Financial Crises and Lending of Last Resort in Open Economies[*]

Luigi Bocola

Northwestern University

and NBER

Guido Lorenzoni

Northwestern University

and NBER

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Abstract

We study a small open economy with flexible exchange rates and financial inter-
mediaries that face a potentially binding leverage constraint. The model features the
possibility of a self-fulfilling crisis with persistent effects on real activity, that produces a
current account reversal and a real devaluation. The presence of dollar debt in the finan-
cial sector makes a crisis of this sort more likely. We show that dollarization can emerge
in equilibrium because of a feed-back loop between risk and the portfolio choices of do-
mestic savers. When domestic savers fear the possibility of a crisis in the future, they
self-insure by saving in dollars. But a reduced supply of peso savings pushes the banks
to issue more dollar debt, exposing the economy to the risk of financial crises in the
future. Domestic authorities can eliminate the crisis equilibrium by acting as a lender
of last resort, but these interventions only work if they are fiscally credible. Holdings
of foreign currency reserves hedge the fiscal position of the government and enhance its
credibility, thus improving financial stability.

Keywords: Financial crises, Dollarization, Lending of Last Resort, Foreign Reserves.

JEL codes: F34, E44, G12, G15

1 Introduction

In the last two decades, emerging economies have experienced remarkable transformations. Financial openness and cross-border flows have broadly increased and have gone together with domestic financial deepening. Many emerging economies have abandoned hard pegs and adopted more flexible exchange rate regimes with some form of inflation targeting. Finally, a striking fact is that emerging economies as a whole have accumulated massive reserves of foreign currency, mostly dollars.

Many observers interpret this combination of facts as follows: increased financial openness and larger domestic financial systems expose emerging economies to more sources of financial instability. Choosing exchange rate flexibility offers some protection from the instability that comes from classic speculative attacks, but not from other forms such as bank runs and sudden stops. Accumulating foreign reserves is a form of protection from these latter risks, as reserves allow countries to act—in Mervyn King's language—as "do it yourself" lenders of last resort in US dollars to their own financial system.

A number of empirical studies provide support to this interpretation: Gourinchas and Obstfeld (2012) show that reserves are effective at reducing the probability of financial crises; Aizenman and Lee (2007) and Obstfeld, Shambaugh, and Taylor (2010) show that stocks of reserves correlate more with the size of the domestic financial sector and with the degree of financial openness than with traditional indicators like the trade deficit.

But why does a country with a flexible exchange rate regime would need dollars to prop up its own financial system? An answer sometimes put forward is that, for emerging economies, domestic financial meltdowns go hand in hand with sudden stops in capital flows, and reserves can help attenuate the required exchange rate adjustment. This answer is based on the idea that emerging economies are generally wary of large devaluations and reserves are a useful tool to smooth them (Ilzetzki, Reinhart, and Rogoff, 2017). Here we explore a related but different answer. Namely, we argue that reserves have beneficial hedging properties which support a credible commitment by the government to stabilize the domestic financial system.

Our main observation is that if a country's financial system is exposed to self-fulfilling panics, the value of the assets in the balance sheets of domestic financial institutions will tend to comove with the exchange rate and with the fiscal capacity of the government. A government that holds dollar reserves is in a better position to provide lending-of-last-resort funding to its financial institutions, because reserves are liquid resources which appreciate in value exactly when the government needs them most.

We study self-fulfilling panics in the context of a fairly canonical, three-period, small open

economy with a financial sector, modeled following recent advances in the literature (Gertler and Kiyotaki, 2010; Brunnermeier and Sannikov, 2014; He and Krishnamurthy, 2015). There are two domestic agents, households and bankers, and risk neutral international investors. Bankers borrow in domestic and foreign currency, and use these resources along with their accumulated net worth to purchase domestic assets, which are used as inputs in production. Households work for domestic firms and save in domestic and foreign currency. The model features two financial frictions: bankers are subject to leverage constraints and foreign investors only borrow and lend in foreign currency.

A financial panic in our model has the features of a "twin crisis" (Kaminsky and Reinhart, 1999), with a drop in domestic asset prices, a real exchange rate depreciation, a current account reversal and low economic growth. The real exchange rate depreciation is driven by a Balassa-Samuelson effect, as low growth makes domestic households feel poorer and depresses their demand for non-tradables.

The possibility of self-fulfilling crises provides a rationale for operations of lending of last resort. We consider a benevolent government that can extend a liquidity facility to banks. This intervention can stimulate investment when the banks are financially constrained and can potentially eliminate panics. Thus, the government has an incentive to promise aggressive interventions when savers have pessimistic expectations. These promises, however, are not necessarily optimal ex-post. In a crisis, households also hold pessimistic expectations about future tax revenues, limiting the government's ability to finance its interventions by issuing debt. The government then faces a trade-off between stabilizing the financial sector and increasing distortionary taxes in the short run, which hampers its role as lender of last resort. We show that dollar reserves hedge the fiscal position of the government, because they appreciate precisely when households hold pessimistic expectations. The ex-ante accumulation of dollar reserves thus allows the government to do lending of last resort more effectively, and eliminates the possibility of financial panics.

Why does the private sector not have the incentive to accumulate on its own the reserves needed to stave off a panic? The answer has to do with the different incentives of households and bankers. To avoid a panic either households have to hold dollar reserves and lend them freely to banks or banks have to hold dollar reserves directly. The first solution does not work because households do not internalize the general equilibrium benefits that lending more to banks has on asset prices and, eventually, on the whole economy. The second solution does not work for a subtler argument, that has to do with the pattern of comovement produced by self-fulfilling panics. To self-insure against a panic banks should limit their exposure in dollars by accumulating reserves and/or reducing their dollar debt ex-ante and, at the same time, borrowing more in domestic currency. However, when the prospect of a

panic looms, domestic households develop a preference to save in dollars because dollar-denominated assets are a good crisis "hedge". They then require a positive risk premium to lend in domestic currency. This makes borrowing in dollars cheaper than borrowing in domestic currency and this incentive can be sufficiently strong to overturn the banks' motive to self-insure against a panic.

Our analysis leads to three somewhat counterintuitive implications. First, if reserves are needed to give credibility to off-the-equilibrium path promises, then reserves may sit there, never be used in equilibrium, and yet play a very useful role. Second, the accumulation of reserves by the official sector does not lead the banking sector to respond by borrowing more in dollars. In fact, when the government can credibly rule out panics, the banking sector is able to borrow at cheaper terms in domestic currency and thus ends up borrowing less in dollars. In this sense, reserves play a form of catalytic role, by encouraging virtuous behavior by the financial sector. Third, the accumulation of reserves by the official sector can have the overall effect of reducing global imbalances, i.e., to reduce the aggregate accumulation of dollar positions by the country as a whole. The reason is that by supporting financial stability, official reserves lead to lower consumption volatility in the future. Households are then induced to consume more ex-ante, thus leading to a smaller trade surplus (or a larger trade deficit).

Literature. Our research is related to several strands of literature. Following the crises of the 1990s, several authors have developed equilibrium models to explain the joint occurrence of financial and currency crises. Burnside, Eichenbaum, and Rebelo (2001b) and Corsetti, Pesenti, and Roubini (1999) emphasize the role of prospective deficits due to bailout guarantees, Chang and Velasco (2000, 2001) points to the role of maturity mismatches and illiquidity in the banking sector, while Aghion, Bacchetta, and Banerjee (2001, 2004) focus on the interactions between the nominal exchange rate and firms' balance sheet in a model with price rigidities. We share with all these papers the emphasis on the self-fulfilling nature of these crises, although we focus on different economic mechanisms.

The closest to our work is the seminal paper by Krugman (1999) who emphasizes how the feedback between investment demand and the real exchange rates can lead to crises equilibria when firms have dollar debt.[1] Dollar debt is not crucial to produce multiple equilibria in our environment, but it plays an important amplifying effect, by making banks' balance sheet further exposed to pessimistic expectations.

A crucial innovation in our paper relative to this literature, is that we endogenize debt

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[1]A recent paper by Céspedes, Chang, and Velasco (2017) uses similar ingredients to discuss non-conventional monetary policy in emerging economies.

denomination ex-ante and show how risk premia can lead banks to endogenously choose currency positions that make multiple equilibria possible. The economic mechanisms that produces dollar-denominated liabilities in our setting are distinct from other explanations offered in the literature and, in particular, from Schneider and Tornell (2004) and Burnside, Eichenbaum, and Rebelo (2001a) who emphasized the role of bailout guarantees.[2] In contrast, we emphasize the portfolio choices of domestic savers and how their demand for safety can lead to dollarized liabilities of the financial sector when these markets are segmented.[3]

The feed-back between risk and portfolio choices as a source of equilibrium multiplicity is shared by other papers, although in different contexts. Bacchetta, Tille, and Van Wincoop (2012) show in a stylized model that volatility in asset prices can be self-fulfilling when investors are risk averse. Heathcote and Perri (2015) and Ravn and Sterk (2017) study the feed-back between unemployment risk and self-insurance motives of households in models with nominal rigidities. See also Chamley (2014) and Broner and Ventura (2016). To the best of our knowledge, we are the first to identify a mechanism of this sort in a macroeconomic model with a financial sector.

Our treatment of lending of last resort builds on Gertler and Kiyotaki (2015). In their environment, providing liquidity to the financial sector during a panic has ex-ante benefits because it reduces the probability of future runs, and it is always optimal ex-post because the government does not face borrowing constraints. Apart from working in a small open economy framework, the main innovation in our paper relative to their approach is that we explicitly formulate a game between the government and private investors, which embeds equilibrium in good and asset markets. This allows us to analyze whether off-the-equilibrium-path promises to intervene in a "bad" equilibrium are credible and to discuss how limited fiscal capacity can interfere with lending of last resort policies. The assumption of limited fiscal capacity appears especially relevant for emerging economies. The only previous work we know that discusses credibility issues in lending of last resort policy is Ennis and Keister (2009), who analyze deposit freezes in the Diamond and Dybvig (1983) model.

There is an important literature studying the role of reserves as self-insurance against various types of shocks (Caballero and Panageas, 2008; Durdu, Mendoza, and Terrones, 2009; Jeanne and Rancière, 2011; Bianchi, Hatchondo, and Martinez, 2012). We share with these papers a precautionary view on the accumulation of foreign reserves. But our focus on

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[2]On the normative side, Caballero and Krishnamurthy (2003) suggest that dollar debt might be excessive relative to the social optimum because of pecuniary externalities.

[3]A mostly empirical literature provides evidence supporting this portfolio approach. For instance, Ize and Levy-Yeyati (2003) and Levy-Yeyati (2006) show that indicators of inflation risk have been historically an important predictor of dollarized liabilities in emerging markets. See also the recent evidence in Du, Pflueger, and Schreger (2017). In our framework, emerging markets might be exposed to dollarization even when inflation is stable, because of volatility in the real exchange rate.

reserves as help in fighting financial panics and our approach to modeling the official sector and the financial sector lead to a distinct set of predictions (see in particular the predictions discussed at page 4).[4]

Finally, our paper relates to recent research aimed at understanding the patterns of global capital flows. See, for example, Gourinchas and Jeanne (2013), Mendoza, Quadrini, and Rios-Rull (2009), Maggiori (2017) and Fahri and Maggiori (2017). Caballero, Farhi, and Gourinchas (2008) show that an increase in crash risk in emerging markets can explain their accumulation of U.S. assets and the observed low equilibrium rates in the world economy over the past twenty years. We contribute to this literature by detailing a model that shed lights on the increased risks faced by emerging economies in presence of deeper and more open financial markets.

Layout. Section 2 presents the model. We then move to characterize the equilibria of the model, proceeding backward in time. Section 3 describes the continuation equilibria of the model from period 1 onward, taking the currency denomination of assets and liabilities as given. Section 4 studies the optimal portfolio choices of households and banks in the initial period. In Section 5 we introduce a government and study operation of lending of last resort and the role of foreign currency reserves. Section 6 concludes.

2 Model

Consider a small open economy that lasts three periods, $t = 0, 1, 2$, populated by two groups of domestic agents, households and bankers, who trade with a large number of foreign investors.

There are two goods: a tradable and a non-tradable good. There are two units of account: the domestic one and the foreign one. We will refer to the first as "pesos", and to the second as "dollars". The price of tradables in dollars is exogenously given by foreign monetary policy. We assume that domestic monetary policy keeps a stable domestic price index, which includes the prices of both tradable and non-tradable goods in pesos. This implies that adjustments in the relative price of tradables vs non-tradables lead to fluctuations in the nominal exchange rate. The model features flexible prices, but movements in the nominal exchange rate matter because agents trade financial claims denominated in pesos and dollars.

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[4]A model that focuses more on equilibrium multiplicity is Hur and Kondo (2016), although in a different context.

The bankers act as intermediaries: they hold all capital goods in the economy and issue liabilities denominated in pesos and dollars. Therefore the price of capital goods and the exchange rate affect bankers' net worth and, due to collateral constraints, bankers' net worth affect real investment in the economy. To allow for the endogenous determination of the price of capital goods, we assume an upward sloping supply of new capital coming from firms producing capital goods subject to convex costs.

We now turn to a detailed description of the environment and to the definition of an equilibrium. Along the way we make a number of simplifying assumptions. Their role is discussed in detail at the end of the section.

2.1 Agents and their decision problems

Households. Household preferences are represented by the utility function

$$\mathbb{E}\left[\sum_{t=0}^{2} \beta^t U(c_t)\right],$$

where c_t is the consumption aggregator

$$c_t = (c_t^T)^\omega (c_t^N)^{1-\omega},$$

and c_t^T and c_t^N are consumption of tradable and non-tradable goods. The prices of tradable and non-tradable goods, in pesos, are p_t^T and p_t^N.

Each period t, households supply a unit of labor inelastically at the wage w_t (in pesos), receive an endowment of non-tradable goods, e^N, and receive the profits of the firms producing capital goods, Π_t (also in pesos), which are described below. Households also trade risk-free, one period claims denominated in pesos and dollars, denoted by a_t and a_t^*. The interest rates in pesos and dollars are i_t and i_t^*. The nominal exchange rate (pesos per dollar) is s_t. Accordingly, the household period t budget constraint is

$$p_t^T c_t^T + p_t^N c_t^N + \frac{1}{1+i_t}a_{t+1} + s_t\frac{1}{1+i_t^*}a_{t+1}^* + \leq w_t + p_t^N e^N + \Pi_t + a_t + s_t a_t^*. \tag{1}$$

The households choose consumption and asset positions in order to maximize expected utility subject to the budget constraints (1) and the terminal conditions $a_3 = a_3^* = 0$.

Bankers. Bankers are agents who own and run banks. Banks have the following assets and liabilities.

On the asset side, banks hold capital k_t which is used as input in the production of tradable goods. The production function for tradable goods is

$$y_t^T = k_t^\alpha l_t^{1-\alpha}, \tag{2}$$

so capital earns the rental rate

$$r_t = p_t^T \alpha k_t^{\alpha-1}, \tag{3}$$

since labor supply is 1 and labor is only employed in the production of tradables. The pesos price of capital is Q_t. Capital does not depreciate in periods 0 and 1 and fully depreciate after production at $t = 2$.

On the liability side, banks enters period t with debt in pesos and in dollars, respectively b_t and b_t^*. The banks' net worth in pesos is then

$$n_t = (Q_t + r_t)k_t - b_t - s_t b_t^*. \tag{4}$$

and the banks' budget constraint is

$$Q_t k_{t+1} = n_t + \frac{1}{1+i_t}b_{t+1} + s_t \frac{1}{1+i_t^*}b_{t+1}^*, \tag{5}$$

as banks use their net worth and new borrowing to purchase the capital good.

There are two important sources of financial frictions in the model. First, only banks can hold capital. Second, banks face limits in their ability to raise outside finance. Namely, banks have to satisfy the following collateral constraint, which requires total end-of-period liabilities to be bounded by a fraction of the capital held by the bank:

$$\frac{1}{1+i_t}b_{t+1} + s_t \frac{1}{1+i_t^*}b_{t+1}^* \leq \theta Q_t k_{t+1}, \tag{6}$$

where θ is a parameter in $[0,1]$.

We assume that bankers only consume tradable goods at date 2, and that they are risk neutral. Therefore, the bankers' problem is to choose $\{k_{t+1}, b_{t+1}, b_{t+1}^*\}$ to maximize the expected value of n_2/p_2^T, subject to the law of motion for net worth (4), the budget constraint (5), the collateral constraint (6), and the terminal condition $b_3 = b_3^* = 0$.

Capital goods production. Competitive firms owned by the households transform tradable goods into capital at date 0 and 1. In order to produce $\iota_t \geq 0$ units of capital, these firms

require $G(\iota_t)$ units of tradable goods. The function $G(\iota_t)$ takes the form

$$G(\iota_t) = \phi_0 \iota_t + \frac{\phi_1}{1+\eta} \iota_t^{1+\eta}.$$

The profits of the capital producing firms are

$$\Pi_t = \max_{\iota_t \geq 0} Q_t \iota_t - p_t^T G(\iota_t). \tag{7}$$

Market clearing in the capital goods market in periods $t = 0, 1$ is given by

$$k_{t+1} = k_t + \iota_t, \tag{8}$$

as the capital inherited from past periods plus the newly produced capital is accumulated by banks for future production. The capital goods market is not active at date 2 because all capital fully depreciates.

Foreign investors. Foreign investors are risk neutral, consume only tradable goods, and discount the future with discount factor β. We assume that foreign investors can only buy claims denominated in dollars. Therefore, equilibrium in the domestic claims market requires $a_t = b_t$. On the other hand, in the foreign claims market the difference $a_t^* - b_t^*$ can be positive or negative, as foreign investors will absorb the difference.

Let p_t^{T*} denote the price of tradable goods in dollars, which is exogenous to the small open economy. The law of one price implies:

$$p_t^T = s_t p_t^{T*}. \tag{9}$$

This price p_t^{T*} is normalized to 1 at date 0, and it is subject to random shocks at date 1 and 2. Specifically, at $t = 1$ the permanent shock ε is realized and the price of non-tradables is

$$p_1^{T*} = p_2^{T*} = \varepsilon.$$

The variable ε is lognormally distributed and satisfies $\mathbb{E}[1/\varepsilon] = 1$. This nominal shock is introduced to have a source of exchange rate volatility that is independent of developments in the domestic economy.

The interest rate in dollars is pinned down by the Euler equation of foreign investors

$$1 = (1 + i_t^*) \beta \mathbb{E}_t \left[\frac{p_t^{T*}}{p_{t+1}^{T*}} \right],$$

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which yields $1 + i_t^* = 1/\beta$ given the assumed properties of p_t^{T*}.

Monetary regime and the exchange rate. Our economy features flexible prices, so the only role of monetary policy is to determine nominal prices and the nominal exchange rate. The reason why these prices matter for the real allocation is that assets and liabilities are denominated in different currencies, so fluctuations in the nominal exchange rate reallocate wealth across agents.

We assume that the monetary authority is only concerned with price stability. Given consumers' preferences, the price index is

$$p_t = \omega^{-\omega} (1 - \omega)^{-(1-\omega)} (p_t^T)^\omega (p_t^N)^{1-\omega}. \tag{10}$$

We assume that the monetary authority successfully targets a constant price index

$$p_t = \bar{p} = \omega^{-\omega}(1 - \omega)^{-(1-\omega)}. \tag{11}$$

Combining this rule with the CPI definition (10) and the law of one price (9), we obtain the nominal exchange rate

$$s_t = \frac{1}{p_t^{T*}} \left(\frac{p_t^T}{p_t^N} \right)^{1-\omega}. \tag{12}$$

Two forces drive the nominal exchange rate: nominal fluctuations in the price level in the rest of the world and movements in the relative price of tradables and non-tradables. Both forces will be relevant for our analysis.

2.2 Equilibrium

There are two sources of uncertainty in this economy, both realized at date 1. The nominal shock ε introduced above, and a sunspot variable ζ uniformly distributed in $[0, 1]$. The sunspot determines which equilibrium is played at date 1 when multiple equilibria are possible. Note that we are leaving implicit in our notation that all variables dated 1 and 2 are function of the state of the world (ζ, ε).

A competitive equilibrium is a vector of prices $\{Q_t, i_t, i_t^*, r_t, w_t, p_t^T, p_t^N, s_t\}$, households' choices $\{a_{t+1}, a_{t+1}^*, c_t^T, c_t^N\}$, bankers' portfolio choices $\{k_{t+1}, b_{t+1}, b_{t+1}^*\}$, and choices of capital good producers $\{\iota_t\}$, such that: **(i)** the choices of households, banks, capital good producers and foreign investors are individually optimal; **(ii)** markets clear; **(iii)** the law of one price holds; **(iv)** and the price index p_t is constant.

2.3 Discussion of assumptions

Let us briefly discuss the main simplifying assumptions made in the model.

First, we are assuming that tradables are produced with capital and labor while non-tradables are in fixed endowment. This assumption simplifies the analysis because we don't have to determine how labor is allocated among the two sectors and we only need to keep track of capital accumulation in one sector. Moreover, it captures in a stylized way the fact that the tradable sector is typically more capital intensive than the non-tradable sector in emerging markets.

Second, we are assuming that foreign investors cannot trade domestic-currency claims. We could have a less stark form of segmentation, by allowing foreign investors to accept domestic-currency claims subject to some friction, as long as we don't have an infinitely elastic demand for domestic claims. Ruling out foreign investors' participation altogether is a useful simplification.

Third, we are representing monetary policy purely as a choice of numeraire and we are assuming the monetary authority can commit to perfect price stability. This is a simple way to model a floating exchange rate regime, where nominal exchange rate volatility is not driven by inflationary choices of the central bank. As we shall see, our main mechanism is based on the relation between the country's real wealth and the real exchange rate, so it is useful to mute other, policy-driven channels of exchange rate instability.

2.4 Roadmap

In the following two sections we analyze the model in two steps, moving backwards in time. First, we analyze how the equilibrium in the last two periods is determined, taking as given the capital stock and the financial claims inherited from date 0. We call this a *continuation equilibrium*, and we show that for a subset of initial conditions there can be multiple continuation equilibria in the model. Next, we go back to period 0 and study the equilibrium determination of investment and financial claims in that period. We show examples in which equilibrium choices ex-ante can lead to equilibrium multiplicity in the following periods.

3 Continuation equilibria

In this section we characterize the behavior of the economy at dates 1 and 2, taking as given the financial positions and the capital stock inherited from the past.

Our approach consists of using a subset of the equilibrium conditions to express all the endogenous variables of the model as a function of the price of capital in terms of tradables

$$q_1 \equiv \frac{Q_1}{p_1^T}.$$

This will allow us to characterize the continuation equilibria of the model using a simple diagram that plots the demand and the supply of capital against q_1.

3.1 The supply of capital goods

Let us start from the supply of capital goods. From the optimization problem of capital producing firms (7) we obtain the supply of capital goods $k_1 + i_1$, given by the function

$$K_S(q_1) = k_1 + \left(\frac{q_1 - \phi_0}{\phi_1} \right)^{1/\eta}, \tag{13}$$

if $q_1 \geq \phi_0$. If $q_1 < \phi_0$, the solution is at the corner $i_1 = 0$ and the supply of capital goods is just k_1.

3.2 The determination of the nominal exchange rate

Before deriving the demand for capital, we need to obtain a relation between the price of capital q_1 and the equilibrium exchange rate.

The following lemma derives useful properties of a continuation equilibrium

Lemma 1. *All continuation equilibria satisfy the following conditions:*

   i. Consumption is constant over time, $c_1 = c_2$;

   ii. The relative price of tradable and non-tradable goods is constant over time,

$$\frac{p_1^N}{p_1^T} = \frac{p_2^N}{p_2^T};$$

   iii. The domestic real interest rate is

$$(1 + i_1) \frac{p_1}{p_2} = \frac{1}{\beta}.$$

The logic of this lemma is simple. At date 1, all uncertainty is revealed, and the households will try to smooth their consumption over time. Tradable consumption is perfectly smoothed by trading with foreign investors. Non-tradable consumption is constant because

the non-tradable endowment is constant. So the relative price of tradables and non-tradables must be constant. The result for the domestic real interest rate comes from the Euler equation for bonds in pesos.

Using the previous lemma and the household intertemporal budget constraint, we can write consumption expenditure at date 1 as

$$p_1 c_1 = \frac{1}{1+\beta} \left(w_1 + \beta w_2 + p_1^N e^N + \beta p_2^N e^N + \Pi_1 + a_1 + s_1 a_1^* \right).$$

Since consumers spend a fraction $1 - \omega$ of their total expenditure on non-tradables, the market clearing condition for non-tradable goods is

$$(1 - \omega) \frac{p_1 c_1}{p_1^N} = e^N.$$

Combining the last two conditions and rearranging, we get

$$\frac{1-\omega}{1+\beta} \left\{ \frac{p_1^T}{p_1^N} \left[(1-\alpha)k_1^\alpha + \beta(1-\alpha)k_2^\alpha + \pi(q_1) + \frac{1}{\varepsilon}a_1^* \right] + \left(\frac{p_1^T}{p_1^N} \right)^\omega a_1 \right\} = \omega e^N, \qquad (14)$$

where we use the fact that real wages in tradables are equal to the marginal product of labor, we use the law of one price to substitute $s_1 = (1/\varepsilon)p_1^T$, the monetary rule and the result in Lemma 1 to express p_1^N and p_2^N in terms of p_1^N/p_1^T,[5] and denote by $\pi(q_1)$ the profits of capital producers in terms of tradables.[6]

Equation (14) defines an implicit relation between p_1^N/p_2^T and (k_2, q_1). More capital invested in the tradable sector or a higher price of capital lead to higher wages and profits for the households. This shifts up the demand for non-tradables and leads to a real appreciation (a higher value of p_1^N/p_1^T). This is just a version of the Balassa-Samuelson effect.

Using the supply of capital (13), we can further express k_2 as a function of q_1. This allows us to express all the variables in a continuation equilibrium as a function of the price of capital q_1.

Lemma 2. *Given the initial conditions $(a_1, a_1^*, b_1, b_1^*, k_1)$, with $a_1^* \geq 0$, the shock ε, and a candidate*

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[5]The monetary rule requires $(p_t^T)^\omega (p_t^N)^{1-\omega} = 1$ which yields $p_t^N = (p_t^N/p_t^T)^\omega$.

[6]The function $\pi(q_1)$ is obtained from profit maximization and is

$$\pi(q_1) = \frac{\eta}{1+\eta} \frac{1}{\phi_1^\eta} \left(\max\{q_1 - \phi_0, 0\} \right)^{\frac{1+\eta}{\eta}}.$$

value of the capital price q_1, there exists a unique vector of prices and quantities

$$(i_1, i_1^*, s_1, s_2, p_1^T, p_2^T, p_1^N, p_2^N, c_1, c_2)$$

consistent with a continuation equilibrium. Let

$$s_1 = \mathcal{S}(q_1, \varepsilon) \tag{15}$$

denote the relation between the capital price q_1 and the nominal exchange rate. The function $\mathcal{S}$ is decreasing in q_1.

An important result in Lemma 2 is the relation between the nominal exchange rate and the price of capital q_1. This relation is constructed by substituting the supply of capital (13) in condition (14) to get p_1^N / p_1^T as a function of q_1. Next, we use (12), which we rewrite as

$$s_1 = \frac{1}{\varepsilon} \left(\frac{p_1^N}{p_1^T} \right)^{-(1-\omega)},$$

to transform the relative price p_1^N / p_1^T into the nominal exchange rate s_1.

When q_1 increases, the economy faces a real appreciation because of the Balassa-Samuelson effect described earlier. An increase in p_1^N / p_1^T is consistent with a stable domestic price index only if p_1^N goes up and p_1^T goes down. But due to the law of one price, p_1^T going down requires a reduction in s_t, that is a nominal appreciation. As we will discuss momentarily, these endogenous fluctuations in the exchange rate play an important role in the model because the equilibrium price of capital depends on the health of the banks' balance sheets which, in turn, depends on the exchange rate through valuation effects.

3.3 The demand for capital goods

From Lemma 2 we can characterize the continuation equilibria of the model by determining q_1, the price that equilibrates the demand and the supply of capital goods. While we have already discussed the supply in Section 3.1, the demand for capital can be obtained using the optimality conditions of the bankers. The rate of return to tradable capital is r_2 / Q_1 because buying capital costs Q_1 at $t = 1$, earns the dividend r_2 at $t = 2$, and then capital fully depreciates. Comparing this rate of return to the interest rate, two cases are possible in equilibrium:

1. *Unconstrained banks.* The marginal gain from borrowing an extra peso and investing it in capital is zero and the collateral constraint is slack,[7]

$$\frac{r_2}{Q_1} = 1 + i_1, \quad Q_1 k_2 - n_1 \le \theta Q_1 k_2.$$

2. *Constrained banks.* The marginal gain from borrowing an extra peso and investing it in capital is positive and the collateral constraint is binding,

$$\frac{r_2}{Q_1} > 1 + i_1, \quad Q_1 k_2 - n_1 = \theta Q_1 k_2.$$

The conditions above can be used to derive the demand schedule for capital goods. Substituting the rental rate from (3) and $1 + i_1 = 1/\beta$, we get the unconstrained demand for capital:

$$K_U(q_1) = \left(\frac{\alpha\beta}{q_1}\right)^{\frac{1}{1-\alpha}}. \tag{16}$$

In the constrained case, we can rewrite the binding collateral constraint in terms of tradables and obtain the constrained demand for capital:

$$K_C(q_1) = \frac{1}{(1-\theta)q_1} \left[q_1 k_1 + \alpha k_1^\alpha - \frac{b_1}{\varepsilon \mathcal{S}(q_1,\varepsilon)} - \frac{b_1^*}{\varepsilon} \right], \tag{17}$$

where debt in pesos b_1 is converted into tradables by dividing by the peso price of tradables $p_1^T = \varepsilon \mathcal{S}(q_1,\varepsilon)$ (from the law of one price and $p_1^{T*} = \varepsilon$) and debt in dollars is converted into tradables by dividing by $p_1^{T*} = \varepsilon$. The demand for capital is given by the lower between the constrained and the unconstrained demand at each q_1,

$$K_D(q_1) = \min\{K_U(q_1), K_C(q_1)\}.$$

The unconstrained portion of the demand curve is always downward sloping. However, the constrained portion of the demand curve can have upward sloping regions. We will return shortly to the determinants of this slope. For now we just show two numerical examples in Figure 1, one in which the demand curve is everywhere downward sloping, in panel (a), and one in which the constrained portion of the curve is upward sloping for some values of q_1, in panel (b).

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[7]Here we use the budget constraint (5) to substitute $\frac{b_1}{1+i_1} + s_1 \frac{b_1^*}{1+i_1^*}$ on the left-hand side of the collateral constraint (6). Moreover, there is no residual uncertainty from $t = 1$ onward, which implies that banks are indifferent between borrowing in pesos or in dollars.
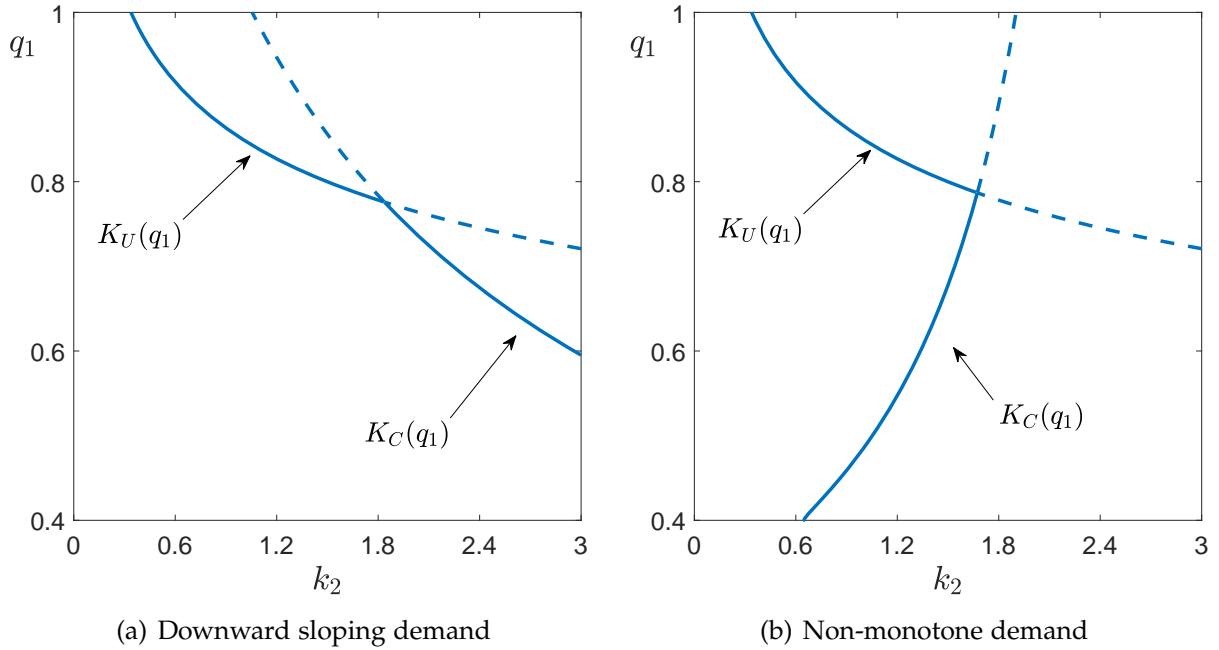
(a) Downward sloping demand   (b) Non-monotone demand

Figure 1: The demand for capital goods

3.4 Equilibrium in the capital goods market

We can now combine the supply and demand curves just derived to find the equilibrium price q_1. First, we establish a sufficient condition for the existence of a continuation equilibrium.

Proposition 1. *Assume the following inequalities are satisfied:*

$$\alpha k_1^{\alpha-1} > \phi_0, \quad \alpha k_1^{\alpha} + \theta \phi_0 k_1 > \frac{1}{\varepsilon \mathcal{S}(\phi_0, \varepsilon)} b_1 + \frac{1}{\varepsilon} b_1^*. \tag{A1}$$

Then there exists a continuation equilibrium with $q_1 > \phi_0$ for all ε. Moreover, there can be at most one equilibrium in which banks are unconstrained.

From now on, we focus on economies that satisfy (A1) and restrict attention to continuation equilibria with $q_1 > \phi_0$. The main advantage of these restrictions is that we do not need to worry about the possibility that banks have negative net worth and so we don't need to specify how banks' bankruptcy is resolved for bond holders.[8]

In Figure 2 we plot the demand and supply for two numerical examples. In panel (a) the equilibrium is unique. In panel (b), instead, there are three equilibria, given by points *A*, *B*

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[8]Of course, individual banks' bankruptcies are commonplace in financial crises. However, our stylized model captures the entire financial system with a single representative bank, and it thus seems reasonable to model a crisis as a severe reduction in the total net worth of the financial sector rather than a complete depletion of its equity.
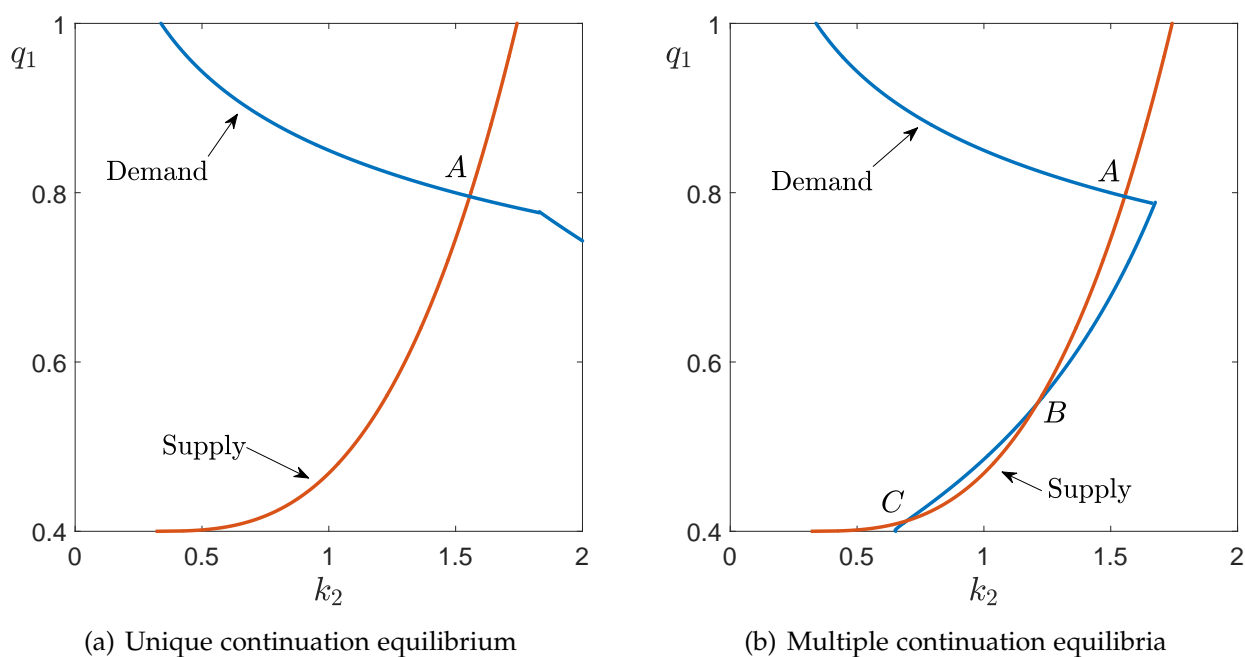
Figure 2: Capital market equilibrium

and C. At equilibrium A banks are unconstrained. Because the unconstrained demand curve is downward sloping, there can be at most one equilibrium of this type. At equilibria B and C, instead, the collateral constraint is binding. Equilibrium B can be ruled out on stability grounds, so we focus on the two stable equilibria A and C.

What economic mechanisms expose the economy to equilibrium multiplicity? As the price of capital q_1 goes down, so does the value of the banks' existing assets $q_1 k_1$, which reduces banks' net worth. Banks' capacity to borrow against new investment is also reduced. If these effects are sufficiently strong, banks become constrained and their demand for capital is lower despite the fact that capital is cheaper. A low demand for capital depresses asset prices, so the drop in asset prices becomes self-fulfilling.

If we interpret a financial crisis as a continuation equilibrium with constrained banks, such as point C in panel (b) of Figure 2, we obtain a number of predictions about the behavior of consumption, investment, the exchange rate, the current account and welfare.

Proposition 2. *If multiple equilibria with a_1^* are possible, and we compare a low q_1 to a high q_1 equilibrium, we obtain the following predictions for the former:*

   i. *The nominal exchange rate s_1 is higher;*

  ii. *Consumption and investment are lower;*

 iii. *The current account balance improves;*

 iv. *The utility of both consumers and bankers is lower.*

The improvement in the current account shows that the domestic financial crisis is associated to a capital flight from the entire country. The capital flight has a double nature: the contraction in investment is driven by the binding collateral constraints of the banks, while the contraction in consumption is driven by the reduction in the country's wealth due to lower future wages. The recent literature is split between papers that emphasize uncertainty about future income growth (Aguiar and Gopinath, 2007) and binding financial constraints (Mendoza, 2010) as sources of fluctuations in emerging markets. Here both channels are operative, because a crisis in our three-period model act as a permanent shock to households' future income.[9] An interesting observation here is that even though some agents in the economy are not forced to borrow less from the rest of the world, spillovers from the financially constrained agents induce them to move in the same direction. That is, unconstrained agents act as amplifiers instead of shock-absorbers.[10]

The proposition also shows that the equilibria are Pareto ranked. Notice that the foreign investors' supply of funds is perfectly elastic at the rate $1 + i_1^*$, so their welfare is unaffected by the equilibrium selected. On the other hand, both households and bankers are hurt by low asset prices in equilibrium. On the households' side, welfare is lower due to lower capital accumulation and hence lower real wages. On the bankers' side, the effects are subtler, as the rate of return on banks' net worth is actually higher in a low q_1 equilibrium, because asset prices are lower and future rental rates are higher (due to lower k_2). However, low asset prices reduce the banks' initial net worth. The proof of the proposition shows that this second effect always dominate.

3.5 Sources of financial instability

The fact that the demand curve is locally upward sloping is crucial for the possibility to obtain multiple equilibria. So let us now go back to the slope of the demand curve. Differentiating the constrained demand curve (17) yields:

$$K_C'(q_1) = \frac{b_1/(\varepsilon \mathcal{S}(q_1, \varepsilon)) + b_1^*/\varepsilon - \alpha k_1^\alpha}{(1-\theta)q_1^2} + \frac{b_1 \mathcal{S}'(q_1, \varepsilon)}{(1-\theta)q_1 \varepsilon \mathcal{S}(q_1, \varepsilon)^2}. \tag{18}$$

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[9] As documented by Cerra and Saxena (2008), financial crises are historically associated to permanent reductions in income. An infinite horizon extension of our model would miss this feature, thus muting the response of consumption and of the exchange rate to a tightening of the bankers' collateral constraint. However, it would not be complicated to further extend that framework to incorporate these empirically relevant effects. See Queralto (2014) for such an example.

[10] The fact that the unconstrained agents here are identified with the household sector is just due to specific modeling assumptions. It would be easy to extend the model to a case where constrained and unconstrained agents are present both in the household and in the business sector.

The first term on the right-hand side of equation (18) shows that leverage makes the curve upward sloping. Namely when total debt is large enough, the expression at the numerator is positive.[11] The second term shows that borrowing in domestic currency can mitigate the effects of leverage because the value of peso obligations depreciates exactly when the price of capital q_1 goes down (recall that $\mathcal{S}'(q_1, \varepsilon) < 0$ from Lemma 2), thus providing hedging against a reduction in asset prices. Note that this is somewhat distinct from the *currency mismatch* channel emphasized, for instance, by Krugman (1999) and Aghion, Bacchetta, and Banerjee (2004). Our bankers have revenues in tradable goods, so matching the balance sheet would require to issue dollar liabilities. Yet, peso debt is an hedge for the bankers because it requires lower payments when the market value of their asset falls.

Figure 3 shows by numerical examples the role of the banks' balance sheet in determining the slope of the demand curve and the possibility of multiple equilibria. In panel (a) we consider an increase in banks' leverage, holding constant their currency exposure. This is achieved by increasing b_1^*. Ceteris paribus, an increase in leverage raises the sensitivity of net worth to asset prices, increasing the elasticity of the demand function in the constrained portion. Comparing the solid and dotted line, we can see that an increase in bankers' leverage makes the economy more prone to multiple equilibria.

A similar result is obtained when considering a change in the currency composition of debt, holding total debt constant. Panel (b) of Figure 3 shows how the demand for capital changes when we increase dollar debt b_1^* and offset such increase by a corresponding reduction in peso debt b_1. We can verify from equation (18) that such a change makes the constrained portion of the demand schedule steeper, raising in this fashion the potential for multiple equilibria.

The economy can thus exhibit confidence crises characterized by low asset prices, a depreciated exchange rate, and capital flights, and these are more likely to arise when the financial sector is more levered and has more dollar debt. These debt positions, however, are determined endogenously at date 0. So we turn now to the decisions of households and bankers at $t = 0$, and study whether they endogenously choose positions that expose the economy to multiple continuation equilibria.

Before continuing, though, we must adopt a rule for selecting among continuation equilibria when we have more than one, as agents at date 0 need to form expectations over future outcomes. First, as argued above, we restrict attention to stable continuation equilibria in the tâtonement sense. Moreover, we focus on economies with at most two stable continuation equilibria. As the equilibria are ranked in terms of welfare, we refer to the one with high

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[11]The effect of leverage on the slope of the capital demand curve has been remarked in closed economy financial accelerator models (e.g. Lorenzoni (2008)), and has been used to generate equilibrium multiplicity in Gai, Kapadia, Millard, and Perez (2008) and Gertler and Kiyotaki (2015).
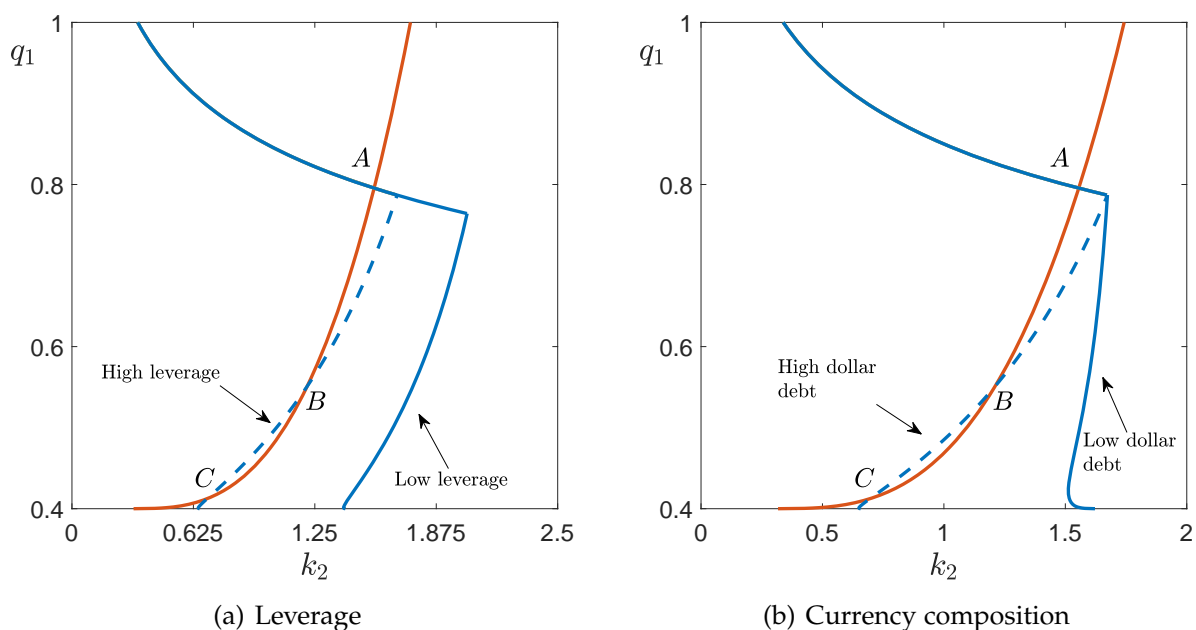
Figure 3: The role of leverage and of the currency composition of debt

asset prices as the "good" equilibrium and to the other as the "bad" equilibrium—see, for example, points A and C in panel (b) of Figure 2. When multiple equilibria are possible, we assume that agents coordinate on the bad equilibrium if the sunspot $\zeta \leq \mu$. Since ζ is uniformly distributed on $[0, 1]$, μ is the probability of the bad continuation equilibrium.

4 The currency denomination of debt

We now go back to date 0 and describe two classes of equilibria that can arise. These equilibria differ in the currency denomination of households' savings and banks' liabilities, and in the exposure of the economy to the confidence crises described earlier. We show, by numerical examples, that these two types of equilibria can coexist for some initial conditions.

We start by characterizing equilibria in which the banks' collateral constraint is always slack and the economy is not exposed to confidence crises at $t = 1$. These equilibria are supported by the households' incentive to save in peso. Because households don't expect a confidence crisis in the future, they have a motive to save in pesos because it insures them against fluctuations in the price of foreign tradables. The households' demand for peso assets allows the banks to borrow in pesos, this making their balance sheet safer. Hence, the financial sector is not exposed to confidence crises at $t = 1$. We call this a "non-dollarized" equilibrium.

We then describe equilibria in which confidence crises can arise with positive probability

at date 1. These equilibria are supported by the incentives of households to save in dollars. When households expect a confidence crisis in the future, they have an incentive to save in dollars because these assets provide insurance against the bad equilibrium at date 1. When sufficiently strong, this precautionary motive of the households induces the banks to issue dollar liabilities at date 0, which exposes the financial sector to confidence crises at date 1. We call this scenario a "dollarized" equibrium.

4.1 Non-dollarized equilibrium

We simplify the analysis further by assuming that at date 0 capital good producing firms are not operative, so market clearing requires $k_1 = k_0$. We can then use equations (4) and (5), along with market clearing, to write the bankers' budget constraint as

$$\frac{b_1}{1+i_0} + s_0 \frac{b_1^*}{1+i_0^*} = b_0 + s_0 b_0^* - r_0 k_0. \tag{19}$$

The total liabilities of the financial sector are given, and the only choice of the bankers regards the currency composition of their debt. The households at $t = 0$ face a portfolio problem and decide how much to consume and save, and in which currency to denominate their savings.

We now state a result that characterizes our first class of equilibria.

Proposition 3 (Non-dollarized equilibrium). *Suppose that there is an equilibrium in which the collateral constraint of the bankers is slack in period* 0 *and is slack almost surely in period* 1. *This equilibrium has the following properties:*

   i. *There is a unique continuation equilibrium from $t = 1$ onward, with (k_2, q_1) solving*

$$k_2 = \left(\frac{\alpha\beta}{q_1}\right)^{\frac{1}{1-\alpha}} \qquad k_2 = k_1 + \left(\frac{q_1 - \phi_0}{\phi_1}\right)^{\frac{1}{\eta}}; \tag{20}$$

   ii. *The prices of tradables and non-tradables are constant over time and equal to p^N and p^T. The domestic real interest rate is constant over time, and equal to $1 + i = 1/\beta$;*

   iii. *Household consumption is constant over time and equal to*

$$\bar{c} = \frac{1}{1 + \beta + \beta^2}\left[p^T\left((1+\beta)(1-\alpha)k_0^\alpha + \beta^2(1-\alpha)k_2^\alpha + \beta\pi(q_1) + a_0^*\right) + a_0\right] + p^N e^N.$$

   At $t = 0$, households set $a_1^* = 0$, and save only in pesos;

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iv. *The banks' debt in pesos and dollars at $t = 1$ are*

$$b_1 = (1 + i) \left(p^T (1 - \alpha) k_0^\alpha + p^N e^N + a_0 + p^T a_0^* - \bar{c} \right),$$

$$b_1^* = (1 + i) \left((b_0 - \beta b_1) / p^T + b_0^* - \alpha k_0^\alpha \right).$$

Because the bankers are unconstrained at date 1, the equilibrium in the capital market is unique, with the quantity and the price of capital independent of ε. As a result, the wages and profits that households obtain in period 1 and 2 are non-stochastic. The households can then achieve perfect consumption smoothing by setting $a_1^* = 0$: in this fashion, their lifetime wealth is non-stochastic, and they can consume $\bar{c}$ in every period. Because consumption is constant over time, we obtain that the domestic real interest rate and the relative price of tradables and non-tradables are also constant. Banks' debt in pesos and dollars in (iv) are then obtained from the market clearing condition $a_1 = b_1$, and from the bankers' budget constraint (19).

In a non-dollarized equilibrium households do not save or borrow in dollars at date 0. To understand this property consider the portfolio choice of households. Rearranging their optimality conditions for bonds in pesos and in dollars at date 0 we obtain the standard asset pricing relation

$$\mathbb{E}_0 \left[(1 + i_0^*) \frac{s_1}{s_0} \right] = 1 + i_0 - \text{Cov}_0 \left[(1 + i_0^*) \frac{s_1}{s_0}, \frac{U'(c_1)}{U'(c_0)} \right], \tag{21}$$

The returns on peso bonds are always perfectly safe for domestic households due to the assumption of a stable price index in pesos. The returns on dollar bonds, instead, are stochastic and equal to $(1 + i_0^*)(s_1/s_0) = 1/(\varepsilon \beta)$. Equation (21) then says that households must expect a premium for holding dollar bonds when these latter are "risky"- when $(1 + i_0^*)(s_1/s_0)$ covaries negatively with the households' marginal utility at $t = 1$.

In the non-dollarized equilibrium dollar bonds are indeed risky for the households because their non-financial income at $t = 1$ is non-stochastic and independent from ε, and setting $a_1^* > 0$ would expose households to volatility in nominal exchange rates.

Hence, households save in dollars only if they expect an excess return. This is, however, not possible in equilibrium. To understand why, note that an asset pricing condition similar to (21) can be derived from the banks' optimality conditions, giving

$$\mathbb{E}_0 \left[(1 + i_0^*) \frac{s_1}{s_0} \right] = 1 + i_0 - \text{Cov}_0 \left[(1 + i_1^*) \frac{s_1}{s_0}, \frac{\lambda_1}{\mathbb{E}_0[\lambda_1]} \right], \tag{22}$$

22

where

$$\lambda_1 = \frac{1}{p_2^T} \frac{r_2 - \theta Q_1/\beta}{(1-\theta)Q_1} \tag{23}$$

is the banks' marginal value of wealth at date $t = 1$.[12]

In equilibrium, equation (21) and (22) must both hold. Because the collateral constraint of the bankers does not bind at $t = 1$, their marginal value of wealth is constant, implying that $\mathbb{E}[(1 + i_0^*)(s_1/s_0)] = (1 + i_0)$ in equilibrium. But at those prices, households optimal choice is to set $a_1^* = 0$.

Importantly, the households' choice to save in pesos is a stabilizing force. From the analysis of the continuation equilibrium, we know that equilibrium multiplicity at $t = 1$ is more likely when banks have dollar debt. Because the leverage of banks is fixed by equation (19), the fact that households are willing to save in pesos minimizes the currency mismatches of the financial sector, and the risk of a bad equilibrium at date 1.

This incentive of households to save in pesos, however, arises because of their expectations about financial stability in the future. In the equilibrium described in Proposition 3, the expectations that the economy will not experience a confidence crisis in the future generate a preference for the households to save in pesos, and the fact that households save in pesos contributes to the stability of the financial sector in the future. We will next see that this feed-back mechanism can also lead to dollarized equilibria.

4.2 Dollarized equilibria

We now ask whether the model can feature equilibria in which confidence crises are possible at date 1, as in the example of Figure 2, panel (b). Given the analysis in Section 3, we know that these equilibria can arise only when the portfolio choices at $t = 0$ makes the bankers' balance sheet sufficiently responsive to asset prices and exchange rates.

We start by studying the portfolio choices of the households at $t = 0$, which are still determined by the asset pricing condition (21). The main difference with the case analyzed earlier is that the possibility of a confidence crisis at date 1 influences the joint distribution of realized returns and households' consumption. Indeed, the $t = 1$ returns on bonds

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[12]A unit of net worth in pesos can be levered to purchase $k_2 = 1/((1-\theta)Q_1)$ units of capital at $t = 1$, as θQ_1 can be borrowed per unit of capital. After paying the interest $1/\beta$ on the borrowed amount $\theta Q_1 k_2$, the return obtained at $t = 2$ is $r_2 k_2 - \theta Q_1 k_2 = (r2 - \theta Q_1/\beta)/((1-\theta)Q_1)$, which, converted in tradables, yields the expression above.

denominated in dollars are now

$$(1 + i_0^*)\frac{s_1}{s_0} = \frac{\left(\frac{p_1^T}{p_1^N}\right)^{1-\omega}}{\varepsilon\beta\left(\frac{p_0^T}{p_0^N}\right)^{1-\omega}}. \tag{24}$$

Differently from before, there is now an endogenous component to these returns which is due to changes in the relative price of tradables and non-tradables between date 0 and 1.

To understand how this component influences the portfolio choices of the households, let's assume for simplicity that $\sigma_\varepsilon \to 0$, so that all uncertainty at $t = 0$ is whether a crisis occurs or not at date 1. Given our selection rule, with probability $(1 - \pi)$ the economy will be in the good continuation equilibrium, while with probability π in the bad continuation equilibrium. From Proposition 2, we know that the real exchange rate is more appreciated in the good than in the bad equilibrium. Therefore, by equation (24) we can see that bonds denominated in dollars have higher returns in the bad continuation equilibrium. This property makes these assets a good hedge for the households, because they pay more in the bad equilibrium, a state where households' consumption is low.

Thus, the anticipation of the bad continuation equilibrium makes households willing to save in dollars. When sufficiently strong, this precautionary motive leads to little peso savings in the economy and might effectively push banks to issue dollar debt in order to finance their operations. Whether this occurs in equilibrium, however, depends also on the behavior of the bankers.

Differently from the case analyzed in the previous section, bankers don't act anymore as risk neutral agents because the possibility of a binding collateral constraint at date 1 makes the bankers' marginal value of wealth state dependent.[13] A unit of net worth at date 1 is worth more when the collateral constraint binds than when it slacks, because in the former case it allows the bankers to partly relax their financial constraints. Because of this property, bankers' have now an incentive to issue peso debt because it requires lower payments in states of of the world in which the bankers' are financially constrained, see equation (22).

This discussion clarifies that the equilibrium currency denomination of assets and liabilities balances the precautionary motive of the households to save in dollars with the precautionary motives of the bankers to borrow in pesos. In what follows, we present a numerical example of an equilibrium of the model in which the households' precautionary motives dominates, and the economy has a dollarized financial sector. Importantly, as this example shows, these dollarized equilibria can coexist with the non-dollarized ones, and

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[13]See Aiyagari and Gertler (1999) and Mendoza and Smith (2006) for a discussion of the asset pricing implications of models with collateral constraints.

might be thus self-fulfilling. Appendix B reports the details of the computation, as well as an algorithm to construct the example.

Table 1 reports several statistics of interest across the two equilibria. First, we can verify that the non-dollarized equilibrium conforms with Proposition 3. In the non-dollarized equilibrium, households' savings are entirely denominated in pesos ($a_1^* = 0.00, a_1 = 0.70$). Banks absorb all the desired pesos savings of the households, and they issue dollar denominated bonds to finance the residual part of their balance sheet. In this example, households savings are more than enough to cover bankers' expenditures in productive assets, and as a result the domestic financial sector accumulates foreign reserves ($b_1^* = -0.28$). Given this balance sheet structure of the banks, the economy is not exposed to confidence crises at date 1. Thus, households' income and consumption have a variance of zero, and the nominal exchange rate fluctuates in period 1 only because of variation in the price of foreign tradables. This also implies that uncovered interest parity holds.

Table 1: Non-dollarized and dollarized equilibria: a numerical illustration

	Non-dollarized	Dollarized
a_1, b_1	0.703	0.050
a_1^*	0.000	0.710
b_1^*	-0.280	0.360
Stdev($\tilde{w}_1$)	0.000	0.043
Stdev($\tilde{s}_1$)	0.005	0.016
Corr($\tilde{w}_1, \tilde{s}_1$)	0.000	-0.969
Stdev($\tilde{c}_1$)	0.000	0.015
$\mathbb{E}[(1 + i_0^*)(s_1/s_0)]$	1.000	0.941
$(1 + i_0)$	1.000	1.034

Notes: Consumption ($\tilde{c}_1$), the nominal exchange rate ($\tilde{s}_1$), and non-financial income ($\tilde{w}_1$) are reported in logs. The parameters used in the example are: $\omega = 0.50$, $\alpha = 0.85$, $\beta = 1.00$, $e^N = 0.70$, $\theta = 0.70$, $\phi_0 = 0.40$, $\phi_1 = 0.20$, $\eta = 3.00$, $\sigma_\varepsilon = 0.005$, $\pi = 0.10$. The initial conditions are $k_0^T = 0.30$, $a_0 = b_0 = 0.00$, $a_0^* = 1.29$, $b_0^* = 0.71$. See Appendix B for additional details.

In the dollarized equilibrium, households save mostly in dollars ($a_1^* = 0.71, a_1 = 0.05$). Bankers, thus, have little access to pesos, and they need to finance their operations at date 0 by issuing dollar debt. This structure of the balance sheet of the bankers makes confidence crises possible at $t = 1$, and it endogenously generates risk in the economy. At date 1, households non-financial income becomes exposed to the realization of the sunspot. Importantly, households' non-financial income and the nominal exchange rate are negatively correlated, because financial crises result in real depreciations. These hedging properties

justify the households' choices of saving in dollars at date 0, and it allows for some insurance, as households' consumption at $t = 1$ is less volatile than their non-financial income. However, the precautionary motive of the households is met in equilibrium with a more risky balance sheet for the banks, which is what exposes the economy to financial instability in the future. Why are banks willing to borrow in dollars and expose themselves to exchange rate risk? The answer is that dollar borrowing is cheaper than peso borrowing, $\mathbb{E}[(1 + i_0^*)(s_1/s_0)] < (1 + i_0)$. That is, the model endogenously generates excess returns on the carry trade strategy in order to accommodate the precautionary motives of households.

4.3   Discussion

Before continuing, it is useful to discuss in more details some properties of the dollarized equilibria. First, the presence of segmentation in the market for peso claims is critical to generate these dollarized equilibria. The segmentation has both an international and a domestic dimension.

At the international level, it is important that some frictions prevent foreign investors to lend to domestic banks in local currency. If foreign lenders were risk neutral toward the small open economy, they would have an incentive to lend to domestic banks in pesos in order to capture the returns from the carry trade emphasized in Table 1, thus making the decisions of domestic savers irrelevant for the balance sheet of the bankers. Empirically, however, there is evidence that foreign lenders do not act as risk neutral toward emerging market economies, see for example the work of Borri and Verdhelan (2013) and Tourre (2017). In our paper, we capture this segmentation in a stark way, by assuming that foreign lenders do not participate to local currency bond markets. We do not expect our results to be qualitatively different if we were to relax this assumption, but model instead foreign lenders as risk averse specialists (Lizarazo, 2013).

At the domestic level, it is important that the bankers in our framework are distinct agents, and do not act on behalf of the households. This assumption, shared by recent papers such as Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2012), gives us some flexibility to make the precautionary motives of the households sufficiently strong relative to the one of the bankers, which is critical to obtain the dollarized equilibrium. More primitive frictions such as limited market participation on the households side, or limited liabilities on the bankers' side, would offer a justification for our assumption.

A second aspect that we wish to emphasize is that the model does not have predictions on whether domestic agents save in dollars by opening a foreign currency deposit with domestic banks, or whether they do so by opening it abroad. After the crises of the 1990s,

several emerging markets imposed restrictions on households' ability to open dollar deposits in domestic banks. To the extent that these regulations do not interfere with an open capital account, they would not discourage domestic agents in our model from saving in dollars.

Finally, our current analysis mutes an interesting aspect of the dollarized equilibrium, the leverage choice of the bankers at date 0. This was done mostly for tractability, as we wanted to isolate how expectations of future crises impact the currency positions of households and bankers. Introducing a leverage choice would add an interesting dimension to the analysis that we leave for future research.[14]

5 Lending of last resort and foreign currency reserves

After having characterized the sources of financial instability in this economy, we turn to study government interventions. We introduce a benevolent government that intervenes at date $t = 1$ and ask what is the optimal intervention and whether it can eliminate the multiplicity identified in the previous sections. Although lending of last resort is typically done by central banks, here we do not distinguish the role of the central bank from the role of the treasury and just call "government" the consolidated public sector.

In this section, we first describe how we model government intervention at date 1, introducing distortionary taxation and the government budget constraint. Then, we study government interventions in the continuation equilibrium. Finally, we move back to date 0 and analyze how reserve accumulation affects the ex-ante choices of the private sector and the ex-post equilibrium.

5.1 Modeling ex-post interventions

Government interventions and timing. The role of a lender of last resort is to provide emergency liquidity to banks and, at the same time, restore confidence in banks' liabilities. We capture these two elements by assuming that at date 1 the government extends a liquidity facility up to a limit $\bar{b}_2^{*g}$ to the banks and provides a guarantee to the loans that the private sector extends to the banks up to the pledgeable value of the banks' collateral. We do not assume that the government has a superior ability to enforce repayment from the banks. However the government, being a large player, internalizes the effects that lending to banks has on the market for capital and that plays a crucial role in its incentives to intervene.

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[14]In a similar framework, Bocola (2016) emphasizes that expectations of binding financial constraints can lead banks to delever, pushing the economy toward a recession today.

We assume that the government is financed with distortionary taxation, so we extend the model by introducing a labor supply decision. For tractability, we assume that households have quasi-linear preferences in tradable goods and hours worked and adopt the utility function

$$\mathbb{E}\left[\sum_t \beta^t U\left(\left(c_t^T - v(l_t)\right)^\omega \left(c_t^N\right)^{1-\omega}\right)\right],$$

where l_t denotes hours worked and $v(l_t) = \chi(1/(1+\phi))l_t^{1+\phi}$. The government levies a proportional tax τ_t on labor income.

The government enters date 1 with net positions in pesos and dollars A_1 and A_1^* inherited from date 0, and it chooses the liquidity facility to extend to the banks $\bar{b}_2^{*g}$, labor taxes and the new dollar positions A_2^*. We denote by b_2^{*g} the loans that the bankers take from the government. The budget constraint of the government at date 1 is then

$$s_1 \frac{1}{1+i_t^*}(b_2^{*g} + A_2^*) = \tau_1 w_1 l_1 + A_1 + s_1 A_1^*,$$

while the date 2 budget constraint is

$$T_2 = \tau_2 w_2 l_2 + s_2(\hat{b}_2^* + A_2^*),$$

where $\hat{b}_2^*$ denotes the repayments of the loans by the banks, and $T_2 \geq 0$ is a lump sum transfer to households. Note that we have assumed that all borrowing at $t = 1$, by the banks and by the government, is done in dollars.[15] Moreover, to simplify on notation, we assume that $\varepsilon = 1$. This implies, by the law of one price, that $p_t^T = s_t$.

Modeling government intervention requires to specify in detail the timing of events. We split period 1 in four stages and assume the following timing:

i. Investors decide the maximum amount of bank and government claims they are willing to buy, $\bar{b}_2^*$ and $-\bar{A}_2^*$, forming expectations about the value of banks' assets and about the government's repayment capacity.

ii. The government decides the size of the liquidity facility extended to the banks, $\bar{b}_2^{*g}$, the tax rate τ_1 and the dollar position $A_2^* \geq \bar{A}_2^*$.

iii. Banks decide how much to borrow from the private sector and from the government to finance purchases of capital, choosing $b_2^* \leq \bar{b}_2^*$ and $b_2^{*g} \leq \bar{b}_2^{*g}$.

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[15]In the previous sections, it was immaterial whether borrowing was in pesos or in dollars at date 1 because there was no further uncertainty. Here, however, debt denomination is potentially important for determining off-the-equilibrium path behavior.

iv. If banks' total liabilities exceed $\theta q_1 k_2$, banks make a take-it-or-leave-it offer to their creditors to renegotiate their debt down to $\theta q_1 k_2$.

In period 2, banks repay their debt and the government sets the tax rate τ_2 and T_2 in order to satisfy its budget constraint.

Step (iv) of the game provides a microfoundation for the collateral constraint used in previous sections. In particular, we are implicitly assuming that creditors expect to only recover a fraction θ of the assets if debt renegotiation fails and that creditors can perfectly coordinate during a renegotiation. It is useful to have an explicit description of the credit friction here, to make sure that we subject the government to the same friction.

We analyze the game described above in three steps. First, we describe how investors form expectations about the repayment capacity of the banks and the government. Second, we study a continuation equilibrium for given debt limits and given a government policy. Finally, we combine the previous steps and formulate a fixed point problem that defines an equilibrium.

Debt limits of banks and government. Investors (domestic and foreign) expect to receive at most $\theta q_1 k_2 (1 + i_1^*)$ dollars from banks in period 2, which is the maximum guaranteed by the government. Moreover, they expect that the government will receive

$$\hat{b}_2^* = \min\{\theta q_1 k_2 (1 + i_1^*) - b_2^*, b_2^{*g}\}$$

from banks in period 2. The maximum that the government can raise in labor income taxes in period 2 is determined by a standard Laffer curve. After some algebra, we can show that the maximum of the Laffer curve (in tradables) is equal to

$$\Xi k_2^{\alpha \frac{1+\phi}{\alpha+\phi}},$$

where Ξ is a constant which depends on the model parameters.[16]

Thus, using the government budget constraint at date 2, we can obtain an upper bound on the government capacity to repay loans contracted in period 1, and set the government debt limit at

$$-\bar{A}_2^* = \min\{\theta q_1 k_2 (1 + i_1^*) - b_2^*, b_2^{*g}\} + \Xi k_2^{\alpha \frac{1+\phi}{\alpha+\phi}}. \tag{25}$$

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[16]Here we are assuming that the government can commit to a future level of taxes and to repaying its debt. In practice, financial crises in emerging markets are often associated to a rise in default risk for the government, which constraints further its ability to borrow. Introducing such feature would be an interesting extension of our framework.

Continuation equilibrium. We now characterize a continuation equilibrium given the debt limits $(\bar{b}_2^*, -\bar{A}_2^*)$, and given a policy of the government $(\bar{b}_2^{*g}, A_2^*, \tau_1, \tau_2, T_2)$. To economize on notation, we collect the debt limits and the policy variables in a vector X,

$$X \equiv [\bar{b}_2^*, -\bar{A}_2^*, \bar{b}_2^{*g}, A_2^*, \tau_1, \tau_2, T_2].$$

Starting from the households, our preference specification implies that labor supply in equilibrium equals

$$l_t = \left[\frac{(1-\tau_t)(1-\alpha)k_t^\alpha}{\chi} \right]^{\frac{1}{\alpha+\phi}}.$$

Moreover, the ability of the households to save at the rate $1/\beta$ implies that they will equalize $c_t^T - v(l_t)$ in periods 1 and 2. This implies that the main results from Lemma 1 and Lemma 2 extend to this setting, although the conditions characterizing the nominal exchange rate are slightly different from the one discussed in Section 3.2 because households' lifetime income is now affected by the government intervention. The details are provided in Appendix C. Slightly abusing notation, we denote the equilibrium exchange rate derived there as $s_1 = \mathcal{S}(q_1; X)$.

The supply of capital is derived as in Section 3.1. With a slight abuse of notation, we denote the capital demand by banks as $K_D(q_1; X) = \min \{K_U(q_1; X), K_C(q_1; X)\}$. When the bankers are constrained, they borrow up to the limit $\bar{b}_2^*$ and $\bar{b}_2^{*g}$,

$$K_C(q_1; X) = \frac{1}{q_1} \left[q_1 k_1 + \alpha k_1^\alpha l_1^{1-\alpha} + \beta(\bar{b}_2^* + \bar{b}_2^{*g}) - b_1^* - \frac{b_1}{\mathcal{S}(q_1; X)} \right]. \tag{26}$$

The unconstrained demand is derived as in Section 3.3, by equating r_2/Q_1 to the risk free rate $1/\beta$. Differently from before, however, r_2 depends on labor in period 2, which is affected by τ_2.

The continuation equilibrium price q_1 is then derived from the market clearing condition

$$K_D(q_1; X) = K_S(q_1),$$

which implicitly ensures also market clearing in goods and factor markets. We assume that this equation has a unique solution denoted as $q_1 = \mathcal{Q}(X)$, and we denote by $\mathcal{K}(X)$ the equilibrium quantity of capital.[17]

Having solved for a continuation equilibrium, we can then compute the continuation

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[17]There is an important difference between the K_D function constructed here and the one constructed in Section 3: here we are conditioning on the debt limit $\bar{b}_2^*$. Therefore, assuming uniqueness of the equilibrium price conditional on $\bar{b}_2^*$ does not rule out multiplicity once $\bar{b}_2^*$ is endogenized, as we shall see in Figure 4.

utility of the households, which we denote by $\mathcal{W}(X)$.

Equilibrium. A Nash equilibrium is given by debt limits $(\bar{b}_2^*, -\bar{A}_2^*)$, government policies $(\bar{b}_2^{*g}, A_2^*, \tau_1, \tau_2, T_2)$, and banks' borrowing from the private sector and the government (b_2^*, b_2^{*g}) such that:

1. Taking $(\bar{b}_2^*, -\bar{A}_2^*)$ as given, the government chooses $(\bar{b}_2^{*g}, A_2^*, \tau_1, \tau_2, T_2)$ to maximize $\mathcal{W}(X)$ subject to its budget constraints, the debt limit $A_2^* \geq \bar{A}_2^*$, and the optimal behavior of the private sector.

2. Banks choose (b_2^*, b_2^{*g}) optimally, taking $(\bar{b}_2^*, -\bar{A}_2^*)$ and $(\bar{b}_2^{*g}, A_2^*, \tau_1, \tau_2, T_2)$ as given.

3. Debt limits satisfy rational expectations,

$$\bar{b}_2^* = \theta \mathcal{Q}(X)\mathcal{K}(X)/\beta, \tag{27}$$

$$-\bar{A}_2^* = \min\{\theta \mathcal{Q}(X)\mathcal{K}(X)/\beta - b_2^*, b_2^{*g}\} + \Xi \mathcal{K}(X)^{\alpha \frac{1+\phi}{\alpha+\phi}}.$$

5.2 Government liquidity and government interventions

To make the analysis interesting, we focus on economies in which parameters' values and initial conditions are such that, absent government intervention, there are three continuation equilibria, as in panel (b) of Figure 2. We use the labels "Good" and "Bad" to denote the two stable equilibria with no intervention.

Since consumers smooth $c_t - v(l_t)$ and non-tradable consumption is equal to the non-tradable endowment, it is easy to show that maximizing consumers' utility is equivalent to maximizing the net present value of their tradable resources (net of labor costs):

$$\frac{(1-\tau_1)w_1}{p_1^T}l_1 + \beta \frac{(1-\tau_2)w_2}{p_2^T}l_2 + \pi(q_1) + \frac{a_1}{s_1} + a_1^* + \beta \frac{T_2}{s_2} - v(l_1) - \beta v(l_2). \tag{28}$$

The following lemma shows that if there is an unconstrained equilibrium with no government intervention and the government has access to enough liquidity—i.e., if $-\bar{A}_2^*$ is large enough—then the government's best response implements uniquely the unconstrained equilibrium.

Lemma 3 (Abundant government liquidity). *Suppose that, absent government intervention, there are three continuation equilibria, one of which unconstrained. For each $\bar{b}_2^* \in [b_2^{*Bad}, b_2^{*Good}]$, if the*

government debt limit $-\bar{A}_2^*$ *satisfies*

$$\frac{A_1}{s_1} + A_1^* + \beta\bar{b}_2^* - \beta\bar{A}_2^* \geq \theta q_1^{Good} k_2^{Good}, \tag{29}$$

for all $s_1 \in [s_1^{Good}, s_1^{Bad}]$*, then the government best response is*

$$\bar{b}_2^g = \theta q_1^{Good} k_2^{Good}/\beta - \bar{b}_2^*, \qquad -A_2^* = \theta q_1^{Good} k_2^{Good}/\beta - \bar{b}_2^* - \frac{1}{\beta}\left(\frac{A_1}{s_1^{Good}} + A_1^*\right),$$

$$\tau_1 = \tau_2 = 0, \qquad T_2 = \frac{1}{\beta}\left(A_1 + s_1^{Good} A_1^*\right).$$

To understand the logic of government intervention, it is useful to represent graphically the equilibrium in the private loans market focusing on equilibrium condition (27).

For simplicity, let's assume that there is no government intervention and taxes are zero. The solid line in panel (a) of Figure 4 plots the relation between the credit limit $\bar{b}_2^*$ and the collateral value of the banks $\theta q_1 k_2/\beta$. The relation is first increasing, then flat. For low levels of $\bar{b}_2^*$ banks are constrained and an increase in $\bar{b}_2^*$ leads to a shift to the right of the capital demand curve (see equation (26)) and to a higher value of $\theta q_1 k_2/\beta$. For higher levels of $\bar{b}_2^*$ banks are unconstrained, so a higher credit limit does not affect their behavior and the value of $\theta q_1 k_2/\beta$ is constant. In Figure 4 this relation crosses the 45° line three times. These crossing points represent three continuation equilibria and correspond to the three equilibria identified in the price space in Figure 2.[18]

Suppose now that $\bar{b}_2^*$ is at $\bar{b}_2^{*\text{Bad}}$ and condition (29) is satisfied. Then the government can borrow and extend the liquidity facility $\bar{b}_2^{*g} = \theta q_1^{Good} k_2^{Good}/\beta - \bar{b}_2^*$ without the need of raising taxes in period 1. Since banks are constrained, they fully use the liquidity facility and the economy moves from point C to point A. At point A, the market value of banks' collateral is

$$\theta q_1^{Good} k_2^{Good}/\beta = \bar{b}_2^* + \bar{b}_2^{*g},$$

so banks can repay both private investors and the government. The government can then repay its debt without raising taxes in period 2.

This argument shows that lending of last resort can always be successfully conducted if the government faces no liquidity constraints. Being a large player, the government understands that by lending enough resources to the banks, it can increase the value of their

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[18]Representing equilibria in the q_1 space is better to explain how initial conditions affect the slope of the demand curve. Representing them in the $\bar{b}_2^*$ space is more convenient to analyze government intervention. That's why we use different graphs.

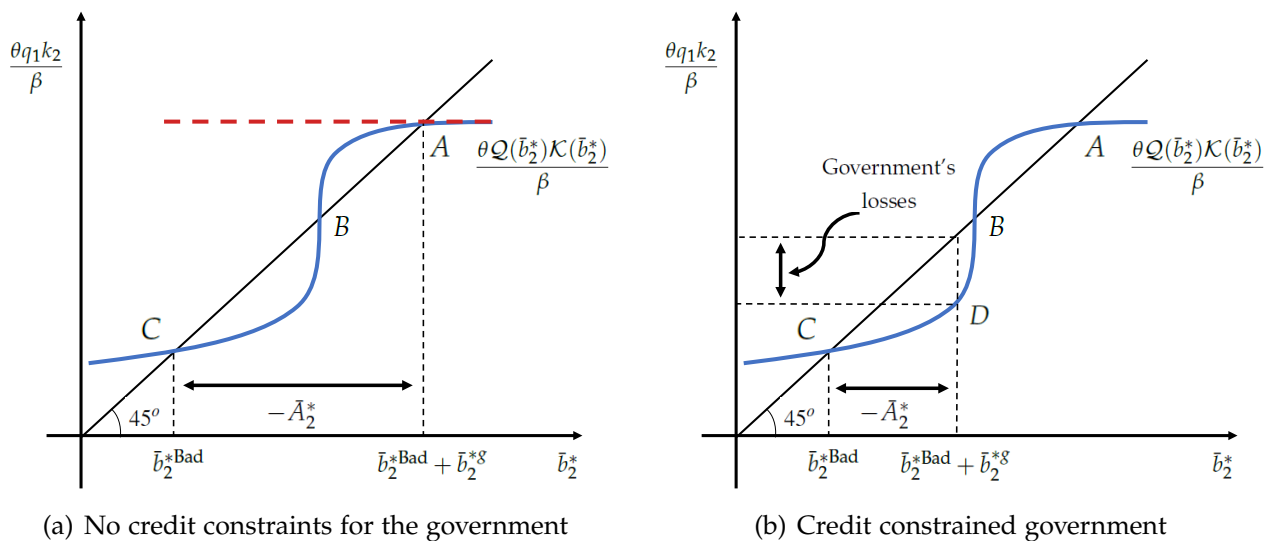| (a) No credit constraints for the government | (b) Credit constrained government |

Figure 4: Ex-post optimality of government's interventions

collateral to that prevailing in the good equilibrium. Intervening has zero costs for the government because in the good equilibrium the banks can meet their debt payments, and the government can use the proceeds to repay government debt. This seems a general proposition that extends beyond the specifics of the model presented here.

Panel (b) of Figure 4 shows an example where the government borrowing limit is tighter and condition (29) is violated. Suppose now that the government sets $\tau_1 = 0$ and finances its interventions only by issuing debt. Under this policy, the economy now reaches point D. The collateral value of banks at D is smaller than $\bar{b}_2^* + \bar{b}_2^{*g}$ so debt is renegotiated and the government makes losses on its loans. As a result, the government needs to raise distortionary taxes at date 2 to satisfy its budget constraint. In this scenario the government faces a trade-off: rasing taxes at date 1 is distortionary, but it allows to lend more to banks, increasing k_2 and future real wages. In a neighborhood of D, lending more to banks also reduces total portfolio losses by the government and so reduces taxation at date 2.

In the Appendix we complete the characterization of the government best response and show examples in which, in spite of optimal government intervention, there are still multiple Nash equilibria. In a bad equilibrium, the private sector holds pessimistic expectations about the ability to repay of both banks *and* the government. Under these expectations, the government best response is a partial intervention, with distortionary taxes at dates 1 and 2, that does not bring the economy all the way back to an unconstrained allocation. The expectations of the private sector are rational because low capital accumulation leads to both a low collateral value for banks *and* low future tax revenues for the government.

5.3 The role of reserves

An important implication of Lemma 3 is that the reserves help the government in conducting its role of a lender of last resort, as they increase the resources that the government can use for lending to banks without having to increase distortionary taxes, and so make it credible a commitment to shore up the banking system.

The following proposition provides a sufficient condition that ensure that there is unique Nash equilibrium in period 1. To state this sufficient condition, we need to define the set $\mathcal{C}$ of all possible triples (q_1, k_2, s_1) that are consistent with a continuation equilibrium when banks' total borrowing $b_2^* + b_2^{*g}$ is in the interval $[\bar{b}_2^{*\text{Bad}}, \bar{b}_2^{*\text{Good}}]$ and there are no taxes. The details of the construction of this set is in the proof of the proposition in the Appendix.

Proposition 4 (Reserves and Lending of Last Resort). *Suppose that, absent government intervention, there are three continuation equilibria, one of which unconstrained. If*

$$\frac{A_1}{s_1} + A_1^* + \beta \Xi (k_2)^{\alpha \frac{1+\phi}{\alpha+\phi}} \geq \theta \left(q_1^{Good} k_2^{Good} - q_1 k_2 \right), \tag{30}$$

for all $(q_1, k_2, s_1) \in \mathcal{C}$ then the Nash equilibrium is unique, and characterized as follows:

- *The debt limits for the banks and the government are*

$$\bar{b}_2^* = \theta q_1^{Good} k_2^{Good} / \beta, \qquad -\bar{A}_2^* = \Xi(k_2^{Good})^{\alpha \frac{1+\phi}{\alpha+\phi}}.$$

- *Banks' borrow $b_2^* = \bar{b}_2^{*Good}$ and $b_2^{*g} = 0$.*

- *The government does not intervene in financial markets ($\bar{b}_2^g = 0$), taxes are zero ($\tau_1 = \tau_2 = 0$), and reserves are rebated to households in the final period:*

$$T_2 = (A_1 + s_1^{Good} A_1^*) / \beta.$$

When reserves satisfy condition (30), the government is always in a position to intervene as in Lemma 3, and uniquely implement the best continuation equilibrium.[19] This threshold has an intuitive meaning. Consider condition (30) at the bad equilibrium. Then, the government fiscal capacity is $A_1/s_1^{\text{Bad}} + A_1^* + \beta \Xi(k_2^{\text{Bad}})^{\alpha \frac{1+\phi}{\alpha+\phi}}$. If condition (30) is satisfied, then the government can extend a liquidity facility of size $\theta(q_1^{\text{Good}} k_2^{\text{Good}} - q_1^{\text{Bad}} k_2^{\text{Bad}})$ to the banks, which is sufficient to bring the economy to the best continuation equilibrium.

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[19]Condition (30) is only sufficient because. When the cost of distortionary taxation is not too large, the government may be able to credibly rule out the bad equilibrium also with a lower level of fiscal resources, by raising τ_1 enough to reach the unconstrained allocation for any $\bar{b}_2^* \in [\bar{b}_2^{*\text{Bad}}, \bar{b}_2^{*\text{Good}}]$.

This proposition helps understanding why dollar reserves can help the government in carrying out these interventions. Consider a government that at date 0 issues peso debt to purchase dollar assets ($A_1 < 0, A_1^* > 0$). These positions generate a transfer to the government in states of the world in which the domestic currency is more depreciated. Through general equilibrium, these are also the states in which (q_1, k_2) are more depressed, which implies a low fiscal capacity for the government and a high required size of the intervention. In this sense, dollar reserves hedge the fiscal position of the government, and they facilitate its role of a lender of last resort.

Proposition 4 leads to two further remarks on reserve accumulation.

Remark 1 (Required reserves, openness, and the size of the financial sector)**.** *An economy with an open capital account and with a more levered financial sector might require a higher amount of dollar reserves to credibly rule out financial panics.*

To understand this remark, consider again condition (30) at the bad equilibrium. The positions necessary to rule out the bad equilibrium depend on the size of the liquidity facility, $\theta(q_1^{\text{Good}} k_2^{\text{Good}} - q_1^{\text{Bad}} k_2^{\text{Bad}})$. From the analysis in Section 3, we know that dollar debt and leverage of the financial sector raise the slope of demand schedule in the constrained region: for a given supply, this increases $\theta(q_1^{\text{Good}} k_2^{\text{Good}} - q_1^{\text{Bad}} k_2^{\text{Bad}})$. Thus, economies with an open capital account and with a more levered banking sector need more resources to rule out financial panics. This observation can help rationalizing the findings in Aizenman and Lee (2007) and Obstfeld, Shambaugh, and Taylor (2010) that financial openness and the size of the banking sector are important predictors for the accumulation of foreign currency reserves by emerging market over the past twenty years.

Remark 2 (Unused reserves)**.** *Reserves may play a useful role to credibly rule out financial panics and yet never be used in equilibrium.*

Under Proposition 4, the government doesn't intervene in equilibrium and rebates back the reserves to the households in period 2. However, the presence of reserves is important to rule out the bad equilibrium. This remark might help rationalizing why emerging markets have been reluctant to "use" their foreign currency reserves. See for example Aizenman and Sun (2012) for evidence related to the global downturn of 2009.[20]

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[20]Furthermore, it might help accounting for some empirical shortcoming of the "self-insurance" hypothesis in models with a unique equilibrium. Jeanne and Sandri (2016) show that these models tend to generate excessive volatility in foreign reserves when calibrated to fit the average reserve-to-gdp ratio observed in emerging markets.

5.4 The ex-ante effects of reserve accumulation

Turning to date $t = 0$, we can now ask how the accumulation of reserves by the government interacts with the private sector choices and study the effects of reserve accumulation from an ex-ante perspective.

Consider the following comparative static exercise. Take an economy like the one in the example of Table 1 in which, absent government intervention, both a dollarized and a non-dollarized equilibrium are possible. Let us now compare two regimes for government policy. In the first regime, which we call "lassez faire", the government holds no reserves and no debt, $A_1 = A_1^* = 0$, and the government does not intervene in financial markets. In the second regime, the government borrows in pesos to finance reserve holdings so $A_1^* > 0$, $A_1 < 0$, with

$$\frac{1}{1 + i_0} A_1 + \frac{s_0}{1 + i_0^*} A_1^* = 0,$$

and condition (30) is satisfied. This latter requirement guarantees that the government can credibly commit to shore up the banking system in the event of a crisis.

Let's assume that in the first regime, a dollarized equilibrium emerges as in the second column of Table 1.[21] In the second regime, the presence of reserves that satisfy (30) means that only the unconstrained equilibrium will be selected ex-post, so the ex-ante equilibrium has to be a non-dollarized equilibrium. Since optimal government intervention ex-post requires no taxation, the non-dollarized equilibrium is analogous to the non-dollarized equilibrium under no intervention, that is, the one characterized in the first column of Table 1. In particular, it is possible to show that the non-dollarized equilibrium with reserves is equivalent to the non-dollarized equilibrium with no intervention with only the following differences:

$$a_1^{\text{No intervention}} = a_1^{\text{With reserves}} + A_1^{\text{With reserves}},$$
$$a_1^{*\text{No intervention}} = a_1^{*\text{With reserves}} + A_1^{*\text{With reserves}}.$$

This construction implies that we can draw implications on the effect of reserves and lending of last resort policies by comparing the two columns of Table 1. In particular, we can make the following two observations.

Remark 3 (Catalytic reserves). *The presence of reserves can reduce the volatility of the exchange rate and induce the banking sector to borrow more in pesos. This reduces the degree of fragility of the financial system.*

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[21]Table 1 was derived in the model with no labor supply, but the model with labor supply can be easily calibrated to yield the same results.

In the dollarized equilibrium that arises under lassez faire, domestic banks have dollar debt equal to 0.36 and that makes the banking sector exposed to crises with peso depreciations. When government intervention successfully eliminates the bad equilibria, domestic agents are more willing to take peso deposits. As a consequence, a non-dollarized equilibrium emerges in which banks hold a creditor position in dollars equal to 0.28. Total bank leverage is similar in the two equilibria, but the different currency composition has a stabilizing role. As we remarked earlier, in the non-dollarized equilibrium of Table 1, continuation equilibria are unique even absent government intervention. So, compounding Remark 2, reserves appear here doubly unnecessary: they aren't used in equilibrium because the bad equilibrium is not played, and in fact multiple equilibria are not even a possibility ex-post. However, reserves still play a useful role, because their presence ex-ante is what keeps the economy away from financial dollarization.

In this economy, stabilizing intervention ex-post instead of encouraging more risk-taking ex-ante, encourages banks to take safer positions. The reason for that has all to do with the response of savers: as households feel safer, they are happier to lend in pesos, which reduces the difference between the costs of borrowing in pesos and dollars and thus reduces the incentive of banks to take risky dollar debt.

Our second remark concerns the consolidated dollar position of the country as a whole.

Remark 4 (Global imbalances). *The presence of reserves can induce the country to accumulate a smaller net position in dollars.*

Adding the position of households, banks and the government, the lassez faire, dollarized equilibrium features total dollar assets equal to 0.35, while the equilibrium under intervention features total dollar assets equal to 0.28. Since foreign investors only trade dollar assets and the initial net dollar position of the country is given, this also means that the country trade balance is smaller under intervention. The intuition here has to do with the precautionary behavior of domestic savers: households, facing a less volatile environment, choose a higher level of tradable consumption and so run a smaller trade surplus.

We then get the counterintuitive result that reserve accumulation by the government leads to less total accumulation of dollar assets by the economy. The idea is that if dollars are accumulated in a way to make the domestic financial system more stable, the private incentive to accumulate dollars for precautionary reasons may be reduced.

Notice that we made our remarks in this section in the form of possibility results that are proved by offering a numerical example. However, the economic forces identified seem robust, as we have experimented with a variety of parameters and, as long as the qualitative features of our main example were preserved, the results were qualitatively in line with

those in Table 1.

The ex-ante analysis could be extended in various directions. First, instead of taking as benchmark a lassez faire regime, one could look at a regime in which policy is ex-post optimal but the reserve position is zero. This makes the analysis more complicated, as positive taxation will arise in equilibrium. But in the Appendix we show numerical examples that extend Remarks 3 and 4 to this comparison. Second, one could study more explicitly the decision to accumulate reserves and see if the government is able to move from a situation of zero reserves to a situation of large enough reserves by trading A_1^* for A_1, in such a way as to rule out multiplicity. The main difficulty there is that the price at which reserves are traded for peso debt depends on the equilibrium itself (as noticed in Section 4). In particular, if investors expect a dollarized equilibrium to emerge, borrowing in pesos will be more expensive for the government. Explicitly analyzing the government ability to make this trade successfully at date 0 would require making more explicit assumptions about the timing of the game at date 0 (as we have done at date 1) and we leave that to future work.

6   Conclusion

This paper has presented a model of a small open economy with a financial sector. The framework appears useful to study a variety of phenomena, such us twin crises, financial dollarization, and the constraints that an open capital account imposes on domestic authorities. Our analysis assumed a floating exchange rate regime with perfect inflation targeting. In future research we plan to enrich our framework with nominal rigidities, and study how crises play out under alternative monetary regimes.

We have conducted our analysis in a three-period version of the model in order to clearly isolate the complex general equilibrium linkages between the financial and the real side of the economy. We believe that considering a more quantitative version of the model represents a fruitful avenue for future research.

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APPENDIX TO "FINANCIAL CRISES AND LENDING OF LAST
RESORT IN OPEN ECONOMIES"

BY LUIGI BOCOLA AND GUIDO LORENZONI

A Proofs of results in Section 3 and Section 4

Proof of Lemma 1

Since there is no uncertainty left after period 1 the Euler equations for domestic and foreign
bonds are

$$U'(c_1) = \frac{P_1}{P_2}\beta(1+i_1)U'(c_2), \tag{A.1}$$

$$U'(c_1) = \frac{s_2}{s_1}\frac{P_1}{P_2}\beta(1+i_1^*)U'(c_2). \tag{A.2}$$

Using market clearing in non-tradables, the definition of the price index (10), and the con-
stant endowment of non-tradables, we get

$$(1-\omega)\left(\frac{p_1^T}{p_1^N}\right)^\omega c_1 = (1-\omega)\left(\frac{p_2^T}{p_2^N}\right)^\omega c_2$$

Using $(1+i_1^*) = 1/\beta$, the law of one price (9), the assumption of constant foreign prices,
and the definition of CPI (10), we can write the Euler equation for foreign bonds (A.2) as

$$U'(c_1) = \frac{\left(p_1^N/p_1^T\right)^{1-\omega}}{\left(p_2^N/p_2^T\right)^{1-\omega}}U'(c_2).$$

Combining these two conditions yields

$$\left(U'(c_1)\right)^{-\frac{\omega}{1-\omega}}c_1 = \left(U'(c_2)\right)^{-\frac{\omega}{1-\omega}}c_2,$$

which, given the concavity of $U(.)$, implies $c_1 = c_2$. The preceding equation implies a
constant relative price of non-tradables. The goods prices in terms of CPI are monotone
transformations of p_t^N/p_t^T, so they are also constant. The domestic interest rate is then
obtained from (A.1).

Proof of Lemma 2

All the steps are in the text except for the comparative statics with respect to q_1, which is straightforward.

Proof of Proposition 1

As $q_1 \to \infty$, the demand of capital goes to 0 and the supply of capital goes to ∞. Because under the conditions (A1) demand exceeds supply at $q_1 = \phi$, continuity ensures existence. The second statement follows because the unconstrained demand of capital is decreasing in q_1 while the supply is increasing.

Proof of Proposition 2

The low q_1 equilibrium is associated with low investment, as we move along an upward sloping supply schedule. The fact the consumption is lower and the exchange rate is more depreciated follows from Lemma 2. The result on the current account follows from the fact that y_1 is the same across the continuation equilibria (as capital is predetermined and labor is inelastically supplied) while investment and consumption of tradables are lower in the low q_1 equilibrium.

Proof of Proposition 3

We first describe the equilibrium conditions of the model, and then verify that the statement of the proposition.

The equilibrium conditions from date 1 onward are discussed in the main text. At date 0 we need to determine the choices of households $\{c_0, a_1, a_1^*\}$, the choices of bankers, $\{b_1, b_1^*\}$, and the prices $\{p_0^T, p_0^N, p_0, s_0, i_0\}$. The monetary rule pins down p_0, while we have $s_0 = p_0^T$ by the law of one price and the normalization of the foreign price of tradables. The remaining variables are determined through households' optimality, bankers' optimality, and market clearing.

Households' optimality at date 0 requires

$$U(c_0) = \beta(1 + i_0)\mathbb{E}_0[U'(c_1)], \tag{A.3}$$

$$s_0 U(c_0) = \beta(1 + i_0^*)\mathbb{E}_0[U'(c_1)s_1], \tag{A.4}$$

$$\frac{1}{1+i_0}a_1 + s_0\frac{1}{1+i_0^*}a_1^* + c_0 = p_0^T(1-\alpha)k_0^\alpha + p_0^N e^N + a_0 + s_0 a_0^*, \tag{A.5}$$

where we have used the assumption of price stability.

The bankers' optimality at date 0 requires that

$$(\lambda_0 + \mu_0) = \beta(1 + i_0)\mathbb{E}_0[\lambda_1], \tag{A.6}$$

$$s_0(\lambda_0 + \mu_0) = \beta(1 + i_0^*)\mathbb{E}_0[\lambda_1 s_1], \tag{A.7}$$

$$\frac{1}{1 + i_0}b_1 + s_0\frac{1}{1 + i_0^*}b_1^* = a_0 + s_0 a_0^* - p_0^T \alpha k_0^\alpha, \tag{A.8}$$

where λ_t is the bankers' marginal value of wealth at date t, and μ_0 is the Lagrange multiplier on the date 0 collateral constraint.

Finally, there are two market clearing conditions,

$$a_1 = b_1, \tag{A.9}$$

$$(1 - \omega)c_0 = e^N p_0^N. \tag{A.10}$$

We can then use Lemma 2 to determine a continuation equilibrium for an arbitrary asset position (a_1, a_1^*, b_1, b_1^*). Because the collateral constraint does not bind at date 1 by assumption, the price and quantity of capital are uniquely pinned down by (20), which implies that w_2/p_2^T and $\pi(q_1)$ are unique and independent of ε. From equation (14) and the monetary rule we can then see that $\{c_t, p_t^T, p_t^N\}$ are also independent of ε if and only if $a_1^* = 0$.

Consider now the bankers' optimality conditions at date 0. Because the collateral constraint does not bind at date 1, we know by equation (23) and equations (A.6)-(A.7) that

$$(1 + i_0) = (1 + i_0^*)\mathbb{E}_0\left[\frac{s_1}{s_0}\right]$$

must hold in equilibrium.

Using the above results and equations (A.3) and (A.4), it then follows that a_1^* must be equal to 0 in equilibrium.[22] This implies that households' future consumption is non-stochastic. Using the same logic of Lemma 1 and the monetary rule at date 0, we can then show that an equilibrium must feature $c_0 = c_1 = \bar{c}$, $p_0^N = p_1^N = p^N$, $p_0^T = p_1^T = p^T$, and $(1 + i_0) = (1 + i) = (1/\beta)$.

From equation (A.5) and the law of one price we then obtain

$$a_1 = (1 + i)\left[p^T(1 - \alpha)k_0^\alpha + p^N e^N + a_0 + p^T a_0^* - \bar{c}\right].$$

The positions of the bankers in point (iv) of the Proposition are obtained by using the

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[22]See the main text for a discussion of this result.

bankers' budget constraint (A.8) and the market clearing condition (A.9).

Note that a non-dollarized equilibrium exists if initial conditions and model parameters are such that the asset positions $\{a_1, a_1^*, b_1, b_1^*\}$ derived here imply a slack collateral constraint in period 1 (almost surely), and the existence of an unconstrained continuation equilibrium in the capital market.

B   Constructing dollarized equilibria

In this section we present an algorithm to "reverse engeneer" a dollarized equilibrium, and to verify whether it coexists with a non-dollarized equilibrium. In what follows, we take as given the structural parameters of the model with the exception of those governing the households' utility function,

$$U(c_t) = \frac{(c_t - \bar{c})^{1-\sigma}}{1 - \sigma}.$$

This utility function allows us to flexibly parametrize households' precautionary behavior.

The algorithm is composed of three main steps. First, we start with a guess for the asset position $[a_1, a_1^*, b_1^*]$ that guarantees that the model features multiple continuation equilibria from date 1. Second, given the continuation equilibria, we solve for $[\sigma, \bar{c}]$ and for the initial conditions $[a_0, a_0^*, b_0^*]$ that guarantees that $[a_1, a_1^*, b_1^*]$ are optimally chosen at date 0 by households and bankers, and that all markets clear. Third, given $[\sigma, \bar{c}]$ and $[a_0, a_0^*, b_0^*]$, we verify whether a non-dollarized equilibrium exists. We now describe in details these steps.

> **Step 1**: Fixing $[a_1, a_1^*, b_1^*]$ and a realization of ε, we use the demand and the supply of capital derived in Section 3 in order to compute the (q_1, k_2) that clear the capital market. We focus on the case in which there are two stable solutions in the capital market, which we label "Good" and "Bad". We next use the results in Lemma 1 and 2 to compute $\{c_1^{\text{Good}}(\varepsilon), s_1^{\text{Good}}(\varepsilon), \lambda_1^{\text{Good}}(\varepsilon)\}$ and $\{c_1^{\text{Bad}}(\varepsilon), s_1^{\text{Bad}}(\varepsilon), \lambda_1^{\text{Bad}}(\varepsilon)\}$, with λ_1 is the marginal value of net worth of the bankers, defined in equation (23). We denote the implied distribution of these variables by $\{c_1(\varepsilon, \xi), s_1(\varepsilon, \xi), \lambda_1(\varepsilon, \xi)\}$, where ξ is the sunspot selecting between the two equilibria.

> **Step 2**: Given $\{c_1(\varepsilon, \xi), s_1(\varepsilon, \xi), \lambda_1(\varepsilon, \xi)\}$, we compute

> $$A = \frac{\mathbb{E}_0[(c_1(\varepsilon, \xi) - \bar{c})^{-\sigma} s_1(\varepsilon, \xi)]}{\mathbb{E}_0[(c_1(\varepsilon, \xi) - \bar{c})^{-\sigma}]}, \qquad B = \frac{\mathbb{E}_0[\lambda_1(\varepsilon, \xi) s_1(\varepsilon, \xi)]}{\mathbb{E}_0[\lambda_1(\varepsilon, \xi)]},$$

> on a grid from $(\sigma, \bar{c})$.[23] We select the $(\sigma, \bar{c})$ that makes sure that $A = B$. In this fashion,

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[23]At this stage, we apply Gauss-Hermite quadrature to integrate over the ε shock.

we are guaranteed that the Euler equations of bankers and households, equation (21) and (22), hold at date 0. We can then solve for $\{c_0, p_0^N/p_0^T, i_0\}$ using the following equations,

$$(c_0 - \bar{c})^{-\sigma} = \beta \frac{(1 + i_0^*)}{(p_0^N/p_0^T)^{-(1-\omega)}} \mathbb{E}_0[(c_1(\varepsilon, \xi) - \bar{c})^{-\sigma} s_1(\varepsilon, \xi)],$$

$$(c_0 - \bar{c})^{-\sigma} = \beta(1 + i_0^*)\mathbb{E}_0[(c_1(\varepsilon, \xi) - \bar{c})^{-\sigma}],$$

$$(1 - \omega)c_0 = \left(\frac{p_0^N}{p_0^T}\right)^{\omega} e^N.$$

Finally, we select initial conditions $\{a_0, a_0^*, b_0^*\}$ such that the budget constraints of the bankers and of the households is satisfied.

Step 3: To verify whether a non-dollarized equilibrium exists at $\{a_0, a_0^*, b_0^*\}$, we can use Proposition 3 to compute the asset positions $\{a_1, a_1^*, b_1^*\}$. The non-dollarized equilibrium exists if at $\{a_1, a_1^*, b_1^*\}$ net-worth at date 0 is positive and the collateral constraint at date 1 is slack for all the realizations of ε in our grid.

C The model with labor and government interventions

[to be added]