1-\theta_2)}_{\leq \frac{\varepsilon}{4}}\,\mathrm{d}F_1(\theta_1) \\
&\leq \frac{\varepsilon}{4}\left(\int_0^{\underline{\theta}_1(\theta_2)} 1\,\mathrm{d}F_1(\theta_1) + \int_{\overline{\theta}_1(\theta_2)}^{1} 1\,\mathrm{d}F_1(\theta_1)\right) \leq \frac{\varepsilon}{4}.
\end{aligned}$$

Hence, the difference in the net expected utility for *all types* of partner 2 can be made arbitrarily small by picking a small enough ε. Therefore, the change in the lump-sum fee partner 2 is willing to pay can never be greater than $\varepsilon/4$ either.

The difference in the expected subsidy can be computed using equation (16):

$$\mathcal{S}(r_1, r_2, V) - \mathcal{S}(r_1, r_2, W) = \int_0^1 \int_0^{\underline{\theta}_2(\theta_1)} \theta_1 - \overline{\theta}_1(\theta_2)\, \mathrm{d}F_2(\theta_2)\, \mathrm{d}F_1(\theta_1)$$
$$- \int_0^1 \int_0^{\underline{\omega}_2(\theta_1)} \theta_1 - \overline{\omega}_1(\theta_2)\, \mathrm{d}F_2(\theta_2)\, \mathrm{d}F_1(\theta_1).$$

Note that this expression involves only threshold functions that are below the diagonal in Figure 4, and is therefore independent of ε. Moreover, this difference is positive since $\overline{\omega}_1(\theta_2) > \overline{\theta}_1(\theta_2)$ for all $\theta_2 > 0$ and $\underline{\theta}_2(\theta_1) \geq \underline{\omega}_2(\theta_1)$. Thus, there is a small enough ε such that the difference in the lump-sum fee is smaller than the difference in the expected subsidy. For such ε, a dispute in a less effective partnership is more costly to resolve. ■

Finally, the next result generalizes the findings of Proposition 1 by showing that a dispute can only be resolved efficiently if the partnership is sufficiently ineffective.

Proposition 5 *Fix a pair of threshold functions $\overline{\theta}_1$ and $\overline{\theta}_2$ not identical to 0, and a pair of type distributions F_1 and F_2. There exists $K > 0$ such that for every ownership structure (r_1, r_2), and for every partnership function V satisfying (i) the dissolution thresholds under V coincide with $\overline{\theta}_1$ and $\overline{\theta}_2$, and (ii) the net ex-post surplus from efficiently resolving the partnership dispute is never greater than K, the partnership dispute (r_1, F_1, F_2, V) cannot be resolved efficiently.*

Proof. Using equation (15) derived for the net expected utility, the largest ex-ante fee that partner 1 is willing to pay for participation can be bounded from above as follows:

$$\min_{\theta_1} U_1(\theta_1; r_1, V) = \min_{\theta_1} \Big(r_1 \mathbb{E}_{\theta_2} \operatorname{Surp}(\theta_1, \theta_2; V) + r_2 \mathbb{E}_{\theta_2} \operatorname{Impr}_1(\theta_1, \theta_2, V) \Big)$$
$$\leq r_1 \max_{\theta_1, \theta_2} \operatorname{Surp}(\theta_1, \theta_2; V) + r_2 \min_{\theta_1} \mathbb{E}_{\theta_2} \operatorname{Impr}_1(\theta_1, \theta_2, V).$$

An analogous bound can be established for $\min_{\theta_2} U_2(\theta_2; r_2, V)$. Therefore, the lump-sum fee satisfies

$$\mathcal{L}(r_1, r_2, V) \leq \max_{\theta_1, \theta_2} \operatorname{Surp}(\theta_1, \theta_2; V) + r_2 \min_{\theta_1} \mathbb{E}_{\theta_2} \operatorname{Impr}_1(\theta_1, \theta_2, V) + r_1 \min_{\theta_2} \mathbb{E}_{\theta_1} \operatorname{Impr}_2(\theta_1, \theta_2, V).$$

The expected subsidy can also be rewritten using Impr_1 and Impr_2:

$$\mathcal{S}(r_1, r_2, V) = r_2 \int_0^1 \int_0^{\underline{\theta}_2(\theta_1)} \theta_1 - \overline{\theta}_1(\theta_2)\, \mathrm{d}F_2(\theta_2)\, \mathrm{d}F_1(\theta_1) + r_1 \int_0^1 \int_0^{\underline{\theta}_1(\theta_2)} \theta_2 - \overline{\theta}_2(\theta_1)\, \mathrm{d}F_1(\theta_1)\, \mathrm{d}F_2(\theta_2)$$
$$= r_2 \mathbb{E}_{\theta_1} \mathbb{E}_{\theta_2} \operatorname{Impr}_1(\theta_1, \theta_2, V) + r_1 \mathbb{E}_{\theta_2} \mathbb{E}_{\theta_1} \operatorname{Impr}_2(\theta_1, \theta_2, V).$$

Define the values

$$\begin{aligned}
\delta_1 &= \mathbb{E}_{\theta_1}\mathbb{E}_{\theta_2}\,\mathrm{Impr}_1(\theta_1,\theta_2,V) - \min_{\theta_1}\mathbb{E}_{\theta_2}\,\mathrm{Impr}_1(\theta_1,\theta_2,V);\\
\delta_2 &= \mathbb{E}_{\theta_2}\mathbb{E}_{\theta_1}\,\mathrm{Impr}_2(\theta_1,\theta_2,V) - \min_{\theta_2}\mathbb{E}_{\theta_1}\,\mathrm{Impr}_2(\theta_1,\theta_2,V);\\
\delta &= \min\{\delta_1,\delta_2\}.
\end{aligned}$$

Since $f_1, f_2 > 0$ and $\overline{\theta}_1$ and $\overline{\theta}_2$ are not identical to 0, the functions $\theta_1 \mapsto \mathbb{E}_{\theta_2}\,\mathrm{Impr}_1(\theta_1,\theta_2)$ and $\theta_2 \mapsto \mathbb{E}_{\theta_1}\,\mathrm{Impr}_2(\theta_1,\theta_2)$ are strictly increasing. Hence, $\delta_1, \delta_2, \delta > 0$, and the budget surplus can be bounded from above as follows:

$$\begin{aligned}
\mathcal{L}(r_1,r_2,V) - \mathcal{S}(r_1,r_2,V) &\le \max_{\theta_1,\theta_2}\mathrm{Surp}(\theta_1,\theta_2;V)\\
&\quad + r_2\min_{\theta_1}\mathbb{E}_{\theta_2}\,\mathrm{Impr}_1(\theta_1,\theta_2,V) + r_1\min_{\theta_2}\mathbb{E}_{\theta_1}\,\mathrm{Impr}_2(\theta_1,\theta_2,V)\\
&\quad - r_2\mathbb{E}_{\theta_1}\mathbb{E}_{\theta_2}\,\mathrm{Impr}_1(\theta_1,\theta_2,V) - r_1\mathbb{E}_{\theta_2}\mathbb{E}_{\theta_1}\,\mathrm{Impr}_2(\theta_1,\theta_2,V)\\
&= \max_{\theta_1,\theta_2}\mathrm{Surp}(\theta_1,\theta_2;V) - r_2\delta_1 - r_1\delta_2\\
&\le \max_{\theta_1,\theta_2}\mathrm{Surp}(\theta_1,\theta_2;V) - \delta.
\end{aligned}$$

Note that the functions Impr_1 and Impr_2 depend only on the threshold functions $\overline{\theta}_1$ and $\overline{\theta}_2$, but not on other properties of V. Therefore, the same holds for the number $\delta > 0$. Hence by choosing $K = \delta/2$, for every ownership structure (r_1, r_2), and for every partnership value function V such that (i) the dissolution thresholds under V coincide with $\overline{\theta}_1$ and $\overline{\theta}_2$, and (ii) $\max_{\theta_1,\theta_2}\mathrm{Surp}(\theta_1,\theta_2;V) \le K$, the partnership dispute (r_1, F_1, F_2, V) cannot be resolved efficiently. ■