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A Simple Model of a Money-Management Market with Rational

and Extrapolative Investors

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A Simple Model of a Money-Management Market with Rational and Extrapolative Investors[*]

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Abstract

I analyze a simple model of competition in fees among mutual funds. The funds are vertically differentiated in terms of the expected return they can generate for investors. Following Berk and Green (2004), I assume that a fund's net return is decreasing in the amount of capital it manages, and that there is an infinite supply of capital by rational investors. Unlike the Berg-Green model, I assume there is also a finite supply of capital by non-rational investors who naively chase recent net returns. Investor behavior and the funds' fee profile induce a long-run average amount of managed capital for each fund. I analyze Nash equilibrium in the game played by the funds, focusing on the implications of fund skill on fees, capital flows and net performance.

1 Introduction

Numerous studies since Jensen (1968) established the empirical regularity that actively managed mutual funds do not outperform passive benchmarks in terms of risk-adjusted returns. This finding gave rise to two conflicting interpretations. One widespread viewpoint, often associated with Eugene Fama, is that active funds are "quacks" - i.e. they are unable to systematically beat the passive benchmark. The underlying idea is that the strong version of the Efficient Market Hypothesis holds, and therefore asset prices are fully revealing. According to this viewpoint, any evidence of superior performance by a fund is illusory and transient, and investors who flock to such a fund are rewarding luck rather than skill.

The other view, proposed by Berk and Green (2004), is that actively managed mutual funds have genuine skill, in the sense that they can systematically beat passive

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benchmarks. At the same time, this ability exhibits diminishing returns to scale: The larger the amount of capital the fund manages, the harder it is for the fund to generate above-normal returns. Assuming a sufficiently large supply of "smart money", in market equilibrium the amount of capital that the fund manages will be such that its observed expected return coincides with the benchmark. Thus, the empirical finding that actively managed funds do not outperform benchmark returns is not evidence of lack of skill; rather, it is a sign of the twin assumptions of diminishing returns to scale and an unlimited supply of capital by rational investors.

These two viewpoints are based on conflicting assumptions regarding investor behavior. However, they are only mutually exclusive if we insist that all investors are the same. Yet just as behavioral-finance researchers have constructed models of asset pricing that admit a mixture of rational and boundedly rational investors, so can our understanding of mutual-fund flow-performance patterns benefit from a model that assume a diversely sophisticated population of investors.

In this paper I present a simple model of a market for money management. Following Berk and Green (2004), I assume that each fund potentially generates above-normal returns but exhibits diminishing returns to scale, such that the fund's net return is decreasing in the amount of capital it manages. Also like Berk and Green (2004), I assume an infinite supply of capital by rational, forward-looking investors. These investors form correct equilibrium expectations regarding expected future return from each market alternative. I depart from the Berk-Green model by assuming that the population of investors also includes a finite measure of backward-looking investors, who naively extrapolate from recent returns. By chasing returns, the naive investor commits two errors. First, he neglects sampling error and extrapolates from a single sample point, thus rewarding funds for luck. Second, he neglects the diminishing-returns feature, thus ignoring the dampening effect that his fellow investors' flocking to high-performing funds has on their future returns.

In the model, funds aim to maximize long-run revenues. They simultaneously choose fees ex-ante. The fee profile induces a Markov process that governs the flow of "dumb money" (i.e. the capital supplied by naive, extrapolative investors) between market alternatives (including a passive benchmark). I assume that all funds face a common shock that is $i.i.d$ across time periods. This drastically simplifies the transition probabilities of the Markov process. The funds' long-run average managed capital (consisting of both "smart money" and "dumb money") is determined by the invariant distribution of this process. I study Nash equilibrium in the induced game among the funds.

The basic insight of the model is that because there are two classes of investors and because shorting funds is impossible, a fund's marginal investor at a given period can be of either type. Specifically, the set of funds can be partitioned into "high-quality" and "low-quality" funds. A high-quality fund generates an expected gross return that is sufficiently high relative to the amount of "dumb money", such that even when extrapolative investors select the fund, it continues to attract a positive amount of "smart money". This means that the high-quality fund's marginal investor is always rational. In contrast, a low-quality fund generates a low gross expected return relative to the amount of "dumb money", such that when extrapolative investors select it, they drive its return below zero, and consequently the "smart money" wants to stay clear of the fund altogether. This means that at a low-quality fund, the marginal investor will be rational in some periods and naive in others. This distinction has implications for equilibrium fees, flow-performance patterns and the net payoff that each fund generates for its investors. Specifically, low-quality funds experience more volatile clientele; they inflict a net expected loss on investors, and may charge higher fees than higher-quality funds.

Related literature

The background to this paper is an empirical literature that has documented a positive and convex relation between mutual funds' lagged returns and capital flows (Chevalier and Ellison (1997), Sirri and Tufano (1998). Christoffersen et al. (2014)), as well as the finding that most funds earn a zero excess returns on average (Jensen (1968), Carhart (1997), Fama and French (2010)). As mentioned, there are two conflicting views on how to interpret these findings. Some papers argue that investors' behavioral biases can explain the twin observation of zero average excess returns and performance-sensitive flows. They do so both by documenting flows that are hard to reconcile with rational investment (Frazzini and Lamont (2008)) and by observing errors in investors' strategies. For example, Bailey et al. (2010) identify biases among individual investors, and show that they are associated with poor performance and "trend chasing". In an experiment with university staff and MBA students, Choi et al. (2010) find that a large share of subjects do not minimize fees and focus mainly on past performance when choosing S&P 500 index funds. Goetzmann and Peles (1997) find evidence of overly optimistic recollection of past performance in a survey of mutual fund investors.

The other strand of the literature argues that zero average excess returns and observed flows are consistent with rational investor behavior, under the additional assumptions of uncertainty regarding funds' quality and diminishing returns as the size of managed capital increases (Berk and Green (2004), Berk and van Binsbergen

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(2017)). In this paper, I build a model with ingredients taken from these two strands: diminishing returns and "trend chasing" by naive investors chasing returns. I abstract away from rational learning, as rational investors in my model know the funds' quality.

Other authors have modelled capital flows in the mutual-fund industry, focusing on the effect of learning from past performance. Lynch and Musto (2003) study a model where both investors and fund managers adjust their strategy after observing performance. Huang et al. (2007) argue that heterogeneous participation cost in funds combined with learning can explain the convex flow-performance relation. Choi et al. (2016) and Brown and Wu (2016) extend the Berk-Green model to allow for learning from other funds and test for the investor sophistication.

2   A Model

A market for mutual funds consists of $n > 2$ funds as well as an outside option, denoted 0, which gives a fixed return of 0 to any investor at any time period. Each fund $i = 1, ..., n$ is characterized by the expected gross return $\mu_i > 0$ it can generate. This measures the fund's intrinsic quality. At time $t = 0$, each fund i independently chooses its fee p_i. At every subsequent time period $t = 1, 2, ...$, the fund's net return for its investors is

$$y_i^t = \mu_i + \varepsilon^t - z_i^{t-1} - p_i$$

where:

(i) z_i^{t-1} is the total amount of capital that the fund managed at period $t - 1$.
(ii) ε^t is a draw from an underlying continuous distribution that is symmetric around zero. The draw is independent across time periods but common to all funds (hence it is not indexed by a subscript i).

Denote $z^t = (z_1^t, ..., z_n^t)$. The stochastic evolution of $(z^t)_{t=1,2,...}$ is driven by the behavior of two groups of investors: *naive* (a.k.a extrapolative) and *sophisticated* (a.k.a rational). Naive investors have a fixed supply of capital of size 1. At each period, they choose one of the $n + 1$ available investment alternatives, denoted $s^t \in \{0, 1, ..., n\}$. Thus, s^t indicates where this "dumb money" parks at period t. I also refer to s^t as the *state* of the system at period t.

The evolution of this state is simple: s^1 is arbitrary; and at any $t > 1$, $s^t \in \arg\max_{i=0,...,n} y_i^t$, with symmetric tie-breaking. In other words, naive investors choose the market alternative with the highest realized return at period t. This is why naive investors are "extrapolative" - they chase recent returns. Thus, the amount of dumb

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money that is invested in fund i at period t is

$$z_{i,0}^t = \begin{cases} 1 & \text{if } s^t = i \\ 0 & \text{if } s^t \neq i \end{cases}$$

Sophisticated investors have an infinite supply of capital. They perfectly understand the process of $(s^t, z^t)_{t=1,2,...}$ - i.e. they anticipate the behavior of naifs as well as the behavior of fellow sophisticates. They invest in fund i at period t as long as its expected net return at period $t+1$ is positive - i.e., if

$$E(y_i^{t+1} \mid z_i^t) = \mu_i - z_i^t - p_i \geq 0$$

Shorting funds is impossible. Therefore, if $E(y_i^{t+1} \mid z_i^t) < 0$, sophisticated investors will offer *zero* capital to the fund at period t. Therefore, the amount of "smart money" that is invested in fund i at period t is

$$z_{i,1}^t = \begin{cases} \max\{0, \mu_i - 1 - p_i\} & \text{if } s^t = i \\ \max\{0, \mu_i - p_i\} & \text{if } s^t \neq i \end{cases}$$

Finally, the total amount of capital that each fund manages at any time period is the sum of smart and dumb money parked in the fund - i.e.,

$$z_i^t = z_{i,0}^t + z_{i,1}^t$$

for every fund $i = 1, ..., n$ and every time period $t = 1, 2, ...$

We have thus fully described the stochastic process that governs $(s^t, z^t)_{t=1,2,...}$: s^1 is arbitrary, and

$$s^t = \arg\max_{i=0,...,n} \left(\mu_i + \varepsilon_i^t - z_i^{t-1} - p_i \right)$$

for every $t > 1$ (where $\mu_0 = p_0 = \varepsilon_0^t = 0$), whereas

$$z_i^t = \begin{cases} \max\{1, \mu_i - p_i\} & \text{if } s^t = i \\ \max\{0, \mu_i - p_i\} & \text{if } s^t \neq i \end{cases}$$

for every $t = 1, 2, ...$

We will later see that the process is ergodic, such that the long-run average of z_i^t, denoted $\bar{z}_i$, is well-defined. Fund i's long-run profit is therefore $p_i \cdot \bar{z}_i$. Maximizing this profit is the fund's objective when it chooses its fee p_i at period $t = 0$. We have thus described a simultaneous-move game that the funds play at period zero: Each fund

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chooses its fee to maximize its long-run profit, where this profit is calculated according to the ergodic distribution of the process $(z^t)_{t=1,2,...}$ that is induced by the funds' fees.

3 Analysis

In this section I analyze Nash equilibrium in the period-zero game played by the funds, and highlight aspects of the process $(z_{i,0}^t, z_{i,1}^t)_t$ that is induced by this equilibrium. For every μ, denote $m_\mu = |\{i \mid \mu_i \geq \mu\}|$ - that is, m_μ is the number of funds that generate a gross return above μ.

Proposition 1 *(i) In any Nash equilibrium of the game, there exists $\mu^* \in [2, \frac{4n-1}{2n-1}]$ such that for every fund i:*

$$
p_i = \begin{cases}
\frac{\mu_i}{2} & \text{if } \mu_i \geq \mu^* \\[2mm]
\frac{\mu_i}{2} + \frac{1}{2} \cdot \frac{n-1}{(2n-1)(n-1)+m_{\mu^*}} & \text{if } \mu_i < \mu^*
\end{cases}
$$

(ii) The invariant distribution over states is as follows: $\lambda_0 = \frac{1}{2}$, whereas for every $i = 1, ..., n$:

$$
\lambda_i = \begin{cases}
\frac{1}{2} \cdot \frac{2n-1}{2n(n-1)+m_{\mu^*}} & \text{if } \mu_i \geq \mu^* \\[2mm]
\frac{1}{2} \cdot \frac{2n-2}{2n(n-1)+m_{\mu^*}} & \text{if } \mu_i < \mu^*
\end{cases}
$$

(iii) The net return of each fund $i = 1, ..., n$ at any period t is

$$
y_i^t = \begin{cases}
\varepsilon^t & \text{if } \mu_i \geq \mu^* \text{ or } s^{t-1} \neq i \\[2mm]
\frac{\mu_i}{2} - \frac{1}{2} \cdot \frac{n-1}{(2n-1)(n-1)+m_{\mu^*}} - 1 + \varepsilon^t & \text{if } \mu_i < \mu^* \text{ and } s^{t-1} = i
\end{cases}
$$

Moreover, the net performance of fund i when $\mu_i < \mu^*$ and $s^{t-1} = i$ is strictly below ε^t.

Proof. The proof proceeds in a series of steps.

Step 1: All states $s = 1, ..., n$ are recurrent in Nash equilibrium. That is, every fund attracts naive investors with positive long-run frequency.

Proof: At every period t, $z_{i,1}^t = \max\{0, \mu_i - p_i - z_{i,0}^t\}$. Therefore, the net return of fund i at period $t+1$ is $y_i^{t+1} = \varepsilon^{t+1} + \min\{0, \mu_i - p_i - z_{i,0}^t\}$. Thus, $y_i^{t+1} \leq \varepsilon^{t+1}$ for every fund i. Let us consider two cases. First, suppose $\mu_i - p_i > 0$. Therefore, whenever

$s^t \notin i$, $y_i^{t+1} = \varepsilon^{t+1}$, such that with positive probability, fund i is one of the top-performing alternatives at period $t+1$. As a result, $s^{t+1} = i$ with positive probability. This means that i must be a recurrent state of the Markov process. Second, suppose $\mu_i - p_i \leq 0$. Then, fund i never attracts sophisticated investors. This means that if i is not a recurrent state, the long-run average capital that the fund attracts from naive investors is zero. Therefore, the fund's expected long-run profit is zero. However, this is contradicted by the fact that the fund can always ensure positive profits (from sophisticated investors) by charging $p_i \in (0, \mu_i)$. The reason is as follows. If $\mu_i > 1$, the fund can choose $p_i \in (0, \mu_i - 1)$ such that sophisticated investors will be attracted to the fund at every period. If $\mu_i \leq 1$ such that $\mu_i - p_i - 1 < 0$, $s^t = i$ implies $s^{t+1} \neq i$ and therefore $z_{i,1}^{t+1} > 0$ - i.e., sophisticated investors are attracted to the fund with a strictly positive frequency.

Step 2: If $(p_1, ..., p_n)$ is a Nash equilibrium, then $\mu_i - p_i > 0$ for every fund i. That is, every fund i attracts a positive measure of sophisticated investors at any period t for which $s^t \neq i$.

Proof: Assume the contrary, and let i be a fund with the lowest $\mu_i - p_i$. There are two cases to consider. First, suppose there exist at least two funds $j \neq i$ such that $\mu_i - p_i < \mu_j - p_j$. Then, at every period t, $s^t \neq j$ for at least one of these fund j. The net return of this fund in the next period is $y_j^{t+1} = \min\{0, \mu_j - p_j\} + \varepsilon^{t+1} > \mu_i - p_i + \varepsilon^{t+1}$, such that $s^{t+1} \neq i$, independently of whether $s^t = i$. This means that i cannot be a recurrent state, contradicting Step 1. Second, suppose $\mu_i - p_i = \mu_j - p_j$ for at least $n - 2$ funds $j \neq i$. By Step 1, all states are recurrent. Therefore, there is a positive frequency of periods t in which $s^t = k \neq i$ such that $\mu_j - p_j$ is the same and (by the definition of i) weakly above $\mu_k - p_k$ (and therefore strictly above $\mu_k - p_k - 1$) for all $j \neq k$. As a result, $s^{t+1} = j$ with positive and equal probability for each $j \neq k$. If firm i deviates by lowering p_i by an arbitrarily small amount, the probability with which the state switches from j to i will jump discontinuously, and so will the long-run probability of the state i. Therefore, the deviation is profitable, a contradiction.

Step 3: Let $(p_1, ..., p_n)$ be a Nash equilibrium. Define $M = \{i \mid \mu_i - p_i \geq 1\}$, and denote $m = |M|$. Then, the invariant distribution over $s = 1, ..., n$ is given by the following equations:

$$\lambda^0 = (1 - (n-m) \cdot \lambda^0) \cdot \frac{1}{2n} + (n - m - 1) \cdot \lambda^0 \cdot \frac{1}{2(n-1)}$$

$$\lambda^1 = (1 - (n-m) \cdot \lambda^0) \cdot \frac{1}{2n} + (n - m) \cdot \lambda^0 \cdot \frac{1}{2(n-1)}$$

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where λ^0 is the probability of any state $i \notin M$, and λ^1 is the probability of any state $i \in M$. The solution to these equations is

$$\lambda^0 = \frac{n-1}{m - 2n + 2n^2}$$

$$\lambda^1 = \frac{2n - 1}{2m - 4n + 4n^2}$$

Note that $\lambda^1 > \lambda^0$ and that λ^0 decreases with m.

Proof: Steps 1 and 2 establish that whenever $s^t \neq i$, sophisticated investors will be attracted to fund i such that $z_{i,1}^t = \mu_i - p_i > 0$, and therefore $y_i^{t+1} = \varepsilon^{t+1}$. Now define M as the set of funds i for which $\mu_i - p_i \geq 1$. Sophisticated investors are attracted to such a fund i even during periods t for which $s^t = 1$. Therefore, $z_{i,1}^t = \mu_i - p_i - 1$ during such periods, and therefore $y_i^t = \varepsilon^t$ at *every* period t (independently of s^t) if $i \in M$. In contrast, if $i \notin M$, then $z_{i,1}^t = 0$ and $y^{t+1} = \mu_i - p_i - 1 + \varepsilon^{t+1} < \varepsilon^{t+1}$ whenever $s^t = i$.

It follows that when $s^t = i \in M$, all funds (including i) generate the same net return $y_i^{t+1} = \varepsilon^{t+1}$ at period $t+1$. Since $\Pr(\varepsilon^{t+1} > 0) = \frac{1}{2}$, $s^{t+1} = j$ with equal probability $\frac{1}{2n}$ for each fund j. This implies the system of equations in the statement of the step.

Step 4: Deriving the formulas in the statement of the result

Proof: We are now able to characterize each fund's optimal price as a function of (λ^0, λ^1). Consider a fund i, and let λ_i denote the long-run frequency of periods t for which $s^t = i$. By Step 2, if $i \in M$, then $z_i^t = \mu_i - p_i$ at every t, regardless of s^t; and if $i \notin M$, then $z_i^t = \mu_i - p_i$ when $s^t \neq i$ and $z_i^t = 1$ when $s^t = 1$. Then, fund i's long-run expected profit is

$$\begin{matrix} p_i \cdot \left[\lambda^0 \cdot 1 + (1 - \lambda^0) \cdot (\mu_i - p_i) \right] & \text{if} & p_i \in (\mu_i - 1, \mu_i) \\ p_i \cdot (\mu_i - p_i) & \text{if} & p_i \in [0, \mu_i - 1] \end{matrix}$$

Denote

$$\mu^* = \frac{2 - \lambda^0}{1 - \lambda^0}$$

and observe that since $\lambda^0 \in (0, \frac{1}{2n})$, $\mu^* \in (2, \frac{4n-1}{2n-1})$. The fund's optimal fee p_i^* is thus

$$\begin{matrix} \frac{\mu_i}{2} + \frac{\lambda^0}{2(1 - \lambda^0)} & \text{if} & \mu_i \leq \mu^* \\ \frac{\mu_i}{2} & \text{if} & \mu_i > \mu^* \end{matrix}$$

By the definition of M, we obtain that $M = \{i = 1, ..., n \mid \mu_i \geq \mu^*\}$. This pins down the values of p_i in the statement of the proposition. The expressions for y_i^t and λ_i follow immediately. Note that by the formula for λ^0, it decreases with m. Therefore,

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there is a range of possible values of μ^*: A higher value implies a lower value of m, which in turn implies a higher value of λ^0, which is consistent with a higher value of μ^*. The range of possible values of μ^* is induced by the range of values that λ^0 can take. This completes the proof. ∎

This result has a number of noteworthy features:

High- and low-quality funds
Funds' equilibrium choice of fees divides them into "high-quality" and "low-quality" categories. The high-quality funds are those that generate a gross return above some threshold μ^*, which is not uniquely determined but can take a narrow range of values in the right-hand neighborhood of 2 - i.e., twice the size of dumb money (this range vanishes when n gets large). What distinguishes these two types of funds is that they face different marginal investors. For a high-quality fund, the marginal investor at each period is always sophisticated, independently of the state s. In contrast, the marginal investor at a low-quality fund i is sophisticated only when $s \neq i$; when $s = i$, the naive investors who flock to the fund crowd out sophisticated investors entirely. The long-run probability that naive investors choose a fund only depends on the size of the two categories, and on which of the two categories the fund belongs to. High-quality funds have a small advantage in this regard, which decreases in their number.

Funds' net returns
When a fund's marginal investor at period t is sophisticated, this means that its expected net return is zero, such that $y^{t+1} = \varepsilon^{t+1}$. In contrast, if the fund's marginal investor at period t is naive, then by definition the sophisticated investors shy away from the fund - which can only happen when naive investors pick the fund and drive its expected net return below zero. By the common-shock assumption, it follows that this fund cannot be the top performer in the next period: Either the shock will be negative and the outside option will be the best-performing alternative, or the shock will be positive and the competing funds will all tie as the top-performers. This means that the flow of dumb money between alternatives does not depend on the fee profile.

Thus, the top-performing alternative at any period t is either the outside option or a fund that did not attract naive investors at period $t-1$. This means that the highest net return in any period t is $\max\{0, \varepsilon^t\}$. Since this quantity is completely invariant to the funds' quality, sophisticated investors will completely disregard it. That is, all return-chasing behavior in equilibrium will be due to the naive investors. Note that this stark result is partly due to the assumption that sophisticated investors know the funds' quality, hence they have nothing to learn from realized returns - unlike the Berk-

Green model, which attempts to explain return-chasing behavior by rational learning in the presence of incomplete information.

Non-monotone fees

Funds' equilibrium fees are not entirely monotone in their quality. Consider two funds of similar quality. If the funds' gross returns are on different sides of the cutoff μ^*, the lower-quality fund will actually charge a higher fee. In contrast, fees are monotone *within* each quality class. The origin of this effect lies in the different effective price elasticities of the two investor types. Sophisticated investors adjust the capital they supply to the *expected* net return, which decreases with the fund's fee. In contrast, naive investors respond to the highest *recent* net return, which is purely a result of the noise realization. Therefore, they are not responsive to prices. Because the marginal investor in a low-quality fund is sometimes naive, the effective demand these funds face is inelastic relative to the demand faced by high-quality funds. This explains the non-monotonicity effect.

Multiple equilibria

If there are funds whose quality is in the range $[2, 2 + \frac{1}{2n-1}]$, then there is scope for multiple Nash equilibria. The reason is that the game exhibits strategic complementarities: When there are more high-quality funds, the probability that naive investors visit an individual low-quality fund goes down. This in turn reduces the fund's incentive to charge a high fee that exploits naive investors' inelastic demand. As a result, it is more likely that the fund's marginal investor will be sophisticated, which is the defining feature of high-quality funds. Therefore, the larger number of high-quality funds is self-sustaining. Note, however that this possible multiplicity disappears if there are no funds whose quality is in the above range.

Let us now characterize the process over $(z_{i,0}^t, z_{i,1}^t)_{i,t}$ that is induced by equilibrium fees. Consider the Markov process that governs the transition of naive investors between market alternatives. Given the equilibrium defined by μ^*, let $M = \{i \mid \mu_i \geq \mu^*\}$ denote the set of high-quality funds.

Remark 1 *Consider a Nash equilibrium as described in the proposition. Then, the transition probabilities $(\pi(s^{t+1} \mid s^t))$ that govern the behavior of naive investors are as*

follows:

$$\pi(s^{t+1} \mid s^t \in M \cup \{0\}) = \begin{cases} \frac{1}{2} & if & s^{t+1} = 0 \\ \frac{1}{2n} & otherwise \end{cases}$$

$$\pi(s^{t+1} \mid s^t \notin M \cup \{0\}) = \begin{cases} \frac{1}{2} & if & s^{t+1} = 0 \\ 0 & if & s^{t+1} = s^t \\ \frac{1}{2(n-1)} & otherwise \end{cases}$$

Thus, after every history, there is a 50% chance that the naive investors will choose the outside option. Otherwise, the transition matrix only distinguishes between high- and low-quality funds, but does not distinguish between funds that belong to the same category. When naive investors are at a low-quality fund at a certain period, they will necessarily abandon it in the next period. In contrast, when they are at a high-quality fund at a certain period, it is as likely as any other fund to attract them at the next period.

Let us turn to the sophisticated investors. Here, too, the distinction between high- and low-quality funds is relevant. Consider a high-quality fund i. Then, at every period its total managed capital will be $\mu_i/2$. When $s^t = i$, this amount will consist of the amount of 1 provided by naive investors and the rest $\mu_i/2 - 1$ will be provided by sophisticates. The "smart money" will thus flow into and out of the fund in response to the behavior of the "dumb money". Now consider a low-quality fund i. When $s^t = i$, there will be no "smart money" at the fund: its managed capital will purely consist of the amount 1 provided by naive investors. In contrast, when $s^t \neq i$, the capital managed by the fund, totalling $\mu_i - p_i$ (which is below $\mu_i/2$) purely consists of "smart money".

The picture of mutual fund flows that emerges from this analysis is a cat-and-mouse dynamic between smart and dumb money. Sophisticated investors always flow into alternatives that have not been top performers in recent experience, because they expect that naive investors will not pick them. As long as the naifs stay out of a fund, the amount of smart money that the fund manages will stay constant. The smart money will flow out of the fund when the fund is a top performer such that dumb money is expected to flow into the fund. When the fund is of low quality, no smart money will be left at the fund when the dumb money flows into it.

The final piece of the analysis concerns investor welfare. Define the net investor payoff at a fund i as the long-run expectation of $z_i^t \cdot (\mu_i - z_i^t - p_i)$. For high-quality funds, the characterization is simple: Since $z_i^t = p_i = \mu_i/2$ for every t whenever $i \in M$,

the fund's net investor payoff is zero. The case of low-quality funds is more involves. When $i \notin M$, $z_i^t = 1$ whenever $s^t = i$ and $z_i^t = \mu_i - p_i$ whenever $s^t \neq i$. Therefore, the fund's net investor payoff is zero whenever $s^t \neq i$, and $\mu_i - 1 - p_i$ whenever $s^t = 1$. Therefore, the fund's expected net is

$$\lambda_i \cdot 1 \cdot (\mu_i - 1 - p_i)$$

Plugging our expressions for p_i and λ_i, we obtain

$$\frac{n-1}{2[2n(n-1) + m_{\mu^*}]} \left[\mu_i - \frac{(4n-1)(n-1) + m_{\mu^*}}{(2n-1)(n-1) + m_{\mu^*}} \right]$$

which is negative, given the definition of μ^*. Thus, low-quality funds induce a negative net payoff for naive investors.

4 Conclusion

The simple model I presented in this paper provides a prototype of models of the market for money management, where the population of investors includes both sophisticates (who correctly predict expected net return, taking into account the effect of the total capital that the funds manage) and naifs (who chase funds with high recent net returns). The picture that emerges from such a model is that sophisticated investors correctly predict when naifs will flow into a fund, and they adjust their supply of capital accordingly. In particular, they will completely pull out of a low-quality fund when they anticipate the arrival of naifs. At those periods, the naifs end up generating a negative net return (relative to the benchmark). In all other instances, sophisticates and naifs achieve zero net return, as in the Berk-Green model.

The assumption that all funds are subjected to the same shock at every period greatly simplified the analysis. A challenge for future modeling is to allow for idiosyncratic shocks. This extension not only complicates the analysis of the Markov process over s^t, but it would also enable the analysis of how funds might strategically choose their investment portfolio in period zero, in order to control the correlation of its returns with the other market alternatives. (For a prototype of such a model, see Spiegler (2006), where "quack forecasters" compete in prices and forecasting strategies.)

Another element that is missing from the simple model is the lack of any inertia in the behavior of naive investors. In reality, it appears that naive investors do not constantly chasing returns - once they have selected a fund, they exhibit inertia and

tend to resume the return-chasing search phase only when their current fund's net performance drops below some threshold (in absolute or relative terms). Incorporating this sort of performance-sensitive inertia would complicate the Markov process because the probability that investors switch out of a fund would depend on its price. (Merely assuming that investors have a constant propensity to stick to their current fund would have no effect on the main result, because it would not change the invariant distribution of the Markov process.)

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