Search, Matching, and Online Platforms*

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Abstract

Recent technological advances offer solutions to alleviate search and information frictions in matching markets. Consequently, online platforms have emerged as dominant players in modern matching markets. We explore the incentives of an online platform to harness further technological advances through the lens of a dynamic two-sided search model with horizontally differentiated agents and platform-mediated search-and-matching. We find that the platform has incentives to invest in reducing search frictions, but lacks incentives to improve the quality of information it collects and shares about the compatibility of potential partners. Moreover, in equilibrium, reductions in search frictions and improvements in the quality of information both reduce the fees charged by the monopolistic platform and enhance consumer welfare.

Keywords: Online Platforms, Decentralized Matching, Learning.

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1 Introduction

Recent technological advances have revolutionized matching markets, giving rise to online matching platforms that facilitate decentralized matching in a wide array of contexts, ranging from dating and labor to commerce and tourism. The main advantages online platforms offer over traditional forms of match-making lie in their ability to mitigate search frictions by creating a thick market, and to mitigate information frictions by effectively utilizing big data to identify promising matches. While the aim of online platforms is to harness these technological advances to improve the service that they provide to their users, their primary objective remains profit maximization. This commercial intent can influence the degree to which technological advances are harnessed, the information platforms provide to market participants, and the "rules of the game" in general. Thus, to understand how modern matching markets operate, we must understand the role and incentives of online platforms in these markets.

In this paper, we study platform-mediated two-sided matching markets. Specifically, we explore the platform's technology adoption and pricing decisions, their effect on the market participants' behavior, and their welfare implications. For concreteness, we present these topics through the lens of a specific application, namely, dating apps and the marriage market.[1] However, in the concluding section, we argue that our analysis and insights apply in other contexts such as the labor market and consumer search.

The unique nature of matching decisions, often driven by idiosyncratic personal preferences, means that not all matches facilitated by these platforms lead to successful partnerships. At first glance, unsuccessful matches might seem like a bad outcome. However, from a platform's perspective, these outcomes can be financially advantageous, as they prompt users to return to the platform in search of better prospects, thereby generating additional revenue. The decision of whether to terminate a match and return to the platform is endogenous, as it depends on the quality of the service provided by the platform, as well as its pricing policy. This creates a dynamic repeated clientele tradeoff whereby, on the one hand, the platform wants to provide high quality service to attract repeated clientele, but on the other hand, it does not want to induce matches that are so good that agents never terminate them. This tradeoff is at the heart of the platform's considerations and affects its incentives to adopt technologies that reduce search frictions and improve

[1] Since the beginning of the 21st century, meeting online has gradually displaced the roles that family and friends once played in bringing couples together, becoming the most popular way couples meet (Rosenfeld, Thomas and Hausen, 2019).

the quality of the information provided to users about potential matches.

We develop a dynamic model of a two-sided market in which horizontally differentiated agents subscribe to a monopolistic platform in order to meet agents on the other side of the market. Agents' tastes are distributed on a circle, where the fit of a match between any two agents is decreasing in the distance between their tastes (à la Salop, 1979). We assume that the payoff from a match is proportional to its fit.

The platform has information that it can use to predict the fit between potential partners, and it uses these predictions to tailor agents' search. Specifically, the platform knows whether the distance between any pair of agents is less or greater than $\tilde{\alpha}_0$, and uses this information to restrict agents' search to those potential partners for whom the distance is less than $\tilde{\alpha}_0$ (in our environment, this is equivalent to the platform simply providing the relevant information about fit, and agents consequently restricting their own search based on this information). Agents on the platform randomly meet relevant partners (i.e., ones from whom the distance is less than $\tilde{\alpha}_0$) at a constant rate of μ. Together, $\tilde{\alpha}_0$ and μ characterize the platform's technology.

The agents in our model are (initially) uninformed about their fit with each potential partner. They search for a partner and, when they meet one, all they know is that their fit is better than $\tilde{\alpha}_0$. The couple then leave the platform and continue learning gradually about their fit. The more time a couple spend together, the more they learn about their fit. At any point, agents can (unilaterally) decide to terminate their match, and return to the platform in search of a better match.

Given its technology, the platform chooses the subscription fee that it charges from its users. The central tradeoff that the platform faces in setting its fee is as follows. On the one hand, a higher fee increases the payment collected from any user who chooses to return to the platform. On the other hand, a higher fee makes returning to the platform less attractive, which means some users may decide to stay with their current partner rather than search for a more promising one. This, in turn, reduces the platform's repeated clientele base.

We study the implications of improvements in the platform's technology. We find that improvements in the speed of search increase the platform's profits, whereas improvements in the quality of information about the fit of a match reduce its profits. The difference between the effects of these two technological improvements arises due to their opposite effects on the size of the platform's repeated clientele base. An increase in the speed of search, for any subscription fee, makes returning to the platform more attractive for the agents but does not affect the fit of the resulting matches. This, in turn, increases the platform's repeated clientele base and increases its profits. On the other hand, improvements in the quality of information about the fit of a match increase both the fit of the resulting matches and the prospects of returning to the platform. We show that the former effect dominates, and so, for any subscription fee, such a technological change reduces the platform's repeated clientele base and decreases its profits.

These results imply that in a richer setting where the platform can invest in improving its technology, it has an incentive to invest in reducing search frictions and a disincentive to invest in improving the information it provides. This suggests that the rise of online platforms is leading to underinvestment in the the quality of information about the fit of a match and, as a result, to a low probability that agents choose to stay with their partner indefinitely. Such a state of affairs is consistent with the phenomenon commonly referred to as the "dating apocalypse" (see, e.g., Sales, 2020), where despite the growing ease of finding dating partners, it is increasingly difficult to form a long-lasting relationship.

Technological advances affect the optimal fee that the platform charges. We show that, under mild parametric assumptions, technological improvements in either the speed of search or the quality of information about the fit of a match reduce the fee charged by the platform. This prediction is in contrary to the basic intuition that higher quality products are associated with higher prices.

We then turn to consider the users' perspective. First, our results show that the increase in consumer surplus due to technological improvements in either the speed of search or the quality of information about the fit of a match is amplified by the platform's response in pricing. Moreover, combining this result with the previous results implies that a reduction in search frictions leads to a Pareto improvement. Nevertheless, the platform chooses to underinvest in reducing search frictions, as it does not internalize the increase in consumer surplus from such investment.

While the main focus of the paper is to analyze the role of platforms in the marriage market, our results have wider implications. At the applied level, our analysis is directly applicable to two-sided matching markets in which utility is transferable, e.g., the labor market. Furthermore, our analysis is also directly applicable to one-sided search markets in which consumers search for experience goods or service providers.[2] We explain why our results apply in such settings in Section 5.

[2]Nelson (1970) defines experience goods as goods whose quality becomes apparent to the consumer only through consuming it.

At the methodological level, we contribute to the literature on platform design by introducing agents' search and learning incentives and considering a dynamic model that features repeated clientele. We also contribute to the search-and-matching literature by proposing a search-and-matching model with horizontal differentiation (and learning about match quality) that can be solved in closed form and that enables the derivation of comparative statics, which are typically difficult to obtain in such models.

The paper proceeds as follows. The remainder of this section discusses the related literature. Section 2 presents the model. Section 3 analyzes the agents' equilibrium behavior for a fixed subscription fee. Section 4 endogenizes the platform's choice of the subscription fee and studies the impacts of technological improvements. Section 5 concludes and discusses extensions and the role of selected modeling assumptions. All proofs are relegated to the Appendix.

1.1 Related Literature

This paper contributes to the matching-with-search-frictions literature, which explores the properties of equilibrium matching under various assumptions on the search technology, match payoffs, search costs, the ability to transfer utility, and agents' rationality.[3] See Chade, Eeckhout and Smith (2017) for a comprehensive review of this literature. While some papers in this literature consider the role of a mediator in the market (e.g., Bloch and Ryder, 2000), by and large, this literature has not studied decentralized search in a platform-mediated market.[4]

Within the matching-with-search-frictions literature, Jovanovic (1984) and Moscarini (2005) incorporate a learning aspect into a two-sided search model, studying the effect of post-match learning on employee turnover. Antler, Bird and Fershtman (2023) explore the effects of pre-match learning on segregation and sorting in marriage. The present paper's contribution is in proposing a model of search-and-matching with horizontal heterogeneity and post-match learning that admits a unique equilibrium with a simple closed form characterization, which we leverage to derive comparative statics results that are typi-

[3]See, e.g., McNamara and Collins (1990), Morgan (1996), Burdett and Coles (1997), Eeckhout (1999), Bloch and Ryder (2000), Shimer and Smith (2000), Chade (2001, 2006), Adachi (2003), Atakan (2006), Smith (2006), Lauermann and Nöldeke (2014), Coles and Francesconi (2019), Antler and Bachi (2022), and Antler, Bird and Fershtman (2023).

[4]Bloch and Ryder (2000) study a market in which agents choose between decentralized search and a matchmaker who immediately matches them with an agent of their own "caliber" on the other side of the market.

cally difficult to obtain in search-and-matching models. Moreover, we analyze how decentralized two-sided matching markets are affected by the presence of a profit-maximizing platform that facilitates the matching process.

In studying the implications of technological improvements in two-sided search markets, this paper is related to Eeckhout (1999), Adachi (2003), Lauermann and Nöldeke (2014), and Antler and Bachi (2022), all of which study the effects of reductions in search frictions. Unlike the present paper, these papers either impose a cloning assumption or consider only the frictionless limit. Moreover, these papers focus only on technological changes that improve the speed of search, but do not consider changes that impact the fit of the match.[5]

Markets where agents purchase access to one another are the focus of a vast literature on two-sided markets pioneered by Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), and Armstrong (2006); see Belleflamme and Peitz (2021) and Jullien, Pavan and Rysman (2021) for recent overviews. Much of this literature is not concerned with the process by which platform users are matched with one another. In particular, all users on one side of the platform interact with users on the opposite side, and the platform, apart from influencing the participation of each side through the choice of its prices, does not actively engage in matching agents.

In recent years, the literature on platform markets and, in particular, matching design has started studying models where platforms match participating agents in a customized manner, with an emphasis on price discrimination (see, e.g., Hałaburda and Yehezkel, 2013; Reisinger, 2014; Gomes and Pavan, 2016, 2018; Fershtman and Pavan, 2017, 2022; Jeon, Kim and Menicucci, 2022). In our model, the platform engages in customized matching, but in contrast to the existing literature, the customization takes the form of the platform using its information to restrict matching based on the level of fit between agents.

While much of the literature on platform markets has focused on static environments, there are several exceptions that, like the present paper, focus on a dynamic setting. Cabral (2019) studies dynamic pricing in a model with a monopoly platform and switching costs where, at random times, consumers outside the platform decide whether to pay a subscription fee to join, and consumers already on the platform decide whether to renew

[5]A parallel strand of the literature studies the implications of declining search frictions on product design, vertical differentiation, and growth in product and labor markets (e.g., Martellini and Menzio, 2021; Albrecht, Menzio and Vroman, 2023; Menzio, 2023).

their subscription to the platform. Peitz, Rady and Trepper (2017) consider a continuous-time model of a monopoly platform learning about demand through experimentation. Jullien and Pavan (2019) study a platform market where the information about users' preferences is dispersed. One of the key differences with respect to our model is that in these papers the platform does not engage in customized matching.[6] Fershtman and Pavan (2017, 2022) consider dynamic platform markets where agents arrive over time and experience shocks to their preferences over specific agents on the other side of the market. In contrast to the present paper, these papers seek to characterize an auction format that allows the platform to maximize profits or welfare.

The incentive created by the possibly of repeated sales has also been studied in other contexts. First, there is an extensive literature that shows how firms can signal their quality in an initial period to attract more consumers or charge higher prices in subsequent periods (see, e.g., Milgrom and Roberts, 1986; Bagwell and Riordan, 1991). An aspect of repeated clientele that is more similar to ours appears in Mekonnen, Murra-Anton and Pakzad-Hurson (2023), who study efficiency and surplus allocation in a McCall (1970) search setting where a principal sells an (uninformed) agent information about the quality of each good that is sampled. Thus, the seller's objective takes into account the probability that the buyer will continue searching and purchasing information.

2 The Model

We consider an environment in which a monopolistic platform facilitates matching between two sets of agents. We assume that the two sets are symmetric and, in the concluding section, explain why this assumption is without loss of generality. Agents are horizontally differentiated: on each side of the market, agents' tastes are uniformly distributed on a circle.[7] We identify each agent by their (clockwise) distance from the top of the circle, and denote this characteristic by x. The length of the arc between two agents determines the *fit* of their match: the shorter the arc, the better the fit.

The market operates in continuous time, and agents discount the future at a rate of $r > 0$. New agents arrive to the platform at a constant rate of $2\eta > 0$, with the inflow equally distributed between both sides of the market. Based on its information,

[6] See also Cabral (2011), Halaburda, Jullien and Yehezkel (2020), and Biglaiser and Crémer (2020), all of whom study dynamic competition in platform markets.

[7] This setting is analogous to product differentiation models à la Salop (1979).

the platform can determine whether the arc between any pair of agents is less than $\tilde{\alpha}_0$. The platform restricts agents' search to those potential partners with whom it predicts they can form a "good match," that is, for which $\alpha(x, y) \leq \tilde{\alpha}_0$, where $\alpha(y, x)$ denotes the length if the arc between points x and y.[8] Agents on the platform randomly meet relevant potential partners at a constant rate of μ. Together, μ and $\tilde{\alpha}_0$ characterize the platform's technology.

After agents x and y meet and decide to match, they leave the platform and begin learning about their fit.[9] Specifically, after spending t units of time together, the agents learn whether or not $\alpha(x, y) \geq \tilde{\alpha}_t$, where $\tilde{\alpha}_t$ evolves according to

$$\frac{d\tilde{\alpha}_t}{\tilde{\alpha}_t} = -\lambda. \tag{1}$$

The parameter λ represents the rate at which a couple learn about their fit.

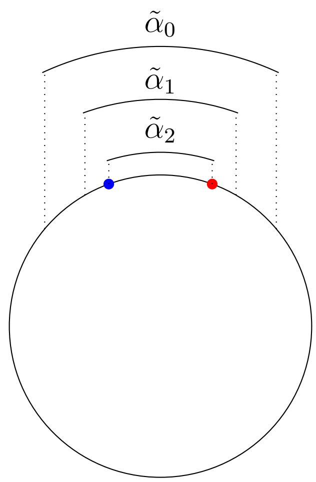

Figure 1: Learning technology.

Figure 1 depicts the dynamics of learning for a couple, whose tastes are represented by the blue and red dots. Upon being matched, the agents infer that the distance between them is at most $\tilde{\alpha}_0$; after being together for 1 unit of time they infer that the distance is $\tilde{\alpha}_1$; and after spending 2 units of time together they learn their fit. Note that the reduction in uncertainty over their fit between time 0 to time 1 is greater than the reduction in uncertainty between time 1 and time 2. That is, the couple learn quickly if their match

[8]This is essentially equivalent to the platform providing relevant information about the fit, and agents restricting their own search based on this information.

[9]As all matches are ex-ante identical, agents either accept all matches, or reject all of them. We later impose a parametric assumption that rules out the latter degenerate case.

is bad, but it takes longer to distinguish between a good match and a great one.

Each agent can unilaterally terminate a match at any moment in time, at which point each of the agents must decide whether to remain unmatched or to pay a fee of $\phi \geq 0$ and (immediately) return to the platform to search for other partners.[10] As is common in the search literature (for a discussion, see, e.g., Stiglitz 1979), we assume that newly arriving agents do not need to pay the fee to join the platform. This assumption is also in line with the widespread practice of platforms offering new users a free trial period. The fee ϕ is set by the platform to maximize its profit.

While agents x and y are together, they obtain an average flow payoff of

$$u(x, y) = 1 - \beta \cdot \alpha(x, y),$$

where $\beta > 0$ measures the importance fit relative to the value of being matched (normalized to one). To simplify the analysis, we assume that while agents are matched, they do not infer their fit from the flow payoffs that they obtain.[11] Finally, we impose the qualitative assumption that agents prefer accepting any match with a fit that is better than $\tilde{\alpha}_0$ to remaining unmatched;[12] that is, we assume that

$$\beta \tilde{\alpha}_0 < 1. \tag{2}$$

We study the steady-state equilibrium of this model. In a steady-state equilibrium, the platform specifies a subscription fee, $\phi \geq 0$, and agents respond to this choice by selecting the optimal stationary strategy. Finally, in a steady-state equilibrium, the measure of agents that are active on the platform must be consistent with the agents' strategies and must not change over time. That is, the flows into and out of the platform must be balanced.

Since all matches are ex-ante symmetric, an agent's strategy specifies two things. First, given their current beliefs about $\alpha(x, y)$, it must specify whether or not to (unilaterally) terminate the match. Second, after a match is terminated, it must specify whether to pay

[10] In reality, fees may be per-usage (e.g., a monthly fee). Such a pricing method is outcome equivalent to the one in our model (see Section 5 for a discussion).

[11] Allowing agents to learn from their payoffs would significantly complicate the agents' decision problem without changing the qualitative results. Since such experimentation is not the focus of the paper, we opted for a simpler learning technology.

[12] This assumption is standard in the matching-with-search-frictions literature. See, e.g., Burdett and Coles (1997), Shimer and Smith (2000), Smith (2006).

the subscription fee and return to the platform.

As in many other two-sided matching models, agents' ability to unilaterally terminate a match can sustain a plethora of equilibria in which agents choose to separate from their partner on the basis of a belief that the partner will choose to separate from them. To abstract away from equilibria that are sustained due to such lack of coordination, the matching-with-search-frictions literature (e.g., Burdett and Coles, 1997; Smith, 2006) typically assumes that agents accept any match that exceeds their reservation value, that is, that agents decide which matches to accept as if their choice were pivotal. In this paper, we make the analogous assumption that agents' termination choices are made as if they were pivotal.

3 Agents' Behavior

We start by analyzing the agents' behavior given an arbitrary fee, ϕ. In Section 4 we endogenize the platform's choice of the fee. In this section we show that there exists a unique steady-state continuation equilibrium. This (continuation) equilibrium can be either a *nontrivial equilibrium* in which agents terminate matches and rejoin the platform with positive probability, or a *trivial equilibrium* in which all agents stay indefinitely with the first partner with whom they are matched. We show that the equilibrium is nontrivial if and only if ϕ is below a critical threshold. Focusing on this case, we further characterize the nontrivial equilibrium in closed form and derive several comparative statics results that we later use to analyze the platform's choices.

3.1 Agents' Behavior in Equilibrium

In our model, a couple's belief about their fit becomes more optimistic over time until their true fit is revealed. It follows that the continuation value of staying in a match of unknown fit, vis-a-vis terminating the match and returning to the platform, increases over time. Therefore, the only time at which agents may find it optimal to terminate a match is when its fit is revealed. Furthermore, if agent x prefers staying in a match with agent y to terminating the match (when its fit is revealed), agent x would prefer to stay with every agent y' such that $\alpha(x, y') < \alpha(x, y)$. Since agents use stationary strategies and act as if they were pivotal, their strategies are therefore given by a separation threshold $\alpha_s \in [0, \tilde{\alpha}_0]$ such that agents terminate the match if they learn that $\alpha(x, y) > \alpha_s$, and remain together indefinitely if $\alpha(x, y) \leq \alpha_s$.

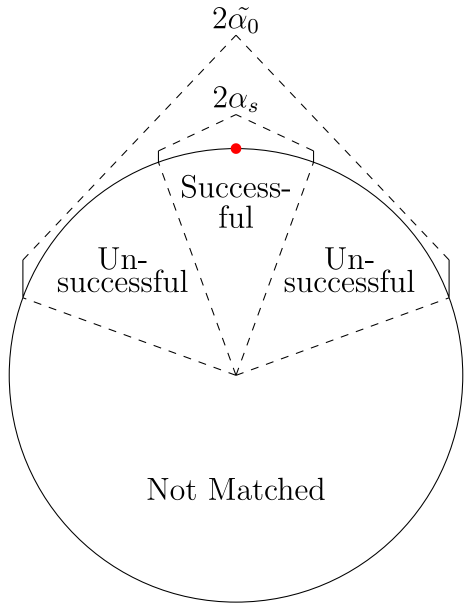

Figure 2: Strategies and outcomes.

Figure 2 depicts how the strategy of an agent located at the top of the circle (red dot) determines the outcome of matches with various partners: a match with a partner on the arc of the "Success" region leads to an indefinite relationship, whereas a match with a partner on the arc of the "Unsuccessful" regions leads to eventual separation and a return to the platform. There are no meetings with potential partners on the arc of the "Not Matched" region.

The continuation value of an indefinite relationship for a couple $\langle x, y\rangle$ is

$$\frac{1 - \beta\alpha(x, y)}{r}.$$

Denote by W_s the endogenous continuation value of an agent that is subscribed to the platform. The continuation value after terminating a match is $\max\{W_s - \phi, 0\}$. By Assumption 2, the continuation value of remaining with any partner is positive. Hence, in a nontrivial equilibrium the continuation value after terminating a match is $W_s - \phi$. In a nontrivial equilibrium, the separation threshold α_s is thus given by the distance α for which the agents are indifferent whether to stay together or terminate their match and rejoin the platform. Clearly, in a nontrivial equilibrium, $\alpha_s \in (0, \tilde{\alpha}_0)$: if $\alpha_s \leq 0$, agents terminate all matches, whereas if $\alpha_s \geq \tilde{\alpha}_0$, no matches are terminated.

The following lemma formalizes these observations.

Lemma 1 (Equilibrium strategies) *In a nontrivial equilibrium, strategies are characterized by a separation threshold* α_s *such that*

$$\alpha_s = \frac{1 - r(W_s - \phi)}{\beta}, \tag{3}$$

where $\alpha_s \in (0, \tilde{\alpha}_0)$*.*

In a steady-state equilibrium the composition of agents that are active on the platform does not change over time. Due to the symmetry of the model and of the agents' strategies, the distribution of tastes of the agents that are active on the platform is uniform. Hence, to characterize the platform's users, it is enough to specify their mass on each side of the market, M.

The outflow from each side of the platform is μM, whereas the inflow is the sum of the arrival rate η and the measure of couples who choose to terminate their match. A couple $\langle x, y \rangle$ terminate their match if $\alpha(x, y) > \alpha_s$, an event that occurs with probability $\frac{\tilde{\alpha}_0 - \alpha_s}{\tilde{\alpha}_0}$. Hence, in the steady state, the measure of couples that separate at each instant is $\mu M \frac{\tilde{\alpha}_0 - \alpha_s}{\tilde{\alpha}_0}$. Thus, the flow into and out of the platform is balanced if

$$\eta + \mu M \frac{\tilde{\alpha}_0 - \alpha_s}{\tilde{\alpha}_0} = \mu M. \tag{4}$$

The steady-state mass of agents that are active on the platform is pinned down by α_s:

$$M^\star = \frac{\eta \tilde{\alpha}_0}{\mu \alpha_s}. \tag{5}$$

To characterize the equilibrium behavior of the agents, we must connect the continuation value of agents that are subscribed to the platform to the separation threshold derived in Lemma 1. To do so, we use a recursive representation of W_s. In a nontrivial equilibrium, matches either last indefinitely or lead to an eventual separation and return to the platform. We refer to the former type of match as a *successful match* and the latter type of match as an *unsuccessful match*. A match between agents x and y is successful if $\alpha(x, y) \leq \alpha_s$ and is unsuccessful otherwise. Denote the probability that a match is successful by $\Pr(succ)$.

For $i \in \{s, n\}$, where s represents a successful match and n represents an unsuccessful one, let EV_i denote the expected payoff while a couple remain together. Note that EV_s is the continuation value after a successful match, whereas EV_n includes the payoff obtained while a couple remain together minus the cost of subscribing to the platform, ϕ, but does not include the value of being single again in the future. Let σ denote the expected discounting between the beginning and end of an unsuccessful match.

W_s can be written recursively as

$$W_s = \frac{\mu}{\mu + r} \left((1 - Pr(succ)) \left(EV_n + \sigma W_s \right) + Pr(succ) EV_s \right), \tag{6}$$

where $\frac{\mu}{\mu+r}$ is the expected discounting until the agent is matched for the first time.

Using this representation, we establish that there exists a unique continuation equilibrium for any choice of ϕ. Moreover, we derive a simple upper bound on ϕ such that this equilibrium is nontrivial if and only if ϕ is below this upper bound. Finally, in such cases, we characterize the equilibrium in closed form. Let $\xi \equiv \frac{r}{\lambda}$.

Proposition 1 *For any given ϕ there exists a unique continuation equilibrium. This equilibrium is nontrivial if and only if*

$$\phi < \frac{1}{2} \left(\frac{\tilde{\alpha}_0 \beta - 2}{\mu + r} + \frac{\tilde{\alpha}_0 \beta}{r} \right) \equiv \overline{\phi}. \tag{7}$$

The nontrivial equilibrium is characterized by the solution to

$$\frac{1}{\tilde{\alpha}_0^{\xi+1} (\xi^2 + 3\xi + 2)} \alpha_s^{\xi+2} + (\frac{\xi}{\xi + 1} + \frac{r}{\mu}) \alpha_s = \frac{\tilde{\alpha}_0 \xi}{2\xi + 4} + \frac{r\phi(\mu + r) + r}{\beta \mu}. \tag{8}$$

For the remainder of this section, we focus on the nontrivial equilibrium. That is, we assume that inequality (7) is satisfied.

3.2 Technological Changes

In this section, we explore how the agents' behavior depends on the platform's technology and pricing decisions, when the latter is taken as an exogenous parameter, i.e., fixing ϕ, $\tilde{\alpha}_0$, and μ. We later endogenize the platform's fee ϕ in Section 4.2. The agents' separation threshold equates the value of remaining with the marginal acceptable partner, $\frac{1-\alpha_s \beta}{r}$, to the value of terminating the match and returning to the platform, $W_s - \phi$. Thus, improvements in the platform's technology that increase W_s – whether through an increase in μ or a reduction in $\tilde{\alpha}_0$ – increase an agent's incentive to terminate a match and return to the platform. Similarly, a reduction in the platform's fee also increases agents' incentives to return to the platform and search for better matches. Formally, we have the following result.

Proposition 2 *Assume that Condition (7) holds. The separation threshold α_s is increasing in ϕ and $\tilde{\alpha}_0$, and decreasing in μ.*

An immediate corollary of Proposition 2 is that technological improvements (or a reduction in the subscription fee) increase the agents' welfare. The negative relation between the separation threshold, α_s and agents' welfare, W_s, can be seen in Equation (3).

Corollary 1 *Agents' welfare increases due to technological improvements or a reduction in the platform's fee.*

Technological changes affect the *conversion rate*, i.e., the probability that a match is successful, which, as established below, is of key importance to the platform's profits. Roughly speaking, the conversion rate determines the probability that an agent returns to the platform, and so it pins down the size of the platform's repeated clientele base.

The conversion rate, which is defined as

$$\gamma \equiv \Pr(\alpha(x,y) < \alpha_s \mid \alpha(x,y) \leq \tilde{\alpha}_0), \tag{9}$$

is directly affected by changes in the platform's technology, and indirectly affected by how these changes – and changes in pricing – impact α_s.

Proposition 3 *Assume that Condition (7) holds. The conversion rate*

1. *decreases due to improvements in the speed of search (an increase in μ);*
2. *increases due to improvements in the quality of information about the fit of a match (a decrease in $\tilde{\alpha}_0$);*
3. *increases due to increases in the platform's fee.*

Proposition 3 highlights the fact that the effects of technological improvements depend on whether the improvement lies in the speed of search or the quality of information the platform provides. To understand the intuition for this result, note that faster meeting rates increase the value of being active on the platform but have no effect on the payoff from a match. This implies that agents become more selective, and results in a lower conversion rate. On the other hand, improvements in the quality of the information directly increase the expected payoff from a match, and indirectly increase the value of being active on the platform. Proposition 3 establishes that the direct effect is the one that dominates. Note that this intuition does not rely on the exact specification of how agents learn about their fit, and hence this key qualitative result holds for a wider range of learning technologies.

4 The Platform's Problem

In this section, we study the platform's problem. The platform chooses its pricing strategy while taking the agents' behavior into account. We start by showing that this problem has a unique solution, i.e., that there is a unique optimal price (Proposition 4). We then use this result to explore the implications of advances in the platform's technology on the platform's profit and the user' welfare.

We first show that improvements in the speed of search increase the platform's profit, whereas improvements in the quality of information about the fit of a match reduce the platform's profit (Proposition 5). This implies that the platform has an incentive to invest in the former type of improvements, but not in the latter one. Second, we consider the consumers' perspective and show that (under mild parametric assumptions) both types of improvements reduce the optimal subscription fee and increase consumer welfare (Proposition 6 and Corollary 2). Thus, the consumers' and the platform's interests are aligned when it comes to reducing search frictions, but not when it comes to reducing information frictions.

To make the subsequent analysis nondegenerate, we must have the platform's technology be such that it can set a strictly positive fee that induces a nontrivial equilibrium. Doing so is feasible if $\overline{\phi}$ (as defined in Equation (7)) is strictly positive, which occurs if

$$\tilde{\alpha}_0\beta\mu + 2r(\tilde{\alpha}_0\beta - 1) > 0. \tag{10}$$

Note that due to Condition (2), this condition is relaxed as agents become more patient. In what follows, we assume that (10) holds. Moreover, to ease the exposition, we also assume that the platform does not discount its future payoffs.

4.1 Optimal Pricing

To analyze the platform's problem, we first derive its profit function. The platform generates profits from the fees paid by the agents that have unsuccessful matches and elect to return to the platform. The probability that a match is successful is given by γ. Thus, on average, an agent has $\frac{1}{\gamma} - 1$ unsuccessful matches before finding a successful one, and so the expected payment that the platform collects from a given agent is $(\frac{1}{\gamma} - 1)\phi$. Since the conversion rate γ is determined endogenously and, in particular, depends on ϕ, the platform's problem is nontrivial.

By Proposition 1, if $\phi > \overline{\phi}$ then agents prefer staying in the worst possible match to paying the subscription fee and returning to the platform. Importantly, for such fees the platform's profit would be zero. Hence, without loss of generality, we can restrict the platform's choice of fee to the interval $[0, \overline{\phi}]$. This, in turn, implies that the platform's objective is

$$\max_{\phi \in [0, \overline{\phi}]} (\frac{1}{\gamma} - 1)\phi \tag{11}$$

subject to (8) and (9).

Let $\alpha_s(\phi)$ denote the solution to (8) as a function of ϕ, and let $\gamma(\phi)$ denote the induced conversion rate of matches. As there is a unique continuation equilibrium strategy for any fixed ϕ (Proposition 1), these functions are well defined.

Proposition 4 *The platform has a unique optimal price that is given by*

$$\frac{1}{\gamma(\phi)} - 1 = \phi \frac{\gamma'(\phi)}{\gamma^2(\phi)}. \tag{12}$$

This optimality condition captures the central tradeoff that arises in a dynamic setting where the platform's pricing choices determine the probability with which agents return to the platform. On the one hand, a marginal increase in ϕ increases the fee that the platform collects from each agent. Since, on average, agents pay the fee $\frac{1}{\gamma(\phi)} - 1$ times, this direct effect is captured by the LHS of Equation (12). On the other hand, an increase in ϕ increases the conversion rate $\gamma(\phi)$ (Proposition 3). Thus, an increase in ϕ decreases the average number of times that an agent pays the fee by

$$-\frac{d}{d\phi}\frac{1}{\gamma(\phi)} = \frac{\gamma'(\phi)}{\gamma^2(\phi)}.$$

As each additional round of search yields the platform a profit of ϕ, this indirect cost of increasing ϕ is captured by the RHS of Equation (12).

4.2 Optimal Pricing and Technology

In this section, we explore the effects of technological changes on equilibrium outcomes, while taking into account the platform's response to such changes. We begin by considering how technological changes impact the platform's profits. We then examine their effect on optimal pricing and consumer welfare.

The Platform's Profit

Since the platform is a monopoly, basic economic reasoning suggests that it should benefit from an improvement in the quality of the services it provides. However, the following proposition establishes that technological improvements can actually reduce the platform's profit.

Proposition 5 *Improvements in the speed of search increase the platform's profits, whereas improvements in the quality of information about the fit of a match reduce its profits.*

To understand the intuition for this result, note that the platform's profit depends on the size of its clientele base and, in particular, on the size of its repeated clientele base: a larger client base shifts up the entire profit function. For any arbitrary fee, the platform's repeated clientele base is inversely related to the induced conversion rate. As the quality of the service improves, the repeated clientele base may decrease, which shifts down the platform's profit function. By Proposition 3, this is exactly what happens when the average fit of matches improves.

The logic behind Proposition 5 can also be used to analyze the impact of multidimensional technological change on the platform's profits, that is, changes that affect both the rate at which agents meet and the quality of information. For example, consider a change in technology that increases μ by ϵ_μ and decreases $\tilde{\alpha}_0$ by ϵ_α. Such a change is profitable for the platform if and only if it reduces the conversion rate

$$\frac{\partial \gamma(\phi, \mu, \tilde{\alpha}_0)}{\partial \mu}\epsilon_\mu - \frac{\partial \gamma(\phi, \mu, \tilde{\alpha}_0)}{\partial \tilde{\alpha}_0}\epsilon_\alpha < 0.$$

Proposition 5 has clear implications for the platform's incentive to invest in its technology. To see this, consider a richer setting in which the platform can invest in improving its technology. Proposition 5 suggests that, regardless of the specific details concerning the cost and feasibility of such technological investments, the platform has neither an incentive to invest in improving the quality of information it has about the fit between users, nor an incentive to use all of the information it does have. On the other hand, if the cost of investing in reducing search frictions is not excessive, the platform may have an incentive to do so. These predictions are in line with changes in dating apps over the past decade: despite the vast improvements in the ability to predict users' preferences using big data and machine-learning algorithms, platforms often still provide users with a vast number of potential matches that are unlikely to be successful.

Optimal Pricing and Consumer Welfare

We conclude this section by analyzing the impact of technological improvements on optimal pricing and consumer welfare. In particular, we show that such changes are beneficial when agents are sufficiently patient.

Proposition 6 *There exists $r^\star > 0$, such that if $r < r^\star$ then the optimal subscription fee decreases following an improvement in the speed of search or the quality of information about the fit of a match.*

The platform chooses its fee to maximize profits, while taking into account that a higher fee reduces the measure of agents returning to the platform. All else being equal, improvements in the matching rate μ increase the agents' incentive to return to the platform and search for better partners. Thus, one might think that such improvements should lead the platform to increase its prices: intuitively, such changes counteract the reduction in the repeated clientele base resulting from a higher fee. However, there is a second effect that may not be as transparent: the marginal effect of increasing prices on the probability that an agent returns to the platform depends on μ. In particular, as μ increases, a marginal increase in ϕ leads to a greater reduction in the probability the an agent returns to the platform. That is, there is a complementarity between reducing fees and reducing search frictions on the size of the platform's repeated clientele base. Proposition 6 shows that the latter effect dominates the former, more transparent one, when agents are sufficiently patient. That is, even though the repeated clientele base increases due to technological advances, the platform further enhances this growth by reducing its fees.

By contrast, improvements in the quality of information about the fit of a match reduce the agents' incentive to return to the platform (Proposition 3). Moreover, in this case it can be shown that the negative impact of a higher fee on the the platform's repeated clientele base becomes stronger as the quality of information improves. Hence, both the direct and indirect effects induce the platform to lower its prices.

Proposition 6 has several noteworthy implications. First, the result shows that faster search leads to a Pareto improvement in welfare. Agents benefit not only from the improvements in technology, but also from the reduction in prices, and the platform's profits increase (Proposition 5). On the other hand, improvements in the fit of proposed matches increase consumer surplus, but reduce the platform's profit. Hence, they do not lead to a Pareto improvement.

Corollary 2 *For any $r < r^{\star}$:*

- *Faster search leads to a Pareto improvement in welfare.*
- *Better information about the fit of a match leads to an increase in consumer surplus.*

Corollary 2 suggests that a platform that is able to invest in its technology will underinvest, as it will not internalize the positive effect of technological improvements on consumer welfare. This, in turn, may imply that such investment should be subsidized.

5 Concluding Remarks

Radical technological changes have turned platforms into major players in two-sided matching markets in recent decades. We analyzed the incentives of such platforms to harness new technological advances in the dimensions of the matching rate and the fit of a match. We argued that such advances increase consumer welfare, whereas, despite the monopolistic nature of the platform, its profits suffer from improvements in the fit of matches and benefit only from improvements in the matching rate.

Our results imply that in a richer setting where the platform can invest in its technology, it has an incentive to invest in increasing the matching rate and a disincentive to invest in improving the quality of information about the fit of a match. Thus, despite recent technological advances, it may be harder for users to find successful matches using a (monopolistic) online platform.

We conclude by discussing several extensions and additional applications.

Revenue from Advertising

Our baseline specification focuses on the case where the platform generates income only from the fees paid by its users. In practice, online platforms may generate additional income by exposing their users to advertisements. Consider a variant of our model in which the platform earns a flow payoff of A per agent that is subscribed to the platform. Moreover, assume that the agents do not suffer any disutility from being exposed to ads.[13]

The agents' preferences in this variant of the model are the same as in the baseline model. Therefore, all the results of Section 3 continue to hold when the platform has additional advertising revenue. If the platform does not discount the future, each round of search – including the first – provides an expected profit of $\frac{A}{\mu}$ from advertising revenue. Hence, the platform's objective is

$$\max_{\phi\in[0,\bar{\phi}]} \frac{1}{\gamma}(\phi + \frac{A}{\mu}) - \phi \tag{13}$$

subject to (8).

Due to the continuity of this problem in A, it can be shown that the results in Section 4 continue to hold as long as A is sufficiently small. Nevertheless, advertising income does impact the platform's optimal fee. In particular, as A increases, the platform's incentive to attract repeated customers increases. Therefore, increased revenue (per unit) from advertising leads to a lower fee. We thus obtain the following result.

Proposition 7 *If the optimal fee is interior, then an increase in A leads to a reduction in the optimal fee.*

Alternative Subscription Fees

Online platforms often charge users a "flow" subscription fee that allows them to be active on the platform so long as they continue paying this fee. Our analysis remains valid when the platform uses such a pricing policy rather than pricing policy we assumed in the main paper of charging agents a single upfront fee that allows them to stay on the platform until they find a match, regardless of how long it takes.

The agents' separation choices depend on their *expected* payment to the platform, and not the exact manner in which the payment is made. Thus, whether ϕ represents an

[13] In some cases, agents strictly benefit from being exposed to (personalized) advertisements. See, e.g., Bird and Neeman (2023) and the references therein.

upfront payment or the expected discounted flow payments until an agent is matched is irrelevant. Furthermore, the expected cost of two such policies does not depend on the fit of matches, and hence all of the results with regard to improvements in the quality of information about the fit of a match do not depend on the platform's pricing policy. On the other hand, an increase in the speed of search reduces the expected discount cost of flow payments but does not alter the cost of an upfront payment. Therefore, to maintain the equivalence between the two types of pricing policies after an increase in the matching rate the platform has to increase the flow cost. Since we consider the case where both the platform and the agents are patient, this modification has a similar impact on both players, and would not have a qualitative impact on our results.

Asymmetry between the Two Sides of the Market

Throughout the paper, we assumed that both sides of the market are symmetric (e.g., use the same discount rate and find the level of fit equally important). This symmetry allowed us to simplify the exposition and present the analysis succinctly.

In reality, there may be various asymmetries between both sides of the market. We now explain why introducing such asymmetries would not change our results. Recall that beliefs about the fit of a match become more optimistic over time. As a result, agents on both sides of the market use a (side-specific) separation threshold whether they are symmetric or not. This, in turn, implies that separation choices are driven entirely by the side with the lower separation threshold. Technically, this implies that Condition (8) (which determines the separation threshold) is derived according to the "pickier" side of the market. The only effect the less picky side of the market has on the analysis is in that it determines an upper bound on the fee that is lower relative to the symmetric case (see Equation (7)). Given these two changes, our analysis can be applied directly to a market with asymmetries between groups, with the caveat that if the asymmetry between the two sides is extreme, the platform may want to forgo generating repeated clientele from the less picky side of the market. We can therefore conclude that the main insights of the paper are relevant also when there are asymmetries between agents on different sides of the market.

Other Matching Markets

While we use the marriage market terminology throughout the paper, our model and insights have implications for a variety of other markets. Consider, for example, the labor market. As in the marriage market, new search technologies have changed the way people search for a job and online platforms are becoming more dominant in the process of matching employers and workers. Our results imply that platforms have an incentive to invest in technologies that help match workers and employers at a faster rate, and a disincentive to invest in technologies that improve the fit of the match.

The main difference between the marriage market and the labor market in terms of modeling is in the ability to transfer utility: models of the marriage market typically assume that utility is nontransferable, whereas models of the labor market assume that utility is transferable (the latter assumption captures the idea that employers and potential hires can negotiate wages). The nontransferable utility model in the present paper is equivalent to a model in which agents' utility is transferable under the assumption that the flow surplus from a match is $2(1-\beta\alpha)$ and that the bargaining over the surplus generated in a match is settled via the Nash bargaining solution (as is typically assumed in the literature). To see the equivalence, note that the symmetry between the agents implies that they have the same outside option, which, under the Nash bargaining solution, implies that their flow payoff from a match is equal to $(1-\beta\alpha)$ as in our model. Furthermore, it is possible to show that even if the bargaining weights were unequal on both sides of the market, all of our results would hold were we to repeat our analysis from the perspective of the side of the market with the greater bargaining power.

The scope of our analysis goes beyond the realm of two-sided matching. The driving force behind our analysis is that after a match occurs on the platform, agents continue learning gradually about its quality and then decide whether to remain with the match or return to the platform in search of a better one. This feature is present in consumer search problems and one-to-many matching problems where the goods that are assigned have an experience good component. I.e., where the consumer learns the quality of the good only through consuming it. Moreover, given that in our model matched agents make symmetric choices, our analysis is directly applicable to matching problems in which one side of the market is passive.

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A Proofs

Proof of Proposition 1. First, we consider nontrivial equilibria in which agents choose to separate from a partner and return to the platform with positive probability, i.e., equilibria in which $\alpha_s < \tilde{\alpha}_0$. To analyze such an equilibrium we assume that $\alpha_s < \tilde{\alpha}_0$, and at the end of this part of the proof we verify that this inequality holds under Condition (7).

Characterization. We begin by expressing σ, W_s, EV_s, and EV_n as functions of α_s and the primitives of the model. This will later enable us to use Equation (6) to pin down the connection between α_s and W_s.

First, we calculate the expected value of a successful match. The average distance between the agents that constitute a successful match is $\frac{\alpha_s}{2}$, which means that the expected payoff from such a match is

$$EV_s = \frac{1 - \beta\alpha_s/2}{r}. \tag{A.1}$$

Next, we calculate the expected payoff (minus the cost of subscribing to the platform) that an agent receives from an unsuccessful match. Fix $\alpha \in (\alpha_s, \tilde{\alpha}_0)$. The match between agents x and y lasts until $\tilde{\alpha}_t$ drops to α. Integrating (1) yields that

$$\tilde{\alpha}_t = e^{-\lambda t}\tilde{\alpha}_0.$$

Hence, that match lasts for

$$T(\alpha) \equiv \frac{1}{\lambda}\log\{\frac{\tilde{\alpha}_0}{\alpha}\}$$

units of time. It follows that the discounted payoff from such a match is

$$\frac{1 - e^{-rT(\alpha)}}{r}(1 - \beta\alpha) - e^{-rT(\alpha)}\phi.$$

The distribution of α in unsuccessful matches is uniform over $[\alpha_s, \tilde{\alpha}_0]$. Taking the expectation over the above payoff yields

$$EV_n = \frac{\tilde{\alpha}_0 - \alpha_s + \frac{\beta}{2}\left(\alpha_s^2 - \tilde{\alpha}_0^2\right) - \alpha_s\lambda\left(\frac{\tilde{\alpha}_0}{\alpha_s}\right)^{-\frac{r}{\lambda}}\left(\frac{\alpha_s\beta}{2\lambda+r} - \frac{r\phi+1}{\lambda+r}\right) + \tilde{\alpha}_0\lambda\left(\frac{\tilde{\alpha}_0\beta}{2\lambda+r} - \frac{r\phi+1}{\lambda+r}\right)}{r(\tilde{\alpha}_0 - \alpha_s)}. \tag{A.2}$$

Next, note that the expected discount factor at the end of an unsuccessful match is

$$\sigma = E\left(e^{-rT_m(\alpha)} | \alpha \sim U[\alpha_s, \tilde{\alpha}_0]\right) = \frac{\lambda\left(\tilde{\alpha}_0 - \alpha_s\left(\frac{\alpha_s}{\tilde{\alpha}_0}\right)^{r/\lambda}\right)}{(\tilde{\alpha}_0 - \alpha_s)(\lambda + r)}. \tag{A.3}$$

Finally, since agents' tastes are distributed uniformly around the circle, we have

$$Pr(succ) = \frac{\alpha_s}{\tilde{\alpha}_0}, \tag{A.4}$$

and by (3) we have

$$W_s = \frac{1 - \alpha_s \beta}{r} + \phi.$$

Plugging this representation of W_s, as well as the representations derived in Equations (A.1)–(A.4), into Equation (6) and rearranging yields Equation (8).

Existence and Uniqueness. To see that the nontrivial equilibrium, if it exists, is unique, note that the LHS of Condition (8) is increasing in α_s, whereas its RHS is constant in α_s.

To show that a nontrivial equilibrium exists we must show that there exists an $\alpha_s \in (0, \tilde{\alpha}_0)$ that solves Equation (8). The RHS of Equation (8) is strictly positive and independent of α_s. On the other hand, the LHS of Equation (8) increases in α_s and equals zero if evaluated at $\alpha_s = 0$. Therefore, a nontrivial equilibrium exists if the LHS that is evaluated at the maximum value of α_s, namely, $\tilde{\alpha}_0$, yields a term that is greater than the RHS. That is, if

$$\tilde{\alpha}_0 \beta(\mu + 2r) > 2r(\phi(\mu + r) + 1), \tag{A.5}$$

which is equivalent to (7). Moreover, under the assumption that agents act as if they were pivotal, it must be that if the above condition holds, then the agents' unique optimal strategy is given by the interior solution of (8). That is, if (A.5) is satisfied, there is no trivial equilibrium.

Next, consider the trivial equilibrium. In the trivial equilibrium agents terminate a match with probability zero. The previous analysis shows that if (7) does not hold, then an agent prefers staying in a match of fit $\tilde{\alpha}_0$ to terminating the match and paying ϕ to return to the platform. By Assumption (10), an agent is better off staying in any match than terminating it and remaining single. Therefore, if (7) does not hold there is only a trivial equilibrium in which agents never terminate a match. ■

Proof of Proposition 2. From Equation (3) it follows that α_s is decreasing in W_s. To prove this proposition we show that, under Condition (7), W_s is increasing in μ and decreasing in $\tilde{\alpha}_0$ and ϕ.

With linear meeting technology, the mass of agents that are active on the platform does not impact the rate at which they meet potential partners on the platform. Hence, we can conduct comparative statics of W_s without considering changes in M^*.

Due to the symmetry of the model and, in particular, the symmetry of the agents' equilibrium strategies, comparative statics for this model can be analyzed as if it were a decision problem. By Condition (7) an agent's optimal strategy is interior and satisfies the first-order condition. Hence, the envelope theorem applies, and so the impact of marginal changes in model parameters on an agent's payoff can be evaluated by how such a change alters the agent's payoff while using the original equilibrium strategy.

Fixing an agents' strategies, their payoffs in a nontrivial equilibrium are decreasing in ϕ as they pay the fee with positive probability. Their payoffs are also decreasing in $\tilde{\alpha}_0$ since increasing in $\tilde{\alpha}_0$ increases the probability of a match being unsuccessful. Finally, since agents do not receive payoffs when they are single and receive positive payoffs while in a match, increasing the meeting rate would increase the payoffs from their equilibrium strategies. ■

Proof of Proposition 3. The only part of this proposition that does not immediately follow from Proposition 2 is the comparative statics of γ with respect to $\tilde{\alpha}_0$. Due to the uniform distribution of agents' tastes, the conversion rate is given by $\gamma = \frac{\alpha_s}{\tilde{\alpha}_0}$. To derive this part of the proposition, perform the replacement $\alpha_s = \gamma\tilde{\alpha}_0$ in Condition (8). Rearranging the resulting condition yields the following implicit characterization of the conversion rate:

$$\frac{\gamma^{\xi+2}}{\xi^2+3\xi+2} + \frac{\gamma\xi}{\xi+1} + \frac{\gamma r}{\mu} - \frac{\xi}{2\xi+4} - \frac{r\phi(\mu+r)+r}{\tilde{\alpha}_0\beta\mu} = 0.$$

Implicit differentiation of this characterization of γ gives that :

$$\frac{\partial\gamma}{\partial\tilde{\alpha}_0} = -\frac{(\xi+1)r(\phi(\mu+r)+1)}{\tilde{\alpha}_0^2\beta\left(\mu\gamma^{\xi+1}+\xi(\mu+r)+r\right)} < 0.$$

■

Proof of Proposition 4. Note that $\alpha_s(\phi)$ is a differentiable function. The platform therefore maximizes a differentiable profit function over a closed interval, an hence there is an optimal fee that is given by a first-order condition.

The derivative of the firm's profit with respect to ϕ is

$$\pi'(\phi) = \frac{\tilde{\alpha}_0}{\alpha_s(\phi)} - 1 - \phi\frac{\tilde{\alpha}_0\alpha_s'(\phi)}{\alpha_s(\phi)^2}. \tag{A.6}$$

Evaluating (A.6) at $\overline{\phi}$ yields

$$\pi'(\overline{\phi}) = -\frac{\overline{\phi}\alpha_s'(\overline{\phi})}{\tilde{\alpha}_0} < 0,$$

where the inequality follows from the fact that $\alpha_s(\phi)$ is increasing in ϕ. It follows that $\phi \geq \overline{\phi}$ are strictly suboptimal. On the other hand, evaluating (A.6) at $\phi = 0$ yields

$$\pi'(0) = \frac{\tilde{\alpha}_0}{\alpha_s(0)} - 1 > 0,$$

where the inequality is due to Assumption (10). Thus, the optimal price is interior and is given by equating (A.6) to zero.

Next, we show that the firm's profit is concave in ϕ. Differentiating (A.6) yields

$$\pi''(\phi) = \frac{2\tilde{\alpha}_0\phi\alpha_s'(\phi)^2 - \tilde{\alpha}_0\alpha_s(\phi)\left(\phi\alpha_s''(\phi) + 2\alpha_s'(\phi)\right)}{\alpha_s(\phi)^3}.$$

By implicit derivation of (8) it follows that

$$\alpha_s'(\phi) = \frac{r(\mu + r)}{\beta\mu\left(\frac{1}{\xi+1}\left(\xi + \left(\frac{\alpha_s(\phi)}{\tilde{\alpha}_0}\right)^{\xi+1}\right) + \frac{r}{\mu}\right)}, \tag{A.7}$$

which, in turn, implies that $\alpha_s''(\phi) < 0$. It follows that

$$\pi''(\phi) \leq \frac{2\tilde{\alpha}_0\alpha_s'(\phi)\left(\phi\alpha_s'(\phi) - \alpha_s(\phi)\right)}{\alpha_s(\phi)^3}.$$

As $\alpha_s'(\cdot) > 0$, the sign of the RHS is equal to the sign of

$$z = \phi\alpha_s'(\phi) - \alpha_s(\phi).$$

Equation (8) implies that

$$\phi = \frac{\beta\left(\alpha_s + \frac{\mu\tilde{\alpha}_0^{-\xi-1}\alpha_s^{\xi+2}}{(\xi^2+3\xi+2)r} - \frac{\tilde{\alpha}_0\mu\xi}{2\xi r+4r} + \frac{\alpha_s\mu\xi}{\xi r+r}\right) - 1}{\mu + r}. \tag{A.8}$$

From this equality and (A.7) it follows that z can be written as

$$z = -\frac{(\xi+1)\left(\tilde{\alpha}_0^{\xi+1}(\tilde{\alpha}_0\beta\mu\xi + 2(\xi+2)r) + 2\beta\mu\alpha_s^{\xi+2}\right)}{2\beta(\xi+2)\left(\mu\alpha_s^{\xi+1} + \tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)\right)} < 0.$$

Consequently $\alpha_s''(\phi) < 0$, and the platform's profit function is concave in ϕ.

To conclude the proof, note that since the platform's problem is concave and has an interior solution, there is a unique optimal price that is given by equating the marginal gain of increasing ϕ, (A.6), to zero. Finally, note that since $\gamma(\phi) = \frac{\alpha_s(\phi)}{\tilde{\alpha}_0}$ this solution is given by (12). ■

Proof of Proposition 5. The platform's profits (11) are decreasing in the conversion rate. By Proposition 3, holding ϕ fixed, we know that the conversion rate is decreasing in both μ and $\tilde{\alpha}_0$. Hence, from a standard revealed preference argument, improvements in the speed of search (an increase in μ) increase the platform's profit, whereas improvements in the quality of information about the fit of a match (a decrease in $\tilde{\alpha}_0$) decrease its profit. ■

Proof of Proposition 6. To establish this result, we consider separately improvements in μ and in $\tilde{\alpha}_0$.

Faster searching— By Proposition 4, the optimal fee is given by the solution to the first-order condition

$$-\tilde{\alpha}_0\phi\alpha_s^{(1,0,0)}(\phi,\mu,\tilde{\alpha}_0) - \alpha_s(\phi,\mu,\tilde{\alpha}_0)^2 + \tilde{\alpha}_0\alpha_s(\phi,\mu,\tilde{\alpha}_0) = 0, \tag{A.9}$$

where $\alpha_s(\phi,\mu,\tilde{\alpha}_0)$ denotes the solution to (8) as a function of ϕ, μ, and $\tilde{\alpha}_0$. To establish the first part of the proposition, we show that the derivative of the left-hand side of this first-order condition with respect to μ is negative when agents are sufficiently patient.

The derivative of the LHS of the first-order condition is

$$(\tilde{\alpha}_0 - 2\alpha_s(\phi,\mu,\tilde{\alpha}_0))\alpha_s^{(0,1,0)}(\phi,\mu,\tilde{\alpha}_0) - \tilde{\alpha}_0\phi\alpha_s^{(1,1,0)}(\phi,\mu,\tilde{\alpha}_0). \tag{A.10}$$

By implicit differentiation, it follows that

$$\alpha_s^{(0,1,0)}(\phi,\mu,\tilde{\alpha}_0) = -\frac{(\xi+1)r\tilde{\alpha}_0^{\xi+1}(-\beta\alpha_s + r\phi + 1)}{\beta\mu\left(\mu\alpha_s^{\xi+1} + \tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)\right)}$$

and that

$$\alpha_s^{(1,1,0)}(\phi,\mu,\tilde{\alpha}_0) = \frac{(\xi+1)r^2\tilde{\alpha}_0^{\xi+1}\left(\beta\mu\left(\tilde{\alpha}_0^{\xi+1} - \alpha_s^{\xi+1}\right)\left(\mu\alpha_s^{\xi+1} + \tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)\right) + \mu(\xi+1)^2\tilde{\alpha}_0^{\xi+1}\alpha_s^{\xi}(\mu+r)(-\alpha_s\beta + r\phi + 1)\right)}{\beta\mu\left(\mu\alpha_s^{\xi+1} + \tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)\right)^2\left(\beta\mu\alpha_s^{\xi+1} + \beta\tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)\right)}.$$

Plugging these derivatives into (A.10) and using the change of variable $\alpha_s = \gamma\tilde{\alpha}_0$ yields that (A.10) is equivalent to

$$\frac{(\xi+1)r\tilde{\alpha}_0^{\xi+2}}{\beta\mu\left(\mu\alpha_s^{\xi+1} + \tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)\right)^2} \times \left(\frac{(\tilde{\alpha}_0 - 2\alpha_s)(\alpha_s\beta - r\phi - 1)\left(\mu\alpha_s^{\xi+1} + \tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)\right)}{\tilde{\alpha}_0} - \frac{r\phi\left(\beta\mu\left(\tilde{\alpha}_0^{\xi+1} - \alpha_s^{\xi+1}\right)\left(\mu\alpha_s^{\xi+1} + \tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)\right) + \mu(\xi+1)^2\tilde{\alpha}_0^{\xi+1}\alpha_s^{\xi}(\mu+r)(-\alpha_s\beta + r\phi + 1)\right)}{\beta\mu\alpha_s^{\xi+1} + \beta\tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)}\right). \tag{A.11}$$

The fraction on the first line is clearly positive. Thus, to establish this part of the proposition, we can normalize $\lambda = 1$, use the change of variable $\alpha_s = \gamma\tilde{\alpha}_0$, and show that the term inside the large parentheses is negative when r is small. That term can be written as

$$(\tilde{\alpha}_0 - 2\tilde{\alpha}_0\gamma)\left(\mu\gamma^{r+1} + r(\mu+r)+r\right)(\tilde{\alpha}_0\beta\gamma - r\phi - 1) + \mu r\phi\left(\tilde{\alpha}_0\left(\gamma^{r+1} - 1\right) - \frac{(r+1)^2\gamma^r(\mu+r)(-\tilde{\alpha}_0\beta\gamma + r\phi + 1)}{\beta\mu\gamma^{r+1} + \beta r(\mu+r+1)}\right). \tag{A.12}$$

Plugging (A.8) and (A.7) into the first-order condition (A.9), and evaluating it at $r = 0$ yields

$$\frac{1}{2}(\tilde{\alpha}_0 - 2\alpha_s)\alpha_s.$$

It follows that $\alpha_s \to \frac{1}{2}\tilde{\alpha}_0$ as r converges to zero.

Finally, plugging (A.8) into (A.12) and evaluating it at $r = 0$ and $\lambda = \frac{1}{2}$ yields

$$\frac{1}{32}\tilde{\alpha}_0\mu(\tilde{\alpha}_0\beta - 8) < 0,$$

where the inequality follows from Assumption 2.

Thus, due to the continuity of the model in r, there exists $r_1^\star > 0$, such that if $r < r_1^\star$, then (A.11) is negative. This completes the first part of the proof.

Better quality of information— The optimal fee is determined by the solution of the first-order condition (A.9), which can be written as

$$-\phi\alpha_s^{(1,0,0)}(\phi,\mu,\tilde{\alpha}_0) - \frac{\alpha_s(\phi,\mu,\tilde{\alpha}_0)^2}{\tilde{\alpha}_0} + \alpha_s(\phi,\mu,\tilde{\alpha}_0). \tag{A.9a}$$

Differentiating this first-order condition with respect to $\tilde{\alpha}_0$ yields

$$\frac{\alpha_s(\phi,\mu,\tilde{\alpha}_0)\left(\alpha_s(\phi,\mu,\tilde{\alpha}_0) - 2\tilde{\alpha}_0\alpha_s^{(0,0,1)}(\phi,\mu,\tilde{\alpha}_0)\right)}{\tilde{\alpha}_0^2} + \alpha_s^{(0,0,1)}(\phi,\mu,\tilde{\alpha}_0) - \phi\alpha_s^{(1,0,1)}(\phi,\mu,\tilde{\alpha}_0). \tag{A.13}$$

To show that this derivative is positive, we follow the same steps as in the first part of the proof: we use implicit differentiation of (8) to calculate the derivatives of α_s, use (A.8) to replace ϕ, normalize $\lambda = 1$, use the change of variable $\alpha_s = \gamma\tilde{\alpha}_0$, and evaluate the resulting condition at $r = 0$ and $\lambda = \frac{1}{2}$.

By implicit differentiation of (8) it follows that

$$\alpha_s^{(0,0,1)}(\phi,\mu,\tilde{\alpha}_0) = \frac{\mu(\xi+1)\left(2\alpha_s(\phi,\mu,\tilde{\alpha}_0)^{\xi+2} + \xi\tilde{\alpha}_0^{\xi+2}\right)}{2\tilde{\alpha}_0(\xi+2)\left(\mu\alpha_s(\phi,\mu,\tilde{\alpha}_0)^{\xi+1} + \tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)\right)}$$

and that

$$\alpha_s^{(1,0,1)}(\phi,\mu,\tilde{\alpha}_0) = \frac{\mu(\xi+1)^2 r\tilde{\alpha}_0^{\xi}(\mu+r)\left(\alpha_s(\phi,\mu,\tilde{\alpha}_0) - \tilde{\alpha}_0\alpha_s^{(0,0,1)}(\phi,\mu,\tilde{\alpha}_0)\right)\alpha_s(\phi,\mu,\tilde{\alpha}_0)^{\xi}}{\beta\left(\mu\alpha_s(\phi,\mu,\tilde{\alpha}_0)^{\xi+1} + \tilde{\alpha}_0^{\xi+1}(\xi(\mu+r)+r)\right)^2}.$$

Plugging these two expressions into (A.9a), using the change of variable $\alpha_s = \gamma\tilde{\alpha}_0$, normalizing $\lambda = 1$, and rearranging yields

$$\frac{\mu(r+1)\left(r\tilde{\alpha_0}^r + 2\gamma^2(\tilde{\alpha}_0\gamma)^r\right)}{2(r+2)\left(r\tilde{\alpha_0}^r(\mu+r+1)+\gamma\mu(\tilde{\alpha}_0\gamma)^r\right)} + \gamma\left(\gamma - \frac{\mu(r+1)\left(r\tilde{\alpha_0}^r + 2\gamma^2(\tilde{\alpha}_0\gamma)^r\right)}{(r+2)\left(r\tilde{\alpha_0}^r(\mu+r+1)+\gamma\mu(\tilde{\alpha}_0\gamma)^r\right)}\right) + \frac{\mu r(r+1)^2\phi\tilde{\alpha_0}^{r-1}(\mu+r)(\tilde{\alpha}_0\gamma)^r\left(-2\gamma r(r+2)\tilde{\alpha_0}^r(\mu+r+1)+\mu r(r+1)\tilde{\alpha_0}^r - 2\gamma^2\mu(\tilde{\alpha}_0\gamma)^r\right)}{2\beta(r+2)\left(r\tilde{\alpha_0}^r(\mu+r+1)+\gamma\mu(\tilde{\alpha}_0\gamma)^r\right)^3}. \tag{A.14}$$

Finally, plugging (A.8) into (A.14) and evaluating at $r = 0$ and $\gamma = \frac{1}{2}$ yields $\frac{1}{4} > 0$. From continuity, there exists $r_2^\star > 0$ such that if $r < r_2^\star$ then the marginal value of increasing ϕ increases with $\tilde{\alpha}_0$. Thus, an improvement in the quality of information about the fit of a match – i.e., a decrease in $\tilde{\alpha}_0$ – leads to a reduction in the optimal fee.

The proposition is established for $r^\star = \min\{r_1^\star, r_2^\star\}$. This completes the second part of the proof. ■

Proof of Proposition 7. If ϕ^* is an interior optimum of the platform's profit, then it must be a solution of the first-order condition associated with (13). This first-order condition is given by

$$\frac{\tilde{\alpha}_0}{\alpha_s(\phi^*, \tilde{\alpha}_0, \mu)} - \frac{\tilde{\alpha}_0\left(\frac{A}{\mu} + \phi^*\right)\alpha_s^{(1,0,0)}(\phi^*, \tilde{\alpha}_0, \mu)}{(\alpha_s(\phi^*, \tilde{\alpha}_0, \mu))^2} - 1 = 0.$$

By Proposition 2 the derivative of α^s with respect to ϕ is positive. Hence, an increase in A leads to the LHS of the first-order condition being negative. Since ϕ^* is a maximum, the profit function is locally concave, and hence, to satisfy the first-order condition, ϕ must be decreased. ■