Putting the Cycle Back into Business Cycle Analysis

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Introduction

- Equilibrium stochastic dynamic modeling in macro came to age in the 1970, and with it came the need to specify business cycles features in a manner appropriate for the framework
- At that point, post war business cycles seemed to be generally less than 8 years, so business cycles were defined as fluctuations arising at periodicities of less that 8 years
- One step in this process of fact finding was to look at the spectrum of important macro-varaibles at this frequencies, ie, at periodicity below 32 quaters, to see if there was evidence of cycles

Introduction

- What was noticed is that the spectral densities of many macro variables over this range resembled that of AR(1) processes, and especially, there were no obvious peak in the spectrum. However, there was substantial coherence.
- This suggested that business cycle theory *should not be about recurrent cycles*, but should mainly be about *co-movements*
  - Co-movement: joint movement of many variables, arriving at irregular intervals,with little linking one boom to the next.
  - Recurrent Cycles: systematic boom and busts process, where a boom sows the seed of the next recession and so on.
- Hence, much of modern macroeconomics developed theories of business cycles that aren't cyclical , with the aim of explaining data where there is no apparent cycle
- All good!

Introduction

- In this paper we question this focus.
- In particular, we begin by re-examining the data and noting that with 40 more years of data, the business cycle –which using the same metric should be defined as slightly longer, at least up to 40 quarters– is "much more cyclical" than generally recognized
- Such evidence suggests that business cycle theory may need to include mechanisms capable of explaining cyclical behavior, as opposed to mechanisms producing near AR(1) behavior.
- So begin by new look at the data, focusing on frequencies at least up to 40 quarters

Motivating Observations

- If there are important recurrent cyclical forces in the economy then
  - This should show up as a distinct peak in the spectrum of the data
- However, this idea requires stationary data which links to the huge debate about trend-cycle decompositions
- To minimize such difficulties, a good starting point is to look at varaibles such as hours worked, employment rates, unemployment rates, capital utilization, since these are close to stationary.

Decompositions and Sprectrums

- A key issue for identifying business cycle features is the capacity to distinguish trend and cycle.
- If the two components are independent, then the spectrum of a sum is the sum of the spectra.
- So a hump in the cyclical component should be observable in the spectrum if either 1) the trend component becomes relevant only far after the hump, or 2) it is smooth across frequencies, like at AR(1). A problematic case would be if trend compoment becomes importand just as the cyclical component is dying away.

Figure : Non Farm Business Hours

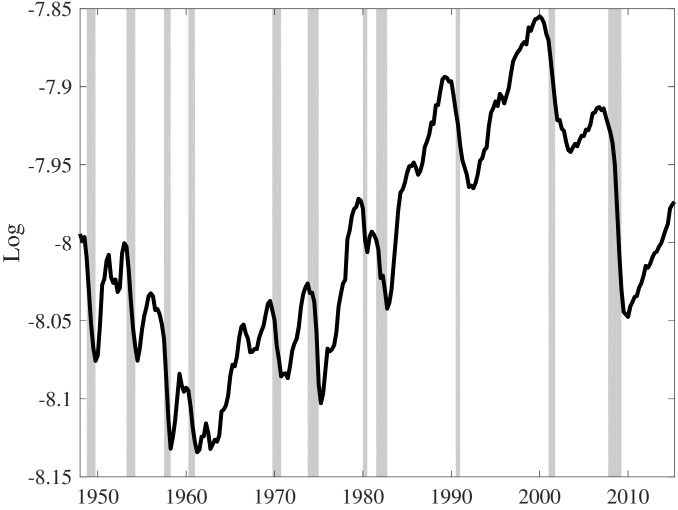


Figure : Spectral Density, Level and Bandpass (2,100) to (2,200), Full Sample - Non Farm Business Hours

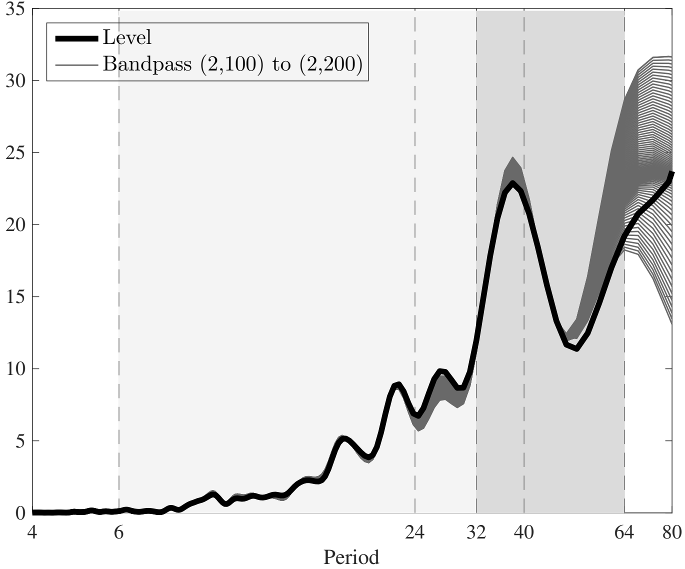


Figure : Spectral Density, Level and Bandpass (2,100) to (2,200), Full Sample - Unemployment Rate

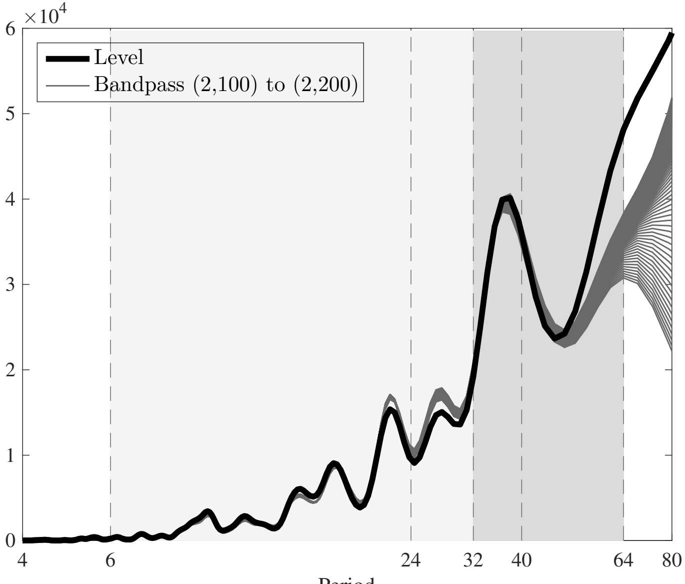


Figure : Spectral Density, Level and Bandpass (2,100) to (2,200), Full Sample - Capacity Utilization Rate

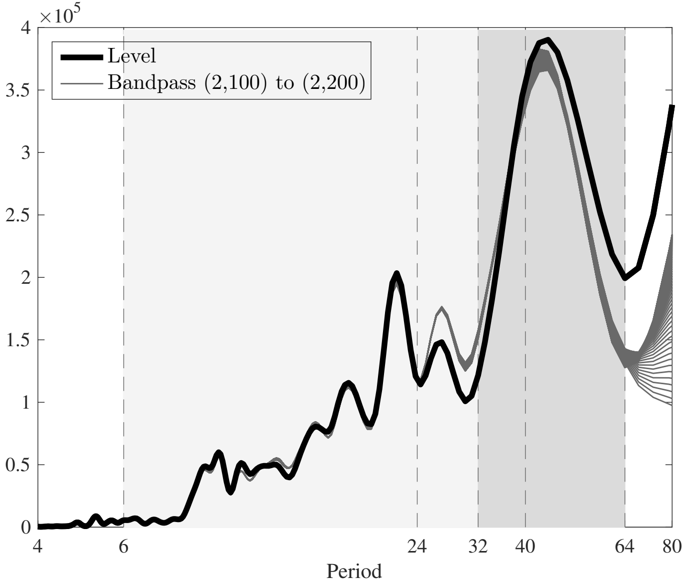


Figure : SW hours spectrum

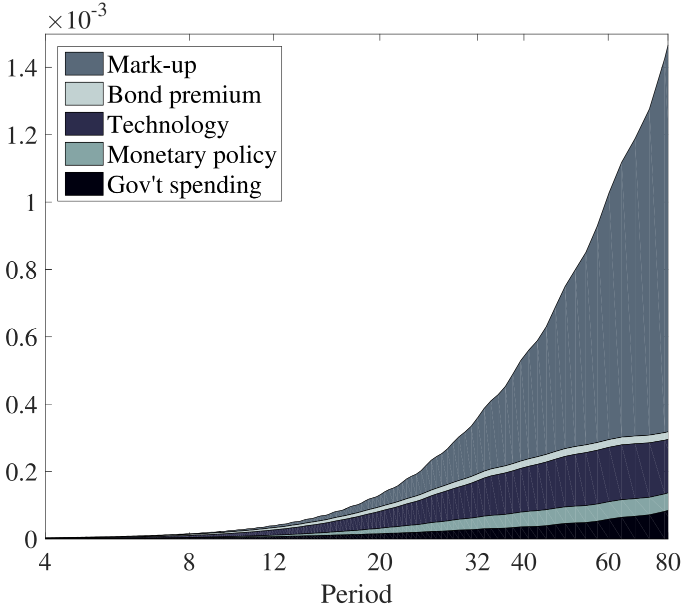


Figure : Spectral Density, Level and Bandpass (2,100) to (2,200), Full Sample - Output

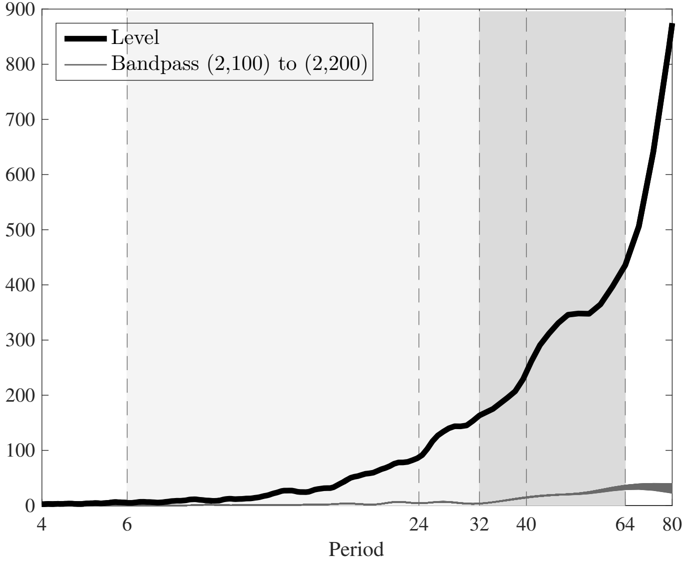
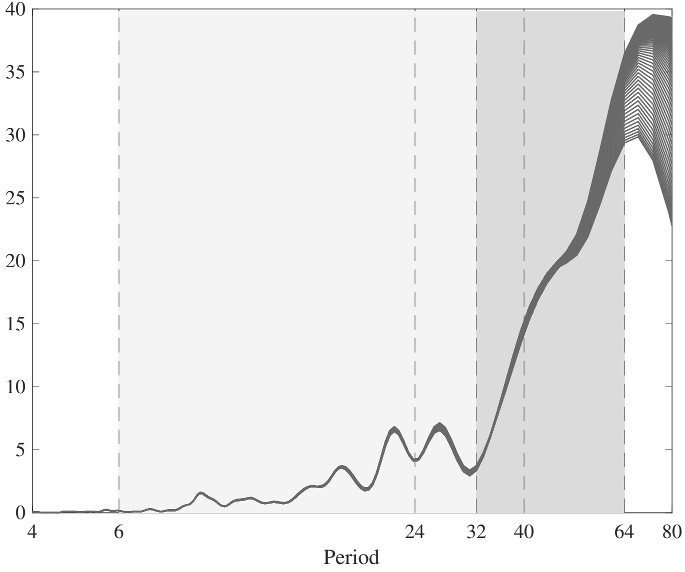


Figure : Spectral Density, Bandpass (2,100) to (2,200), Full Sample - Output



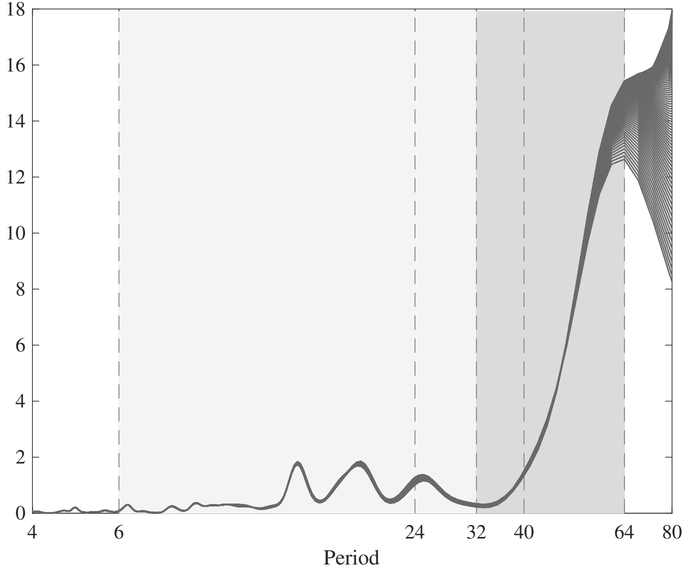
Explaining cyclical behavior

- So when can there be a peak in hours and not one in output?
- Output is approximately given by

$$\log Y_t \approx \log H_t + \log TFP$$

- with the spectrum of a sum being the sum of the two specta if they are un-corrlated
- So this pattern can arise if the spectrum of TFP picks up strongly after around 40 quarters
- Let's look

Figure : Spectral Density, Bandpass (2,100) to (2,200), Full Sample - TFP



Explaining cyclical behavior

- To explain a distinct peak in the spectrum at around 38-40 quarters we may need strong "‘cyclical"mechanisms: where should we look?
- Idea: model with complementarities in the process of accumulation; where the complementarities could be due to financial frictions.
- However, before examining specifics of this type of model, we want to first highlight some general principles and important challenges.

A Framework for explaining recurrent cyclical behavior

- We want to begin by showing how interaction between individual level accumulation decisions and weak complementarities
  - Creates powerful forces capable of explaining recurrent cycles.
  - In particular, it can gernerate endogenous cyclical forces where, when augmented with minor shocks, has the potential to explain BC patterns.
  - However, this type of model also has the potential of creating local instability (well before creating indeterminancy).
  - If such local instability arises, it may create limit cycles, and this envolves new challenges in estimation.
- Approach: give some general principles and then explore issue empirically within an extented NK type model

Agent Interaction

- Goal: show that cyclical behavior arises quite readily as the unique equilibrium outcome when agent interact in dynamic envronment in the presence of weak complemenrities. But this also creates new challenges!
- environment is one which will combine'stable" accumulation behavior on part of indivuduals, with 'stable' complementaities
- Staring point: In the absence of agent interaction, suppose behavior be described by

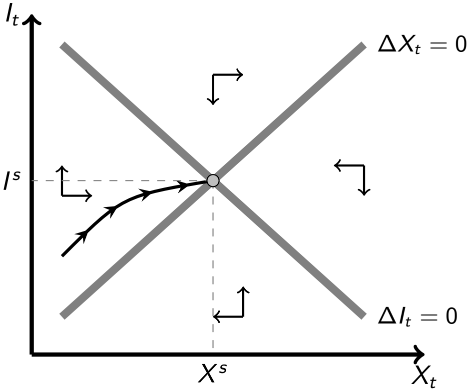
$$I_{it} = \alpha_0 + \alpha_3 E_t I_{it+1} + \alpha_2 I_{it-1} - \alpha_1 X_{it} \quad (+\epsilon_t)$$

- with accumulation satisfying

$$X_{it} = (1-\delta)X_{it} + I_{it}$$

- Restrict attention to situations where α are such to ensure this system is saddle path stable (with non-cyclical convergence).

Figure : Phase diagram in the model without demand complementarities



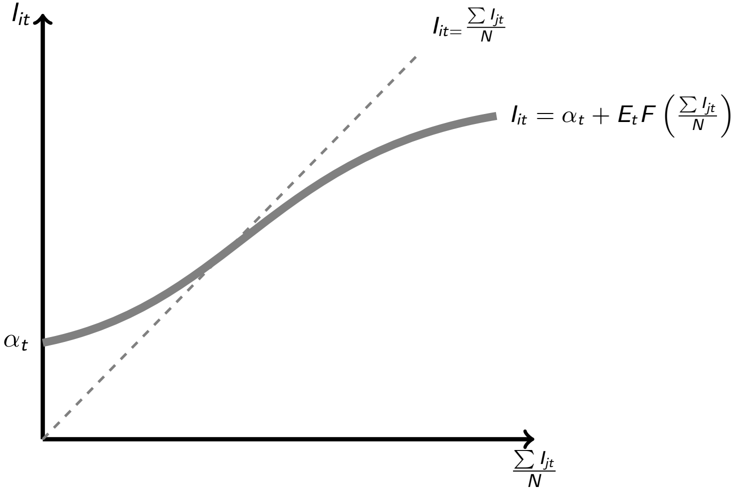
Allowing for interaction

- Now consider allowing for interactions among agents in the form.

$$I_{it} = \alpha_0 + \alpha_3 E_t I_{it+1} + \alpha_2 I_{it-1} - \alpha_1 X_{it} + E_t F(I_t, \psi)$$

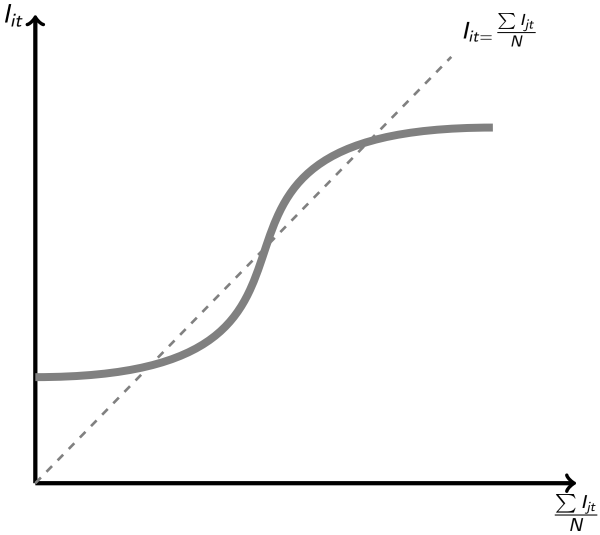
- When $F'(\frac{\sum I_{it}}{N}) > 0$, implies strategic complementarities; if $F'(\frac{\sum I_{it}}{N}) < 0$, implies strategic substitutes
- $F' < 1$ guarantees no multiple static equilibrium. We will maintain this assumption.

Figure : Best response rule for a given history



$$\alpha_t = \alpha_0 - \alpha_1 X_{it} + \alpha_2 I_{it-1} = \alpha_0 - \alpha_1 \sum_0^{\infty} (1-\delta)^{\tau} I_{jt-1-\tau} + \alpha_2 I_{it-1}$$

Figure : Best response rule for a given history - Multiple Equilibria



Complementarities and limit cycles

- We now want to examine how dynamics change with complementarities. Think of parameterizing $F(I, \psi)$, and having changes in ψ capture changes in $F'(I^{ss})$.
- As $F'(I^{ss})$ grows, dynamics of system will change. Question: how do they change.
- Focus on systems with only one steady state.

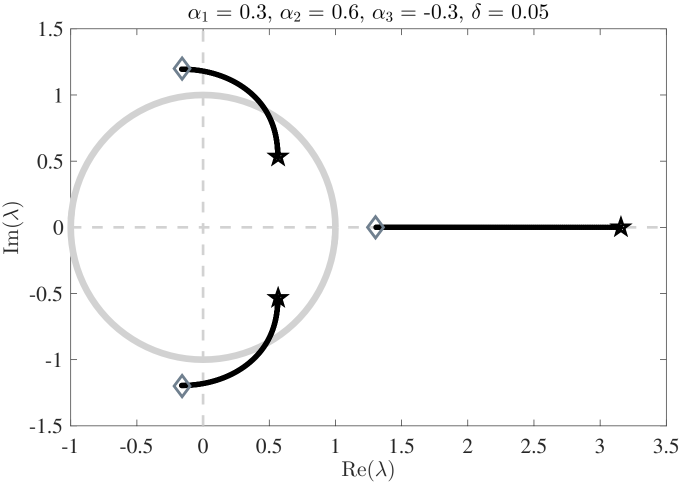
Complementarities and cycles

- As $F'(I^{ss})$ grows two things generally happen
  1. System stops having simple convergence, and instead starts to produce cycles (complex roots)
  2. However, the system – while maintaining determinancy– may loose local stability.
  3. If it looses local stability, strcuture suggests the possible emergence of limit cycles.
- So when one builds on such a framework (ie accumulation with complementarities), should be aware of possible local instability and should be equiped to explore such a possibility.

Set of potential changes in dynamic properties (including bifurcations)

- If the initial situation has two stable (real) roots and one unstable: many changes are conceptually are possible:
  1. The real roots become complex.
  2. The unstable root enters the unit circle: local indeterminacy arises
  3. One stable root leaves the unit circle: instability arises with a flip or fold type bifurcation
  4. Two stable roots leave the unit circle simultaneous because they are complex: this is a Hopf bifurcation

Figure : Eigenvalues of Model

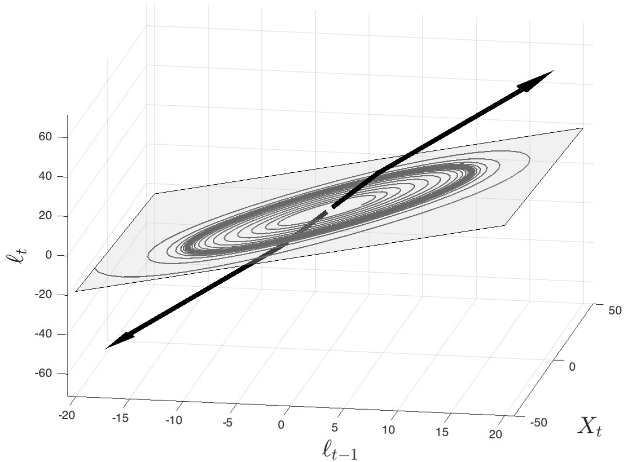


Notes: Complementarities parameter τ grows from 0 at the stars to .99 at the diamonds.

Set of results

- Restricting attention to systems with unique steady state, then no indeterminacy inducing bifurcations nor Fold bifurcations (Prop 5)
- If α_2 is not too small then no Flip Bifurcations.(Prop 6)
- Hence, under quite simple conditions only relevant bifurcation is a determinate Hopf.
- Prop 7 and 8 given necessary ($(1-\delta)\alpha_2 > \alpha_3$)) and suffiicient conditions for determinate Hopf Backward part at least as important as than forward part.
- If a hopf bifurcation arises, then we have the possibility of (saddle path) limit cycles.

Figure : A Saddle Limit Cycle

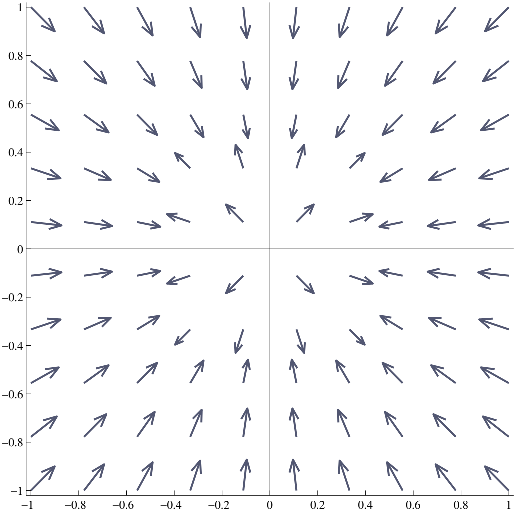


The grey lines are two paths that converge to the limit cyle. The grey zone is the saddle-stable manifold (which is here locally drawn as a linear plane, but which is not). The black lines are two paths for which the jump variable has not jumped on the stable manifold, and which are therefore explosive.

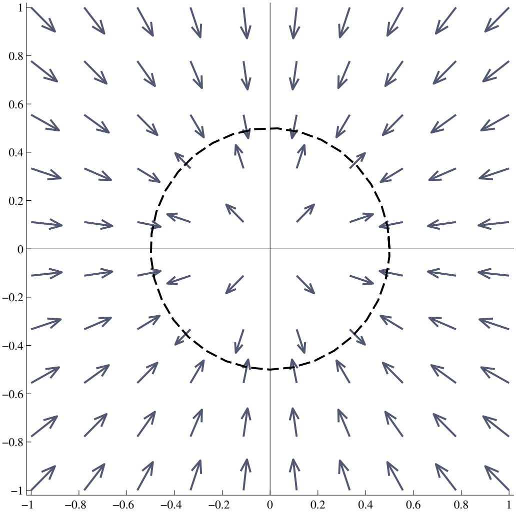
Basic notion of limit cycles and stochastic limit cycles

- Consider a stochastic dynamic environment, it has a stochastic limit cycle if:
- The deterministic structure of the model admits a cycliclical solution, admits a solution of the form $X_t = X_{t+i} = X_{t+2i}...$, where i is the length of the cycle.
- So phase diagram of transitions dynamics look something like the following

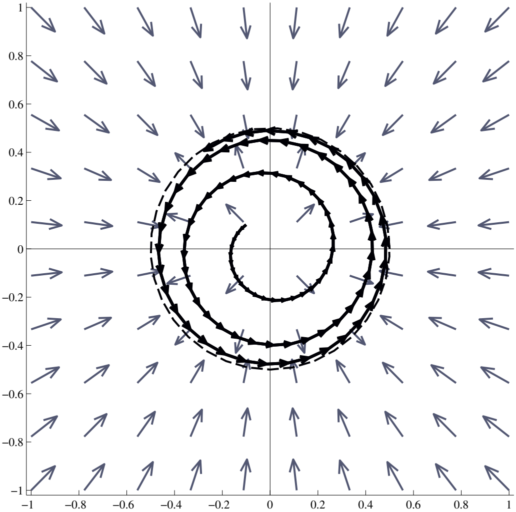
Stable limit cycle



Stable limit cycle



Stable limit cycle



Stable limit cycle

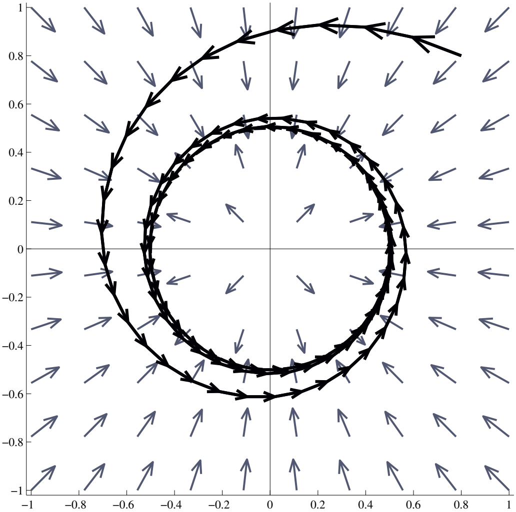
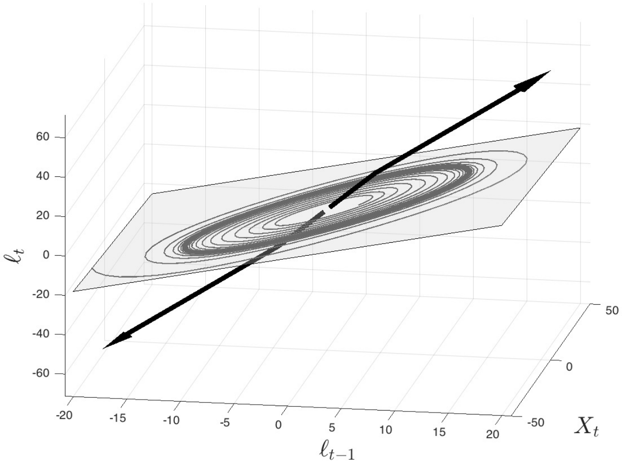


Figure : A Saddle Limit Cycle



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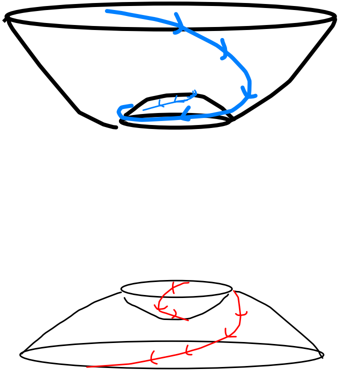
3D version of phase diagram



Large previous literature on limit cycles (deterministic)

- Old non-optimization based models associated with Hicks, Kalecki and Goodwin
- Then 1970s-80s, sizeable literature on optimization and mostly Walrasian based LC models.
  - Benhabib & Nishimura (1979; 1985), Day (1982; 1983), Benhabib & Day (1982), Grandmont (1985), Boldrin & Montrucchio (1986), Day & Shafer (1987), Diamond & Fudenberg (1989), etc.

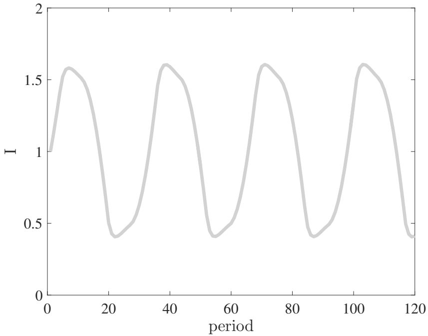
  - Related literature, on growth and innovations cycles. Example Shleifer (1986), Matsuyama (1999), (2001).
  - Also literature on credit cycles in OLG models. Ex: Azariadis and Smith (1998), Matsuyama (2007, 20013) and Myerson (2014).
  - More recent literature in money-search tradition

Common arguments against a limit cycle view

- There are several reasons why limit cycles play very little role in the modern theory of fluctuations.
- EX: Cycles too regular, not empirically relevant...Within optimizing agent based models, most previous mechanisms that have been shown to generate limit cycles (explored intensively in 80s) have not generally been very convincing.

1. Determnistic versus stochastic Limit cycles

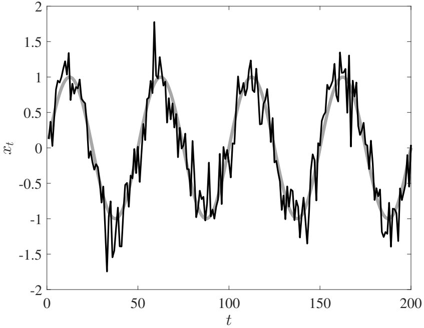
Figure : Deterministic simulation



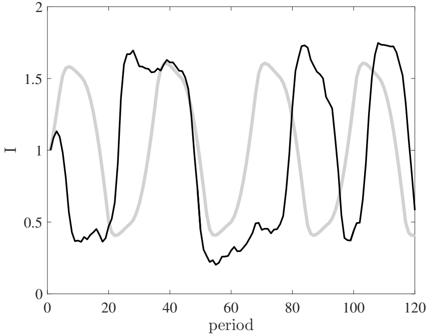
Stochastic limit cycles

- In the above we discussed limit cycles in a deterministic setting
- However, if such forces are to be helpful to understand business cycles, stochastic elements need to be included
- This can be done easily in the above setup, and this is what will do when we explore the idea empirically.
- Important: when extending a deterministic model with a limit cycle to a stochastic elements does NOT simply lead to disturbances around the cycle. The stocks actually move the whole cycle, rendering the cycle hard to predict.
- In the bowl analogy, shocks are like shaking the bowl.

Wrong interpretation of the effects of shocks



Effects of Shocks



Limit Cycles and Quasi-limit cycles.

- In a stochastic environment, in generally, nothing drastic happens in terms of empirical observations when one passes a bifurcation from a situations with a quasi-limit cycle versus a situation with a limit cycle.
- The degree of non-linearity to have the emergence of a limit cycle may be very minor (if the instability is weak)
- However, if one is not aware of the possibility and and analyses the data from a linear perspective, one may bias inference (will not see it!).
- The importance of the bias is an empirical issue: it may or may not be important (ie if there is a LC and one looks at it with a linear framework, one may have a large or small bias).

Exploring an accumulation-credit model of the business cycle.

- Let's go back to the original observations: what forces may explain observed spectrum peaks?
- Could it reflect boom-bust forces associated with an accumulation-credit cycle.
- Old/common Idea: interaction between financial conditions and accumulation decisions. When agents buy more, labor market is tight, when it labor tight, less risky to lend, this favors boom. But eventually, investment opportunities are exploited, system turns around, and recession takes place...then it repeats after liquidation.
- Try to explore this empirically within a slightly modified NK. Set of questions: are complementarities important to explain the data, and if so could they be the source of local instability ans possible hidden limit cycles

Basic Elements

- Aim: build tractable model with accumulation and financial market imperfections
  1. Household buy consumption services to maximise utility. Members borrow to buy goods before getting wage payments. Because of possibly costly recourse, this creates risky credit.
  2. Firms supply consumption services to the market where the services can come from existng durable goods or new production. These firms have sticky prices.
  3. Central Bank set policy rate according to a type of Taylor rule
  4. Interest rate faced by households is the policy rate plus a risk premium. Where the risk premium is endogenously determined by the bank in response to default risk by borrowers.

Default risk and borrowing rate

- Key element is the household-banking sector intereaction (source of complementarity)
- The framework is that of a large household whose members can go out and borrow with imperfect backing from the household.
- The backing is imperfect since it may be costly for banks to pursue a household when a household member can't pay back a loan.
- The level of imperfection is captured by a parameter $\bar{f}$ which is the probability that it is costly to pursue a household.
- Individual consumption is decided and shared at household level.

Timing

- Household members go out and get loans to place orders to firms and to possibly pay back existing household debt.
- Firms make hiring decisions based on orders, and pay workers. Because of labor market frictions not all workers get jobs.
- Consumption goods are returned to households and shared equally.
- Loans are repayed if either a worker is employed or if unemployed and bank can pursue household. Otherwise, there is default.
- Borrowing rates are set so the banks make zero profits.

Default risk and borrowing rate

- With the risk free/policy rate given by i_t, this framework gives rise to a household Euler equation of form

$$U'(C_{jt}) = \beta U'(C_{jt+1})((1+i_t) + b(1 - Prob(E)_t)(1-\bar{f}))$$

- or with external habit

$$U'(C_{jt} - hC_{t-1}) = \beta U'(C_{jt+1} - hC_{t+1})((1+i_t) + b(1 - Prob(E)_t)(1-\bar{f}))$$

- Framework delivers an endogenous risk premium on borrowing, where $Prob(E)$ is the probability of employment, b is cost of default and $\bar{f}$ is probability of easy recourse.

Allowing for durable goods and accumulation

- Households consumption services are supplied by firms and come from either the production of new goods or from the service flow of existing durables.

$$C_t = sX_t + F(L_t\theta_t)$$

where X_t is the stock of durable goods and $F(L_t\theta_t)$ is the current production of goods. The accumulation of durables is

$$X_{t+1} = (1-\delta)X_t + \psi F(L_t\theta_t)$$

- where a fraction $\psi \leq (1-\delta)$ of newly produced goods become durables after one period.
- Households own the stock of durable-housing, rent it to firms, who supply back the consumption services as a composite good.

Risk premium

- The interest rate which household face is assume to be equal to the policy rate plus a risk premium

$$(1 + r_t) = (1 + i_t + r_t^p)$$

- where r_t^p is $b(1 - Prob(E)_t)(1 - \bar{f})$.
- The probability of employment depends on firms' decision to hire l_t. Hence the risk premium becomes of the form

$$r_t^p = g(l_t)$$

- Have Taylor rule of form

$$i_t = \Phi_1 E_t \Pi_{t+1} + \Phi_2 E_t l_{t+1}$$

Extended 3 equation NK model

- Under a set of functional form assumption (and assuming technology grows linearly), then the real side of this model can be reduced to a system of three equation: log employment (output gap) , interest spread and the real interest rate.

$$l_t = \frac{1}{1+\gamma - s\psi}(E_t l_{t+1} + \gamma(1 - \frac{s\psi}{1-\delta})l_{t-1} - \kappa X_t - \sigma(R_t - E_t\Pi_{t+1}) + \beta_t)$$

$$R_t = i_t + r_t^p = i_t + g(l_t)$$

$$i_t - E_t\Pi_{t+1} = \phi_0\bar{\beta} + \phi E_t l_{t+1}$$

- source of shocks is an AR(1) demand-β shock.
- want to consider three cases: 1) $\bar{f} = 1$, no risk premium. 2) Risk premium linear in l_t and 3) allow for non-linearity.

Recap Narrative: Accumulation-credit cycle

- Agents' purchasing decisions depend on policy rates and risk premium.
- Risk premium on loans tends to be favorable when the labor market is tight.
- Labor market condition tends to be good when people are purchasing. This is the complementarity.
- If policy rate is not too pro-cyclical, this creates endogenous cyclical forces, with shocks causing it to be irregular.
- DR to consumption important in mechanism

Preliminary on rates and spreads

- Substantial evidence that interest rate spreads are countercyclical
- But are movements in the spread at right frequency for our story?

Figure : Spectral Density, Level and Bandpass (2,100) to (2,200), Full Sample (1954-2015) - Spread

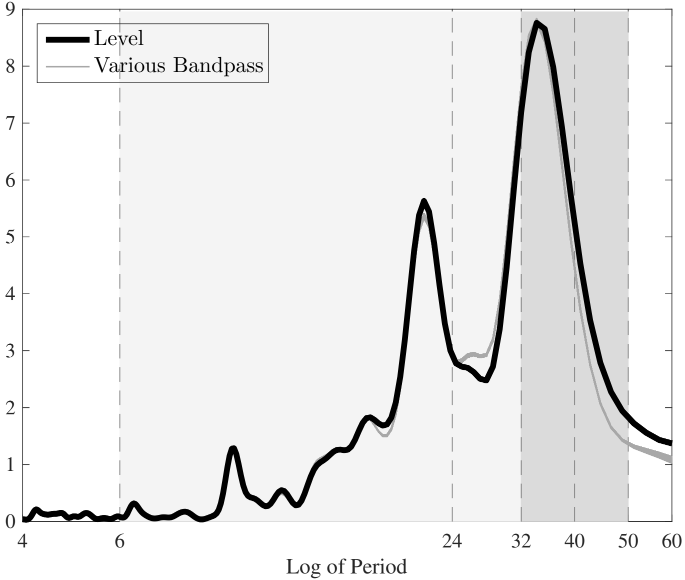
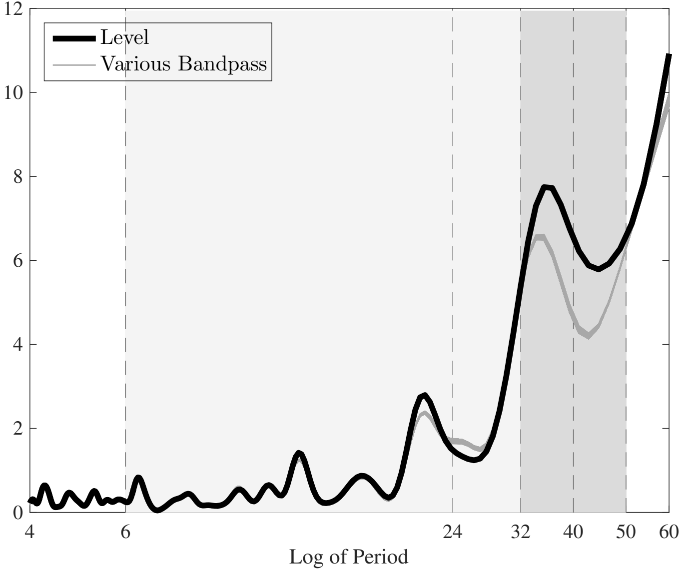


Figure : Spectral Density, Level and Bandpass (2,100) to (2,200), Full Sample (1954-2015) - Real Rate



Estimation

- Estimate parameters of model so that the model aims to reproduce the spectrum of hours worked and the interest rate spread on the frequencies 2-50, plus some higher moments in non-linear case. Check whether model is also consistent with interest rate observations over this range.
- Calibrate two parameters: $\delta = .05$ and $\psi = .4$.
- compare fit of hours spectrum.

Estimation

- Estimating by SMS three version of (with increasing targets)

$$I_t = \frac{1}{1+\gamma - s\psi}(E_t I_{t+1} + \gamma(1 - \frac{s\psi}{1-\delta})I_{t-1} - \kappa X_t - \sigma(i_t + r_t^o - E_t \Pi_{t+1}) + \beta_t)$$

$$r_t^p = g(I_t)$$

$$i_t - E_t \Pi_{t+1} = \phi_0 \bar{\beta} + \phi E_t I_{t+1}$$

$$X_t = (1-\delta)X_t + \psi I_t$$

Figure : Spectrum fit hours: no endogenous risk premium

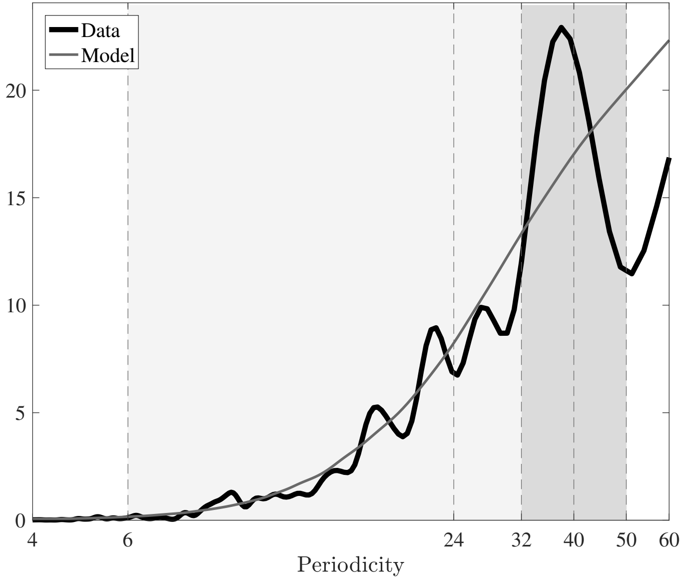
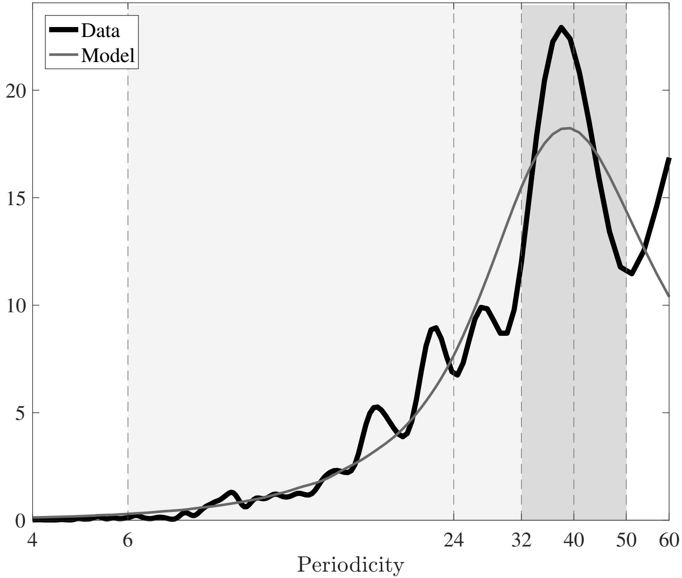


Figure : Spectrum fit hours: risk premium linear in unemployment risk



Higher moments

- Brief word on other properties of data.
- If look at the distribution of hours worked (or unemployment) around cyclical peak, we observe.
  - Distribution is not normal
  - It has thin tails, and skewed
- suggests linear guassian model may not fit well data

Figure : Spectrum fit hours: non-linear version

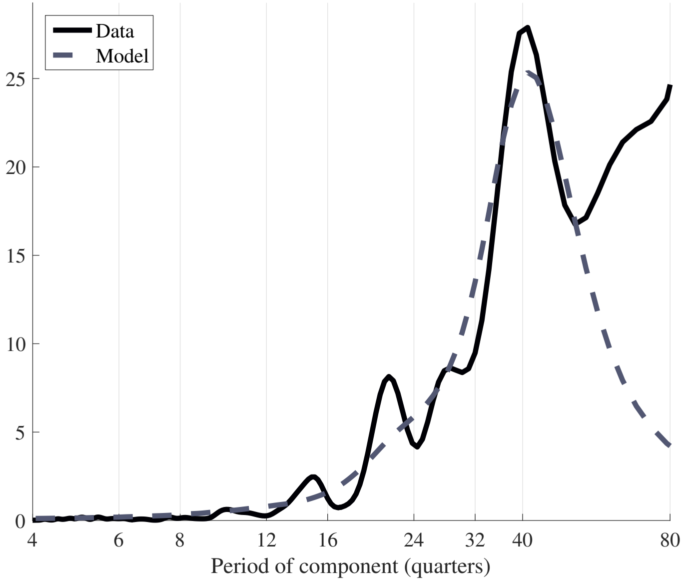


Figure : Spectrum fit spread

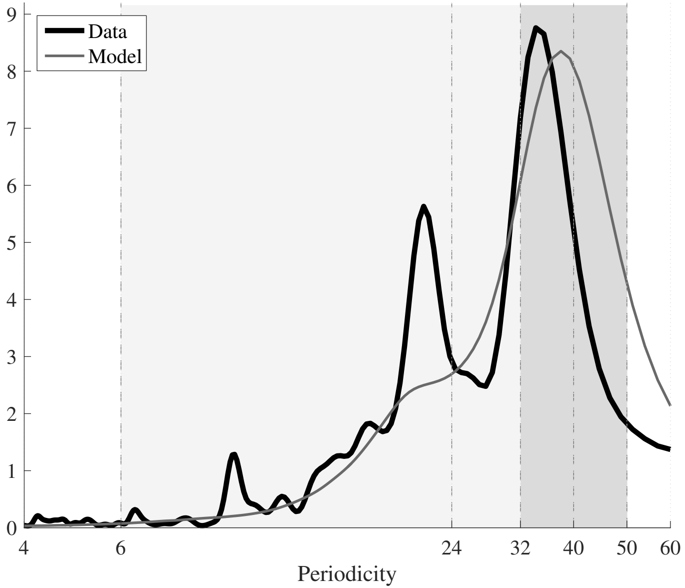


Figure : Spectrum fit real rate

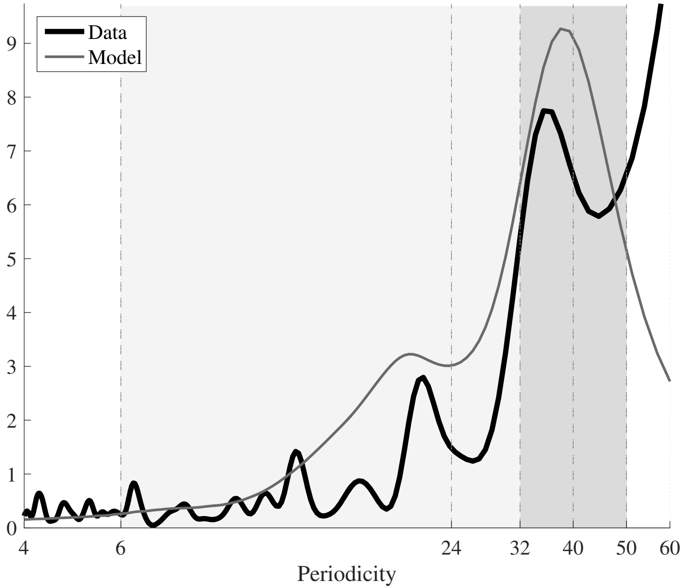


Figure : Sample Draw for Hours

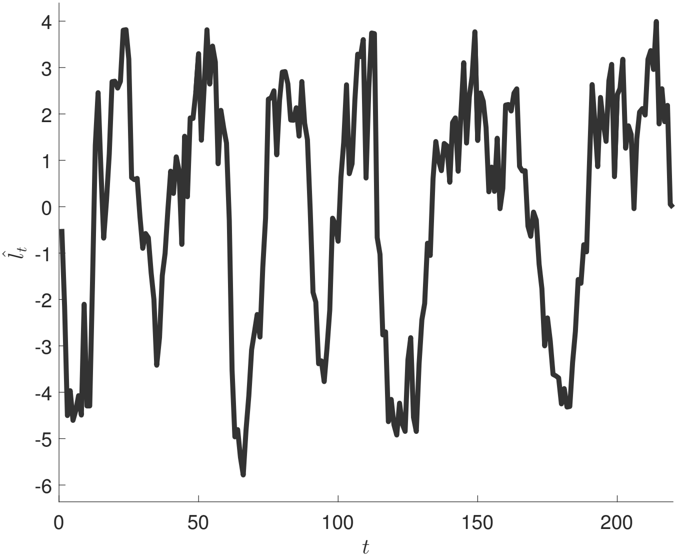
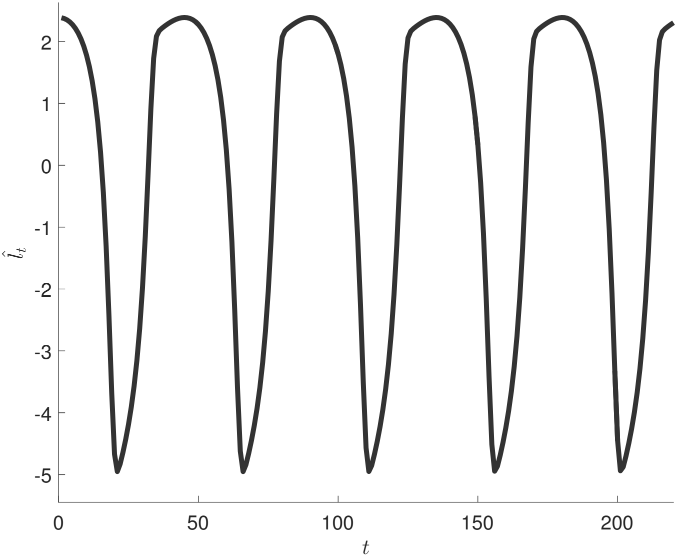
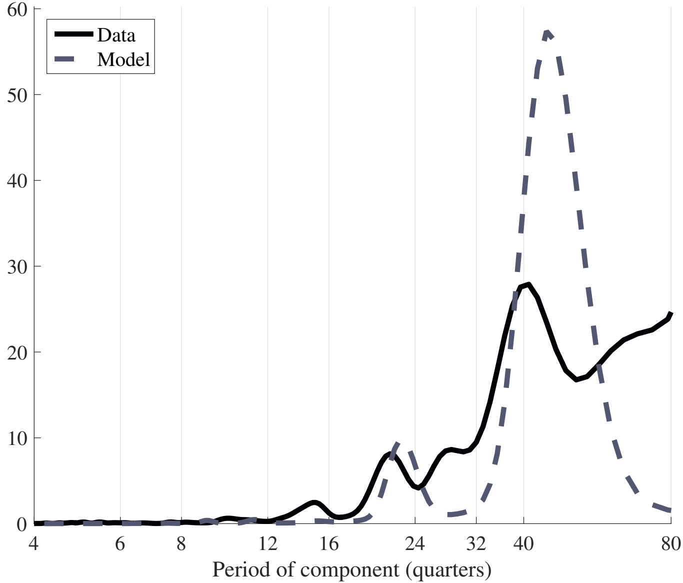


Figure : Sample Draw for Hours, no shocks



(no shk).pdf

Figure : Spectrum, no shocks



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Shocks

- Shocks are important for explaining the data
- However, we estimate them to be almost iid in case with LC; Hence, almost all dynamics are internal.
- Moreover, reducing the the variance of shocks has essentailly no effects of the variance of the system.

Why common estimations approach could miss the cycle

- Either the data is filtered, which usually removes what we think is the main part of the cycle
- Or one uses the full spectrum, in which case, it models need to match three components; missing out on what we believe is a key aspect of the cycle.
- Advanatages of spectrum matching

Conclusion

- Most mainstream macro models do not produce or aim to produce cyclical behavior, in the sense of a peak in the spectrum at business cycle frequencies for variable such as employment and spreads.
- However, when we look as such variables over the last 70 years, we see such a peak around 38 quarters.
- Contributions:
    1. Shown that models with 'weak' complementarities and accumulation offer a framework which may help explain such observations. However, such models may exhibit local instability and LC, and researcher should be aware and address appropriately.
    2. When extending a simple NK model to include complementarity forces at paly over an accumulation cycles we note: 1) complementarities captured by endogenous change in financial conditions are key to explain spectral features of the data 2) an linear model may do a decent job, but this may hide the strength of the endogenous forces. When we allow for non-linearities, we find more endoegnous forces, actually enough to support LC.