

# Risk and Exchange Rates<sup>\*,†</sup>

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†Earlier drafts of this paper were presented in 1998 at Harvard, Wharton, MIT, Stanford, LSE, and at the 1998 National Bureau of Economic Research Summer Institute. Following our initial presentations of this paper, its approach has furnished the workhorse framework for an important subsequent literature that has made substantial advances. See, for example, Bacchetta and van Wincoop (2000), Devereux and Engel (2000), Engel (1999), and Obstfeld and Rogoff (2000a, 2000b). We do not attempt to survey that literature here; see Obstfeld (2000) for a recent discussion. This version of our original paper, however, adds only slight revisions and corrections to National Bureau of Economic Research Working Paper 6694 (August 1998). We have benefited from helpful comments from Paolo Pesenti, Alan Stockman, and Cedric Tille. Remaining errors are ours alone. Support from a National Science Foundation grant to the NBER is acknowledged with thanks.

Nominal exchange risk is a ubiquitous factor in international economic policy analysis. For example, sudden appreciations of the dollar following financial crises outside the United States often are ascribed to “safe haven” portfolio shifts. The elimination of national currencies in Europe has been rationalized in part by the claim that uncertain exchange rates discourage trade, and thereby hamper the full realization of the gains from removing other obstacles to commodity and asset-market integration.

Unfortunately, the analytical underpinnings of such widely discussed phenomena have received scant attention. In analyzing the properties of stochastic general-equilibrium monetary models, researchers typically rely on a certainty equivalence assumption to approximate exact equilibrium relationships. This practice, as Kimball (1995, p. 1243) remarks, “precludes a serious welfare analysis of changes that affect the variance of output.” In the relatively rare cases in which higher moments are considered theoretically, tractability usually has required the assumption of instantaneously flexible commodity prices and wages.<sup>1</sup> That modeling choice not only assumes away much of the real effect of nominal exchange rate uncertainty. It simultaneously precludes discussion of the feedback from monetary non-neutralities to market risks, and instead imposes exogenously the covariances between monetary shocks and consumption levels. And it is unrealistic. There is strong, indeed overwhelming, empirical evidence that the nominal prices of domestically-produced goods tend to adjust far more sluggishly than exchange rates.<sup>2</sup> But the implications of product price setting in general-equilibrium models with uncertainty remains largely unexplored, despite being at the core of the debate over the impact of exchange-rate volatility.

The model we propose in this paper extends the “new open-economy macroeconomics” framework of Obstfeld and Rogoff (1995, 1996), Corsetti

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<sup>1</sup>General-equilibrium models of exchange-rate risk typically follow Lucas (1982) in assuming that all prices are fully flexible. See Svensson (1985), Hodrick (1989), Engel (1992), and Obstfeld and Rogoff (1996, ch. 8) for relevant extensions.

<sup>2</sup>See, for example, Mussa (1986), Baxter and Stockman (1989), Flood and Rose (1995), and Obstfeld (1998).

and Pesenti (2001), and others to an explicitly stochastic environment.<sup>3</sup> We analyze a sticky-price monetary model in which risk has an impact not only on asset prices and short-term interest rates, but also on the price-setting decisions of individual producers, and thus on expected output and international trade flows.

Our approach allows one to quantify explicitly the welfare tradeoff between alternative exchange-rate regimes, and to relate that tradeoff to country size. In its emphasis on the exact welfare analysis of alternative exchange-rate regimes, this paper follows in the pioneering footsteps of Assaf Razin and his co-author Elhanan Helpman (see, for example, Helpman and Razin 1979). Interestingly, we find in this paper’s model that even in cases where uncertainty induces substantial heterogeneity across countries both *ex ante* and *ex post*, there may be a strong, even perfect, convergence of interests in choosing a global monetary system.

Although the main thrust of our approach is normative, the model also yields some potentially important positive results. For example, we show how exchange risk affects the *level* of the exchange rate, and not just the predictable return to forward speculation that has been studied extensively in earlier literature. While these two effects of exchange risk turn out to be proportional, the multiplier linking them can be quite large. Indeed, our analysis suggests that fluctuations in the “level” risk premium may be a very significant source of exchange-rate volatility, one that is missing or inadequately captured in standard empirical exchange rate equations (e.g.,

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<sup>3</sup>Obstfeld (1998) discusses other recent contributions to this literature. Rankin (1998) develops a very interesting analysis of a small open economy with complete asset markets and competitive production, in which monopolistic labor suppliers preset money wages. While (like us) he examines the positive effects of monetary uncertainty, he does not systematically explore the welfare effects of policies. An earlier complete-markets model with nominal rigidities is that of Svensson and van Wijnbergen (1989). Svensson and van Wijnbergen (1989), building on Svensson’s (1986) closed-economy model, provide an early discussion of price-setting in advance by maximizing firms facing uncertainty. The appendix to Svensson (1986) briefly discusses the welfare impact of an infinitesimal degree of money-supply variability, but such higher-moment effects are not the main focus of his paper.

Meese and Rogoff 1983).<sup>4</sup>

Finally, under specified conditions, we can solve the model explicitly for equilibrium second as well as first moments. The solution yields novel insights about the exchange risk premium in a sticky-price setting within which the covariance between monetary shocks and consumption is endogenous, unlike in models such as Lucas's (1982). For example, we find that when home monetary volatility is an important source of uncertainty, home-currency assets actually may serve as a hedge against consumption risk. The reason is that under price stickiness, positive home monetary surprises lead home output and consumption to covary positively with the home-currency price of foreign exchange. If so, domestic nominal interest rates are *lower* than would be the case under risk neutrality. We show, however, that greater domestic monetary variability, despite causing lower home nominal interest rates and an appreciation of the home currency (all else equal), must reduce expected welfare both at home and abroad.

Section 1 of the paper presents a basic model with monetary and productivity shocks, while section 2 employs a key simplifying feature which turns out to imply that current accounts are always zero in equilibrium. Section 3 shows how the presetting of nominal goods prices can be analyzed without a certainty-equivalence assumption, and discusses some implications. In section 4 we complete the derivation of the model's equilibrium, and in section 5 we show how to calculate the exchange risk premium explicitly as a function of underlying money-supply shocks. Except for the money demand function (which we generally must log-linearize), the model is naturally log-linear provided the underlying monetary and productivity shocks are lognormally distributed. Section 6 offers a guide to quantifying the "amplification effect"

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<sup>4</sup>A large effect of risk factors on exchange rate levels was suggested by Frankel and Meese (1987), based on partial-equilibrium intuition. Hodrick (1989), using a version of Svensson's (1985) cash-in-advance model with a variable velocity of money, showed the effect of higher-moment fundamentals on exchange rates in a flexible-price, general equilibrium setting, but his restrictive money-demand specification implied a generally moderate effect of monetary risk factors. In our setup, in which the interest semi-elasticity of money demand may be any negative number, the exchange-rate level effect of exchange risk can plausibly be an order of magnitude higher than in Hodrick's.

that links the level exchange rate risk premium to the standard forward risk premium that characterizes excess returns to currency speculation.

Section 7 discusses the link between policy uncertainty and ex ante welfare, taking up results on country size (which can be surprising) and developing a quantitative example illustrating that the costs of exchange volatility can be big. Importantly, we find that by explicitly treating price setting under uncertainty, we obtain much more general and elegant welfare results than would be possible under the usual assumption of certainty equivalence. It is important to note that many of the key welfare results derived in this section do not depend on the ancillary linearization of the money-demand function needed to get a closed-form solution in sections 4 and 5. Finally, section 8 summarizes; a variety of extensions and technical derivations are relegated to appendices.

## 1 A Stochastic Two-Country Model

The model is a stochastic version of the one in Obstfeld and Rogoff (1995, 1996), modified along lines proposed by Corsetti and Pesenti (2001), who present a model in which current account imbalances are always zero *in equilibrium*. The general issue of current accounts is quite important to any complete model of international policy transmission, but allowing for imbalances here would pose some very subtle and difficult technical issues, issues that we prefer to abstract from in a first pass at a stochastic sticky-price model.

### 1.1 Preferences and Technology

There are two countries, Home and Foreign. Home agents are indexed by numbers in the interval  $[0, n]$ , while Foreign agents reside on  $(n, 1]$ . Every individual is a “yeoman farmer” and is presumed to have a monopoly in producing a single good, also indexed by  $n$ . Preferences of the representative

Home agent are given by

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \left( \frac{1}{1+\delta} \right)^{s-t} \left[ \frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left( \frac{M_s}{P_s} \right)^{1-\varepsilon} - \frac{\kappa_s}{2} Y_s^2 \right] \right\}, \quad (1)$$

where  $C$  is an index of per capita consumptions of Home and Foreign commodity bundles,

$$C = \frac{C_H^n C_F^{1-n}}{n^n (1-n)^{1-n}}, \quad (2)$$

with

$$C_H = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n C(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad C_F = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 C(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1. \quad (3)$$

Thus, across goods produced within a country the elasticity of substitution is  $\theta$ , while the elasticity of substitution between the composite Home and Foreign goods is 1. Utility depends negatively on individual output,  $Y$ , because production requires irksome labor effort. Utility depends positively on individual domestic real money balances,  $M/P$ , because of the role of money in reducing transaction costs. Foreign agents have identical preferences except that  $\kappa^*$  may differ from  $\kappa$ ,  $Y^*$  may differ from  $Y$ , and Foreign agents hold their own national currency  $M^*$ , which is deflated in their utility function by the Foreign general consumer price index  $P^*$ .

The coefficient  $\kappa$ —which multiplies  $Y^2$  in the utility function (1) and can be viewed as inversely related to productivity—may be a random variable. All random shocks in the model are assumed to be lognormally distributed.

## 1.2 Prices, Demand, and Budget Constraints

The overall Home-currency consumption-based price index is given by

$$P = P_H^n P_F^{1-n} \quad (4)$$

where the subindexes for Home and Foreign products are, respectively,

$$P_H = \left[ \frac{1}{n} \int_0^n P(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \quad P_F = \left[ \frac{1}{1-n} \int_n^1 P(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

The law of one price is assumed to hold across all individual goods, so that  $P(z) = \mathcal{E}P^*(z)$ ,  $\forall z \in [0, 1]$ , where asterisks denote Foreign values of the corresponding Home variables, and  $\mathcal{E}$  is the nominal exchange rate (the Home price of Foreign currency). Because Home and Foreign agents have identical preferences, the law of one price implies that purchasing power parity must hold for overall consumer price indexes:

$$P = \mathcal{E}P^*. \quad (5)$$

Under the subutility functions in (3), the allocation of a representative individual's demand across each of the goods produced within a country is given by

$$C(h) = \frac{1}{n} \left[ \frac{P(h)}{P_H} \right]^{-\theta} C_H, \quad C(f) = \frac{1}{1-n} \left[ \frac{P(f)}{P_F} \right]^{-\theta} C_F, \quad (6)$$

(where  $h$  denotes the representative Home good and  $f$  the representative Foreign good). The Cobb-Douglas *total* consumption index, eq. (2), implies that demands for the composite Home and Foreign goods,  $C_H$  and  $C_F$ , are given by

$$C_H = n \left( \frac{P_H}{P} \right)^{-1} C, \quad C_F = (1-n) \left( \frac{P_F}{P} \right)^{-1} C. \quad (7)$$

Define world consumption as

$$C^w \equiv nC + (1-n)C^*, \quad (8)$$

where  $C$  is the total consumption of a representative Home resident and  $C^*$  that of a representative Foreign resident. (Since world population is 1,  $C^w$  is per capita as well as total world consumption.) Then, combining eqs. (6) and (7), and aggregating the result with the identical Foreign demand functions, we see that the global demand for individual goods is given by

$$C(h) = \left[ \frac{P(h)}{P_H} \right]^{-\theta} \left( \frac{P_H}{P} \right)^{-1} C^w, \quad C(f) = \left[ \frac{P(f)}{P_F} \right]^{-\theta} \left( \frac{P_F}{P} \right)^{-1} C^w. \quad (9)$$

Home and Foreign agents can trade riskless real bonds that are indexed to total consumption  $C$ . Let  $r_t$  denote the consumption-based real interest rate

between dates  $t - 1$  and  $t$  (the own-rate of interest on the total consumption basket). Written in terms of Home money, the intertemporal budget constraint for the representative Home agent is

$$P_t B_{t+1} + M_t = P_t(1 + r_t)B_t + M_{t-1} + P_t(h)Y_t(h) - P_t C_t - P_t \tau_t, \quad (10)$$

where  $\tau$  denotes lump-sum taxes and  $B_{t+1}$  denotes end of period  $t$  bond holdings. [In contrast, our assumption on money holdings is that  $M_t$  denotes the end of period  $t$  stock; recall also eq. (1).] It is important to note that although we do not explicitly allow for international trade in equity in this model, such trade will turn out to be redundant. In equilibrium (as we will show), each country's share of world income is constant due to the assumption of a unit elasticity of intratemporal demand across the Home and Foreign composite goods.

We simplify by setting government spending equal to zero throughout, so that the Home government budget constraint, for example, is given by

$$0 = \tau_t + \frac{M_t - M_{t-1}}{P_t}. \quad (11)$$

Appendix A indicates how government spending shocks could be introduced. The (gross) rate of growth of the money supply is assumed to be a lognormally distributed random variable.

We also assume that initial net international asset holdings  $B = 0$ .

## 2 Goods Market Clearing and the Redundancy of Securities Markets

Whether flexible or preset prices prevail, the goods market clears. Even before examining the first-order optimality conditions of consumer/producers, we can infer the key relationships linking national to global consumption levels, and global consumption to national outputs. We explore these relationships first because they imply a key (though special) property of the model: securities markets are redundant and, as a result, current accounts always balance exactly in equilibrium.

Taking account of the differing populations in the two countries, total output supplies equal demands when

$$\begin{aligned} n[nPC + (1-n)PC^*] &= nP_H Y, \\ (1-n)[nPC + (1-n)PC^*] &= (1-n)P_F Y^*. \end{aligned}$$

These equations imply that

$$\frac{P_H}{P_F} = \frac{Y^*}{Y}. \quad (12)$$

As in Corsetti-Pesenti (2001), this relation, together with our assumption that initial net international asset holdings  $B = 0$ , implies that current accounts always are zero. The intuition for this result, of course, is that eq. (12) gives countries constant (indeed equal) shares of per capita world real income, regardless of the pattern of shocks. Given constant real income shares (and our assumption of isoelastic preferences over total consumption  $C$ ), countries always consume exactly their real incomes:

$$C = \frac{P_H Y}{P}, \quad C^* = \frac{P_F^* Y^*}{P^*}. \quad (13)$$

As Cole and Obstfeld (1991) point out, price responses can make international trade in securities redundant with Cobb-Douglas preferences.<sup>5</sup>

An immediate corollary of eqs. (12) and (13) is that

$$C^w = C = C^*. \quad (14)$$

Per capita consumption shares for Home and Foreign are not only constant, but equal.<sup>6,7</sup>

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<sup>5</sup>More formally, notice that for the allocation in eq. (13), Home and Foreign intertemporal marginal rates of consumption substitution are equal for every future state of nature. Since leisure is a nontraded good, there are no unexploited gains from trade and international trade in securities therefore is redundant. Without the assumption that initially  $B = 0$ , this would not necessarily be the case. Shocks that led to temporary changes in the real interest rate could then induce current account movements. [See eq. (26) on p. 78 of Obstfeld and Rogoff (1996)].

<sup>6</sup>One reason for this equality, of course, is that we have chosen the utility function so that expenditure shares are the same as population shares. That assumption is plausible and convenient but easily relaxed.

<sup>7</sup>Recall from (5) that  $P = \mathcal{E}P^*$  always holds. Note then, by eqs. (4), (8), (12), and

### 3 Producer Behavior under Preset Prices

We now look at how monopolistic producers plan output and prices when commodity prices are set a period in advance in producers' currency and cannot be revised until the following period.<sup>8</sup> Thus, we assume that  $P_t(h)$  and  $P_t^*(f)$  are set at the end of period  $t - 1$  and cannot be changed during period  $t$ .<sup>9</sup> With preset prices, supply is no longer determined by the leisure-consumption tradeoff that governs behavior under flexible prices. (Appendix A describes the flexible-price solution.) Instead, because price initially exceeds marginal cost, supply moves to accommodate any unanticipated shock to demand (provided the shock is not so large that full accommodation of demand would raise marginal cost above price).<sup>10</sup>

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(13), that

$$\begin{aligned} C^w &= n \frac{P_H Y}{P} + (1 - n) \frac{P_F^* Y^*}{P^*} \\ &= n \left( \frac{P_H}{P_F} \right)^{1-n} Y + (1 - n) \left( \frac{P_F^*}{P_H^*} \right)^n Y^* \\ &= Y^n (Y^*)^{1-n}. \end{aligned}$$

<sup>8</sup>While this paper assumes the law of one price and pricing in producers' currencies, extensions such as Devereux and Engel (2000) model the failure of the law of one price at the consumer level as well as the apparent rigidity of all consumer prices (even those of imported retail goods) in local currency terms. A debate has arisen over the importance of distinguishing between actual import prices and the retail prices consumers pay for goods that originate abroad. Obstfeld (2000) offers a more detailed discussion, and argues that failure of the law of one price at the consumer level is consistent with expenditure-switching effects of exchange rate changes such as those we analyze below.

<sup>9</sup>More realistic dynamics would result from the assumption of Calvo-style multiperiod staggered price setting; see Kimball (1995), for example. We forgo that realism here to obtain simpler and more accessible results, and leave the inclusion of Calvo contracts for future research.

<sup>10</sup>One must be careful in interpreting a stochastic version of the model if one assumes that supply always accommodates demand under sticky prices. For large enough shocks the voluntary participation constraint will be violated, as Corsetti and Pesenti (1998) stress. Thus, the results of our stochastic model under sticky prices should be viewed as approximate. The approximation can be made arbitrarily precise by looking at ever-

In a certainty-equivalence setup, equilibria with preset prices differ from ones with flexible prices only because of the effects of unanticipated shocks. But that is not the case in an explicitly stochastic version of the model. When  $P_t(h)$  and  $P_t^*(f)$  are set at the end of period  $t - 1$ , they are not generally set at their certainty-equivalent values. Prices are set with a view toward hedging the risks the producer faces. This nuance is quite important, both in understanding the effects of risk on the exchange rate and other macro variables and in using the model to ascertain the welfare effects of alternative macroeconomic policies.

### 3.1 The Price Setting Problem

Consider the pricing decision of the representative Home agent. On date  $t - 1$ ,  $P_t(h)$  is set to maximize the objective function (1), but with the expected value conditional on date  $t - 1$ , instead of date  $t$ , information. Using the individual's intertemporal budget constraint (10) to substitute out for  $C_t$  in the utility function (1), and then using the demand function (9) to substitute out for  $Y_t(h)$ , and finally taking  $t - 1$  expectations over both sides, we obtain the maximand  $E_{t-1}U_t$  as the infinite sum (starting with  $s = t$ ) of terms of the form:

$$E_{t-1} \left( \frac{1}{1 + \delta} \right)^{s-t} \left\{ \frac{1}{1 - \rho} \left[ \left( \frac{P_s(h)}{P_{H,s}} \right)^{1-\theta} C_s^w + (1 + r_s)B_s - B_{s+1} \right. \right. \quad (15)$$

$$\left. \left. + \frac{M_{s-1} - M_s}{P_s} - \tau_s \right]^{1-\rho} + \frac{\chi}{1 - \varepsilon} \left( \frac{M_s}{P_s} \right)^{1-\varepsilon} - \frac{\kappa}{2} \left[ \left( \frac{P_s(h)}{P_{H,s}} \right)^{-\theta} \left( \frac{P_{H,s}}{P_s} \right)^{-1} C_s^w \right]^2 \right\}.$$

Optimal price setting in period  $t - 1$  reflects minimization of the *expected* discrepancy between the marginal utility of marginal revenue and the marginal disutility of effort.

Differentiating the above expression with respect to  $P_t(h)$  yields

$$E \left\{ C^{-\rho} (\theta - 1) P(h)^{-\theta} P_H^{\theta-1} C^w \right\}$$

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smaller variances for the exogenous shocks.

$$= \mathbb{E} \left\{ \kappa \theta \left[ P(h)^{-\theta-1} P_H^{\theta-1} P C^w \right] \left[ \left( \frac{P(h)}{P_H} \right)^{-\theta} \left( \frac{P_H}{P} \right)^{-1} C^w \right] \right\},$$

where we suppress  $t - 1$  subscripts on the expectations operator and  $t$  subscripts on all variables when there is no risk of confusion. Noting that  $P(h) = P_H$  (in a symmetric equilibrium), that  $P_H$  is known in advance, and finally that  $C = C^w$ —eq. (14) applies—we can rewrite this expression as

$$\mathbb{E} \left\{ C^{1-\rho} \right\} = \mathbb{E} \left\{ \kappa \left( \frac{\theta}{\theta-1} \right) \left( \frac{PC}{P_H} \right)^2 \right\}. \quad (16)$$

The parallel Foreign relation is

$$\mathbb{E} \left\{ C^{1-\rho} \right\} = \mathbb{E} \left\{ \kappa^* \left( \frac{\theta}{\theta-1} \right) \left( \frac{P^*C}{P_F^*} \right)^2 \right\}, \quad (17)$$

where  $C = C^*$  has been used.

Notice that the (flexible) nominal Home wage implicit in this model is given by

$$\frac{W}{P} = \kappa \frac{Y}{C^{-\rho}}.$$

Since  $PC/P_H = Y$  in equilibrium (the national budget constraint), eq. (16) would reduce to the constant-elasticity markup equation  $P_H = \left( \frac{\theta}{\theta-1} \right) W$  under certainty (with the corresponding equation for  $P_F^*$ ).<sup>11</sup> Equation (16) and its Foreign analog modify those familiar relationships to account for uncertainty subsequent to the setting of nominal product prices.

## 3.2 Implications for Ex Ante Terms of Trade

Assuming that  $C$  and  $\mathcal{E}$  are jointly lognormally distributed, we can express the solution for the ex ante terms of trade and ex ante consumption in logs. (We shall show later that  $C$  and  $\mathcal{E}$  indeed have lognormal distributions in equilibrium if the exogenous shocks hitting the world economy are lognormal as well.) The solution procedure is not especially illuminating, so it is relegated to Appendix B.

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<sup>11</sup>Obstfeld and Rogoff (2000a) and Obstfeld (2000) explore related models with sticky nominal wages.

To gain some preliminary intuition about the solution described by eqs. (16) and (17), we simplify and adopt the convenient assumptions that  $\kappa$  and  $\kappa^*$  have identical lognormal distributions and are serially uncorrelated, so that  $E_{t-1} \log \kappa_t = E_{t-1} \log \kappa_t^*$ . Notice that if these shocks to “productivity” are purely temporary, then they have no effect on consumption or on the exchange rate, *ceteris paribus* (since output is demand-determined in the short run).<sup>12</sup> If monetary policy reacts systematically to productivity shocks, however, productivity shocks *may* affect consumption and exchange rates through the induced effects on money supplies, so the *ceteris paribus* caveat in the last sentence is essential.

As Appendix B shows, the ex ante terms of trade are given by:

$$p_H - p_F^* - Ee = (1 - 2n)\sigma_e^2 + 2\sigma_{ce} + (1 - n)\sigma_{\kappa c} + n\sigma_{\kappa^* c} + \sigma_{\kappa c} - \sigma_{\kappa^* c}, \quad (18)$$

where we use lower-case letters to denote natural logarithms (except for interest rates and the country-size parameter  $n$ ). Here,  $\sigma_e^2$  stands for the date  $t - 1$  conditional variance  $\sigma_{e,t-1}^2 \equiv \text{Var}_{t-1}\{e_t\}$ ,  $\sigma_{ce}$  stands for  $\sigma_{ce,t-1} \equiv \text{Cov}_{t-1}\{c_t, e_t\}$ , and so on. If consumption is unexpectedly high when the domestic currency is unexpectedly weak, meaning that  $\sigma_{ce} > 0$ , Home producers will find themselves with a highly variable marginal disutility of effort because exchange-rate and world-consumption effects on demand will tend to reinforce each other. Foreign producers will be in the opposite situation. Accordingly, Home producers will set relatively high prices ex ante, and Foreign producers relatively low prices. If Home is relatively small ( $n < \frac{1}{2}$ ), greater currency variability  $\sigma_e^2$  will raise its ex ante terms of trade. In this case, exchange-rate fluctuations have a bigger effect on the Home than on the Foreign demand curves, making world demand for Home goods relatively more variable and making Home’s expected disutility of effort higher at internationally equal ex ante production levels. This asymmetry causes a relative preference of Home producers for leisure, improving Home’s ex ante terms of

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<sup>12</sup>While under sticky prices the output effect of a purely temporary positive productivity shock (a fall in  $\kappa$  or  $\kappa^*$ ) is nil, the disutility from the previously planned level of labor effort falls. If instead the shock were somewhat persistent, consumption would rise, raising demand and with it, current output. See Obstfeld and Rogoff (1995) for similar results.

trade. If  $n = \frac{1}{2}$ ,  $\sigma_e^2$  naturally does not affect the ex ante terms of trade. Note, finally, that if, for example, Home's disutility-of-effort parameter  $\kappa$  tends to be unexpectedly high when the exchange rate  $e$  is unexpectedly high and Home's products therefore are cheap relative to Foreign's, Home producers will on average be in the position of supplying extra labor when it is most painful to do so. Accordingly, they will set higher product prices ex ante, lowering their planned labor supply, and Foreign producers will adjust their own prices downward.

### 3.3 Implications for Ex Ante Consumption

Appendix B also shows that the expected value of the log of (world) consumption is:

$$\begin{aligned} Ec = & \frac{1}{1+\rho} \left\{ \log\left(\frac{\theta-1}{\theta}\right) - E \log \kappa - \frac{1}{2}\sigma_\kappa^2 - 2n(1-n)\sigma_e^2 \right. \\ & \left. - \left[2 - \frac{1}{2}(1-\rho)^2\right] \sigma_c^2 - 2n(1-n)(\sigma_{\kappa e} - \sigma_{\kappa^*e}) - 2[n\sigma_{\kappa c} + (1-n)\sigma_{\kappa^*c}] \right\}. \end{aligned} \quad (19)$$

Certainty-equivalent expected log consumption is  $Ec = \frac{1}{1+\rho} \left[ \log\left(\frac{\theta-1}{\theta}\right) - E \log \kappa \right]$  (see Appendix A). However, uncertainty plainly affects expected log (world) consumption, and, hence, pricing and ex ante log output levels. The relationship between consumption variability, as measured by  $\sigma_c^2$ , and expected log consumption is ambiguous. According to eq. (19),  $\partial Ec / \partial \sigma_c^2$  is negative for  $\rho < 3$ , and nonnegative otherwise.

What do eqs. (18) and (19) imply for producers' date  $t-1$  decisions about date  $t$  prices? Observe that the expectation of the logarithm of the individual monopolist's demand curve (9) is

$$Ey(h) = -\theta p(h) + (\theta - 1)p_H + Ep + Ec^w$$

when domestic-currency prices are preset [in which case  $Ep(h) = p(h)$  and  $Ep_H = p_H$ ]. A change in  $\sigma_c^2$  affects Home and Foreign producers symmetrically, and hence has identical effects on  $Ey(h)$ ,  $Ey(f)$ , and  $Ec^w$ . The last equation therefore shows (because the individual producer takes  $p_H$ ,  $Ep$ , and

$Ec^w$  as given) that if a rise in  $\sigma_c^2$  depresses  $Ec^w$  in equilibrium, for example, it must induce every producer in the world to raise price and thereby lower the expected log of his output. Accordingly, higher  $\sigma_c^2$  is associated with higher nominal product prices when  $\rho < 3$  and with lower prices when producers are so risk averse that  $\rho > 3$ .<sup>13</sup>

What explains the ambiguity? It results from opposition of two effects. Greater consumption variability implies greater demand variability in equilibrium, which in itself induces producers to raise prices so as to limit the ex post variability in labor supply. Roughly speaking, the elasticity of this effect is given by the coefficient on labor supply in the utility function (1), namely, 2.

On the other hand, an increase in  $\sigma_c^2$  alters the expectation of  $C^{-\rho} \times C = C^{1-\rho}$ , which is proportional to the equilibrium marginal utility value of sales; see eq. (16). Since

$$EC^{1-\rho} = \exp \left[ (1 - \rho) Ec + \frac{(1-\rho)^2}{2} \sigma_c^2 \right],$$

an increase in  $\sigma_c^2$  raises the expected marginal utility value of sales at given prices (i.e., given  $Ec$ ) with elasticity  $\frac{1}{2}(1 - \rho)^2$ . Plainly the second effect will dominate the first, inducing lower producer prices ex ante, only when  $\rho > 3$ .

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<sup>13</sup>Because  $\log EC = Ec + \frac{1}{2}\sigma_c^2$ , a rise in  $\sigma_c^2$  raises the expected *level* of consumption  $EC$  mechanically, with  $Ec$  and producer prices held constant, simply because  $C = \exp c$  is a convex function of  $c$ . We can compute the sign of the relation between  $\sigma_c^2$  and  $EC$  by using eq. (19) and calculating

$$\frac{\partial \log EC}{\partial \sigma_c^2} = \frac{- \left[ 2 - \frac{1}{2} (1 - \rho)^2 \right]}{1 + \rho} + \frac{1}{2} = \frac{\rho - 2}{2}.$$

According to the last expression, the expected consumption *level* rises with  $\sigma_c^2$  when  $\rho > 2$  (but, as noted, for  $2 < \rho < 3$ ,  $EC$  rises due to a convexity effect even though producers raise their prices and lower the expected *log* of consumption). Notice also that the variance of the *level* of consumption,  $\sigma_C^2$ , is given by

$$\sigma_C^2 = \exp (2Ec + \sigma_c^2) [\exp (\sigma_c^2) - 1]$$

(due to lognormality of  $C$ ), so an increase in  $\sigma_c^2$ , holding producer prices (i.e.,  $Ec$ ) constant, implies an increase in  $\sigma_C^2$ .

One can view this second case as reflecting a sufficiently strong precautionary saving motive, under which higher future consumption variability leads producers to choose a higher mean level of future (log) consumption despite higher expected disutility from effort.

Equation (19) also shows that greater exchange volatility, other things equal, unambiguously lowers expected consumption. Exchange rate volatility operates through its effect on the volatility of demand for a country's good, which alters the expected marginal disutility of work. Productivity volatility,  $\sigma_{\kappa}^2$ , works the same way.<sup>14</sup> The covariance terms in this equation are intuitive as well. If, for example, the covariance between the real disutility shocks and world consumption is high, individuals expect that world demand and hence labor supply will be unexpectedly high on average precisely when effort is unexpectedly costly, so they will curtail their planned labor supply. That reduces planned output and planned consumption worldwide.

Of course, the relationships in eq. (19) between expected consumption and the displayed variances and covariances are relationships between endogenous variables. While suggestive, they do not reveal how exogenous changes will affect the economy. To determine that, we must fully solve for the model's equilibrium.

## 4 Equilibrium

An equilibrium is a path for consumption, output, and prices that satisfies the conditions for individual optimality, given the preset producer prices of the last section. We compute the equilibrium in steps.

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<sup>14</sup>Interestingly, for equally sized countries, higher exchange rate variability will, *ceteris paribus*, reduce expected output in both countries, and therefore will also reduce the volume of trade (though not trade's output share in GDP). Trade could fall as a share of total GDP in a model with nontraded goods (other than leisure). Bacchetta and van Wincoop (2000) model the impact of exchange rate variability on trade, but in their model the sign of the effect is theoretically ambiguous.

## 4.1 First-Order Conditions for Consumption and Money

The first step is to add consumers' first-order conditions with respect to the dynamic consumption path and money holdings. Since these relationships are standard (see, for example, Obstfeld and Rogoff 1996), we do not include detailed derivations. The following individual optimality conditions hold regardless of whether nominal domestic goods prices are flexible or sticky.

The intertemporal Euler equation for total real consumption is

$$C_t^{-\rho} = \frac{1 + r_{t+1}}{1 + \delta} \mathbf{E}_t \{ C_{t+1}^{-\rho} \}, \quad (20)$$

while the intertemporal Euler equation for money is

$$1 - \frac{\chi P_t^\varepsilon (C_t)^\rho}{M_t^\varepsilon} = \frac{1}{1 + \delta} \mathbf{E}_t \left\{ \frac{P_t}{P_{t+1}} \left( \frac{C_t}{C_{t+1}} \right)^\rho \right\}. \quad (21)$$

The nominal interest rate is (easily shown to be) given by the consumption-based Fisher equation

$$1 + r_{t+1} = \frac{P_t(1 + i_{t+1}) \mathbf{E}_t \left\{ \frac{C_{t+1}^{-\rho}}{P_{t+1}} \right\}}{\mathbf{E}_t \{ C_{t+1}^{-\rho} \}}. \quad (22)$$

Combining the three equations immediately above, we can write the money demand equation as

$$\left( \frac{M_t}{P_t} \right)^\varepsilon = \chi \left( \frac{1 + i_{t+1}}{i_{t+1}} \right) C_t^\rho. \quad (23)$$

## 4.2 Log-linearization and the Exchange-Rate Risk Premium

As noted in the last section, we are going to assume that all the shocks to the model are lognormally distributed. That assumption, as we shall see later in this section, will turn out to imply that consumption is lognormally distributed as well. In that case, one can write eq. (20) for Home as

$$-\rho c_t = \ln \left( \frac{1 + r_{t+1}}{1 + \delta} \right) - \rho \mathbf{E}_t c_{t+1} + \frac{\rho^2}{2} \sigma_{c,t}^2 \quad (24)$$

[where  $\sigma_{c,t}^2 \equiv \text{Var}_t(c_{t+1})$ ], with a parallel relation for Foreign. Of course  $\sigma_{c,t}^2 = \sigma_{c^*,t}^2$  [recall eq. (14)]. We allow for a time-varying variance to capture the possibility of changes in the distributions of the exogenous shocks hitting the economy. We will illustrate later how to compute  $\sigma_c^2$  and other second moments in terms of the variance-covariance structure of the underlying exogenous economic disturbances.

Taking logs of eq. (22) yields

$$\ln(1 + i_{t+1}) = \ln(1 + r_{t+1}) - \ln \mathbb{E}_t \left\{ \frac{C_{t+1}^{-\rho}}{P_{t+1}} \right\} - p_t + \ln \mathbb{E}_t \{ C_{t+1}^{-\rho} \}.$$

Lognormality of  $C$  and  $P$  allows us to write the preceding equation as

$$\ln(1 + i_{t+1}) = \ln(1 + r_{t+1}) + \mathbb{E}_t p_{t+1} - p_t + v_t, \quad (25)$$

with  $v_t$  given by

$$v_t = -\frac{1}{2}\sigma_{p,t}^2 - \rho\sigma_{cp,t} \quad (26)$$

[where  $\sigma_{p,t}^2 \equiv \text{Var}_t(p_{t+1})$ , and  $\sigma_{cp,t} \equiv \text{Cov}_t(c_{t+1}, p_{t+1})$ ]. Note that the first component of  $v_t$ , involving the variance of prices, comes entirely from Jensen's inequality and therefore does not depend on any characteristics of the individual's utility function. It reflects that a mean-preserving rise in expected future price-level variability mechanically raises the expected future real value of money (which is a convex function of the price level). The nominal interest rate falls as a result, other things equal.

Now consider the money market equilibrium condition. It is at this point that we need to resort to an approximation, since the left-hand side of the money Euler equation (21) is not log-linear.<sup>15</sup> We approximate it in the neighborhood of a nonstochastic steady state with a constant rate of growth in consumption and in the money supply. In the steady state, the left-hand side of (21) is

$$1 - \frac{\chi \bar{P}_t^\varepsilon (\bar{C}_t)^\rho}{\bar{M}_t^\varepsilon} = 1 - \frac{\bar{i}}{1 + \bar{i}} = \frac{1}{1 + \bar{i}},$$

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<sup>15</sup>As we illustrate in Appendix C, no approximation is needed for the special case in which  $\varepsilon = 1$  and money supplies (but not necessarily other exogenous shocks) follow a random walk. The approximation we use below may not be a close one outside a neighborhood of  $\varepsilon = 1$ .

where overbars denote the nonstochastic steady state. Log linearizing the left-hand side of (21) therefore gives

$$\begin{aligned} & \bar{i}\varepsilon(m_t - \bar{m}_t) - \bar{i}\varepsilon(p_t - \bar{p}_t) - \bar{\rho}\bar{i}(c_t - \bar{c}_t) - \log(1 + \bar{i}) \\ = & \bar{i}\varepsilon(m_t - p_t) - \bar{\rho}\bar{i}c_t - \bar{i}\log\left[\frac{\chi(1 + \bar{i})}{\bar{i}}\right] - \log(1 + \bar{i}). \end{aligned}$$

The log of the right-hand side of (21) follows with no approximation from properties of the lognormal distribution:

$$-\log(1 + \delta) - \mathbb{E}_t\{p_{t+1} - p_t\} - \rho\mathbb{E}_t\{c_{t+1} - c_t\} + \frac{1}{2}\text{Var}_t\{p_{t+1} + \rho c_{t+1}\}.$$

Thus, eq. (21) can be approximated as

$$\begin{aligned} \varepsilon(m_t - p_t) = & \log\left[\frac{\chi(1 + \bar{i})^{\frac{1+\bar{i}}{\bar{i}}}}{\bar{i}(1 + \delta)^{\frac{1}{\bar{i}}}}\right] - \frac{1}{\bar{i}}\mathbb{E}_t\{p_{t+1} - p_t + v_t\} \\ & - \frac{\rho}{\bar{i}}\mathbb{E}_t\left\{c_{t+1} - c_t - \frac{\rho}{2}\sigma_{c,t}^2\right\} + \rho c_t, \end{aligned} \quad (27)$$

where the substitution  $\text{Var}_t\{p_{t+1} + \rho c_{t+1}\} = \rho^2\sigma_{c,t}^2 - 2v_t$  follows from eq. (26).

### 4.3 Equilibrium Exchange Rates and the “Level” Risk Premium

Assume that Home and Foreign have equal trend inflation rates, and therefore (in this model) equal long-run nominal interest rates in the nonstochastic steady state. Notice that, following the discussion of eq. (24), the term  $\mathbb{E}_t\{c_{t+1} - c_t - \frac{\rho}{2}\sigma_{c,t}^2\}$  is identical for Home and Foreign. Taking eq. (27), subtracting its foreign counterpart, and making use of the logarithmic PPP relation  $e = p - p^*$  implied by (5), we therefore obtain

$$\varepsilon(m_t - m_t^* - e_t) = -\frac{1}{\bar{i}}(\mathbb{E}_t e_{t+1} - e_t + v_t - v_t^*) + \rho(c_t - c_t^*).$$

Except for the risk premium term,

$$v_t - v_t^* = \frac{1}{2}\left(\sigma_{p^*,t}^2 - \sigma_{p,t}^2\right) + \rho(\sigma_{cp^*,t} - \sigma_{cp,t}), \quad (28)$$

this exchange rate equation is the same as in the certainty-equivalence model of Obstfeld and Rogoff (1995).<sup>16</sup> Here, however, because  $c_t - c_t^* = 0$ , it simplifies to

$$\varepsilon(m_t - m_t^* - e_t) = -\frac{1}{\bar{i}}(E_t e_{t+1} - e_t + v_t - v_t^*). \quad (29)$$

Equations (27) and (29) can both be solved in the usual way (assuming there are no speculative bubbles). The solution to (29) is

$$e_t = \frac{\bar{i}\varepsilon}{1 + \bar{i}\varepsilon} \sum_{s=t}^{\infty} \left(\frac{1}{1 + \bar{i}\varepsilon}\right)^{s-t} E_t \left\{ m_s - m_s^* + \frac{v_s - v_s^*}{\bar{i}\varepsilon} \right\}. \quad (30)$$

The term involving  $\{v_s - v_s^*\}_{s=t}^{\infty}$  contributes a “*level*” risk premium to the exchange rate. This term is not precisely equal to the conventional forward exchange rate risk premium, which relates the forward rate to the expected future spot rate, but only because it is multiplied by the (possibly large) number  $1/\bar{i}\varepsilon$ .<sup>17</sup>

Hodrick (1989) showed the presence of related variability effects in the closed-form solution to a flexible-price exchange rate model. However, the

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<sup>16</sup>Equation (28) is derived from eq. (26) and its Foreign counterpart, recalling that  $c = c^*$  in equilibrium.

As we were careful to point out immediately after eq. (26), the term

$$\frac{1}{2} (\sigma_{p^*,t}^2 - \sigma_{p,t}^2)$$

arises entirely from Jensen’s inequality, whereas it is the term

$$\rho (\sigma_{cp^*,t} - \sigma_{cp,t})$$

alone that depends on risk aversion (given the covariances in parentheses). Thus, it is the latter term that is more properly labeled “risk premium.” A section of Engel (1999) is devoted to discussion of this point. As is often done in the literature, however, we will sometimes abuse language and refer to the sum of the two preceding terms as a risk premium, since that is the term examined in many empirical studies of foreign exchange market excess returns. See Engel (1992) for a theoretical context within which the distinction is important.

<sup>17</sup>The exchange rate risk premium is conventionally defined as the difference between the log forward exchange rate and the expected future log spot exchange rate (although we have already noted Engel’s 1992 correction to the conventional definition). Covered

cash-in-advance specification he employed to model money demand, while allowing a variable velocity of money, still implies a fairly low elasticity of the exchange rate level with respect to *monetary* risk factors such as those in eq. (28). And it is the conditional variance of monetary factors that is likely to be most volatile, and thus likely to have the best chance of explaining exchange-rate fluctuations. In contrast, the monetary specification we have used allows for an unrestricted Cagan semi-elasticity of money demand  $1/\bar{i}\varepsilon$ , which—see (29)—determines the response of the spot exchange rate to expectations and risk premia. Equation (30) thus raises the possibility that higher moments of economic variables, not just first moments, could have significant exchange-rate impacts.<sup>18</sup>

By eq. (25), a fall in  $v$  (the result, e.g., of a rise in the covariance of  $c$  and  $p$ ) is associated with a lower Home nominal interest rate and, by eq. (30), with an appreciation of Home’s currency (a fall in  $e$ , resulting from higher Home money demand as  $i$  falls). Thus, the reduced relative riskiness of investments in the Home currency leads simultaneously to a fall in its nominal interest rate and an appreciation in the foreign exchange market. This exchange rate parity ensures that the forward exchange rate level  $\mathcal{F}_t$  obeys the arbitrage relation

$$1 + i_{t+1} = \frac{\mathcal{F}_t}{\mathcal{E}_t} (1 + i_{t+1}^*).$$

Taking logs of both sides of the covered interest parity relation, substituting eq. (25) and its Foreign counterpart, and making use of the PPP relationship (5) yields the logarithmic risk premium as

$$f_t - E_t(e_{t+1}) = v_t - v_t^*, \tag{31}$$

where  $v_t - v_t^*$  is given by eq. (28). Note that this is exactly the same term that enters into the exchange rate *level* risk premium in eq. (30), except that the term’s effect on the exchange rate level is multiplied by a factor of  $1/\bar{i}\varepsilon$ . We argue later that this multiplicative factor is likely to be significantly greater than 1. While eqs. (28), (30), (31), and (32) all hold whether output prices are sticky or flexible, the value of the risk premium may, of course, depend on the degree of price flexibility.

<sup>18</sup>Hodrick (1989) found little support in the data for a model in which exchange rate levels depend on the conditional variances of money supply and industrial output, modeled as generalized ARCH processes. However, his tests comprise only a small subset of the possible risk factors that could be at work.

periment captures the idea of a portfolio shift toward the Home currency or, equivalently, of a “safe haven” effect on the Home currency.

One may similarly solve eq. (27) forward for the price level. Ignoring the fixed constant term (which is irrelevant for calculating the effects of unanticipated shocks), the result is<sup>19</sup>

$$p_t = \frac{\bar{i}\varepsilon}{1 + \bar{i}\varepsilon} \sum_{s=t}^{\infty} \left( \frac{1}{1 + \bar{i}\varepsilon} \right)^{s-t} \mathbf{E}_t \left\{ m_s + \frac{v_s - \frac{\rho^2}{2} \sigma_{c,s}^2}{\bar{i}\varepsilon} \right\} + \frac{\bar{i}\varepsilon}{1 + \bar{i}\varepsilon} \sum_{s=t+1}^{\infty} \left( \frac{1}{1 + \bar{i}\varepsilon} \right)^{s-t} \left( 1 - \frac{1}{\varepsilon} \right) \rho \mathbf{E}_t \{ c_s \} - \left[ \frac{\rho(1 + \bar{i})}{1 + \bar{i}\varepsilon} \right] c_t. \quad (32)$$

#### 4.4 The Short-Run Effects of Monetary Shocks

We next proceed to solve for current consumption. The easiest way to proceed is by multiplying eq. (27) by  $n$  and its Foreign equivalent by  $1 - n$ , and then adding the two equations. The result is

$$\varepsilon (m_t^w - p_t^w) = \log \left[ \frac{\chi(1 + \bar{i})^{\frac{1+\bar{i}}{\bar{i}}}}{\bar{i}(1 + \delta)^{\frac{1}{\bar{i}}}} \right] - \frac{1}{\bar{i}} \mathbf{E}_t \{ p_{t+1}^w - p_t^w + v_t^w \} - \frac{\rho}{\bar{i}} \mathbf{E}_t \left\{ c_{t+1} - c_t - \frac{\rho}{2} \sigma_{c,t}^2 \right\} + \rho c_t,$$

where  $m^w \equiv nm + (1 - n)m^*$ ,  $p^w \equiv np_H + (1 - n)p_F^*$ , and  $v^w \equiv nv + (1 - n)v^*$ . (Recall again that  $c = c^w$  is the same at home and abroad.) Solving forward yields an equation isomorphic to eq. (32) for the domestic price level (where again we ignore the uninformative constant). Rearranging it in the form of

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<sup>19</sup>The ambiguously-signed terms  $(1 - \frac{1}{\varepsilon}) \rho \mathbf{E}_t \{ c_s \}$  in this equation deserve comment. Higher expected future (log) consumption has two countervailing effects on the equilibrium price level, other things equal. By reducing desired saving and thereby increasing the real (and nominal) interest rate, it reduces money demand and puts upward pressure on the current price level. On the other hand, higher expected future consumption raises expected future money demand and hence lowers expected inflation, incipiently reducing the current nominal interest rate and putting downward pressure on the current price level. If  $\varepsilon > 1$ —implying a relatively inelastic response of money demand to the nominal interest rate—the first effect is dominant.

an equation for world consumption gives:

$$\begin{aligned} \left[ \frac{\rho(1 + \bar{i})}{1 + \bar{i}\varepsilon} \right] c_t &= -p_t^w + \frac{\bar{i}\varepsilon}{1 + \bar{i}\varepsilon} \sum_{s=t}^{\infty} \left( \frac{1}{1 + \bar{i}\varepsilon} \right)^{s-t} \mathbb{E}_t \left\{ m_s^w + \frac{v_s^w - \frac{\rho^2}{2} \sigma_{c,s}^2}{\bar{i}\varepsilon} \right\} \\ &+ \frac{\bar{i}\varepsilon}{1 + \bar{i}\varepsilon} \sum_{s=t+1}^{\infty} \left( \frac{1}{1 + \bar{i}\varepsilon} \right)^{s-t} \left( 1 - \frac{1}{\varepsilon} \right) \rho \mathbb{E}_t \{ c_s \}. \end{aligned} \quad (33)$$

One derives the full solution by substituting into the final right-hand side term above the earlier formula for expected future consumption, eq. (19). Because  $p_t^w$  is a predetermined variable as of date  $t$ , eq. (33) shows that, other things equal, higher than expected (as of date  $t - 1$ ) current/expected future money raises current consumption.

While the consumption effects of innovations in current or expected future money are the same internationally regardless of where in the world the shock originates, other effects do depend on which country generates the monetary impulse. By eq. (30), a positive innovation in Home money depreciates its currency, worsening its terms of trade and inducing a demand shift toward Home products.

Because of the Home currency depreciation, Home producers work harder than they would absent the exchange-rate change, Foreign producers enjoy more leisure. Because the short-run Home and Foreign consumption responses necessarily are equal, we know that the surprise Home monetary expansion benefits Foreign more than Home.<sup>20</sup> We examine these ex post welfare effects formally in Appendix D.

While the preceding discussion is suggestive, it is also possibly misleading for thinking about general-equilibrium effects. Equation (33) is not a reduced-form expression, of course, because the consumption and exchange variances it includes are endogenous variables that depend on the interaction among the model's exogenous shocks. We now illustrate how to solve

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<sup>20</sup>The money shock has no real effects beyond the period it occurs, in contrast to the model of Obstfeld and Rogoff (1995), because the current account is zero in equilibrium in this model. For the same reason as in the model of Obstfeld and Rogoff (1995), there is no exchange-rate overshooting in response to monetary shocks.

explicitly for the model's covariance structure, focusing on implications for the foreign exchange risk premium.

## 5 Solving for Exchange Risk Premia

Suppose, for example, that the Foreign money supply is constant and the Home money supply follows a random walk,

$$m_t = m_{t-1} + \mu_t,$$

where  $\mu_t \sim \mathcal{N}(0, \sigma_\mu^2)$  for every date  $t$ . Because the distribution from which money shocks are drawn is time-invariant and lognormal, all of the variances and covariances in the model also will be constant over time.<sup>21</sup> Consider the Home nominal interest rate risk premium in eq. (26), which can be written as  $v_t = -\frac{1}{2}\text{Var}_t(p_{t+1}) - \rho\text{Cov}_t(c_{t+1}^w, p_{t+1})$ . Because  $p_H$  and  $p_F^*$  are preset,

$$\text{Var}_t(p_{t+1}) = (1 - n)^2 \text{Var}_t(e_{t+1}).$$

Also, we can compute the innovation in  $e_{t+1}$  easily from (30) because  $v$  and  $v^*$  are constants.<sup>22</sup> The innovation in  $e_{t+1}$  is

$$\left(\frac{\bar{i}\varepsilon}{1 + \bar{i}\varepsilon}\right) \sum_{s=t+1}^{\infty} \left(\frac{1}{1 + \bar{i}\varepsilon}\right)^{s-t-1} \mu_{t+1} = \mu_{t+1},$$

implying that

$$\text{Var}_t(p_{t+1}) = (1 - n)^2 \sigma_\mu^2.$$

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<sup>21</sup>Under the assumptions made in section 3, productivity shocks are correlated with exchange rates and consumption only to the extent that monetary policy reacts systematically to them. Given that the money-supply process assumed in this section allows no feedback from productivity shocks to policy, we therefore need no further assumptions regarding the shocks  $\kappa$  and  $\kappa^*$ : they do not influence risk premia, expected consumption, or the expected terms of trade.

<sup>22</sup>The analysis becomes more complicated when the covariances, rather than being constants, can evolve stochastically over time. In that case innovations in second moments influence the exchange rate and consumption. Closed-form solutions for dynamic models with time-varying second moments are offered by Abel (1988) and Hodrick (1989).

By (33) the innovation in  $c_{t+1}^w$  is

$$\left[ \frac{1 + \bar{i}\varepsilon}{\rho(1 + \bar{i})} \right] \left( \frac{\bar{i}\varepsilon}{1 + \bar{i}\varepsilon} \right) \mathbb{E}_t \left\{ \sum_{s=t+1}^{\infty} \left( \frac{1}{1 + \bar{i}\varepsilon} \right)^{s-t-1} n\mu_{t+1} \right\} = \left[ \frac{n(1 + \bar{i}\varepsilon)}{\rho(1 + \bar{i})} \right] \mu_{t+1}, \quad (34)$$

and so

$$\begin{aligned} \text{Cov}_t(c_{t+1}^w, p_{t+1}) &= \text{Cov}_t \left\{ \left[ \frac{n(1 + \bar{i}\varepsilon)}{\rho(1 + \bar{i})} \right] \mu_{t+1}, (1 - n)\mu_{t+1} \right\} \\ &= \left[ \frac{(1 - n)n(1 + \bar{i}\varepsilon)}{\rho(1 + \bar{i})} \right] \sigma_\mu^2. \end{aligned} \quad (35)$$

Collecting terms, we see that the Home currency's risk premium (including the convexity term) is

$$v = -\sigma_\mu^2 \left\{ \frac{1}{2}(1 - n)^2 + \left[ \frac{(1 - n)n(1 + \bar{i}\varepsilon)}{1 + \bar{i}} \right] \right\}.$$

If the only uncertainty is Home money-supply uncertainty, then the Home nominal interest rate will be below the level suggested by Fisher nominal-real parity because world consumption rises when the Home currency depreciates. Note also that the degree of risk aversion  $\rho$  *drops out of the expression* because  $\rho$  enters the consumption response to a fall in world interest rates as well as the calculation of the marginal utility of consumption.<sup>23</sup>

We can similarly calculate  $v^*$ . With sticky prices, the innovation to  $p_t^*$  is given by  $-n\mu_{t+1}$ , so that

$$\text{Var}_t(p_{t+1}^*) = n^2 \sigma_\mu^2.$$

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<sup>23</sup>With more general preferences, both the degree of risk aversion and the elasticity of intertemporal substitution would appear separately in the solution for  $v$ . A ceteris paribus rise in risk aversion would raise the absolute value of  $v$ . A rise in intertemporal substitutability, by magnifying the current consumption response to a fall in the world real interest rate, would also raise the absolute value of  $v$ .

Note that for a closed economy ( $n = 1$ ), the price level is fully predictable with sticky prices and therefore  $v = 0$ .

At the same time,

$$\begin{aligned}\text{Cov}_t(c_{t+1}^w, p_{t+1}^*) &= \text{Cov}_t \left\{ \left[ \frac{n(1 + \bar{i}\varepsilon)}{\rho(1 + \bar{i})} \right] \mu_{t+1}, -n\mu_{t+1} \right\} \\ &= -n^2 \sigma_\mu^2 \left[ \frac{1 + \bar{i}\varepsilon}{\rho(1 + \bar{i})} \right].\end{aligned}$$

Thus (again adding in the convexity term),

$$v^* = -n^2 \sigma_\mu^2 \left[ \frac{1}{2} - \left( \frac{1 + \bar{i}\varepsilon}{1 + \bar{i}} \right) \right],$$

and

$$v - v^* = -\sigma_\mu^2 \left[ \frac{1 - 2n}{2} + \frac{n(1 + \bar{i}\varepsilon)}{1 + \bar{i}} \right].$$

is the risk premium.<sup>24</sup>

Thus, recalling eq. (30), we see that for empirically reasonable nominal interest rates (and in all cases for  $n = \frac{1}{2}$ ), a rise level of Home monetary variability leads to both a *fall* in the “level” exchange-rate risk premium for the Home currency and a *fall* in its forward exchange rate risk premium. For plausible values of  $\bar{i}$  and  $\varepsilon$ , the former effect is potentially much larger than the latter effect [because the coefficient  $1/\bar{i}\varepsilon$  multiplying the risk premium in eq. (30) can be large].

Interestingly, our analysis contradicts the common casual presumption that financial markets will attach a positive risk premium to the currency of a country with high monetary volatility. Controlling for expected trend inflation differentials, that presumption is by no means necessarily borne out in a sticky-price world. *Ceteris paribus*, a rise in Home monetary volatility may lead to a fall in the forward premium, even holding expected exchange rate changes constant. Why? If positive domestic monetary shocks lead to

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<sup>24</sup>*Excluding* convexity terms (as arguably is more appropriate for a theoretical examination of the effects of risk), the risk premium would instead be

$$\rho [\text{Cov}_t(c_{t+1}^w, p_{t+1}^*) - \text{Cov}_t(c_{t+1}^w, p_{t+1})] = -\sigma_\mu^2 \left[ \frac{n(1 + \bar{i}\varepsilon)}{1 + \bar{i}} \right].$$

The distinction is irrelevant here for  $n = \frac{1}{2}$ , as the convexity terms cancel out.

increases in global consumption, then domestic money can be a hedge, in real terms, against shocks to consumption. (The real value of Home money will tend to be unexpectedly high in states of nature where the marginal utility of consumption is high.) Furthermore—and this effect also operates in a flexible-price model—higher monetary variability raises the expectation of the future real value of money, other things equal (the convexity term).<sup>25</sup>

By the same logic that we have applied to a currency’s excess return, we also see that higher domestic monetary policy volatility can lead to an appreciation of the domestic currency’s *level* in the foreign exchange market. This effect would result from a *decline* in the “level” risk premium. Indeed, for plausible parameters, this effect can be big, as we have noted.

Although high Home monetary policy volatility may tend to strengthen the nominal value of the Home currency (since a fall in  $e$  is an appreciation), it does not necessarily improve Home’s terms of trade. By the same logic we used to derive eq. (35), we infer that

$$\sigma_{ce} = \left[ \frac{n(1 + \bar{i}\varepsilon)}{\rho(1 + \bar{i})} \right] \sigma_{\mu}^2.$$

Then, using eq. (18), we find that

$$E\{p_H - p_F^* - e\} = \left\{ (1 - 2n) + \left[ \frac{2n(1 + \bar{i}\varepsilon)}{\rho(1 + \bar{i})} \right] \right\} \sigma_{\mu}^2,$$

so that the effect of  $\sigma_{\mu}^2$  is theoretically ambiguous. The intuition is straightforward. First, consider the case where  $n = 1/2$ , so that the first term on the

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<sup>25</sup>This effect offers a possible insight into the “forward premium puzzle.” The puzzle is the empirical regularity that (across the major currencies with floating exchange rates), high interest rates do not seem to be associated with expected depreciation. (If anything, the opposite is true.) Suppose that countries with higher trend inflation tend to experience greater volatility in monetary policy (a fairly well-documented fact; see, e.g., Alesina and Summers 1993). Then, across countries with relatively similar inflation rates (e.g., the main industrialized countries), it is at least theoretically possible that the forward premium is opposite in sign to the expected rate of depreciation of the exchange rate. Empirically, measured money-supply variability probably is too small to explain forward-rate bias based on a model like ours. In reality, however, monetary shocks also result from hard-to-measure money-demand factors, such as unpredictable shifts in the transactions technology.

right-hand side above drops out. Then, a rise in Home monetary volatility strengthens Home's expected terms of trade, because Home producers contract planned output in the hope of limiting the rise in their labor-supply volatility (and Foreign producers do the reverse). On the other hand, if  $n$  and  $\rho$  are relatively large, a rise in  $\sigma_\mu^2$  may worsen Home's ex ante terms of trade. If the Home country is larger, exchange rate variability creates relatively greater demand variability for Foreign agents, leading them to substitute into leisure by setting prices higher.

Finally, our discussion has suggested that some of the results on the sign of risk premia carry over to the case of flexible prices. Plainly the results involving the expected terms of trade do not, since these were derived from price-setting behavior under risk. But the qualitative results concerning monetary variability and the nominal exchange rate risk premium do carry over. In fact, in the case  $\varepsilon = 1$  (log utility for real balances), both the forward exchange rate risk premium and the level exchange rate risk premium are identical for sticky- or flexible-price models (provided all shocks are monetary); see Appendix C. There are two offsetting factors involved in the result. Under flexible prices, the covariance between consumption and prices is zero (rather than a negative number) when all shocks are monetary. However, the overall price level is more volatile when it is flexible, and this makes the Jensen's inequality component of the risk premium larger. In the  $\varepsilon = 1$  case, these two changes in moving from sticky to flexible prices offset each other exactly. Of course, under more general assumptions ( $\varepsilon \neq 1$ , nonmonetary shocks, and so on), the attributes of risk premia can depend on whether the model has flexible or sticky goods prices.

## 6 The Magnitude of the “Level” Exchange Rate Risk Premium

Is the level exchange rate risk premium potentially very large and volatile? Its magnitude depends on that of the forward risk premium and of model parameters. Little is known empirically about the magnitude of the forward

risk premium. Fama (1984) has argued that the small coefficients one usually gets in forward premia regressions are a strong indication that the forward risk premium must be highly volatile, and probably more volatile than the expected rate of change of the exchange rate itself. Magnitudes of 0.5 to 1 percent for the mean absolute value of the one-year forward exchange rate risk premium seem conservative, given the evidence surveyed in Lewis (1995).<sup>26</sup>

The *level* exchange rate risk premium is larger than the forward exchange rate risk premium by a factor of  $1/\bar{i}\varepsilon$  in our model [recall eq. (30)]. Assuming time is measured in years (on the same scale as the risk premium number we have just discussed), then a value between 0.04 and 0.08 seems plausible for  $\bar{i}$ . It is usually thought that  $\varepsilon$  is higher than one, though not necessarily by a large margin. Thus, based on a priori reasoning, it is not implausible to assume that  $1/\bar{i}\varepsilon = 15$ , suggesting that even a 1/2 percent risk premium in the forward rate could translate into an effect of 7.5 percent on the level of the exchange rate. One can also try to quantify  $1/\bar{i}\varepsilon$  by noticing the interpretation of  $1/\varepsilon$  as the interest elasticity of money demand, and then drawing on the theoretical and empirical literature on that topic. Both the Miller-Orr (1966) and Whalen (1966) models of money demand predict an interest elasticity of  $-0.33$ , which, with an average interest rate of 5 percent, suggests a value for  $1/\bar{i}\varepsilon$  of more than 5. Goldfeld's (1973) estimates of the interest elasticity of money demand are lower, on the order of  $-0.1$  to  $-0.2$ , so an estimate of  $-0.33$  may be on the high side. Nevertheless, it remains plausible that exchange risk can have a significantly larger effect on the level of the exchange rate than on the forward risk premium. Correspondingly, if the forward risk premium is quite volatile, as many studies indicate, the analysis here shows that such volatility could significantly heighten exchange-rate volatility.

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<sup>26</sup>Engel (1999) shows that when import prices are set in domestic-currency terms under pricing to market, and a cash-in-advance constraint makes money demand completely interest inelastic, a model of the type set out here can easily generate (realistically) large forward risk premia. In the model Engel describes, however, risk factors do not contribute to the conditional volatility of exchange rates.

## 7 Volatility and Welfare

Can a fully anticipated rise in Home monetary volatility potentially be welfare-improving? What is the effect on Foreign? We have seen that an increase in Home monetary volatility not only leads to a surprising rise in the Home currency's nominal exchange value, but it can also lead to an improvement in Home's expected terms of trade. In addition, expected consumption may rise if agents are sufficiently risk averse. Given that expected global consumption is too low in the nonstochastic steady-state equilibrium (due to the existence of monopoly power), this last effect, taken by itself, would appear offer a potential improvement in global welfare. In section 4.4 we examined how monetary shocks are transmitted between countries *ex post*. Now we ask about the *ex ante* welfare effects of different policy *rules*.

### 7.1 Calculating Ex Ante Utility

Answering such questions turns out to be remarkably straightforward. In comparing the systematic effects of alternative policies, the relevant measure of welfare is *ex ante* welfare  $E_{t-1}U_t$ . We will temporarily ignore the empirically small money-services component of utility, in which case the Home representative agent's *period* objective reduces to

$$E_{t-1}u_t^R \equiv E_{t-1} \left\{ \frac{C_t^{1-\rho}}{1-\rho} - \frac{\kappa_t}{2} Y_t^2 \right\}.$$

Observe, however, that the first-order condition for optimal price setting, eq. (16), implies that in a symmetric equilibrium,

$$E_{t-1} \left\{ \kappa_t Y_t^2 \right\} = \left( \frac{\theta-1}{\theta} \right) E_{t-1} \left\{ C_t^{1-\rho} \right\}, \quad (36)$$

since by eq. (13),  $Y = PC/P_H$  (where, recall,  $C = C^w$ ). Therefore, suppressing time subscripts,

$$E \left\{ \frac{C^{1-\rho}}{1-\rho} - \frac{\kappa}{2} Y^2 \right\} = E \left\{ \frac{C^{1-\rho}}{1-\rho} - \frac{\theta-1}{2\theta} C^{1-\rho} \right\} = E \left\{ \frac{2\theta - (1-\rho)(\theta-1)}{2\theta(1-\rho)} C^{1-\rho} \right\},$$

an expression we can easily evaluate given that  $C$  is lognormally distributed and that we have already solved for  $Ec$ ,  $\sigma_c^2$  and  $\sigma_e^2$ .

To simplify matters slightly and without affecting our main qualitative results, we will abstract from productivity shocks. Then, making use of eq. (19), the final term of the last equation can be solved as<sup>27</sup>

$$\begin{aligned} Eu^R &= E \left\{ \frac{2\theta - (1 - \rho)(\theta - 1)}{2\theta(1 - \rho)} C^{1-\rho} \right\} \\ &= \left[ \frac{2\theta - (1 - \rho)(\theta - 1)}{2\theta(1 - \rho)} \right] \left( \frac{\theta - 1}{\theta\kappa} \right)^{\frac{1-\rho}{1+\rho}} \exp(1 - \rho) \left[ -\frac{2n(1 - n)}{1 + \rho} \sigma_e^2 - \sigma_c^2 \right]. \end{aligned} \quad (37)$$

From this equality we deduce that

$$\frac{\partial (Eu^R)}{\partial \sigma_c^2} = -(1 - \rho)Eu^R < 0$$

(since the sign of  $u^R$  is the same as that of  $1 - \rho$ ). One can similarly calculate that

$$\frac{\partial (Eu^R)}{\partial \sigma_e^2} = -\frac{(1 - \rho)}{2(1 + \rho)}Eu^R < 0.$$

Thus, expected welfare is unambiguously decreasing in both the variability of consumption and exchange rates.

Let us assume that the only shocks are Home monetary shocks drawn from a time-invariant distribution. In that case we can draw on the last section's results. Since

$$\sigma_c^2 = \left[ \frac{n(1 + \bar{i}\varepsilon)}{\rho(1 + \bar{i})} \right]^2 \sigma_\mu^2$$

[recall (34)] and  $\sigma_e^2 = \sigma_\mu^2$  are both increasing in  $\sigma_\mu^2$ , it follows that Home welfare unambiguously falls as monetary variability rises.

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<sup>27</sup>The text expression makes use of the fact that the expression

$$-\frac{1 - \rho}{1 + \rho} \left\{ \left[ 2 - \frac{1}{2}(1 - \rho)^2 \right] \sigma_c^2 \right\} + \frac{(1 - \rho)^2}{2} \sigma_c^2$$

simplifies to  $-(1 - \rho)\sigma_c^2$ .

What is the intuition for this unambiguous result, despite the fact that higher volatility can induce greater work effort, raising world consumption closer to the first-best competitive level? In this monopolistically distorted world, why wouldn't introducing a small amount of uncertainty have a first-order welfare benefit (if it raises consumption) and only a second-order cost (starting from a position of zero volatility)? The answer, loosely speaking, is that because the consumption shift itself is induced only by the change in uncertainty, the countervailing welfare effects must be of the same order of magnitude.<sup>28</sup> Intuitively, the output, consumption, and terms-of-trade effects of greater money-supply variability are all side effects of individual's imperfectly successful attempts to shield themselves from a higher level of outside risk.

## 7.2 Equality of Expected Home and Foreign Utilities

The ex post effects of shocks are not necessarily the same at home and abroad in this model. As we discussed in section 4.4, macroeconomic shocks can lead to internationally asymmetric labor-supply responses even though their consumption effects are symmetric.<sup>29</sup> In contrast to knowing about the ex post international effects of specific shocks, however, we are often interested in seeing how alternative *regimes* affect the ex ante distribution of global welfare. For example, how does a fully anticipated increase in Home's monetary variability affect Home and Foreign welfare? This regime change entails internationally asymmetric changes in several macro variables. Remarkably, however, the overall ex ante welfare effects on Home and Foreign are identical because

$$Eu^R = Eu^{*R} \tag{38}$$

always.

Equality (38) follows directly from the Home and Foreign first-order conditions for price setting, eqs. (16) and (17). Indeed, Home and Foreign

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<sup>28</sup>We thank Meg Mayer for this intuition.

<sup>29</sup>See also Appendix D.

expected utility will still be proportional even if output enters with a different exponent into Home versus Foreign utility. Even though the ex post effects of shocks need not be symmetric, *expected* utility is still equal across the two countries because producers formulate production plans to ensure that equality.<sup>30</sup>

We have not taken into account the terms in  $M/P$  in the utility function, but even these are identical if money shocks are permanent. (In the period of the shock Home real balances rise by  $n$  percent since the price level rises by  $1 - n$  percent. Foreign real balances also rise by  $n$  percent as the Foreign currency appreciates.) Provided the term  $\chi$  in (1) is realistically small, any differences due to real balance effects are presumably third-order in any event.

The model therefore provides an intriguing example in which there is no conflict between Home and Foreign objectives in choosing the exchange-rate *regime*, despite asymmetries in both ex ante price setting behavior and ex post outcomes. Observe that if we relaxed the assumption in eq. (2) that commodity-preference weights equal population weights, eq. (38) would no longer hold, but it would still be true that ex ante Home and Foreign utilities are *proportional*. In that case, too, countries would always agree on the choice of the exchange-rate regime.

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<sup>30</sup>Under perfectly flexible prices, with the disutility of labor in both countries of the form  $\kappa Y^\nu/\nu$  ( $\nu > 1$ ), we would have

$$\kappa Y^\nu = \left( \frac{\theta - 1}{\theta} \right) C^{1-\rho}$$

in every state of nature. From this relationship it follows that  $u^R = u^{*R}$  ex post. With optimally preset prices, however, eq. (36) holds instead of the preceding displayed equation, and so  $u^R = u^{*R}$  holds in expectation—eq. (38). Obviously this is special result that depends on the power form of the utility components, but it is still an interesting and important case. Deviations from purchasing power parity can cause the equality of expected utilities to break down, as in Obstfeld and Rogoff (2000a), but need not, see Obstfeld (2000).

### 7.3 Country Size and the Cost of Currency Volatility

Equation (37) has implications for the relation between country size and the cost of exchange volatility. When one of the countries, say Home, is so big that it occupies nearly the entire world,  $n \approx 1$ , then exchange rate volatility obviously has only a negligible effect on its welfare (because the price level is nearly perfectly predictable in that case). Surprisingly, however, exchange rate volatility also has no effect on the welfare of Foreign, which is of size  $1 - n \approx 0$ . Equation (38) yields the same implication: if country size shields Home from the effects of exchange-rate variability, minuscule Foreign must gain commensurately.<sup>31</sup> This result seems to contradict the conventional wisdom that small countries are hurt relatively more by currency volatility, and therefore would do well to fix their exchange rates.

What explains the result? In this case, Foreign's reduction in planned output raises its terms of trade just enough to compensate it entirely for exchange risk; see eq. (18). And since Foreign goods make up an infinitesimal part of Home's consumption basket, this terms of trade change has essentially no effect on Home. Thus, a very small country may have little or no incentive to peg its currency to that of a very large trading partner.<sup>32</sup>

### 7.4 Quantifying the Cost of Currency Volatility

One can use expression (37) to quantify the gain from moving to a fixed exchange rate regime. Assume for this purpose that all shocks are monetary. The experiment we consider is a monetary regime change that reduces  $\sigma_e^2$  to zero by pegging the exchange rate, while maintaining the variance of world monetary growth (and hence  $\sigma_c^2$ ) at its prior level. The calculation we offer is meant only as an illustration, but it suggests that welfare losses due to monetary shocks' exchange-rate effects could be large.

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<sup>31</sup>Similarly, eq. (19) implies that  $\sigma_e^2$  does not affect the expected log of consumption in either country when Home is nearly the whole world.

<sup>32</sup>On the other hand, in a model with many equally-sized monetary unions, welfare would be enhanced (given an absence of real shocks) by a reduction in the number of independently fluctuating currencies.

We compute the percentage increase  $\lambda$  in output under flexible prices,  $\left(\frac{\theta-1}{\theta\kappa}\right)^{\frac{1}{1+\rho}}$ , that makes the consumer as well off with exchange variability as with  $\lambda = 0$  but  $\sigma_e^2 = 0$ . (We hold  $\sigma_c^2$  constant across regimes as explained above, and refer to the equivalent variation  $\lambda$  as the “cost of exchange-rate variability.”) The cost  $\lambda$  is the solution to

$$(1 + \lambda)^{1-\rho} \exp \left[ -\frac{2n(1-n)(1-\rho)}{1+\rho} \sigma_e^2 \right] = 1,$$

or

$$\lambda = \exp \left[ \frac{2n(1-n)}{1+\rho} \sigma_e^2 \right] - 1.$$

Imagine that the time interval during which prices are set is a year, that  $n = \frac{1}{2}$ , that  $\rho = 1$ , and that  $\sigma_e$  equals 0.20, or 20 percent per year. Then the cost of exchange-rate variability would be a full 1 percent of GDP per year ( $\lambda = 0.01$ ), a substantial number given the low degree of risk aversion that was assumed. Raising the degree of risk aversion—while holding intertemporal substitutability constant—would raise this estimated cost.

## 8 Summary

This paper develops an explicitly stochastic treatment of a “new open economy macroeconomics” model along the lines of the one suggested by Obstfeld and Rogoff (1995). A main contribution of the paper is to introduce powerful new tools for investigating the allocational effects of macroeconomic uncertainties. Our log-linear model allows the explicit computation of risk premia and underscores the factors that determine risk premia when monetary surprises systematically cause output to change. The model admits possibly large effects of risk premia on exchange-rate levels—even when the risk premia themselves are of moderate size. By modeling nominal price setting by monopolistic producers under uncertainty, this paper shows that there will be a risk premium in the expected terms of trade that may differ in sign from the conventional currency risk premium.

The most compelling application of the model is, however, to welfare analysis. Explicit modeling of price setting behavior under uncertainty—rather than the assumption of certainty-equivalence behavior that is common in the literature—leads to very simple and powerful welfare results. Another contribution of this paper is to provide the first exact general-equilibrium account of the welfare costs of exchange-rate volatility, an issue absolutely central to the concept of optimum currency areas, but one that has not been adequately modeled to date (Krugman 1995). Empirically, these welfare costs can be substantial. Our model also provides an intriguing case in which Home and Foreign have the same incentives in designing an optimal world exchange rate system, despite potentially large asymmetries in both ex ante price setting and ex post welfare effects. Though there are many differences in outcomes for the two countries, these wash away in ex ante welfare calculations if producers set their money prices optimally.<sup>33</sup>

Needless to say, while our model is highly suggestive of effects and channels that had long proven elusive to international macro-modelers, it is only a special case. The literature subsequent to this paper has pursued a broad range of pertinent extensions and policy applications.

## **A Flexible-Price Output Levels and Government Spending Shocks**

The first section of this appendix describes flexible-price output level implied by the model. The second shows how government spending shocks could be introduced.

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<sup>33</sup>Appendices explore some additional issues, including the ex post international transmission of monetary shocks and the bearing of national monopoly power in trade on the size and sign of international transmission.

## A.1 Equilibrium Output with Flexible Prices

Under flexible prices, the first-order condition governing aggregate supply is given by<sup>34</sup>

$$Y(h)^{\frac{1+\theta}{\theta}} = \left(\frac{\theta-1}{\theta\kappa}\right) \left(\frac{P_H}{P}\right)^{\frac{\theta-1}{\theta}} (C^w)^{\frac{1}{\theta}} C^{-\rho}. \quad (39)$$

We derive from eqs. (39) and (13) the log-linear relations:

$$\begin{aligned} (1+\rho)y &= \ln\left(\frac{\theta-1}{\theta\kappa}\right) + (1-\rho)(1-n)(p_H - p_F), \\ (1+\rho)y^* &= \ln\left(\frac{\theta-1}{\theta\kappa^*}\right) - (1-\rho)n(p_H - p_F). \end{aligned} \quad (40)$$

In logs, eq. (12) is

$$p_H - p_F = y^* - y.$$

We can use this equation, together with those in (40), to solve for  $p_H - p_F$ ,  $y$ , and  $y^*$ . The equilibrium terms of trade under flexible prices are

$$p_H - p_F = \frac{1}{2} \log\left(\frac{\kappa}{\kappa^*}\right),$$

so that the Home and Foreign flex-price output levels are

$$\begin{aligned} y &= \frac{1}{1+\rho} \log\left(\frac{\theta-1}{\theta\kappa}\right) + \frac{(1-n)(1-\rho)}{2(1+\rho)} \log\left(\frac{\kappa}{\kappa^*}\right), \\ y^* &= \frac{1}{1+\rho} \log\left(\frac{\theta-1}{\theta\kappa^*}\right) - \frac{n(1-\rho)}{2(1+\rho)} \log\left(\frac{\kappa}{\kappa^*}\right). \end{aligned}$$

Observe that because  $p = np_H + (1-n)p_F$ ,

$$\begin{aligned} c &= c^* = c^w = ny + (1-n)y^* \\ &= \frac{1}{1+\rho} \left[ n \log\left(\frac{\theta-1}{\theta\kappa}\right) + (1-n) \frac{1}{1+\rho} \log\left(\frac{\theta-1}{\theta\kappa^*}\right) \right]. \end{aligned}$$

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<sup>34</sup>This is the same as eq. (15) from Chapter 10 of Obstfeld-Rogoff (1996), except that here  $\rho \neq 1$  and supply responds positively when the general price of Home goods,  $P_H$ , rises relative to the overall CPI,  $P$ . Obstfeld and Rogoff (1995) and Corsetti and Pesenti (1998) treat the case  $\rho \neq 1$ .

## A.2 Government Spending Shocks

Introducing government spending shocks into the model is straightforward. Suppose Home government spending falls entirely on Home output and that

$$Y - G = Y \exp(-\gamma)$$

and similarly that  $Y^* - G^* = Y^* \exp(-\gamma^*)$ . In equilibrium  $PC_H^w \exp(\gamma) = P_H Y$ , or

$$\left(\frac{P_H}{P}\right)^{-1} C^w \exp(\gamma) = Y.$$

The implication is that the spending shock can be viewed as shifting the demand curve facing a country.

Thus, with flexible prices, each home producer faces a demand curve

$$Y^d(h) = \left[\frac{P(h)}{P_H}\right]^{-\theta} \left(\frac{P_H}{P}\right)^{-1} C^w \exp(\gamma).$$

Clearly the only change from the corresponding text equation is replacement of  $C^w$  by  $C^w \exp(\gamma)$ , so we can write the producer's first-order condition. as

$$Y(h)^{\frac{1+\theta}{\theta}} = \left(\frac{\theta-1}{\theta\kappa}\right) \left(\frac{P_H}{P}\right)^{\frac{\theta-1}{\theta}} [C^w \exp(\gamma)]^{\frac{1}{\theta}} C^{-\rho},$$

from which we can derive

$$(1+\rho)y = \ln\left(\frac{\theta-1}{\theta\kappa}\right) + (1-\rho)(1-n)(p_H - p_F) + \rho\gamma,$$

$$(1+\rho)y^* = \ln\left(\frac{\theta-1}{\theta\kappa}\right) - (1-\rho)n(p_H - p_F) + \rho\gamma^*.$$

Because  $p_H - p_F = y^* - \gamma^* - (y - \gamma)$  [by the obvious generalization of (12)], we can use the preceding two equations to derive

$$p_H - p_F = \frac{1}{2} \left[ (\gamma - \gamma^*) + \log\left(\frac{\kappa}{\kappa^*}\right) \right].$$

Combining the preceding three equations (and simplifying by assuming that  $\kappa = \kappa^*$ ) yields

$$y = \frac{1}{2}\gamma - \frac{1}{2} \left(\frac{1-\rho}{1+\rho}\right) \gamma^w + \left(\frac{1}{1+\rho}\right) \ln\left(\frac{\theta-1}{\theta\kappa}\right),$$

$$y^* = \frac{1}{2}\gamma^* - \frac{1}{2} \left( \frac{1-\rho}{1+\rho} \right) \gamma^w + \left( \frac{1}{1+\rho} \right) \ln \left( \frac{\theta-1}{\theta\kappa} \right).$$

One can also derive:

$$c = c^* = c^w = \left( \frac{1}{1+\rho} \right) \left[ \ln \left( \frac{\theta-1}{\theta\kappa} \right) - \gamma^w \right].$$

The analysis of the sticky-price case is similarly straightforward.

## B Optimal Price Setting by Producers Facing Monetary and Productivity Shocks

This appendix presents the detailed derivation of the ex ante logarithmic terms of trade and consumption solutions reported as eqs. (18) and (19). In section 3 we derived the first-order condition

$$\mathbb{E} \left\{ C^{1-\rho} \right\} = \mathbb{E} \left\{ \kappa \left( \frac{\theta}{\theta-1} \right) \left( \frac{PC}{P_H} \right)^2 \right\}$$

[eq. (16)]. Because  $P_H$  is predetermined, the latter can be written as

$$P_H = \sqrt{\left( \frac{\theta}{\theta-1} \right) \frac{\mathbb{E} \{ \kappa P^2 C^2 \}}{\mathbb{E} \{ C^{1-\rho} \}}}.$$

The parallel Foreign relation,

$$\mathbb{E} \left\{ C^{1-\rho} (\theta-1) \right\} = \mathbb{E} \left\{ \kappa \theta \left( \frac{P^* C}{P_F^*} \right)^2 \right\},$$

[eq. (17)], can be written as

$$P_F^* = \sqrt{\left( \frac{\theta}{\theta-1} \right) \frac{\mathbb{E} \{ \kappa^* P^{*2} C^2 \}}{\mathbb{E} \{ C^{1-\rho} \}}}.$$

Recall that the Home and Foreign price indices are  $P = P_H^n (\mathcal{E} P_F^*)^{1-n}$ ,  $P^* = (P_H/\mathcal{E})^n (P_F^*)^{1-n}$ . Using these definitions, we infer from the two preceding price-setting equations that

$$\left( \frac{P_H}{P_F^*} \right)^{1-n} = \sqrt{\left( \frac{\theta}{\theta-1} \right) \frac{\mathbb{E} \{ \kappa \mathcal{E}^{2(1-n)} C^2 \}}{\mathbb{E} \{ C^{1-\rho} \}}},$$

$$\left(\frac{P_F^*}{P_H}\right)^n = \sqrt{\left(\frac{\theta}{\theta-1}\right) \frac{\mathbb{E}\{\kappa^* \mathcal{E}^{-2n} C^2\}}{\mathbb{E}\{C^{1-\rho}\}}}.$$

We assume that  $C$  and  $\mathcal{E}$  are lognormally distributed (an assumption consistent with general equilibrium, as the text shows). Note that under lognormality,

$$\mathbb{E}\{\kappa \mathcal{E}^{2(1-n)} C^2\} = \mathbb{E}\{\exp[\log \kappa + 2(1-n)e + 2c]\}.$$

Thus

$$\begin{aligned} \mathbb{E}\{\kappa \mathcal{E}^{2(1-n)} C^2\} &= \exp\left[\mathbb{E}\log \kappa + 2(1-n)\mathbb{E}e + 2\mathbb{E}c + \frac{1}{2}\sigma_\kappa^2\right. \\ &\quad \left.+ 2(1-n)^2\sigma_e^2 + 2\sigma_c^2 + 2(1-n)\sigma_{\kappa e} + 2\sigma_{\kappa c} + 4(1-n)\sigma_{ce}\right]. \end{aligned}$$

Similarly,

$$\mathbb{E}\{C^{1-\rho}\} = \exp\left[(1-\rho)\mathbb{E}c + \frac{1}{2}(1-\rho)^2\sigma_c^2\right].$$

The equation for  $(P_H/P_F^*)^{1-n}$  above therefore becomes

$$\begin{aligned} \left(\frac{P_H}{P_F^*}\right)^{1-n} &= \left(\frac{\theta}{\theta-1}\right)^{\frac{1}{2}} \exp\left\{\frac{1}{2}\mathbb{E}\log \kappa + (1-n)\mathbb{E}e + \frac{1}{2}(1+\rho)\mathbb{E}c + \frac{1}{4}\sigma_\kappa^2\right. \\ &\quad \left.+ (1-n)^2\sigma_e^2 + \left[1 - \frac{1}{4}(1-\rho)^2\right]\sigma_c^2 + (1-n)\sigma_{\kappa e} + \sigma_{\kappa c} + 2(1-n)\sigma_{ce}\right\}. \end{aligned}$$

Likewise,

$$\begin{aligned} \left(\frac{P_F^*}{P_H}\right)^n &= \left(\frac{\theta}{\theta-1}\right)^{\frac{1}{2}} \exp\left\{\frac{1}{2}\mathbb{E}\log \kappa^* - n\mathbb{E}e + \frac{1}{2}(1+\rho)\mathbb{E}c + \frac{1}{4}\sigma_{\kappa^*}^2\right. \\ &\quad \left.+ n^2\sigma_e^2 + \left[1 - \frac{1}{4}(1-\rho)^2\right]\sigma_c^2 - n\sigma_{\kappa^* e} + \sigma_{\kappa^* c} - 2n\sigma_{ce}\right\}. \end{aligned}$$

Taking logs of these two equations leads to

$$\begin{aligned} (1-n)(p_H - p_F^* - \mathbb{E}e) - \frac{1}{2}\log\left(\frac{\theta}{\theta-1}\right) &= \frac{1}{2}\mathbb{E}\log \kappa + \frac{1}{2}(1+\rho)\mathbb{E}c \\ + \frac{1}{4}\sigma_\kappa^2 + (1-n)^2\sigma_e^2 + \left[1 - \frac{1}{4}(1-\rho)^2\right]\sigma_c^2 &+ (1-n)\sigma_{\kappa e} + \sigma_{\kappa c} + 2(1-n)\sigma_{ce} \end{aligned}$$

and

$$\begin{aligned} -n(p_H - p_F^* - \mathbb{E}e) - \frac{1}{2}\log\left(\frac{\theta}{\theta-1}\right) &= \frac{1}{2}\mathbb{E}\log \kappa^* + \frac{1}{2}(1+\rho)\mathbb{E}c \\ + \frac{1}{4}\sigma_{\kappa^*}^2 + n^2\sigma_e^2 + \left[1 - \frac{1}{4}(1-\rho)^2\right]\sigma_c^2 &- n\sigma_{\kappa^* e} + \sigma_{\kappa^* c} - 2n\sigma_{ce}. \end{aligned}$$

As noted in the text, we make the simplifying assumptions that  $\kappa$  and  $\kappa^*$  have identical lognormal distributions and are serially uncorrelated, so that  $E_{t-1} \log \kappa_t = E_{t-1} \log \kappa_t^*$ , and also  $\sigma_{\kappa e} = \sigma_{\kappa^* e} = \sigma_{\kappa c} = \sigma_{\kappa^* c} = 0$ . Subtracting the second from the first of the last two equations gives the expected terms of trade under the simplifying assumptions, eq. (18):

$$p_H - p_F^* - Ee = (1 - 2n)\sigma_e^2 + 2\sigma_{ce} + (1 - n)\sigma_{\kappa c} + n\sigma_{\kappa^* c} + \sigma_{\kappa c} - \sigma_{\kappa^* c}.$$

Finally, the preceding equations yield the expected log of (world) consumption, eq. (19):

$$\begin{aligned} Ec = & \frac{1}{1 + \rho} \left\{ \log \left( \frac{\theta - 1}{\theta} \right) - E \log \kappa - \frac{1}{2} \sigma_\kappa^2 - 2n(1 - n)\sigma_e^2 \right. \\ & \left. - \left[ 2 - \frac{1}{2}(1 - \rho)^2 \right] \sigma_c^2 - 2n(1 - n)(\sigma_{\kappa e} - \sigma_{\kappa^* e}) - 2[n\sigma_{\kappa c} + (1 - n)\sigma_{\kappa^* c}] \right\}. \end{aligned}$$

## C An Important Special Case with an Exact Solution

Because the interest rate enters the money demand equation (23), it is not possible, in general, to write that equation in logs without resorting to a linearization. There is, however, one important special case where a closed-form solution exists. This case is of some interest not only in understanding the ramifications of the model for the exchange-rate risk premium but also because this simple case may be useful in various potential applications of the model.<sup>35</sup>

Suppose that  $\varepsilon = 1$  so that money enters in the utility function (1) as  $\chi \log \left( \frac{M_s}{P_s} \right)$ . Then the intertemporal Euler equation for money (21) becomes

$$1 = \frac{\chi P_t C_t^\rho}{M_t} + \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left( \frac{C_t}{C_{t+1}} \right)^\rho \right\}, \quad (41)$$

where  $\beta \equiv 1/(1 + \delta)$ .

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<sup>35</sup>The example here is developed for the flexible-price case in Obstfeld and Rogoff (1996, section 8.7.3).

Assume that money follows a random walk with drift,

$$\frac{M_{t+1}}{M_t} = (1 + \mu)\epsilon_t, \quad (42)$$

where  $\epsilon_t$  is a positive serially uncorrelated shock with mean 1. (Lognormality is not needed for the special case.) Foreign is symmetric with trend money-growth parameter  $\mu^*$ . Consumption may also be stochastic, due to sticky prices, productivity shocks, etc. We require only weak stationarity assumptions on the consumption process, which may be correlated with money.

The basic trick to solving the model is to rewrite eq. (41) as

$$1 = \frac{\chi P_t C_t^\rho}{M_t} + \beta \frac{\chi P_t C_t^\rho}{M_t} \mathbb{E}_t \left\{ \frac{M_t}{M_{t+1}} \cdot \frac{M_{t+1}}{\chi P_{t+1} C_{t+1}^\rho} \right\} \quad (43)$$

Let us try a candidate solution in which

$$\frac{\chi P_t C_t^\rho}{M_t} \equiv \psi$$

is constant for all  $t$ . In this case, eq. (43) simplifies to

$$1 = \frac{\chi P_t C_t^\rho}{M_t} + \beta \mathbb{E}_t \left\{ \frac{M_t}{M_{t+1}} \right\},$$

or, taking advantage of (42),

$$\frac{\chi P_t C_t^\rho}{M_t} = 1 - \frac{\beta \mathbb{E}_t(1/\epsilon_{t+1})}{1 + \mu}. \quad (44)$$

Note that in deriving this expression, we only required that  $\mu$  and  $\mathbb{E}_t(1/\epsilon_{t+1})$  be constant over time. (Obviously, we are implicitly imposing no speculative bubbles.) If we take eq. (44) and divide it by its foreign counterpart, using the fact  $P_t = \mathcal{E}_t P_t^*$ , we arrive at a reduced-form expression for the exchange rate:

$$\mathcal{E}_t = \frac{M_t}{M_t^*} \left[ \frac{1 - \frac{\beta \mathbb{E}_t(1/\epsilon_{t+1})}{1 + \mu}}{1 - \frac{\beta \mathbb{E}_t(1/\epsilon_{t+1}^*)}{1 + \mu^*}} \right]. \quad (45)$$

An increase in the variance of Home money growth rate raises  $E_t(1/\epsilon_{t+1})$  since the inverse is a convex function; the change therefore lowers the exchange rate. Note that eq. (45) holds under either flexible or fixed prices. (One can easily check that in the log-linear model developed in the previous part of the paper, the value of  $v - v^*$  is the same under sticky or flexible prices also when  $\varepsilon = 1$ .)

How is it possible that money-growth uncertainty can affect the exchange rate equally under either sticky or flexible prices? Recall from eq. (26), that  $v$  is the sum of two components: a term involving the covariance of prices and consumption, and a convexity term deriving from the fact that inflation enters inversely into the money Euler condition. Under flexible prices, the overall price level is more volatile. Therefore, the convexity component of  $v$  is correspondingly larger. When  $\varepsilon = 1$ , these two effects cancel exactly. When  $\varepsilon < 1$ , the convexity effect is actually larger. One can easily solve for the forward exchange rate risk premium and show that for  $\varepsilon = 1$ , it has the same value under fixed and flexible prices.

## D Ex Post Welfare Effects of Monetary Shocks

The first section of this appendix analyzes in detail the global ex post welfare effects of monetary disturbances. The second shows in a general setting how terms-of-trade shifts can bring about asymmetric domestic and foreign effects of domestic monetary shocks.

### D.1 Calculating Ex Post Welfare Effects

The effects of a monetary innovation on the Home representative agent all occur in the initial period of the shock. For concreteness, but with no loss of generality, consider a permanent (small) percentage increase  $dm$  in Home's money supply. Let us assume temporarily that  $\kappa$  is nonrandom and ignore the (empirically small) money-services component of utility. We thus focus

on the period utility component

$$u^R \equiv \frac{C^{1-\rho}}{1-\rho} - \frac{\kappa}{2} Y^2;$$

see eq. (1).

Assume the shock moves the world economy away from an initial path where  $C^{1-\rho}$  and  $Y^2$  equal their expected values as of the period before. Then the utility effect of the shock  $dm$  is

$$\frac{du^R}{dm} = EC^{1-\rho} \frac{dc}{dm} - \kappa EY^2 \frac{dy}{dm}.$$

Recall, however, eq. (36):

$$E\kappa Y^2 = \left( \frac{\theta - 1}{\theta} \right) EC^{1-\rho}.$$

Observe further that by eqs. (4), (5), and (9), in a symmetric sticky-price equilibrium,

$$dy = (1 - n)de + dc.$$

Putting the last three equations together, we infer that the ex post welfare effect on Home is the sum of two separate effects:

$$\frac{du^R}{dm} = EC^{1-\rho} \left( \frac{1}{\theta} \frac{dc}{dm} \right) - EC^{1-\rho} \left( \frac{\theta - 1}{\theta} \right) (1 - n) \frac{de}{dm}.$$

The first term here represents the increase in welfare resulting from a rise in output when price exceeds marginal cost. This welfare gain accrues equally to Home and Foreign, as in the analysis of Obstfeld and Rogoff (1995). The second welfare term arises because the Home currency depreciation switches global demand from Foreign to Home. That term, which reduces Home's gain from its own monetary expansion and augments Foreign's, was absent from the model in Obstfeld and Rogoff (1995). [The next section of this appendix explains why, and shows that in models more general than this one, the expenditure switching exchange-rate effect on Home welfare can be negative (as here), positive, or nil (as in Obstfeld and Rogoff 1995).]

Using eqs. (30) and (33) and our assumption of a permanent Home money-supply increase, we compute that

$$\frac{du^R}{dm} = \frac{EC^{1-\rho}}{\theta} \left\{ \frac{(1 + \bar{i}\varepsilon)n}{\rho(1 + \bar{i})} - (\theta - 1)(1 - n) \right\}.$$

This expression shows plainly that even a small money-supply increase, despite expanding Home consumption and output, need not have a positive welfare effect on Home itself (Home residents work harder). Foreign always gains, however. If the model contained an additional distortion making Home work effort lower—such as a domestic income tax—a positive welfare effect on Home would be more likely.

## D.2 The Terms of Trade and Asymmetric Transmission

What explains the asymmetric ex post international transmission of monetary shocks in the present model, as compared with the symmetric transmission in that of Obstfeld and Rogoff (1995)? The answer is clearest if we generalize the model to allow for a possibly *nonunitary* (but still constant) elasticity of substitution,  $\omega$ , between Home and Foreign goods.<sup>36</sup> In that case the index of total consumption becomes

$$C = \left[ n^{\frac{1}{\omega}} C_H^{\frac{\omega-1}{\omega}} + (1-n)^{\frac{1}{\omega}} C_F^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \quad \omega > 0,$$

instead of eq. (2) (which is the  $\omega = 1$  case). However, (3) still holds, and is repeated below for convenience:

$$C_H = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n C(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad C_F = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 C(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1.$$

Consider the special case in which  $\omega = \theta$ , so that substitutability in consumption does not depend on production locale. In that case we have

$$C = \left[ \int_0^1 C(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}},$$

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<sup>36</sup>Similar analyses have been developed independently by Lombardo (1999) and Tille (2000).

as in Obstfeld and Rogoff (1995). In the general ( $\theta \neq \omega$ ) case, we have the price indexes

$$P_H = \left[ \frac{1}{n} \int_0^n P(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \quad P_F = \left[ \frac{1}{1-n} \int_n^1 P(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}$$

and the demand functions

$$C(h) = \frac{1}{n} \left[ \frac{P(h)}{P_H} \right]^{-\theta} C_H, \quad C(f) = \frac{1}{1-n} \left[ \frac{P(f)}{P_F} \right]^{-\theta} C_F.$$

The demand for the aggregates  $C_H$  and  $C_F$  is given by

$$C_H = n \left( \frac{P_H}{P} \right)^{-\omega} C, \quad C_F = (1-n) \left( \frac{P_F}{P} \right)^{-\omega} C,$$

where

$$P = \left[ nP_H^{1-\omega} + (1-n)P_F^{1-\omega} \right]^{\frac{1}{1-\omega}}.$$

Because

$$C_H = n \left( \frac{P_H}{P} \right)^{-\omega} C, \quad C_F = (1-n) \left( \frac{P_F}{P} \right)^{-\omega} C,$$

we see that individual Home and Foreign producers will face the world demand curves:

$$C(h) = \left[ \frac{P(h)}{P_H} \right]^{-\theta} \left( \frac{P_H}{P} \right)^{-\omega} C^w, \quad C(f) = \left[ \frac{P(f)}{P_F} \right]^{-\theta} \left( \frac{P_F}{P} \right)^{-\omega} C^w. \quad (46)$$

The intuition behind this specification is quite important. In the model of this paper  $\omega = 1 < \theta$ , so the Home and Foreign output baskets are less substitutable than are different Home (or Foreign) commodities for each other. This means (as we show in a moment) that if all Home producers lower price simultaneously, the global demand response for a single producer is smaller than it would be if that producer unilaterally lowered price. From a national standpoint, therefore, producers overestimate the price elasticity of demand. (There is a pecuniary externality.) This is the key reason for asymmetric effects of small exchange-rate changes in this paper's model. When instead  $\omega = \theta$  (as in Obstfeld and Rogoff 1995), each individual producer perceives a demand curve that accurately reflects the country's monopoly power in

trade. This feature lies behind the symmetric effects of small exchange-rate changes in Obstfeld and Rogoff (1995).

More formally, observe that for a representative Home good (say), eq. (46) can be solved for  $P(h)$  to show that real revenue is

$$\begin{aligned}
\frac{P(h)Y(h)}{P} &= Y(h)^{1-\frac{1}{\theta}} \left(\frac{P_H}{P}\right)^{1-\frac{\omega}{\theta}} (C^w)^{\frac{1}{\theta}} \\
&= Y(h)^{1-\frac{1}{\theta}} \left\{ \frac{P_H}{[nP_H^{1-\omega} + (1-n)P_F^{1-\omega}]^{\frac{1}{1-\omega}}} \right\}^{1-\frac{\omega}{\theta}} (C^w)^{\frac{1}{\theta}} \\
&= Y(h)^{1-\frac{1}{\theta}} \Phi \left(\frac{P_H}{P_F}\right)^{1-\frac{\omega}{\theta}} (C^w)^{\frac{1}{\theta}}, \tag{47}
\end{aligned}$$

where  $\Phi'(P_H/P_F) > 0$ .

The experiment we consider is a small increase in Home output, holding world consumption  $C^w$  constant. (This corresponds to a pure expenditure switching effect.) Notice that marginal national revenue, computed using eq. (47), is

$$NR = \left(\frac{\theta-1}{\theta}\right) \frac{P_H}{P} + \left(\frac{\theta-\omega}{\theta}\right) (\eta_\Phi) (\eta_{P_H/P_F}) \frac{P_H}{P}, \tag{48}$$

where

$$\eta_\Phi \equiv \frac{d \log \Phi(P_H/P_F)}{d \log(P_H/P_F)} > 0$$

and

$$\eta_{P_H/P_F} \equiv \frac{d \log(P_H/P_F)}{d \log Y(h)} < 0.$$

Marginal *private* revenue (as perceived by the individual producer) is just the first summand of the right-hand side of eq. (48),

$$PR = \left(\frac{\theta-1}{\theta}\right) \frac{P(h)}{P},$$

that is,  $PR$  is the derivative of total real revenue holding the aggregate terms of trade  $P_H/P_F$  constant.

In a symmetric equilibrium with  $P(h) = P_H$  for all  $h$ , individual monopolistic producers produce too much (from the parochial perspective of

maximizing real national revenue) when  $\theta > \omega$  and  $PR > NR$ . Such is the case in this paper, where  $\omega = 1$  is assumed. In the standard competitive model of the optimal tariff,  $\theta = \infty$  and so the country overproduces absent a protective tariff. Interestingly, however,  $NR > PR$  when  $\theta < \omega$ , so in that case national real income is raised if producers jointly expand output. In the case  $\theta < \omega$  a country expanding its money supply would reap a secondary short-run gain from the depreciation of its currency, rather than the more conventional secondary terms-of-trade loss. The case analyzed in Obstfeld and Rogoff (1995) is the one in which  $\theta = \omega$ , so that  $NR = PR$  and the expenditure switching effect of a currency depreciation has no first-order welfare consequence.

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