# Risky Insurance: Life-cycle Insurance Portfolio Choice with Incomplete Markets

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Tel Aviv University 31 May 2021

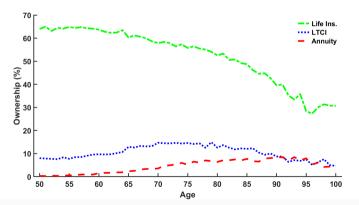
Note: Preliminary Results (active work in progress)

## Outline

- Introduction
- Original Survey
- Structural Model

## **Background**

- Despite significant financial risks and spending needs late in life, most people choose not to purchase insurance
  - Annuity and long-term care insurance (LTCI) are typically owned by less than 10 percent of older Americans



## **Background: Literature**

Understanding consumer insurance demand has been the subject of a large body of research:

- Annuities: Yaari (1965); Brown (2001); Davidoff, Brown, Diamond (2005); Inkmann, Lopes, Michaelides (2011); Peijnenburg, Nijman, Werker (2016); etc.
- Life Insurance: Bernheim (1991); Chambers, Schlagenhauf, Young (2004); Inkmann, Michaelides (2012); Hong Rios-Rull (2012); etc.
- LTCI: Brown, Finkelstein (2008); Lockwood (2012); Ameriks, Briggs, Caplin, Shapiro, Tonetti (2018); Mommaerts (2016); etc.
- Insurance Portfolio: Hubener, Maurer, and Rogalla (2013); Koijen, Van Nieuwerburgh, Yogo (2016)

Many "puzzles." Generally find that consumer insurance holdings are suboptimal and suboptimal holdings impose large welfare costs

## **Background: Economic Forces that Influence Insurance Demand**

- Fact: Slow decrease in wealth at older ages (for those with wealth)
- Motive: Uncertain death timing
  - Don't want to run out of wealth if live longer than expected (consumption and bequests)
- Motive: Long-term-care Risks
  - Want to be able to fund high-quality care if needed
  - ullet  $\sim 1/3$  of 65 year olds will enter nursing home; 1/6 will need help with ADLs for  $\geq 3$  years
  - ullet Average cost at nursing home  $\sim$  \$100K per year
- Environment (in U.S.):
  - · Medicaid is means tested, limits private demand for insurance for lower wealth individuals
  - Most Americans already annuitized via Social Security (less replacement income for high-wealth individuals)
  - Insurance market with high loads, complicated contract structures, and quantity restrictions

Reasons to expect purchase of private insurance and reasons to expect no purchase. Function of preferences, states, and environment.

## **Existing Research Approaches**

Typically, studies that use structural models take one of two approaches:

- 1. Very Incomplete Markets with Stylized Products: Introduce a one-time option to purchase a single state contingent asset (i.e., insurance) and compare demand to actual insurance holdings
- 2. Complete Markets with Stylized Products: Assume markets are complete and calculate life-cycle profiles of demand for portfolio of state contingent assets

#### Our approach:

- Portfolio choice of multiple insurance products
- Model key features of insurance products to make them better match real-world products

## Quote About Real-World Insurance: Nonpayment Risk

"There is really no mystery about [why people don't buy] long term health insurance. The reason it seems to defy reason is because your assumptions are flawed. .... My father had emphysema and the insurance company fought tooth and nail to prevent paying for years. ... And of course, only paying 50 to 80% of what they owed him. Not that they were stupid, but that they were greedy. If we believed they would pay what they should when they should, we'd buy. It's not what the odds are on that lottery ticket, it is what are the odds you'll get paid if you win."

- Anonymous email from reader of my previous paper
- One key dimension of real-world products may be nonpayment risk
- We measure and model (perceived) nonpayment risk. No measure from administrative data, so design a survey
- In model will simulate 2 ways: rational expectations or zero nonpayment risk in DGP

## Risky Insurance: Nonpayment Risk Interpretations

- Difficult state verification of qualifying event
  - LTCI: LTC need difficult to verify
  - Annuity/Life Insurance: death easy to verify
- Contractual complication/paperwork as barrier to receiving payments
  - Interacts with cognitive ability
  - LTCI: reimbursement model for qualifying expenses
  - Real-time paperwork risk, but also historical paperwork risk (e.g., omitted smoking history)
- Financial health of insurer
  - Lack of trust or knowledge of government insurance of insurers
  - When used, government insurance may lead to haircuts???

## This Paper: Question and Research Design

How do properties of the available insurance products affect the demand for insurance and the welfare gains from buying insurance against late-in-life risks?

• In this paper we study portfolio choice of annuities, life insurance, LTCI, and liquid wealth (a stock-bond mutual fund)

### Approach:

- New data:
  - Measured beliefs about nonpayment risk
- New model:
  - Life-cycle model of joint demand for insurance with exogenously incomplete markets
  - We model products as they are in the market and as they are perceived by consumers
  - Buy/Sell price wedges, nonpayment risk, quantity limits (age, no short-selling)

## This Paper: Key Results

- 1. Perceived nonpayment risk is large in annuity, life insurance, and LTCI markets
- 2. Perceived nonpayment risk is predictive of actual insurance holdings
- 3. Nonpayment risk and buy/sell price wedges have large affect on insurance ownership
- 4. After accounting for nonpayment risk and other sources of incomplete markets, welfare costs associated with deviations from optimal insurance portfolios are much smaller
- Incomplete markets and beliefs about nonpayment risks are important determinants of insurance holdings
- Measuring and modeling actual product features is important when studying consumer choices and welfare

#### **Overview**

- Survey: US representative sample of 1,040 people linked to standard survey data by UAS
  - Survey Description
  - Overview of Results
  - Credibility

- Model
  - Model Description
  - Model Solution
  - Model Predicted Demand
  - Welfare Analysis

## **Survey Overview**

- Insurance product ownership
  - For each type of insurance each respondent has a product in mind that promises to pay a certain quantity per qualifying event
  - Either one they own or the best one they think they could buy if they don't own one
- Nonpayment risk measures
  - Adapted from Luttmer-Samwick (2018)
  - Probability of full default on contract value
  - Distribution of annual payment conditional on qualifying event
  - Repeat for different aggregate economic state
- Certainty equivalent measure
- Other supplementary measures

## Measuring Nonpayment Risk - Full Default

Suppose that you own an annuity that promises to pay \$5,000 each year for the rest of your life. Suppose further that you never trade this annuity for cash and hold the contract until the end of your life.

We are now interested in the percent chance that the annuity becomes worthless due to no fault of your own at any point before the end of your life. This means that the annuity permanently stops making payments. This might occur if the insurance company goes out of business, they claim you violated a clause in the contract, or they ruled the policy void for some other reason.

What is the percent chance this occurs?

•		10
Or type in:		

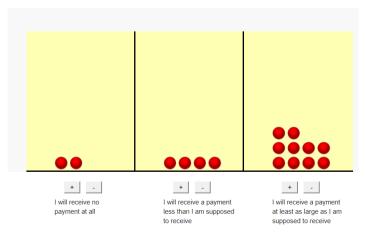
You think that there is a 0% chance that the annuity becomes worthless at some point before the end of your life

## Measuring Nonpayment Risk - Annual Payout Default (1/2)

Suppose that you own an annuity that promises to pay \$24,000 each year for the rest of your life. We would now like to focus on what might happen just during the next calendar year.

You have been given 20 balls to put in the following bins. Each bin describes a scenario that involves the annuity payment that you are supposed to receive next year. The more likely you think a bin is, the more balls you should put in that bin.

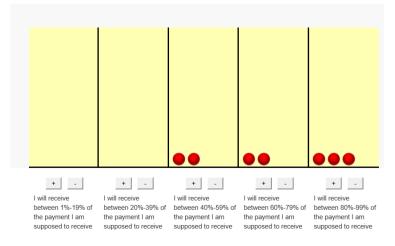
What do you think will happen to the annuity payment next year?



## Measuring Nonpayment Risk - Annual Payout Default (2/2)

You put 8 ball(s) in the bin marked "I will receive a payment less than I am supposed to receive." Please distribute those balls in the following bins. The more likely you think a bin is, the more balls you should put in that bin.

If you do receive a payment that is less than you are supposed to receive, how much do you think you would get?



## Measuring Nonpayment Risk - Aggregate Risk Scenario

Suppose that you own an annuity that promises to pay \$10,000 each year for the rest of your life.
Suppose that the stock market <b>decreases by 20%</b> next year.
We are now interested in the percent chance that during this next year the annuity becomes worthless due to no fault of your own. This means that the annuity permanently stops making payments. This might occur if the insurance company goes out of business, they claim you violated a clause in the contract, or they ruled the policy void for some other reason.
What is the percent chance this occurs?
0100
Or type in: 25
You think that there is a <b>25% chance</b> that the annuity becomes worthless next year.

## **Measuring Certainty Equivalence**

The way you put balls into various bins shows that you expect to receive about 83% of your annuity payment next year. It also shows that you could receive more or less than 83% of the promised payment.

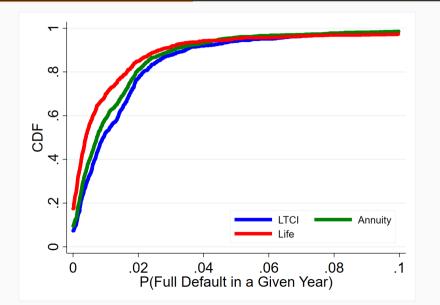
Let's call this distribution of possible payments, as described by you using the bins and balls, your "uncertain payments." So, your uncertain payments are whatever payments you think you might receive next year.

We are now interested in how you value having a contract with no uncertainty. Imagine a contract that is guaranteed to pay 62% of your annuity payment with no risk of the insurance company not paying out as promised. This is like having all 20 balls on this certain percentage. This contract is unbreakable and cannot be changed by anybody. This contract has no risk, but is guaranteed to pay less than the full promised amount of your original contract.

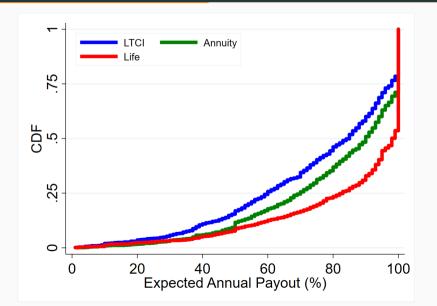
#### Would you rather have:

- O Guaranteed payment equal to 62% of the annuity payment you are supposed to receive
- Uncertain payments around an expectation of 83% of the annuity payment you are supposed to receive

## Distribution of Full Default Probability



## Distribution of Expected Value of Annual Payments



## Average Expected Payouts, Certainty Equivalents, and Implied Risk Premia

	Population Mean Expected Value (1)	Population Mean Certainty Equivalent (2)	Risk Premia (3)
Life	87.16	81.43	5.72
Annuity	81.51	73.79	7.72
LTCI	76.17	72.90	3.27

## Annuity and Life Expected Payouts Vary with Aggregate Risk, but LTCI Payouts Do Not



## Nonpayment Risk Measures Predict Insurance Ownership

Annuity Payment Exp. Value	Own Annuity (1)	Own Life (2)	Own LTCI	Own Annuity (4) -0.0005	Own Life (5)	Own LTCI (6)
Annuity Payment Exp. Value	(0.212)			(0.373)		
Annuity Full Def. Prob	-0.0021*** (0.000)			-0.0020*** (0.000)		
Annuity Payment SD	-0.0043** (0.002)			-0.0029*** (0.000)		
Life Payment Exp. Value		0.0046*** (0.001)			0.0045** (0.003)	
Life Full Default Prob		-0.0015 (0.129)			-0.0013 (0.142)	
Life Payment SD		-0.0006 (0.686)			-0.0002 (0.896)	
LTCI Payment Exp. Value			0.0007			0.0006 (0.181)
LTCI Full Default Prob			-0.0023*** (0.000)			-0.0022*** (0.000)
LTCI Payment SD			-0.0009 (0.195)			-0.0010 (0.136)
Trust				0.0188 (0.091)	-0.0063 (0.758)	0.0162 (0.241)
Cognitive Score				-0.0007 (0.747)	-0.0033 (0.271)	0.0004 (0.852)
Financial Literacy Score				-0.0112 (0.459)	-0.0662° (0.019)	-0.0083 (0.609)
Numeracy Score				-0.0079 (0.560)	0.0207 (0.319)	-0.0240 (0.101)
Experienced Fraud				0.0298 (0.549)	0.0545 (0.375)	-0.0031 (0.941)
Risk Aversion				-0.0072 (0.252)	-0.0160 (0.072)	-0.0015 (0.776)
Propensity to Plan				0.0137 (0.243)	-0.0013 (0.947)	0.0016 (0.888)
Early Stock Returns				0.1474 (0.757)	-0.5123 (0.441)	-0.7936 (0.122)
N	1055	1046	1040	1055	1046	1040
R <sup>2</sup>	0.170	0.132	0.129	0.268	0.218	0.179
Demographic Controls	Yes	Yes	Yes	Yes	Yes	Yes
p-values in parentheses						

p-values in parentheses \* p < 0.05, \*\* p < 0.01, \*\*\* p < 0.001

## **Extrapolation of Regression Suggests Nonpayment Risk Limits Market Size**

#### Counterfactual Predictions of Probit Regressions Under Various Specifications of Risk Perception

			Maginal Effects, 1 Std. Dev. Increase		
	P(Own)	P(Own—No Risk)	Exp. Value	Full Default	Std. Dev.
	<u>(1)</u>	<u>(2)</u>	<u>(3)</u>	<u>(4)</u>	<u>(5)</u>
Annuity	.12	.24	010	017	030
Life	.57	.66	.111	042	010
LTCI	.10	.23	.039	046	003

#### **Model Overview**

Life-cycle, heterogeneous agent model where agents choose how much to:

- Consume  $(C_t)$
- Save in one-period bonds  $(B_{t+1})$
- Invest in insurance products  $(W_t^k)$

#### subject to

- Liquid wealth  $(B_t)$
- Insurance holdings  $(D_t^k)$
- Income  $(Y_t)$
- Age (t)
- Health status  $(s_t)$
- Sex (f)
- Aggregate economic state  $(G_t)$
- Government consumption floor  $(Tr_t^s)$

## **Demographics**

- Age:
  - t = 45, ..., 100
- Sex:
  - f = 1 if female
  - f = 0 if male
- Health:
  - $s = \{0,1\}$  if  $\{good,bad\}$
  - s = 2 if LTC
  - s = 3 if dead
  - Transition matrix  $\Gamma_{t,f}$ .
- Health cost shocks:
  - Transitory random variable  $HC_{t,s,f}$
  - Significantly larger in LTC state

#### **Preferences**

• Households have time-separable, health-state dependent non-homothetic preferences defined over a consumption good  $C_t$  and a warm-glow bequest motive. Flow utility  $\nu_s$  is:

$$\nu_s(C_t) = \frac{\theta_s}{1 - \sigma} (C_t + \kappa_s)^{1 - 1/\sigma}$$

- Specification from Ameriks et. al. (JPE 2020)
- Key functional-form innovation is nonhomotheticity ( $\kappa_2 \neq 0$ ) in long-term-care health state
- With state-dependent utility, insurance demand is nuanced (e.g., risk-averse agent might not buy actuarially fair insurance)

## Aggregate State, Bonds, and Income (to be improved...work in progress)

• Aggregate State:  $G \in \{0,1\}$  evolves according to Markov matrix

$$G' \sim \Lambda_{|G}$$

$$\Lambda_{|G} = \begin{bmatrix} .5 & .5 \\ .8 & .2 \end{bmatrix}$$

• Bonds: Agents save in liquid asset  $(B_t)$  with return rate  $(r_G)$  that varies with G

$$\begin{cases} r_0 = .06 & if \quad G = 0 \text{ (expansion)} \\ r_1 = .02 & if \quad G = 1 \text{ (recession)} \end{cases}$$

- Income:  $Y_t$  includes labor income, social security, and DB pensions
  - Income path over life cycle is deterministic, given by one of five income quintiles paths

#### **Insurance Products: Introduction**

- Three insurance products (indexed by k):
  - Life Annuities (ANN)
  - Life Insurance Policies (LI)
  - Long-Term Care Insurance (LTCI).
- $\bar{D}^k$  vector defines the payout for asset k in state s:

$$\begin{split} \bar{\mathcal{D}}^{Ann} &= [1,1,1,0] \\ \bar{\mathcal{D}}^{LI} &= [0,0,0,1] \\ \bar{\mathcal{D}}^{LTC} &= [0,0,1,0]. \end{split}$$

## **Insurance Products: Pricing and Dividends**

- Base price for 1 unit of insurance  $p_{t_0,s_0,f,G_0}^k$
- Base price is actuarial fair price from risk neutral insurance company
  - $\bar{D}^k$  dividend
  - r interest rate
  - $\Gamma_{t,f}$  stochastic process for health state
- Modifiers on base price to obtain market price to buy and sell:  $\lambda_+^k, \lambda_-^k$  next slide
- LI, ANN: insurance units paid for lump-sum; LTCI paid for with annual premium
- $D_t^k$  is quantity insurance k owned by agent. Then the value of an agent's contract is:

$$A_t^k = p_{t_0,s_0,f,G_0}^k D_t^k$$

#### **Insurance Products: Transactions**

- $W_t^k$  denotes net transactions in insurance product k
- $\lambda_{+}^{k}(\lambda_{-}^{k})$  is the % transaction cost to buying (selling) product k
- Lump-sum cost is:

$$W_t^k p_{t,s,f,G}^k \left(1 - \lambda_-^k \mathbb{I}_{W_t^k < 0} + \lambda_+^k \mathbb{I}_{W_t^k > 0}\right)$$

• No new purchases after age  $t^{max,k}$ :  $W_t^k \leq 0$  if  $t > t^{max,k}$ 

## **Intertemporal Budget Constraints**

• Insurance product k exhibits annual payout  $(q^{k,D})$  and full default  $(q^{k,FD})$  probabilities

$$D_{t+1}^{k} = \begin{cases} D_{t}^{k} + W_{t}^{k} & \text{with prob } \mathbf{1} - \mathbf{q}^{k,FD} \\ 0 & \text{with prob } \mathbf{q}^{k,FD} \end{cases}$$

$$\hat{D}_{t+1}^{k} = \begin{cases} \bar{D}^{k} & \text{with prob } \mathbf{1} - \mathbf{q}^{k,D} \\ 0 & \text{with prob } \mathbf{q}^{k,D} \end{cases}$$

• Bonds:

$$\begin{split} B_{t+1} = & (1+r) \left( B_t - C_t - \sum_{k \in ANN, LIFE} \left[ W_t^k p_{t,s,f,G}^k \left( 1 - \lambda_-^k \mathbb{I}_{W_t^k < 0} + \lambda_+^k \mathbb{I}_{W_t^k > 0} \right) \right] \right) \\ & + Y_{t+1} (1 - \Upsilon_t^{LTCI}) - HC_{t+1,s,f} + v_{s_{t+1}}' \sum_k D_{t+1}^k \hat{D}_{t+1}^k + Tr_{t+1}^s \end{split}$$

• No borrowing  $(B_{t+1} \geq 0)$  and no negative insurance holdings  $(D_{t+1}^k \geq 0)$ 

## **Summary of Incomplete Market Features**

- Nonpayment risk:  $q^{k,FD}q^{k,D}$
- Maximum purchase age
- Price wedges:  $\lambda_+^k, \lambda_-^k$
- Uninsurable medical expense: HC
- Uninsurable aggregate asset-return risk: r

## **Preliminary Calibration**

**Table 1: Baseline Calibration - Insurance products** 

	<u>Annuities</u>	<u>Life</u>	<u>LTCI</u>
Full Default $(q^{k,FD})$	.018	.012	.023
Annual Payout Default $(q^{k,D})$	.195	.128	.238
Price Wedge, buying $(\lambda_+^k)$	.2	.25	.32
Price Wedge, selling $(\lambda_{-}^{k})$	.15	.25	-
Max Purchase age $(t^{max,k})$	70	70	70

- Feed in measured values, as opposed to calibration to match insurance ownership
- $q^{k,FD}$ ,  $q^{k,D}$ : Original survey in this paper (average values)
- $\lambda_{+}^{ANN}, \lambda_{+}^{LTCI}$ : Brown and Finkelstein (JEP 2011)
- $\lambda_+^{LI}$ : Hong and Rios-Rull (AER 2012)
- $\lambda_{-}^{ANN}, \lambda_{-}^{LI}$ : Industry Reports

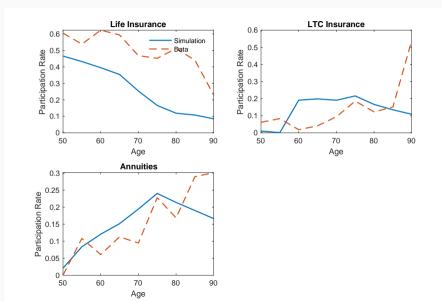
## **Preliminary Calibration**

Table 2: Baseline Calibration - Preferences

Time Preference -  $\beta=.92$ Bequest motive -  $\theta_3=1.09$ LTC motive -  $\theta_2=0.67$  Risk Aversion -  $\sigma = 5.27$ Bequest motive -  $\kappa_3 = 7.83$ LTC motive -  $\kappa_2 = -37.44$ 

- Ameriks et. al. (JPE 2020)
- Strategic Survey Questions + Wealth data (no insurance data)
- ullet  $\beta$  still work in progress

## Insurance Ownership: Model and Data



## **Preliminary Model Predictions**

	No Insurance	Baseline	No Nonpayment Risk	No Price Wedges	No Price Wedges or Nonpayment Risk
A. Insurance Ownership					
Annuity	0	16%	51%	43%	53%
Life	0	39%	40%	52%	48%
LTCI	0	25%	36%	31%	40%
B. Welfare Gains					
Consumption Equivalent	0	0.8%	3.7%	2.9%	7.0%

- Real-world asset features have strong effect on ownership
  - especially for annuities and LTCI
- Welfare costs of "under-insurance" much smaller than complete market analysis suggests

## Empirical Nonpayment Beliefs, but Payments Always Made

	No Insurance	Baseline	No Nonpayment Risk	No Price Wedges	No Price Wedges or Nonpayment Risk
B. Welfare Gains Consumption Equivalent	0	1.9%	3.7%	5.6%	7.0%

- Hold fixed empirical payment beliefs, change payouts in simulation
- Welfare Gains: Rational Expectations vs. Payments Always Made
  - Baseline: 1.9% vs. 0.8% payouts are better than defaults
  - No Price Wedges/Agg Risk: 5.6% vs. 2.9%
- Incorrect beliefs would have large welfare costs: 3.7% vs. 1.9%
  - Even when all payments are made, only 1.9% welfare gain in baseline compared to 3.7% if beliefs correctly reflected zero non-payment risk

#### **Conclusion**

- Incomplete markets and perceived risks are important determinants of insurance holdings, and measuring and modeling actual product features is important when studying consumer choices and welfare
- Perceived nonpayment risk is large in annuity, life insurance, and LTCI markets
- Perceived nonpayment risk is highly predictive of actual insurance holdings
- After accounting for nonpayment risk and other sources of incomplete markets, welfare costs associated with deviations from optimal insurance portfolios are much smaller
- Valuable to study supply and demand of insurance products together, but deeper understanding of one side of the market valuable in and of itself

## Koijen, Van Nieuwerburgh, Yogo (JF 2016): Insurance Portfolio Choice with Complete Markets

- Major advance in literature for studying portfolio choice instead of one asset at a time
- Represented optimal insurance in low-dimensional health and mortality deltas
- Welfare cost of suboptimal insurance holdings order of magnitude larger than underdiversification in stocks
- Differences in our approach and their approach
  - Utility functional form and parameter estimation
  - Incomplete markets
- Difference in findings
  - Prefs: Long-term-care risk is very important to consumers (Ameriks et. al. 2020, 2018)
  - Ownership: When products are not so good, people want to own less of them
  - Welfare: When products are not so good, welfare cost of not owning them is not so large

## Survey Measurement

- Understanding America Study (UAS)
  - Internet panel run by team at USC Dornsife CESR
  - Representative of US population (sampling weights provided )
  - HRS modules (health/labor/income/wealth/etc.) recorded every two years
- Our module (UAS 118)
  - Fielded in May 2018
  - $\approx$  45 questions
  - Average 16 minutes long
  - 1040 usable responses (82% response rate)

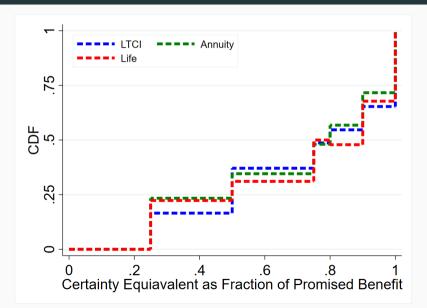
## Measuring Insurance Ownership

- Summary: For each type of insurance each respondent has a product in mind that promises to pay a certain quantity per qualifying event
  - Either one they own or one we describe to them if they don't own one
- For each insurance product, do you own it and if so how much does it promise to pay?
  - Measurement details differ for annuity, LI, and LTCI. Survey publicly available
  - We focus the survey on immediate annuities and whole life insurance
- For survey about nonpayment, we focus on largest policy owned for each type of insurance
- If respondent doesn't own a particular type of insurance, we ask them to imagine they owned the best product they think they could buy that promises to pay \$X per payout
  - X randomized

## Survey Sample is Broadly Comparable to HRS

	HRS (1)	UAS (2)
Male	.47	.51
Age	59.2	61.4
Retired	.28	.36
Education		
High School	.46	.52
Some College	.29	.26
College & Above	.25	.18
Married	.69	.59
Race		
White	.75	.88
Black	.16	.09
Hispanic & other	.09	.04
Health		
Good	.72	.81
Bad	.24	.16
LTC	.04	.03
Income (K \$)	64	130
Wealth (K \$)	280	573
Insurance Ownership		
Annuity	.06	.11
Life Insurance	.61	.56
LTCI	.09	.11
N	10,234	1,040

## Distribution of Certainty Equivalent Measures



## **Insurance Products: Pricing and Dividends**

 Insurance products are priced by a risk neutral insurance company as expected value of payments for a unit of insurance:

$$\rho_{t_0,s_0,f,G_0}^k = \sum_{t=t_0+1}^T \nu_{G_0}' \Lambda^{t-t_0} \left[ \frac{1}{(1+r_0)^{t-t_0}}, \frac{1}{(1+r_1)^{t-t_0}} \right]' \times \nu_{s_0}' \Gamma_{t,f}^{t-t_0} \left[ \bar{\mathcal{D}}^k \right]$$

- $v_{G_0}$ : 2 x 1 vector with one in row  $G_0$ , zero otherwise
- $v_{s_0}$ : 4 x 1 vector with one in row  $s_0$ , zero otherwise
- LI, ANN: insurance units paid for lump-sum; LTCI paid for with annual premium equaling some fraction  $\Upsilon_{\star}^{LTCI}$  of lifetime income
- Let  $D_t^k$  denote the number of units of product k the agent holds. Then the value of an agent's contract can be expressed as:

$$A_t^k(D_t^k) = p_{t_0,s_0,f,G_0}^k D_t^k$$

$$D_t^{LIFE,ANN} \in [0,\infty]$$

$$D_t^{LTCI} \in \{0,H^{LTC}\}$$