Platform Ownership

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Abstract

We analyze a model of two-sided markets, where buyers and sellers interact on a platform. Network effects are two-sided: buyers benefit from a larger number of sellers, and sellers benefit from a larger number of buyers. The platform may be owned by a single large intermediary or many small intermediaries. Our focus of analysis is on the impact of platform ownership structure on platform size, and on the welfare ranking of different ownership structures. In particular, we give conditions under which monopoly ownership is socially preferable to atomistic ownership.

Keywords: Platform, Two-sided markets, Network effects, Competition, Intermediation

JEL-Classification: D23, D40, L10, L22

1 Introduction

Most markets do not form spontaneously, but need to be organized. In a market place, which may be a physical or virtual location, buyers and sellers interact on a trading platform. Examples of market places in physical location are abundant in economic geography, while an example of a market place in virtual location is the internet platform. The interesting central features common to many market places are two-sided platform (or network) effects. Buyers are

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attracted to market places or platforms that house many sellers; and vice versa, sellers are attracted to market places that draw many buyers. Such markets are *two-sided*.

We observe widely differing institutional arrangements or ownership structures. The platform may be owned by a monopoly intermediary, by competitive intermediaries, or by buyers or sellers active on the platform. Examples for monopoly platform ownership are that of a shopping center developer and the independent supplier of an internet platform. An example for dispersed ownership by intermediaries is the retail space supplied by landlords within a downtown shopping district. An example for a vertically integrated platform is *Covisint*, which is owned by automotive producers to procure upstream supplies. Other internet B2B platforms such as supply-on are organized by the sellers of commodities. Dispersed platform ownership may further be distinguished by contractual arrangements and property rights: Incumbent platform owners may or may not have the right to restrict entry onto the platform. (Moreover, platform owners may or may not have to share platform development costs.) In final section we discuss a large number of examples for monopoly platform ownership and vertically integrated platforms.

In this paper, we analyze and evaluate in detail the impact of ownership structure on platform size or, equivalently, on the number of varieties offered by sellers. We consider a market in which buyers and sellers interact exclusively on one trading platform. A large number of heterogeneous sellers each rent a platform slot to sell a differentiated product. The platform slots are let by either a large number of intermediaries or by a monopoly platform owner. A large number of heterogeneous buyers decide whether or not to visit the platform to purchase the products offered by the sellers. On one hand, the larger the number of sellers, i.e., the larger the product variety, the more attractive is the platform to buyers. Consequently, consumer surplus is increasing in platform size. On the other hand, for a given platform size, the larger the number of buyers, the more attractive it is for sellers to rent a platform slot.

The basic ownership structures we consider are: two modes of competitive ownership, namely an open access platform and a closed platform or club, where access is restricted by the incumbent intermediaries; and monopoly platform ownership. Furthermore, we analyze vertically integrated platform ownership, where each platform slot is owned by one seller, and access to the platform may be either open or closed. Finally, we allow for the redistribution of platform development costs amongst platform owners.

One might think that giving market power to a monopoly platform owner leads to high rental charges, and thus drives down seller profits. Under free entry of sellers, this would reduce equilibrium platform size. However, an increase in platform size makes a platform more attractive from the point of view of buyers. A monopoly platform owner may therefore be willing to subsidize sellers at the margin in order to exploit platform effects. Such a subsidization at the margin cannot occur if platform ownership is decentralized and entry of intermediaries is unrestricted. We give conditions under which platform size is larger under monopoly than under decentralized ownership, in which case monopoly ownership is socially preferable. However, the socially optimal platform size always exceeds the monopoly platform size. We also provide conditions in which open competitive platform ownership leads to a socially excessive platform size.

An important question for our comparative analysis is to predict the emergence of particular ownership structures. For this we allow parties to make take-it-or-leave-it offers to the owner(s) of a platform. In particular, a single outside party may make offers to all platform owners. If they accept this offer we predict monopoly platform ownership, and thus *horizontal integration* at the platform level. Alternatively, each seller may make an offer to an owner of a platform slot. If all platform owners accept the offers we predict vertically integrated platform ownership, and thus *vertical integration*. We show that vertically integrated closed platforms or monopoly platform can emerge. This provides a theoretical underpinning to the prevalence of exactly these ownership structures in the real world examples to be discussed in the final section.

We derive all our results in a reduced form model to avoid functional form assumptions. However, we exemplify matters within a model in which sellers are monopolistically competitive, and buyers have CES-demand for the differentiated products and are heterogeneous with respect to their outside option not to participate in the market.

The plan of the paper is as follows. In section 2, we present the model. In the subsequent section, we analyze equilibrium platform size under the basic ownership structures. In section 4, we consider vertically integrated ownership structures. This is followed in section 5 by a welfare comparison of ownership structures and an analysis of the incentives to integrate either horizontally to a monopoly platform or vertically into a vertically integrated ownership structure. In section 6 we extend the analysis to include an entrance or subscription fee on the consumer side. Finally, we conclude in section 7. In the appendix, we extend the analysis to allow for redistribution of platform development costs amongst intermediaries.

Related literature. [preliminary and incomplete, some references still missing] Our model is related to a large and dispersed literature. Two strands of literature are of particular importance. First, the literature on geographical market places and on other organizational forms of intermediation. Second, the literature on network effects, in particular two-sided markets.

The formation of geographical market places was modelled by Stahl (1982 a,b) as a result of consumers' economies of scope in purchasing many commodities on one site, and of economies of scale in searching for the best variant, respectively, and the reaction of oligopolistic or a monopoly supplier thereto. Extensions are provided in Stahl (1987) and Schulz and Stahl (1996).[1] Legros

[1] Only loosely related, Gehrig (1993) shows that buyers and sellers benefit from an intermediary's price commitment in a matching market. Pagano (1989a,b) develops models on the formation of financial intermediaries on the basis of thick market externalities on both sides of the market, that lead to price stabilisation. Spulber (1999) reviews in parts III and IV of his book approaches to alleviate via intermediation search and matching problems, as well as problems of asymmetric information.

and Stahl (2002) model competition between such a market place against an increasingly competitive outside option. Gehrig (1998) considers competition between two market places. Smith and Hay (2003), different from the other cited work, share our objective to compare different ownership structures. They consider a very special model of competing market places where a seller's profit for a given number of participating buyers is independent of platform size. To summarize, the literature on the formation of geographical market places has been dominated by specific funcional-form examples in which market outcomes are analyzed for a given ownership structure. In this paper we present a reduced form approach so that our results do not rely on functional form assumptions and provide a comparative analysis of different ownership structures.

There also exists an empirical literature on shopping malls which gives insights into the presence of various external effects at work, as well as pricing and institutional issues, see, for example, Benjamin, Boyle and Sirmans (1992), Brueckner (1993), Pashigian and Gould (1998), and Gould, Pashigian and Prendergast (2002).

Many of the effects arising in market places, platforms, or two-sided markets are akin to direct or indirect network effects. The literature on network effects goes back to Leibenstein (1950) and has been strongly influenced by Katz and Shapiro (1984). Katz and Shapiro (1994) and Economides (1996) review the concepts and the literature to date.

Amstrong (2002) and Rochet and Tirole (2003) provide an integrating view of two-sided markets and provide models for the systematic analysis of such markets. Both show that an amazing number and variety of real economic markets can usefully be captured and analyzed under this concept. While these authors focus on pricing issues under monopoly platform ownership, and in particular, on the question of how much each side of the market has to pay for access to the platform, we focus on platform size or, equivalently, product variety under different platform ownership structures.

Recent contributions have focussed on particular industries. An interesting pure internet example of a two-sided market is given by Baye and Morgan (2001). Schmalensee (2002), Rochet and Tirole (2002), and Wright (2003) analyze payment systems. In the case of the credit card industry, Rochet and Tirole (2002) analyze the different implications of two different ownership structure, namely credit card associations such as MasterCard and Visa and a proprietary system such as Amex. Rysman (2001) considers the market for yellow pages; Gabszewicz, Laussel, and Sonnac (2001), and Anderson and Coate (2001) consider media markets.

2 A Formal Model of Trading on a Platform

We consider a model of trading on one platform. There are three types of economic agents: sellers (retailers), buyers (consumers), and intermediaries who own the trading platform (landlords or a developer). Sellers and buyers are assumed to be atomistic, while we allow for various ownership structures of the trading platform. Here, we describe the model for the case of a platform owned by independent intermediaries.

Timing. We consider the following sequence of decisions, involving first the intermediaries, then the sellers, and finally the buyers:

Stage 1a The measure of active intermediaries $\overline{\mu}$ is determined and development costs are sunk.

Stage 1b The rental charge r is set by the intermediaries.

Stage 2 Sellers decide whether or not to participate in the platform.

Stage 3 Buyers decide whether or not to participate in the platform.

Buyers. Buyers decide whether or not to visit the market place. If a buyer takes up the outside option, he obtains zero utility. If a buyer of type ω visits the market place, he derives utility $u(\overline{\theta}) - g(\omega)$, where $\overline{\theta}$ is the measure of sellers at the market place. For example, $g(\omega)$ may be interpreted as transport costs. More generally, it reflects the value of the outside option not to participate in the market for a buyer of type ω. We assume that $g(\omega)$ is continuously differentiable and strictly increasing in the buyer's type ω. W.l.o.g., let for any $\widetilde{\omega}$, the measure of buyers of type $\omega \in [0, \widetilde{\omega}]$ be given by $\widetilde{\omega}$. Since $g(\omega)$ is strictly increasing in ω, there will be a marginal type $z(\overline{\theta})$ such that a buyer of type ω visits the market place if and only if $\omega \leq z(\overline{\theta})$. Hence, $z(\overline{\theta}) = g^{-1}(u(\overline{\theta}))$.

Sellers. Sellers decide whether or not to rent a platform slot. If not, they obtain zero payoff. The entering seller has to rent a platform slot at rental charge r. Sellers differ in their fixed costs of running their business, but are symmetric in all other respects. A type-θ seller faces fixed costs $f(\theta)$. We assume that $f(\theta)$ is continuously differentiable and strictly increasing in θ. W.l.o.g., let for any $\widetilde{\theta}$, the measure of potential sellers of type $\theta \in \left[0, \widetilde{\theta}\right]$ be given by $\widetilde{\theta}$. A seller's profit (gross of fixed costs) per unit mass of buyers is given by $\pi(\overline{\theta})$, where $\overline{\theta}$ is the measure of entering sellers. We assume that sellers have constant marginal costs of production, and since all buyers (who decide to visit the market place) have identical demand, a seller's gross profit is proportional to the mass $z(\overline{\theta})$ of buyers visiting the market place. Hence, the net profit of seller θ is given by $z(\overline{\theta})\pi(\overline{\theta}) - f(\theta) - r$. Optimal entry decisions imply existence of a marginal type $\overline{\theta}$ such that a sellers enters if and only if $\theta \leq \overline{\theta}$. Hence, the measure of entering sellers $\overline{\theta}$ is implicitly defined by $z(\overline{\theta})\pi(\overline{\theta}) - f(\overline{\theta}) - r = 0$.

The platform. A slot for a single seller on the platform can be developed at a cost $c(\mu)$, where μ is the type of the platform slot. Entering intermediaries will then optimally develop the available platform slots with the minimum development costs. We assume that $c(\mu)$ is continuous and strictly increasing in μ. W.l.o.g., let for any $\widetilde{\mu}$, the measure of platform slots of type $\mu \in [0, \widetilde{\mu}]$ be given by $\widetilde{\mu}$.[2] Hence, sellers of measure $\overline{\mu}$ need retail space of measure $\overline{\mu}$. Retail space

[2] An alternative interpretation for $c'(\mu) > 0$ is that it becomes increasingly costly to offer the same services or convenience to consumers as more retailers are active on the platform.

a seller θ's payoff	$z(m)\pi(m) - f(\theta) - r$
aggregate platform profits	$mr - C(m)$
a buyer ω's payoff	$u(m) - g(\omega)$

Table 1: Equilibrium payoffs

of measure $\overline{\mu}$ is provided at a minimum development cost of $C(\overline{\mu}) \equiv \int_0^{\overline{\mu}} c(\mu)d\mu$. Since $c(\mu)$ is strictly increasing in μ, $C(\overline{\mu})$ is strictly convex in $\overline{\mu}$.

Market clearing. Under any ownership structure, the rental market for platform slots will clear in equilibrium. Since a single slot accommodates a single retailer, in equilibrium, the measure of entering retailers must thus be equal to the measure of developed platform slots, i.e., $\overline{\theta} = \overline{\mu} = m$. We will refer to m as the platform size.

Payoffs. To summarize, in equilibrium, we can write the payoff of a seller of type θ as $z(m)\pi(m) - f(\theta) - r$. The payoff or surplus of a consumer of type ω is $u(m) - g(\omega)$. Aggregate profits for the platform are the revenues collected through rental charges, mr, minus the accumulated platform development costs $C(m)$. These expressions are collected in Table 1.

Reduced-form assumptions. The functions u, π, and z depend on the interaction between buyers and sellers. We make the following assumptions.

Assumption 1 *Utility $u(m)$ is continuously differentiable and increasing in m.*

An immediate consequence of this assumption is that $z'(m) > 0$. Hence, an increase in platform size (or product variety) has a *market size effect.*

Assumption 2 *Profit per unit mass of buyers $\pi(m)$ is continuously differentiable and weakly decreasing in m.*

According to assumption 2 a larger platform size makes competition on the platform more intense. Hence, an increase in platform size (or product variety) has a *competition effect* (which is possibly zero). Summarizing assumptions 1 and 2, an increase in platform size has two countervailing effects on a seller's revenue $z(m)\pi(m)$: a positive market size effect and a negative competition effect. We speak of positive *platform effects* if the overall effect is positive, i.e. $z'(m)\pi(m) + z(m)\pi'(m) > 0$. The condition can be rewritten as

$$m\frac{z'(m)}{z(m)} + m\frac{\pi'(m)}{\pi(m)} > 0 \tag{1}$$

saying that the demand elasticity dominates the profit elasticity. Clearly, if the competition effect is zero platform effects are globally positive.

Assumption 3 *There exists a unique $\widehat{m} \in [0, \infty) \cup \{\infty\}$ such that the marginal seller's net profit $z(m)\pi(m) - f(m)$ is strictly increasing on $[0, \widehat{m})$ and strictly decreasing on $(\widehat{m}, \infty)$.*

If $\widehat{m} = \infty$, then $z(m)\pi(m) - f(m)$ is monotonically increasing in m. In this case platform effects are very strong, that is, platform effects are globally positive and globally dominate the sellers' cost effect. If $\widehat{m} = 0$, then the marginal seller's net profit is monotonically decreasing in m. In this case platform effects are very weak or negative. If $0 < \widehat{m} < \infty$, the maximizer is implicitly defined by the first-order condition

$$z'(\widehat{m})\pi(\widehat{m}) + z(\widehat{m})\pi'(\widehat{m}) - f'(\widehat{m}) = 0. \tag{2}$$

In this case platform effects, measured by marginal revenue, initially dominate the marginal seller's change in cost so that the marginal seller's net profit is increasing for the platform size sufficiently small. In this case platform effects are moderate. Whether platform effects are very weak, moderate, or very strong depends on whether or not the platform effect or the seller's cost effect dominates globally and if yes, which one.

Assumption 4 *There exists a unique $m^* > 0$ such that the sum of marginal net profits of sellers and platform owners, $\int_0^m [z(\theta)\pi(\theta) - f(\theta)]\, d\theta - C(m)$, is strictly increasing in m on $[0, m^*)$ and strictly decreasing on (m^*, ∞), and strictly positive at m^*.*

The maximizer m^* is thus implicitly defined by the first-order condition

$$z(m^*)\pi(m^*) - f(m^*) - c(m^*) = 0 \tag{3}$$

Platform sizes $\widehat{m}$ and m^* are illustrated in figure 1. If the marginal development cost function $c(m)$ intersects (from below) the marginal seller's net profit function $z(m)\pi(m) - f(m)$ to the left of the latter's peak, then $m^* < \widehat{m}$. In this case we say that platform effects are *strong*. If the intersection is to the right of the peak, then $m^* > \widehat{m}$. In this case we say that platform effects are *weak*. In terms of elasticities, platform effects are strong if at m^*, the demand elasticity dominates profit elasticity and the seller's cost elasticity times the revenue share of costs,

$$m\frac{z'(m)}{z(m)} + m\frac{\pi'(m)}{\pi(m)}m\frac{\pi'(m)}{\pi(m)} - m\frac{f'(m)}{f(m)}\frac{f(m)}{z(m)\pi(m)} > 0.$$

If the reverse inequality holds platform effects are weak. This provides us with a taxonomy of platform effects Figure 1 illustrates the different regimes of platform effects. Table 2 summarizes the four different regimes and the values m^* and $\widehat{m}$ take in each regime.

Assumption 5 *There exists a unique $m^S \geq 0$ such that the total surplus of sellers and platform owners, $mz(m)\pi(m) - \int_0^m f(\theta)d\theta - C(m)$, is strictly increasing in m on $[0, m^S)$ and strictly decreasing on (m^S, ∞), and strictly positive at m^S.*

These assumptions can be better understood if we consider the underlying micro structure we have in mind.

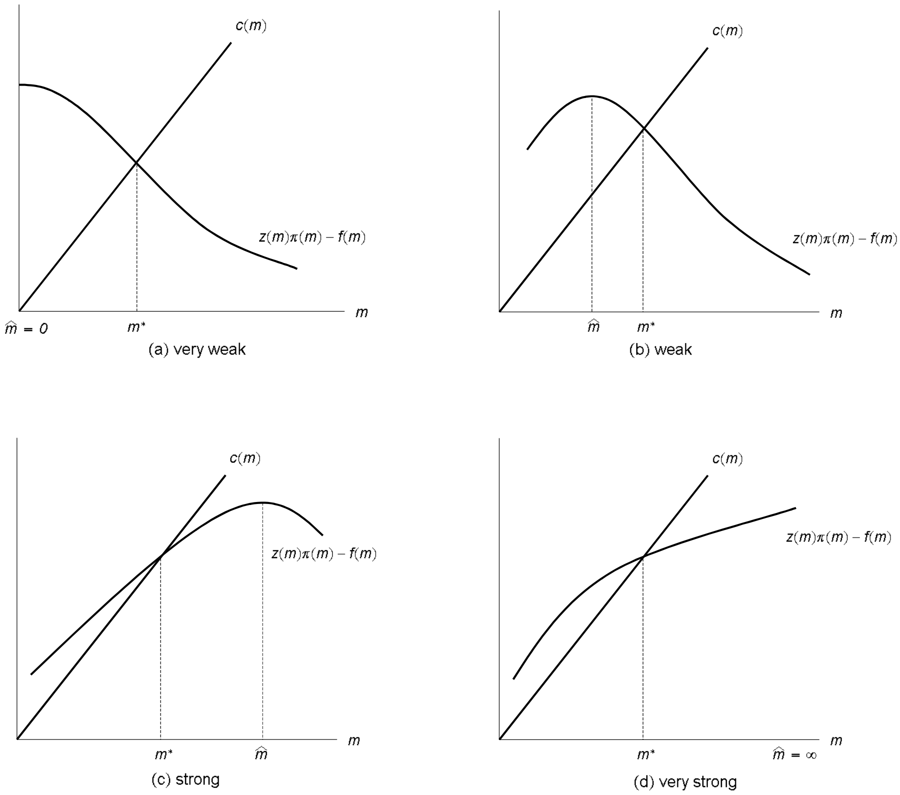

Figure 1: A taxonomy of platform effects

very weak	$m^* > \widehat{m} = 0$
weak	$m^* > \widehat{m} > 0$
strong	$m^* < \widehat{m} < \infty$
very strong	$m^* < \widehat{m} = \infty$

Table 2: Platform effects and critical platform size

Underlying micro structure. Each seller offers a unique variant of a differentiated good. Although sellers are atomistic, each seller faces a downward-sloping demand curve. Hence, there is monopolistic competition between sellers. Since the variants are symmetric, the measure m of entering sellers is the measure of variety offered by sellers. We expect the profit (per unit mass of buyers), as given by $\pi(m)$, to decrease with m for two reasons. First, for given prices, buyers purchase less from each seller as the number of sellers increases.[3] This may be dubbed the *market share effect*. Second, as the number of sellers increases, competition becomes more intense and prices fall (assuming that the goods offered by sellers are substitutes). This may be dubbed the *price effect*. Likewise, buyers' utility $u(m)$ should be increasing in variety m for two reasons. First, buyers have a taste for variety. Second, as the number of sellers increases, prices fall.

In the following we consider an example that is flexible enough to generate very weak, weak, strong, and very strong platform effects.

Example 1 A CES model with an outside option and power cost functions. *Consumers (buyers) make a discrete choice between visiting the market place and an outside option. Consumer type ω is uniformly distributed over some interval $[0, \overline{\omega}]$, where $\overline{\omega}$ is sufficiently large so that the market is never covered; the measure of consumers is equal to $\overline{\omega}$. The disutility of travelling to the market place is given by $g(\omega) = t\omega$. Conditional on visiting the market place, consumers have CES preferences over the variants offered by the retailers. Demand for variant j is*

$$x(j) = \frac{p(j)^{-\frac{1}{1-\rho}} E}{\int_0^m p(i)^{-\frac{\rho}{1-\rho}} di},$$

where $p(j)$ is the price of variant j, E is income spent on the differentiated goods industry, and $\rho \in (0,1)$ measures the degree of product differentiation. Each retailer j maximizes his profits $\pi = (p-c)p^{-\frac{1}{1-\rho}} A$, where c is the marginal cost of production, and $A = E / \int_0^m p(i)^{-\frac{\rho}{1-\rho}} di$. Using symmetry, the first-order conditions of profit maximization yield the equilibrium price $p = c/\rho$. Here, an increase in product variety has no price effect: $\partial p / \partial m = 0$. Equilibrium quantity is given by $x(m) = (E/m)(\rho/c)$, so there is a market share effect, and thus equilibrium profit $\pi(m) = (1-\rho)E/m$ is decreasing in m, as assumed. In equilibrium, utility $u(m) = E(\rho/c)m^{\frac{1-\rho}{\rho}}$ is increasing in m, and so our assumption on u is satisfied. The marginal consumer z who is indifferent between visiting the market place and taking up the outside option is defined by $u(m) - tz = 0$, and thus $z(m) = u(m)/t$. A retailer's gross profit is then given by

$$z(m)\pi(m) = \frac{E^2}{t} \frac{\rho(1-\rho)}{c} m^{\frac{1-2\rho}{\rho}}.$$

[3] There may be a countervailing effect as variety increases, however, namely that variety-seeking consumers may optimally decide to spend a larger fraction of their income on the goods produced in the differentiated goods industry.

Here, $z(m)\pi(m)$ is monotone in m: it is decreasing if the variants are sufficiently good substitutes, $\rho > 1/2$, and increasing if the reverse inequality holds. Hence, a necessary condition for platform effects to become relevant in that $m^* > 0$ is that $\rho < 1/2$.

With respect to costs, we assume that sellers' fixed costs take the form $f(\theta) = a\theta^b$, where $a, b > 0$; and that the development cost function takes the form $C(\mu) = \alpha\mu^\beta$, where $\alpha > 0$ and $\beta > 1$.

Assumption 3 concerns the marginal retailers' net *profit. If $\rho > 1/2$, then the marginal seller's net profit $z(m)\pi(m) - f(m)$ is monotonically decreasing in m, and so $\widehat{m} = 0$. If $\frac{1}{b+2} < \rho < 1/2$, then there exists a unique interior maximum $\widehat{m}$. If $\rho < \frac{1}{b+2}$, then the marginal seller's net profit is monotonically increasing in m, and so $\widehat{m} = \infty$.*

Assumption 4 concerns the sum of marginal net profits of sellers and platform owners. The sum of marginal net profits of sellers and platform owners $\int_0^m [z(\theta)\pi(\theta) - f(\theta)]\, d\theta - C(m)$ is single-peaked in m and has a unique positive maximizer, m^*, provided assumption 3 holds as well. Specifically, if $\rho > 1/2$, $z(m)\pi(m) - f(m)$ is monotonically decreasing, while $c(m)$ is increasing, and so the assumption holds trivially. If $\min\left\{\frac{1}{\beta+1}, \frac{1}{b+2}\right\} < \rho < 1/2$, then $\int_0^m [z(\theta)\pi(\theta) - f(\theta)]\, d\theta - C(m)$ is single-peaked with an interior maximum.

Assumption 5 concerns the total surplus of sellers and platform owners. The assumption holds if $\min\left\{\frac{1}{\beta+1}, \frac{1}{b+2}\right\} < \rho$.

To sum up, all of our assumptions hold in this example under the weak restriction that $\min\left\{\frac{1}{\beta+1}, \frac{1}{b+2}\right\} < \rho$, which requires a minimum degree of convexity of either one of the two aggregate cost functions.

3 Independently-owned Platforms

We begin our equilibrium analysis by considering three basic ownership structures that are all vertically disintegrated: open platform ownership (e.g., high street), closed platform ownership (e.g., landlord club), and monopoly platform ownership (e.g., shopping mall).

- Open platform ownership (O). Potential intermediaries can freely enter the market for intermediation on the platform. Each of them is constrained to competitively develop one slot at exogenously given cost, and to offer it to one seller at the competitive rental price.

- Closed platform ownership (C). The intermediaries form a club to develop the platform, and admit members by maximizing the rental price accruing to each member, net of development costs.

- Monopoly platform ownership (M). The monopoly platform owner decides how many platform slots to develop, and sets the rental price that each seller has to pay for access to a platform slot.

Ownership is fragmented under open and closed platform ownership whereas it is concentrated under monopoly ownership. Under open platform ownership incumbent intermediaries do not have the property right for the platform. That, is they cannot deny access to other intermediaries. By contrast, under closed and monopoly platform ownership, incumbent owners can deny access to other intermediaries because they have the property right for the platform. We call these ownership structures proprietary.

Open platform ownership: the high street. Under atomistic open platform ownership, free entry of (rental-price taking) landlords implies that the cost of developing a platform slot for the marginal landlord of type $\overline{\mu}$ is equal to the rental price r:

$$r = c(\overline{\mu}).$$

Free entry at the seller level implies that the marginal seller's net profit is equal to zero,

$$z(\overline{\theta})\pi(\overline{\theta}) - f(\overline{\theta}) = r.$$

The competitive rental price r clears the rental market, $\overline{\mu} = \overline{\theta}$. The equilibrium platform size m^O is thus determined by

$$z(m^O)\pi(m^O) - f(m^O) - c(m^O) = 0. \tag{4}$$

Comparing (4) and (3), we obtain the following lemma.

Lemma 1 *Under open platform ownership, the equilibrium platform size m^O is given by $m^O = m^*$. From assumption 4, it follows that m^O is unique.*

Closed platform ownership: the landlord club. Under closed platform ownership, a group of intermediaries (landlords) can deny other potential intermediaries access to the platform. Since the most efficient platform slots will be rented out first, the optimization problem for existing intermediaries consists in maximizing each active landlord's profit with respect to the number of active intermediaries, under the free entry constraint for sellers, the market clearing condition for platform slots, and the condition that each landlord makes non-negative profits. Formally,

$$\begin{aligned} &\max_{\overline{\mu}} r - c(\mu) \\ \text{s.t.} \quad & z(\overline{\theta})\pi(\overline{\theta}) - f(\overline{\theta}) = r, \\ & \overline{\theta} = \overline{\mu}, \\ & r \geq c(\overline{\mu}). \end{aligned}$$

Note that, conditional on being admitted to the club, all intermediaries on the platform agree on the optimal number of club members. Market clearing implies a platform size $m = \overline{\theta} = \overline{\mu}$. The optimization problem can thus be rewritten as

$$\begin{aligned} &\max_{m} z(m)\pi(m) - f(m) \\ \text{s.t.} \quad & z(m)\pi(m) - f(m) \geq c(m). \end{aligned}$$

If the constraint is non-binding, the equilibrium platform size $\widetilde{m}^C$ is implicitly defined by

$$z'(\widetilde{m}^C)\pi(\widetilde{m}^C) + z(\widetilde{m}^C)\pi'(\widetilde{m}^C) - f'(\widetilde{m}^C) = 0.$$

Comparing this equation with (2), we conclude that $\widetilde{m}^C = \widehat{m}$. If the constraint is binding, the equilibrium platform size is $\widehat{m}^C$, which is determined by

$$z(\widehat{m}^C)\pi(\widehat{m}^C) - f(\widehat{m}^C) = c(\widehat{m}^C).$$

From (3), it follows that $\widehat{m}^C = m^*$. We summarize our results in the following lemma.

Lemma 2 *Under closed platform ownership, equilibrium platform size m^C is given by*

$$m^C = \min\{\widehat{m}, m^*\}. \tag{5}$$

From assumptions 3 and 4, it follows that m^C is unique.

If platform effects are weak, i.e. $m^* > \widehat{m}$, then the marginal development cost function $c(m)$ intersects the marginal seller's net profit function to the right of the latter's peak; see figure 1. Since existing club members' interests with respect to platform size are perfectly aligned (provided each member makes nonnegative profits), each club member's profit is maximized at the platform size $\widehat{m}$, where the marginal seller's net profit $z(m)\pi(m) - f(m)$ is maximized. The marginal club member at platform size $\widehat{m}$ obtains a strictly positive profit, $r - c(\widehat{m}) > 0$, where $r = z(\widehat{m})\pi(\widehat{m}) - f(\widehat{m})$. If platform effects are weak, i.e. $\widehat{m} = 0$, the equilibrium platform size is of measure zero since an existing club member's profit is decreasing in the number of club members. If, by contrast, platform effects are strong, $m^* < \widehat{m}$, then the marginal club member would make a loss at platform size $\widehat{m}$. In this case, incumbent club members would like to admit more members than the number of entering intermediaries under free entry, and so the equilibrium platform size is equal to that under open platform membership, $m^C = m^O = m^*$.

Monopoly platform ownership. Suppose now that the platform is owned by a monopolist. The monopolist's optimization problem consists in maximizing his profit with respect to total number of platform slots $\overline{\mu}$ (or, alternatively, rental charge r), subject to the free-entry condition for sellers and the rental market clearing condition. Formally,

$$\begin{aligned} & \max_{\overline{\mu}} \overline{\mu} r - C(\overline{\mu}) \\ \text{s.t.} \quad & z(\overline{\theta})\pi(\overline{\theta}) - f(\overline{\theta}) = r, \\ & \overline{\theta} = \overline{\mu}. \end{aligned}$$

Market clearing implies a platform size $m = \overline{\theta} = \overline{\mu}$. The optimization problem can thus be rewritten as

$$\max_m m\left[z(m)\pi(m) - f(m)\right] - C(m).$$

The equilibrium platform size under monopoly ownership, m^M, must satisfy the first-order condition

$$\begin{aligned} & z(m^M)\pi(m^M) - f(m^M) \\ + \quad & m^M[z'(m^M)\pi(m^M) + z(m^M)\pi'(m^M) - f'(m^M)] - c(m^M) = 0 \end{aligned} \quad (6)$$

Note that there may be more than one platform size which satisfies the first-order and second-order conditions of profit maximization. In general, our assumption do not imply that the monopolist's objective function is single-peaked. Nevertheless, assumption 5 ensures that generically there exists a unique global maximizer, which means that equilibrium platform size is generically unique.[4]

Lemma 3 *Under monopoly platform ownership, the equilibrium platform size* m^M *satisfies*

$$m^M \in (\min\{\widehat{m}, m^*\}, \max\{\widehat{m}, m^*\})$$

if $\widehat{m} \neq m^*$. *Otherwise,* $m^M = \widehat{m} = m^*$.

Proof. Observe that we can rewrite the first-order condition for m^M as follows:

$$\varphi(m^M) + m^M \psi(m^M) = 0,$$

where

$$\begin{aligned} \varphi(m) &\equiv z(m)\pi(m) - f(m) - c(m), \\ \text{and } \psi(m) &\equiv z'(m)\pi(m) + z(m)\pi'(m) - f'(m). \end{aligned}$$

By assumption 3, $\psi(m) > 0$ if $m < \widehat{m}$, and $\psi(m) < 0$ if the reverse inequality holds. Similarly, by assumption 4, $\varphi(m) > 0$ if $m < m^*$, and $\varphi(m) < 0$ if the reverse inequality holds. Consequently, if $\widehat{m} \neq m^*$, we have $\varphi(m) + m\psi(m) > 0$ for $m \leq \min\{\widehat{m}, m^*\}$ and $\varphi(m) + m\psi(m) < 0$ for $m \geq \max\{\widehat{m}, m^*\}$. If $\widehat{m} = m^*$, then $\varphi(m) + m\psi(m) > 0$ for $m < \widehat{m} = m^*$ and $\varphi(m) + m\psi(m) < 0$ for $m > \widehat{m} = m^*$. ■

If platform effects are weak, i.e. $\widehat{m} < m^*$, then $m^M \in (\widehat{m}, m^*)$. To see that $m^M > \widehat{m}$, note that at platform size $\widehat{m}$, a marginal increase in platform size does not affect net profits for inframarginal platform slots, but the marginal slot makes a positive contribution to total profits as $z(\widehat{m})\pi(\widehat{m}) - f(\widehat{m}) > c(\widehat{m})$. To see that $m^M < m^*$, note that at platform size m^*, a marginal increase in platform size reduces the net profits for inframarginal platform slots, while the marginal slot makes a contribution of zero to total profits as $z(m^*)\pi(m^*) - f(m^*) = c(m^*)$.

On the other hand, if platform effects are strong, i.e. $\widehat{m} > m^*$, then $m^M \in (m^*, \widehat{m})$. To see that $m^M > m^*$, note that at platform size m^*, a marginal increase in platform size increases the net profits for inframarginal

[4] In the CES example, the monopolist's has a unique solution to the first-order condition under the same parameter restriction that ensures that assumption 5 is satisfied in the example.

platform slots, while the marginal slot makes a contribution of zero to total profits as $z(m^*)\pi(m^*) - f(m^*) = c(m^*)$. To see that $m^M < \widehat{m}$, note that at platform size $\widehat{m}$, a marginal increase in platform size does not affect net profits for inframarginal platform slots, but the marginal slot makes a negative contribution to total profits as $z(\widehat{m})\pi(\widehat{m}) - f(\widehat{m}) < c(\widehat{m})$. Hence, if $\widehat{m} > m^*$, the monopoly platform owner optimally subsidizes the marginal seller and sets a rental price below marginal development cost.

Platform size under basic ownership structures. We now turn to a comparison of the three basic ownership structures considered above. Monopoly ownership leads unambiguously to a larger platform size than the other proprietary ownership, the closed platform ownership. Compared to the open platform, the monopoly platform has a larger platform size if and only if platform effects are strong. This result is summarized by the following proposition:

Proposition 1 *The equilibrium platform size under the three basic ownership structures can be ranked as follows. If platform effects are weak, i.e.* $m^* > \widehat{m}$, *then* $m^O > m^M > m^C$. *If platform effects are strong, i.e.* $m^* < \widehat{m}$, *then* $m^M > m^O = m^C$.

Proof. This follows immediately from lemmas 1 to 3. ■

It may be helpful to interpret platform size m as the "output" of the intermediaries, the "price" of which is the rental charge r. Under this interpretation, intermediaries are "producers" (of platform slots), who face the industry production cost function $C(m)$. Under an atomistic ownership structure, each potential producer has a capacity of one unit of output. The "inverse demand function" (for platform slots) is then given by the marginal seller's willingness to pay for a platform slot, $z(m)\pi(m) - f(m)$. By assumption 3, the inverse demand function is single-peaked and obtains a unique maximum at $\widehat{m} \geq 0$. When comparing open and monopoly platform ownerships, we are thus comparing a competitive market with a monopoly.

Consider first the case where $\widehat{m} = 0$. In this case, inverse demand is downward-sloping. As is well known from intermediate microeconomics, equilibrium output will be lower under monopoly than in a competitive market, since the monopolist has an incentive to restrict output so as to be able to charge a higher price. Under closed platform ownership, members have an incentive to deny access since further entry will reduce the price members can charge. Since each (atomistic) member produces a single unit of output (which is a slot of measure zero), closed platform ownership leads to a measure zero of output. Equilibrium output is thus lower than under monopoly.

Consider now the case where the inverse demand function is first increasing and obtains its maximum at $\widehat{m} > 0$. If $\widehat{m} < m^*$, the comparison between the three basic ownership structures is similar to the case where $\widehat{m} = 0$, the main difference being that the equilibrium platform size under closed platform ownership is of positive measure. Note that if the marginal development cost function

$c(m)$ were constant (as typically assumed in the literature on network effects; see Economides, 1996), then $\widehat{m} < m^*$ would necessarily hold. If $\widehat{m} > m^*$, the industry (production) cost function $C(m)$ intersects the inverse demand function (from below) in the latter's increasing part. In this case, the equilibrium output levels of the open and closed competitive markets coincide: under closed platform ownership, existing producers would like to increase industry output above the level provided under open platform ownership, but cannot force production of additional producers at a price below cost. In contrast, the monopolist can internalize network effects, which cause the inverse demand function to be locally upward-sloping, and subsidize production at the margin. Hence, the ranking of output levels between competitive markets and monopoly is reversed if the relevant part of the inverse demand function is upward-sloping.

4 Vertically integrated platforms

In this section, we consider the case where each seller owns his own platform slot. As before, we distinguish between and open and closed platform ownership.

- Open integrated platform ownership (OI). Potential seller-intermediaries can freely enter the platform. Each of them is constrained to competitively develop one slot at exogenously given cost.
- Closed integrated platform ownership (CI). The seller-intermediaries form a club and admit members by maximizing the rental price per member, net of development costs.

Since in our approach we take as given that the seller side is atomistic, the vertical integration of a monopoly platform owner cannot be considered.

Open integrated platform. In an open integrated platform each seller owns a platform slot. Clearly, active sellers fill up the most efficient slots. Hence the marginal seller m faces slot development costs $c(m)$. Suppose that the market operates under production efficiency, that is, sellers with the lowest costs are active. In other word, across the pairs of sellers and intermediaries the seller's fixed cost and slot development costs are perfectly positively correlated. This implies that the marginal seller faces fixed costs $f(m)$. The equilibrium platform size m^{OI} is thus determined by

$$z(m^{OI})\pi(m^{OI}) - f(m^{OI}) - c(m^{OI}) = 0. \tag{7}$$

Hence the open vertically separate ownership structure decentralizes the outcome under an open integrated platform, $m^O = m^{OI} = m^*$.

Lemma 4 *Under open integrated platform ownership, the equilibrium platform size m^{OI} is given by $m^{OI} = m^*$. From assumption 4, it follows that m^{OI} is unique.*

Closed integrated platform. In a closed integrated platform the maximization problem for each seller-intermediary is

$$\begin{aligned} &\max_{m\geq 0} z(m)\pi(m) - f(\theta) - c(\mu) \\ &\text{s.t. } \theta = \mu \\ &z(m)\pi(m) - f(m) - c(m) \geq 0. \end{aligned}$$

The first-order condition of the unconstrained problem may be written as

$$z'(\widetilde{m}^{CI})\pi(\widetilde{m}^{CI}) + z(\widetilde{m}^{CI})\pi'(\widetilde{m}^{CI}) = 0.$$

The equilibrium platform size is thus given by $m^{CI} = 0$ if $z(m)\pi(m)$ is monotonically decreasing for all $m \geq 0$; otherwise, $m^{CI} = \widetilde{m}^{CI}$ if $z(\widetilde{m}^{CI})\pi(\widetilde{m}^{CI}) - f(\widetilde{m}^{CI}) - c(\widetilde{m}^{CI}) \geq 0$, and $m^{CI} = m^*$ else. Hence, $m^{CI} \leq m^*$. Furthermore, if $m^* \geq \widehat{m}$, then $\widetilde{m}^{CI} \geq \widehat{m}$.

Lemma 5 *Under closed integrated platform ownership, the equilibrium platform size is given by $m^{CI} = 0$ if the platform effects are very weak, $m^{CI} \in (\widehat{m}, m^*]$ if platform effects are weak, and $m^{CI} = m^*$ otherwise.*

Recall that in the CES example, $z(m)\pi(m)$ is either monotonically increasing or decreasing. In this case, $m^{CI} \in \{0, m^*\}$.

Platform size under proprietary ownership structures. What is the effect of vertical integration on the equilibrium platform size under closed ownership? In general, we have $m^{CI} \geq m^C$, where the inequality is strict if and only if $m^{CI} = \widetilde{m}^{CI}$. Under non-integration, the intermediaries do not capture all the inframarginal rents of sellers (due to the uniform rental charge). The intermediaries thus have less incentives to increase platform size than under vertical integration, where the intermediaries *do* capture all the inframarginal rents of sellers.

Note that vertical integration unambiguously (weakly) increases consumer surplus as it increases platform size. This is a novel explanation of a positive effect of vertical integration on consumer surplus. Clearly, it is different from previous arguments in the literature which relied on the elimination of double marginalization or on efficiency gains through vertical integration.

Comparing the platform size with monopoly, we obtain the following. If $m^* < \widehat{m}$, then $m^{CI} < m^M$. The comparison is ambiguous when platform effects are weak, i.e. $m^* > \widehat{m} > 0$. In this case under both ownership structures, different parts of the surplus are internalized that are not internalized under closed non-integrated ownership. Consequently, $m^{CI} > m^C = \widehat{m}$ and $m^M > m^C = \widehat{m}$. The monopoly platform owner internalizes the effect of platform size on the rental price for inframarginal slots. Under closed integrated platform ownership, owners obtain the full surplus for each slot and thus have an incentive to allow more entry than under non-integrated ownership. Our results are summarized by the following proposition.

Proposition 2 *The equilibrium platform size under the three proprietary ownership structures can be ranked as follows. If platform effects are very weak, i.e.* $m^* > \widehat{m} = 0$*, then* $m^M > m^{CI} = m^C$*. If platform effects are weak, i.e.* $m^* > \widehat{m} > 0$ *then* $\min\{m^M, m^{CI}\} > m^C$*. If platform effects are strong (or very strong), i.e.* $m^* < \widehat{m}$*, then* $m^M > m^{CI} = m^C$*.*

5 Welfare Analysis and Incentives to Integrate

5.1 Welfare Analysis

In this subsection, we compare welfare under the different ownership structures.

The planner's problem. We now consider the problem of a benevolent social planner whose objective it is to maximize total surplus with respect to platform size m. We assume that the planner cannot directly control the measure of buyers, z; instead, z is determined by buyers' participation decisions.[5] The planner's objective function is given by

$$\begin{aligned} W(m) &= g^{-1}(u(m))u(m) - \int_0^{g^{-1}(u(m))} g(\omega)d\omega \\ &\quad + mg^{-1}(u(m))\pi(m) - \int_0^m f(\theta)d\theta - C(m), \end{aligned}$$

where $g^{-1}(u(m)) = z(m)$. The planner then solves $\max_m W(m)$. The socially optimal platform size m^W satisfies the first-order condition, which is given by

$$\begin{aligned} & z(m^W)u'(m^W) + \{z(m^W)\pi(m^W) - f(m^W) - c(m^W) \\ & + m^W[z'(m^W)\pi(m^W) + z(m^W)\pi'(m^W)]\} = 0. \end{aligned} \tag{8}$$

Welfare comparison: first results. We now compare the socially optimal platform size m^W with the platform size m^*, which maximizes the sum of marginal net profits of sellers and platform owners. From (3) and (8), it follows that $m^W > m^*$ if and only if

$$z(m^W)u'(m^W) + m^W[z'(m^W)\pi(m^W) + z(m^W)\pi'(m^W)] > 0. \tag{9}$$

A necessary condition for $m^W < m^*$ is thus that $z(m)\pi(m)$ is decreasing in m at $m = m^W$, which can only occur if platform effects are weak, i.e. $m^* > \widehat{m}$.

Amongst the ownership structures considered here, O, C, M, OI, and CI, ownership structure M induces the largest platform size if platform effects are

[5] In our reduced-form model where price-setting of sellers is not explicitly modelled, the planner's problem consists in choosing the platform size m. As regards our underlying micro structure, however, we implicitly assume that the price setting stage is beyond the control of the planner. As is common in the I.O. literature on socially optimal entry (e.g., Weizsäcker, 1980, Mankiw and Whinston, 1986), the planner thus chooses the "second-best solution" by only determining the number of entrants (both sellers and intermediaries), taking consumer behavior and sellers' pricing as given.

strong, i.e. $m^* < \widehat{m}$. Consequently, since $m^W > m^M$, all ownership imply insufficient platform size from a social point of view. If platform effects are weak, i.e. $m^* > \widehat{m}$, then either O or M induce the largest platform size amongst the ownership structures considered here. Since $m^W > m^M$, the only ownership structure that may imply a socially excessive platform size is the open platform ownership structure O.

Proposition 3 *Amongst the ownership structure O, C, M, OI, and CI the only one that* may *lead to socially excessive platform size is the open platform ownership structure O, and only if platform effects are weak, i.e. $m^* > \widehat{m}$. All other ownership structures necessarily induce a socially insufficient platform size.*

Surplus-maximizing platform size. By assumption 5, the total surplus of sellers and intermediaries is single-peaked in m, and attains its maximum at m^S. By assumption 1, consumer surplus increases monotonically in platform size m. As an immediate consequence $m^S < m^W$.[6] Hence, if two ownership structures both induce equilibrium platform sizes that are less than m^S, the ownership structure that induces the larger platform size is socially preferable.

The platform size m^S that maximizes the total surplus of sellers and platform owners is given by

$$m^S = \arg\max_m mz(m)\pi(m) - \int_0^m f(\theta)d\theta - C(m).$$

By assumption 5, there exists a unique m^S. Moreover, if $m^S > 0$, m^S it is the unique solution to the first-order condition

$$\begin{aligned} & z(m^S)\pi(m^S) - f(m^S) - c(m^S) \\ + \quad & m^S[z'(m^S)\pi(m^S) + z(m^S)\pi'(m^S)] = 0. \end{aligned} \tag{10}$$

Comparing m^S with m^*, we obtain that $m^S > m^*$ if the platform externality effect, evaluated at m^*, is positive, i.e.,

$$\frac{m^* z'(m^*)}{z(m^*)} + \frac{m^* \pi'(m^*)}{\pi(m^*)} > 0.$$

This necessarily holds if platform effects are strong or very strong. We have $m^S < m^*$ if the reverse inequality holds. Comparing m^S with $\widehat{m}$, we obtain that $m^S > \widehat{m}$ if

$$\begin{aligned} & z(\widehat{m})\pi(\widehat{m}) - f(\widehat{m}) - c(\widehat{m}) \\ + \quad & \widehat{m}[z'(\widehat{m})\pi(\widehat{m}) + z(\widehat{m})\pi'(\widehat{m})] > 0. \end{aligned}$$

[6] Formally, the reason is that marginal welfare is greater than the marginal total surplus of sellers and buyers. Note that this implies that $m^W > m^S$ independent of whether or not the welfare function is single-peaked.

This necessarily holds if platform effects are weak or very weak. We have $m^S < \widehat{m}$ if the reverse inequality holds. This holds trivially if platform effects are very strong.

Lemma 6 *If platform effects are weak, i.e.* $m^* > \widehat{m}$*, the surplus-maximizing platform size satisfies* $m^S > \widehat{m}$*. If platform effects are strong, i.e.* $m^* < \widehat{m}$*, the surplus-maximizing platform size satisfies* $m^S > m^*$*.*

Note that the surplus-maximizing platform size would be implemented in equilibrium by a monopoly platform owner if she were able to perfectly price discriminate between sellers (i.e., if the rental charge r could be conditioned on the seller's type θ).

Welfare comparison: additional results. We now compare the surplus-maximizing platform size with the equilibrium platform size under different ownership structures.

First, consider open platform ownership. From lemma 1 we know that $m^O = m^*$. Hence $m^S > m^O$ if platform effects are strong or very strong. Also, $m^S > m^O$ if platform effects are weak or very weak but positive at m^*, i.e.

$$\frac{m^* z'(m^*)}{z(m^*)} + \frac{m^* \pi'(m^*)}{\pi(m^*)} > 0. \tag{11}$$

Otherwise, $m^O > m^S$. Since $m^{OI} = m^O$ the same result holds under open integrated platform ownership. Recall that if π is constant, i.e. there is not competition effect, inequality (11) is neccessarily satisfied. This implies that platform size is always socially insufficient.

Second, consider closed platform ownership. From lemma 2 we know that $m^C = \min\{\widehat{m}, m^*\}$. That is, if platform effects are weak, platform size under closed platform ownership is $m^C = \widehat{m}$; if they are strong, platform size is $m^C = m^*$. Hence, $m^S > m^C$ independently of the strength of platform externality. Under closed integrated platform ownership, the platform size coincides with that under non-integrated ownership if platform effects are very weak, strong, or very strong. If they are weak then m^{CI} is determined by $z'(m^{CI})\pi(m^{CI}) + z(m^{CI})\pi'(m^{CI}) = 0$. Since in this case $z(m^{CI})\pi(m^{CI}) - f(m^{CI}) - c(m^{CI}) > 0$ generically, we again obtain that (generically) $m^S > m^{CI}$. Third, consider monopoly platform ownership. Comparing the first order conditions (6) and (10) we immediately observe $m^S > m^M$. Since the non-discriminating monopolist does not extract all of the surplus from the sellers, she has less incentive to develop the platform than if she obtained all of the surplus.

To sum up, the only ownership structures that may induce an equilibrium platform larger than m^S are the open integrated and nonintegrated ownership structures, and this only if platform effects are weak. Under all other ownership structures, the induced equilibrium platform size is smaller than m^S. Since total surplus of sellers and platform owners is single-peaked and consumer surplus is increasing in m, we obtain the following proposition with regards to the welfare ranking.

Proposition 4 *With regard to the different ownership structures the following welfare ranking holds. If platform effects are very weak, either* $W(m^M) > W(m^O) = W(m^{OI})$ *or* $W(m^M) < W(m^O) = W(m^{OI})$*. In addition,* $W(m^M) > W(m^{CI}) = W(m^C)$*. If platform effects are weak,* $W(m^M)$*,* $W(m^O) = W(m^{OI})$*, or* $W(m^{CI})$ *is maximal. In addition,* $\min\{W(m^M), W(m^{CI})\} > W(m^C)$*. If platform effects are strong or very strong,* $W(m^M) > W(m^O) = W(m^{OI}) = W(m^{CI}) = W(m^C)$*.*

5.2 Incentives to Integrate

TBW

6 Platforms with a Subscription Fee

So far, we have assumed that an intermediary's only source of revenue is the rental charge for a platform slot, r. In many real world situations, intermediaries may also charge buyers for their services. In particular, suppose that each buyer who participates in the market place has to pay a subscription fee s to the platform owner(s). In the case of dispersed platform ownership, subscription fees are determined and collected by an agent who represents the intermediaries' interests, and redistributed among the active intermediaries. To simplify the analysis, we assume in this section that the type-ω buyer's disutility of accessing the market place takes the form $g(\omega) = \omega$. This is satisfied in our example

In this section, we consider the three basic ownership structures. Under all ownership structures, the sequence of moves is as follows:

Stage 1a The measure of active intermediaries $\overline{\mu}$ is determined and development costs are sunk.

Stage 1b The subscription fee s is set on behalf of the active intermediaries.

Stage 1c The rental charge r is set by the intermediaries.

Stage 2 Sellers decide whether or not to participate in the platform.

Stage 3 Buyers decide whether or not to participate in the platform.

Determination of the subscription fee. At stage 1b, the monopoly platform owner will set the subscription fee s so as to maximize his profit, given the number of platform slots $\overline{\mu}$. Under dispersed platform ownership, the proceeds from the subscription fee are assumed to be uniformly distributed amongst the active intermediaries. Consequently, the active intermediaries' interests as regards the level of the subscription fee s are perfectly aligned. The representative agent sets the subscription fee s so as to maximize each intermediary's profit. Since the number of active intermediaries is predetermined at this stage, this amounts to maximizing the sum of intermediaries' profits. Hence, for a given number of slots $\overline{\mu}$, the equilibrium subscription fee coincides for all ownership structures, provided the monopolist finds it ex post optimal to rent out all developed platform slots.[7]

If participating in the market place, a type-ω buyer receives net utility $u(\overline{\theta}) - \omega - s$, where $\overline{\theta}$ is the number of active sellers. The measure of buyers on the platform is thus given by $z(\overline{\theta}, s) = u(\overline{\theta}) - s$.

If all developed slots will be rented out, $\overline{\mu} = \overline{\theta} = m$, the equilibrium subscription fee under all ownership structure is the solution to the following program.

$$\max_s m[z(m, s)\pi(m) - F(m)] + sz(m, s).$$

The first-order condition is given by

$$mz_s(m, s)\pi(m) + z(m, s) + sz_s(m, s) = 0$$

for a fixed platform size m. Since $g(\omega) = \omega$ and hence $z_s(m, s) = -1$, the first-order condition can be rewritten as $-m\pi(m) + z(m, s) - s = 0$. The optimal subscription fee thus satisfies

$$s = z(m, s) - m\pi(m).$$

Since $z(m, s) = u(m) - s$, the profit-maximizing subscription fee as a function of platform size m is given by

$$s(m) = \frac{u(m) - m\pi(m)}{2}. \tag{12}$$

Observe that the subscription fee s is positive if the gross utility derived by a unit mass of buyers, $u(m)$, is larger than the sum of sellers' gross profits per unit mass of buyers, $m\pi(m)$, and negative otherwise.

Suppose that $\lim_{m\to 0} u(m) - m\pi(m) < 0$, which holds in our example. Then, the subscription fee is negative if the platform size is sufficiently small. Suppose further that $m\pi(m)$ is weakly decreasing in m, which holds in our example as $m\pi(m)$ is constant in the CES formulation. Then, the subscription fee is strictly increasing in platform size m. Finally, suppose that utility $u(m)$ is not bounded from above, which also holds in our example. Then, the subscription fee is positive if the platform size is sufficiently large. Taken together, these assumptions imply that there exists a unique platform size $\widetilde{m}$ such that $s(m) < 0$ if $m < \widetilde{m}$ and $s(m) > 0$ if the reverse inequality holds. Below, we assume that these properties hold.

These properties imply that under an ownership structure that tends to provide a large platform size it is likely that consumers are charged, provided that this is feasible; whereas under an ownership structure that tends to provide a small platform size it is likely that consumers receive a subsidy, provided that this is feasible.

[7] Clearly, along the equilibrium path, the monopolist will only develop slots that he will rent out later.

Open platform ownership with a shared subscription fee. At stage 1a, free entry of intermediaries implies

$$z(m^{OF}, s(m^{OF}))\pi(m^{OF}) - F(m^{OF}) + s(m^{OF})\frac{z(m^{OF}, s(m^{OF}))}{m^{OF}} - c(m^{OF}) = 0.$$

Since $g(\omega) = \omega$, this equation becomes

$$[u(m^{OF}) - s(m^{OF})]\pi(m^{OF}) - F(m^{OF}) + s(m^{OF})\frac{u(m^{OF}) - s(m^{OF})}{m^{OF}} - c(m^{OF}) = 0, \tag{13}$$

which, substituting for $s(m^{OF})$, becomes

$$\frac{1}{4m^{HF}}[u(m^{OF}) + m^{OF}\pi(m^{OF})]^2 - F(m^{OF}) = c(m^{OF}).$$

Comparing (4) and (13), we find that allowing platform owners to charge a subscription fee unambiguously increases platform size: $m^{OF} \geq m^O = m^*$, where the inequality is strict if $s(m^{OF}) \neq 0$, i.e., $m^{OF} \neq \widetilde{m}$. To see this, suppose the number of entering intermediaries $\overline{\mu}$ is less than m^O, i.e., $\overline{\mu} < m^O$. Since the agent representing the active intermediaries is free to choose, at stage 1b, a subscription fee different from zero, the profit with a zero subscription fee constitutes a lower bound on the active intermediaries' equilibrium profit in the induced subgame. Recall that if the subscription fee is always zero, a number m^O of intermediaries will find it profitable to develop platform slots at stage 1a. Hence, if subscription fees are chosen to maximize each active intermediary's profit, we cannot have $\overline{\mu} < m^O$.

Observe that if $m^* > \widetilde{m}$, and so $s(m^*) > 0$, we must have $s(m^{OF}) > 0$. In words, if platform effects are weak, consumers are charged a positive entrance or subscription fee. If platform effects are strong, consumers may receive a entrance subsidy.

Closed platform ownership with a shared subscription fee.

$$\max_m z(m, s(m))\pi(m) - F(m) + \frac{s(m)z(m, s(m))}{m} \tag{14}$$

$$\text{s.t.} \quad z(m, s(m))\pi(m) - F(m) + \frac{s(m)z(m, s(m))}{m} \geq c(m)$$

If the constraint is non-binding, the first-order condition is given by

$$\begin{aligned} & m^{CF}\left[z_m(m^{CF}, s(m^{CF}))\pi(m^{CF}) + z(m^{CF}, s(m^{CF}))\pi'(m^{CF}) - F'(m^{CF})\right] \\ + \quad & s(m^{CF})z_m(m^{CF}, s(m^{CF})) - \frac{s(m^{CF})z(m^{CF}, s(m^{CF}))}{m^{CF}} = 0, \end{aligned}$$

where we have used the envelope theorem.

Suppose the objective function (14) is single-peaked. How will the chosen platform size be affected when the agent on behalf of the active intermediaries can charge a subscription fee? If

$$\frac{2\left(s(m^C)\right)^2}{m^C}\left[\frac{s'(m^C)m^C}{s(m^C)}-\frac{1}{2}\right]>0,$$

then $m^{CF} < m^C$. Otherwise, $m^{CF} > m^C$. The expression on the l.h.s. is positive if the elasticity of the optimal subscription fee with respect to platform size, evaluated at m^C, is greater than 1/2, i.e.,[8]

$$\varepsilon_{s,m}(m^C) \equiv \frac{s'(m^C)m^C}{s(m^C)} > \frac{1}{2}.$$

In the CES example, this condition is satisfied if and only if $\rho < 2/3$, which means that the utility function $u(m)$ is "not too concave". [CHECK AGAIN] Hence, in the example, if $\rho < 2/3$ then a closed platform will provide a smaller platform size if it can charge consumers than if it cannot. Otherwise, it will provide a larger platform size.

Monopoly platform ownership with a subscription fee. The monopoly platform owner's maximization problem is

$$\max_m m\left[z(m,s(m))\pi(m)-F(m)\right]+s(m)z(m,s(m))-C(m). \tag{15}$$

Using the envelope theorem, the first-order condition of profit maximization reduces to

$$\begin{aligned} z(m^{MF},s(m^{MF}))\pi(m^{MF})-F(m^{MF})+m^{MF}[z_m(m^{MF},s(m^{MF}))\pi(m^{MF}) \\ +z(m^{MF},s(m^{MF}))\pi'(m^{MF})-F'(m^{MF})]+s(m^{MF})z_m(m^{MF},s(m^{MF}))-c(m^{MF}) &= 0. \end{aligned}$$

Since $g(\omega)=\omega$, the first-order condition can be rewritten as

$$\begin{aligned} &[u(m^{MF})-s(m^{MF})]\pi(m^{MF})-F(m^{MF})+m^{MF}\left[u'(m^{MF})\pi(m^{MF})\right. \\ &\left.+[u(m^{MF})-s(m^{MF})]\pi'(m^{MF})-F'(m^{MF})\right]+s(m^{MF})u'(m^{MF})=c(m^{MF}) \end{aligned} \tag{16}$$

Substituting (**??**) into the other first-order condition, we obtain

$$\begin{aligned} &\frac{u(m^{MF})+m^{MF}\pi(m^{MF})}{2}\pi(m^{MF})-F(m^{MF}) \\ &+m^{MF}\left[u'(m^{MF})\pi(m^{MF})+\frac{u(m^{MF})+m^{MF}\pi(m^{MF})}{2}\pi'(m^{MF})-F'(m^{MF})\right] \\ &+\frac{u(m^{MF})-m^{MF}\pi(m^{MF})}{2}u'(m^{MF})=c(m^{MF}) \end{aligned}$$

Suppose the objective function (15) is single-peaked in platform size m. Comparing (6) and (16), we then obtain the following result. If $m^M > \widetilde{m}$,

[8] In terms of the fundamentals of our model, u and π, this conditions reads $m(u(m) - m\pi(m))' > (u(m) - \pi(m))/2$.

which holds if platform effects are weak, and so $s(m^M) > 0$, then allowing the monopoly platform owner to charge a subsciption fee leads to a larger platform size: $m^{MF} > m^M$. If, on the other hand, $m^M < \widetilde{m}$, which holds if platform effects are weak, and so $s(m^M) < 0$, then $m^{MF} < m^M$.

To get an intuition for this result, suppose that $s(m^M) > 0$. For a given platform size m^M, the monopolist obtains revenues $s(m^M)z(m^M, s(m^M))$ from the buyers. Recall that m^M is optimal for $s \equiv 0$ but not if the monopolist can choose s. Just considering revenues from buyers, it thus pays to increase platform size m as this further increases the number of buyers (as well as the optimal subscription fee). Partially counteracting this effect, an increase in platform size m reduces the rent the monopolist can charge to suppliers. The reverse argument holds if $s(m^M) < 0$.

Comparison between ownership structures.

O vs. MF. In the absence of an agent acting on behalf of the intermediaries, intermediaries cannot levy the subscription fee if ownership is open and dispersed. Hence, it seems relevant to compare equilibrium under ownership structure O with that under MF. Recall that in the absence of a subscription fee, platform size under monopoly ownership M is smaller than under open and dispersed ownership O if $m^* > \widehat{m}$. We now show that with the additional instrument s, the monopoly platform owner may offer a larger platform than an open and dispersed platform ownership O (which cannot levy s), even if $m^* > \widehat{m}$. Suppose $m^* = \widehat{m}$. In the absence of a service charge, $m^O = m^M = \widehat{m}$. Comparing (4) and (16), we find that $m^{MF} > m^O = \widehat{m}$ if $s(\widehat{m}) > 0$, i.e., $\widehat{m} > \widetilde{m}$. By continuity, we may have $m^{MF} > m^O$ even if $m^* > \widehat{m}$, i.e., even if the "inverse demand function" of sellers is downward-sloping at m^*. This means that even with weak platform effects, the monopolist, who charges both sides of the market, may provide a larger platform than an open platform which does not have the ability to charge the consumer side. A larger monopoly platform implies a positive entrance or subscription fee for consumers. If, by contrast, it is optimal to subsidize consumers the monopoly platform is always smaller in size than the open platform.

OF vs. MF. We now compare OF and MF, assuming that the objective functions are single-peaked. Since

$$u'(m^{OF})\pi(m^{OF}) + u(m^{OF})\pi'(m^{OF}) - F'(m^{OF}) \leq 0,$$

as we have shown above, a necessary condition for $m^{MF} > m^{OF}$ is that

$$\varepsilon_{s,m}(m^{OF}) \equiv \frac{s'(m^{OF})m^{OF}}{s(m^{OF})} > \frac{1}{2}.$$

TO BE DONE: COMPARISON TO *CF*

TO BE DONE: VERTICALLY INTEGRATED CLOSED PLATFORM

Monopoly platform	**Vertically integrated closed platform**
shopping mall (developer)	covisint (B2B)
factory outlet center	
port/airport	wine producer organization (e.g. VDP)
trade fairs	
many exchanges	property brokerage (multiple listing service)
yellow pages	cooperative buying societies (Rewe, Edeka,...)
ticket restaurant, clean way, ...	
expedia, travelocity,...	
amazon.de	
ISPs (for online shopping)	
operating systems (Windows, Palm, ...)	

Table 3: Examples for Monopoly and Vertically Integrated Platforms

7 Discussion and Conclusions

[to be written]

- competing platforms

Appendix: Cost redistribution

In the two cases of atomistic, non-integrated platform ownership considered in section 3, we have assumed that each intermediary has to pay for the development costs of his platform. Atomistic intermediaries may instead decide to form a redistributive cooperative, where the development costs are (equally) shared. For simplicity of exposition, we make an assumption analogous to assumption 4, namely that

$$\Phi(m) \equiv z(m)\pi(m) - f(m) - \frac{C(m)}{m} \tag{17}$$

is single-peaked in m.

Open redistributive platform. Suppose first that there is open membership to the cooperative formed by intermediaries. The only difference to the case of the open non-redistributive platform ownership (high street) is thus that each active intermediary faces the development cost $C(\overline{\mu})/\overline{\mu}$ if all platform slots $\mu \in [0, \overline{\mu}]$ are developed. For a given rental price of r, free entry of intermediaries thus implies

$$r = \frac{C(\overline{\mu})}{\overline{\mu}},$$

while free entry of retailers implies again

$$z(\overline{\theta})\pi(\overline{\theta}) - f(\overline{\theta}) = r.$$

The competitive rental price r clears the rental market, $\overline{\mu} = \overline{\theta}$. The equilibrium platform size m^{OR} is thus determined by

$$\Phi(m^{OR}) = z(m^{OR})\pi(m^{OR}) - f(m^{OR}) - \frac{C(m^{OR})}{m^{OR}} = 0. \tag{18}$$

Single-peakedness of Φ implies that there is at most one m^{OR} in the declining part of Φ which satisfies $\Phi(m^{OR}) = 0$. Since

$$\frac{C(\overline{\mu})}{\overline{\mu}} < c(\overline{\mu}),$$

we have $\Phi(m^*) > 0$. We then obtain the following lemma.

Lemma 7 *In an open redistributive cooperative, the equilibrium platform size m^{OR} satisfies $m^{OR} > m^*$.*

Note that in a redistributive cooperative, all intermediaries make the same level of profit. Free entry drives these profits down to zero. Hence, if differences in the development costs mirror differences in the efficiency of intermediaries (rather than differences in the slot-specific development costs), low-cost intermediaries would only be willing to join such a cooperative if the cooperative is endowed with the property rights over the platform.

Closed redistributive cooperative. Suppose now that existing members of the cooperative can deny access to the platform. The only difference to the case of the closed non-redistributive platform ownership is thus that each active intermediary faces the development cost $C(\overline{\mu})/\overline{\mu}$ if all platform slots $\mu \in [0, \overline{\mu}]$ are developed.

The optimization problem can be written as

$$\begin{aligned} & \max_{\overline{\mu}} r - \frac{C(\overline{\mu})}{\overline{\mu}} \\ \text{s.t.} \quad & z(\overline{\theta})\pi(\overline{\theta}) - f(\overline{\theta}) = r, \\ & \overline{\theta} = \overline{\mu}, \\ & r \geq \frac{C(\overline{\mu})}{\overline{\mu}}. \end{aligned}$$

Note that, conditional on being admitted to the club, all intermediaries on the platform agree on the optimal number of club members. Market clearing implies a platform size $m = \overline{\theta} = \overline{\mu}$. The optimization problem can thus be rewritten as

$$\begin{aligned} & \max_{m} z(m)\pi(m) - f(m) - \frac{C(m)}{m} \\ \text{s.t.} \quad & z(m)\pi(m) - f(m) \geq \frac{C(m)}{m}. \end{aligned}$$

Since $C(m)/m < c(m)$, it follows from assumption 4 that the objective function Φ is strictly positive at m^*, i.e., $\Phi(m^*) > 0$. Hence, the constraint $\Phi(m) \geq 0$ will not be binding. Any positive equilibrium platform size m^{CR} satisfies the first-order condition

$$z'\left(m^{CR}\right)\pi\left(m^{CR}\right)+z\left(m^{CR}\right)\pi'\left(m^{CR}\right)-f'\left(m^{CR}\right)-\frac{1}{m^{CR}}\left\{c\left(m^{CR}\right)-\frac{C\left(m^{CR}\right)}{m^{CR}}\right\}=0. \tag{19}$$

Since the expression in curly brackets is strictly positive for any $m > 0$, the l.h.s. of (19), evaluated at $m = \widehat{m} > 0$, is negative. Hence, if $\widehat{m} > 0$, the equilibrium platform size m^{CR} must satisfy $m^{CR} < \widehat{m}$. If $\widehat{m} = 0$, then $m^{CR} = 0$. Since Φ is single-peaked in m, and $\Phi(m^{CR}) > \Phi(m^{OR}) = 0$, the equilibrium platform size m^{CR} must also satisfy $m^{CR} < m^{OR}$.

Lemma 8 *In a closed redistributive cooperative, the equilibrium platform size m^{CR} satisfies $m^{CR} < \widehat{m}$ if $\widehat{m} > 0$, and $m^{CR} = \widehat{m} = 0$ otherwise. Moreover, the equilibrium platform size m^{CR} is strictly smaller than in an open redistributive cooperative, $m^{CR} < m^{OR}$.*

Comparison of platform sizes. We now turn to a comparison of the equilibrium platform sizes in open and closed redistributive cooperatives with those under the three basic ownership structures.

Proposition 5 *If platform effects are weak, i.e.* $m^* > \widehat{m}$*, then the equilibrium platform sizes can be ranked as follows:*

$$m^{OR} > m^* = m^O > m^M > \widehat{m} = m^C \geq m^{CR},$$

where the last inequality is strict if $\widehat{m} > 0$.

If platform effects are strong, i.e. $m^* < \widehat{m}$*, then*

$$\min\left\{m^{OR}, m^M\right\} > \max\left\{m^{CR}, m^O = m^C\right\}.$$

Proof. The first part follows immediately from proposition 1 and lemmas 7 and 8. Proposition 1 and lemmas 7 and 8 imply the second part of the proposition as well, except for the comparison between m^C and m^M. To see that $m^C < m^M$, compare (6) and (19), and note that the monopoly platform owner obtains a positive profit, $z(m^M)\pi(m^M) - F(m^M) > C(m^M)/m^M$. Hence, the l.h.s. of (19), evaluated at $m = m^M$, is strictly negative: $\Phi(m^M) < 0$. The assertion then follows from the single-peakedness of Φ. ■

Open redistributive integrated platform. The same conditions hold as under non-integration.

Closed redistributive integrated platform. The maximization for each seller-intermediary becomes

$$\max_{m \geq 0} z(m)\pi(m) - f(\theta) - \frac{C(m)}{m}$$
$$\text{s.t.} z(m)\pi(m) - f(m) - \frac{C(m)}{m} \geq 0.$$

The first-order condition of the unconstrained problem may be written as

$$z'(\widetilde{m}^{CIR})\pi(\widetilde{m}^{CIR}) + z(\widetilde{m}^{CIR})\pi'(\widetilde{m}^{CIR}) - \frac{1}{\widetilde{m}^{CIR}}\left[c\left(\widetilde{m}^{CIR}\right) - \frac{C\left(\widetilde{m}^{CIR}\right)}{\widetilde{m}^{CIR}}\right] = 0,$$

where the term in square brackets is non-negative.

Compared to non-integration, one obtains that unambiguously, $m^{CIR} \geq m^{CR}$. The argument is the same as without redistribution. Compared to non-redistribution we have the following results. If $m^{CI} < m^*$, then $m^{CIR} < m^{CI}$. If $m^{CIR} = m^{OR}$, then $m^{CIR} > m^{CI}$.

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