Sharing risk through social assets
(Preliminary and Incomplete)

Dana Heller
Berglas School of Economics
Tel-Aviv University

Abstract

We present a generational model in which an asset has value in equilibrium as a result of social conventions. The asset need not be tangible but its property rights must be transferable. Transactions and bequest of the asset serve as an instrument to transfer wealth across generations and buffer future income shocks. This social mechanism generates value for the asset and is welfare improving.

In the event that the asset has fundamental value, either through generating income or as a source of direct utility, its additional role as a social asset, while welfare improving, has an ambiguous effect on inequality.

We show that social coordination on the asset can provide agents with the necessary incentives to elect a risky more-productive action over a safer less productive one.

We demonstrate the existence of an asymmetric equilibrium where agents with identical ex-ante opportunities choose different actions; owners of the social asset choosing the riskier more productive action while only some of the non-owners do the same. This causes differences in the ex-post income distribution which could be wrongly attributed to differences in the fundamentals.

1 Introduction

We explore the idea that certain assets may emerge as means of sharing risk within a society. These social assets can range from tangible assets, with or without intrinsic value, to heritable attributes. As a result, their is a component in the price of these assets which is not derived from their objective characteristics but rather from the social convention concerning their value. In the extreme case, these assets may not have any intrinsic value — neither as means of production nor as consumption goods. Notwithstanding, in equilibrium, the assets (or their property rights) are traded for a positive price. This idea goes back to theories of money, e.g., Bewely [3], Kiyotaki and Wright [5] and many papers to follow. In these models, fiat money which is a worthless object receives value in equilibrium. We argue that social arrangements, e.g., conventions as to who possesses social status and an associated mechanism of social mobility, may serve as instruments of sharing risk among members of the population, much like fiat money.

While the exact nature of social status is hard to define, there is much of an agreement within a group as to who has it and who does not. Status is often associated with the possession (or consumption) of certain tangible assets, with behavior according to specific codes or with the possession of certain physical attributes. When goods are concerned, trade of these assets can be easily implemented in a marketplace. However, in order to include acquired behavior and heritable traits as social assets, we need to broaden our definition of a transaction to allow for the possibility of trade through matching between two agents as means of acquiring the asset for one's offspring. The asset is passed on either genetically or through socialization.[1] Payment in exchange for the asset is implemented by sharing joint consumption. Whatever form of transaction we define, the crucial characteristic of the asset is that it could be acquired by spending resources and it could be cashed in return for consumption. Ball, Eckel, Grossman and Zame [1] provide experimental evidence suggesting that status translates into material gains in bilateral trade.

Alternatively, owning the asset may be identified with being a member in an elite group or some other exclusive club. Transactions over the asset are then interpreted as a reduced form for the process of mobility in and out of these social clubs. If membership is acquired by purchasing a share in the club then in effect there is a tangible asset to exchange. However, even when this is not the case, the act of acquiring membership in the club is often achieved by spending resources on activities related to the club, ranging from explicit entrance fees to more indirect measures such as philanthropic activity, and so it can easily be captured by a price. The means by which one sells his membership in the club may be less obvious though we want to think of it as obtaining a one-time gain by exploiting the club, by "defecting" or free-riding on the effort others. This behavior results in an instantaneous gain to the individual though at the cost of future membership in the club.

As in the case of physical assets, membership in the club may or may not be productive by itself and it may or may not generate direct utility. An asset is productive if it is associated with a better income process. If the asset is human capital it can obviously imply its owner has access to higher paid jobs and if it could also be the outcome when the asset is membership in an elite group and so it provides its owner with better access, through social connections, to such jobs. The asset may also generate a stream of direct utility from owning it, e.g., one may enjoy owning a rare painting or being a member in the board of directors of a non-profit organization. Nonetheless, even in the absence of intrinsic value, the asset will be valuable in equilibrium as a result of trade it facilitates. The amount of the asset and its price are determined endogenously in equilibrium.[2]

[1] Mailath and Postlewaite [8] looks at this mechanism.

[2] This is yet another distinction from models of fiat money, where often its supply is assumed to be determined exogenously, while its prices can adjust.

Existing theories of money identify two sources for the value of fiat money. First, its introduction creates a storable mean of exchange, and so it attenuates the problem of double coincidence — it allows trade between two people where one is interested in a good the other has to offer while the latter desires a good held by a third party. Having a medium of exchange that can be traded in exchange for the good improves welfare. The other source of value comes in the presence of risk. When agents are risk averse but they are subject to idiosyncratic risk in the economy, a vehicle of transferring wealth across time and across generations provides a way of insuring against future fluctuations in income. The latter will be the source of the social value in the model presented here. More specifically, since agents are risk averse and they care about their offspring, they are interested in smoothing future income shocks. The possibility of trading the social assets creates an instrument of transferring money across generations. It will be owners of the asset who happen to be poor today that will trade the asset and consumer the additional amount of income. The buyers, rich agents who do not own the asset, forego some current consumption, acquire the asset and bequeath it to their children. The resulting stream of expected utilities to a dynasty, whether it is initially endowed with the asset or not, is higher in the presence of the social asset and the mechanism of trading it than in its absence.

The question of much insurance against risk can be implemented when financial markets are incomplete is pursued for example in Levine and Zame [7]. Kocherlakota [6] explores a mechanism of self insurance through mutual transfers among agents, without a central lender, with the constraint that agents cannot commit to future actions. These features are common with our model. Another closely related paper is Mailath and Postlewaite (2002) [8], M&P hereafter. M&P investigate a generational model where couples are formed, share joint income and have offspring. The matching structure can make a certain heritable attribute — either non-productive such as beauty posture or height or productive, e.g., ability — serve as social assets. While these attributes may not have a direct effect on a person's income process, they may have an indirect effect on one's welfare. This indirect effect comes from their role in coordinating a non-market behavior which is the matching arrangements. In particular, they focus on the existence of a mixed equilibrium where rich agents without the attribute match with poor agents with the attribute. The "transaction" is one where the rich person without the attribute acquires (a chance of having) the trait for his offspring in exchange for current consumption, while the poor agent does the opposite. Mixed matching, when stable, pareto dominates assortative matching on income and the attribute. The source of value, as in our model, is the insurance achieved through the trade of the social asset, in the absence of other financial instruments.[3] Cozzi **??** considers a model where culture serves as means of transferring wealth. While culture is unproductive at the individual level, it is acquired by agents earlier in life so that they can teach it to others later in life as a source of income.[4]

[3] This model can be shown to be a private case of the model proposed in this paper where we allow agents to trade lotteries over the propoerty rights of the asset at stake.

[4] To support investment in culture along with a positive risk-free asset, it is required that the culture has a positive

We identify conditions for the existence of an equilibrium with a social asset traded at a positive price when the asset has no intrinsic value, when it is productive and when it is counter-productive. While the equilibrium amount of the asset is unique there is a range of equilibrium prices. In particular, the equilibrium price may provide under-insurance or over-insurance relative to the price that maximizes average welfare.[5] Whenever the asset is traded at a positive price in equilibrium it increases the welfare of both types of agents. As in all models involving coordination, there exists an equilibrium where the asset does not have value in equilibrium.

The basic set-up illustrates the mechanism by which the asset provides insurance. Next, we explore how the presence of a social asset alters the incentives to undertake certain actions. We extend the model to allow each agent the choice between two actions. One is a risky action and the other is safe, where the former generates higher expected income but lower expected utility than the safe one. By construction, in the absence of risk sharing arrangements, all agents prefer the safe action. Introducing the possibility of endogenous coordination on a social asset gives rise to the following results: (i) For a range of the parameters, there exists a symmetric equilibrium where both types of agents, those in possession of the social asset and those without, choose the risky more productive action. Trade of the asset provides the necessary insurance to facilitate this. The amount of the asset at circulation is determined uniquely while there is a range of equilibrium prices; (ii) There does not exist an equilibrium where the safe action is chosen and the asset is traded at a positive price or an equilibrium where owners of the asset choose the safe action while the others choose the risky assets. This complies with the intuition that ownership of the asset provides agents with insurance and hence it could not be that those without it are willing to bare more risk than those owning the asset; (iii) There is a range of the parameters where an asymmetric equilibrium exists — the elite group, those agents owning the social asset, choose the risky more productive action while members of the non-elite mix between choosing the safe action and the risky action. Therefore, although the agents have the same available actions, they choose differently leading to ex-post differences in expected income across the two groups as well as differences in welfare. For this class of equilibria, the price of the asset is unique while there is a range of values for its amount in circulation, which then determines the percentage of agents from the non-elite who choose the risky action over the safe one. Therefore, the equilibria differ in the amount of ex-post asymmetry they generate in terms of the distribution of income as well as welfare.

The basic mechanism of insurance through social mobility can be applied to investigate differences across social groups when mobility is endogenous. The simple model suggests that the incentives to take different actions may depend on the social hierarchy. In particular, it is not the differences in rewards in the event of success that drive the asymmetry but rather the differences in the fall-back options that

aggregate effect, i.e., that it facilitates growth.

[5] The equilibrium price which maximizes the average welfare implements a symmetric first-best allocation that can be supported by a subgame perfect equilibrium as demonstrated in in Kocherlakota [6].

push those without the asset towards the inferior action.

The paper is structured as follows. Section 2 presents the model. Section 3 analyzes the base-line model with different characteristics of the asset. Section 4 constructs equilibria when there is choice among actions, and Section 5 concludes.

2 Model

Consider an infinite sequence of one-period lived agents. There is a continuum of measure one of agents, with each agent having one child. Income is realized in each generation identically and independently across agents and periods according to the lottery $(H, q; L, 1-q)$, i.e., high income with probability q and otherwise low income.An agent cares about his own utility and the discounted stream of utility to his offspring, with a discount factor β.

Utility is a function of consumption only. The utility function u is twice continuously differentiable and concave, i.e., agents are risk averse. Denote the certainty equivalent of the lottery over income by $CE(H, q; L, 1-q)$, i.e., $u(CE(H, q; L, 1-q)) = qu(H) + (1-q)u(L)$.

Suppose there is an (indivisible) asset or attribute that has no effect on an agent's income distribution. We identify groups in the population with ownership of this asset. In particular, we refer to the group of agents who own the asset as the "elite" and to the remaining agents as the "non-elite". After income is realized, agents transact over the asset and consume their remaining income. The asset is storable and could be transferred from parent to child as bequest.

While we obscure from the details of price formation in the market for the asset we are assuming the following competitiveness structure:

Definition 1 *The market is stable at a price r^* if:*

1. *No potential seller can do better by offering to sell the asset for a price $r < r^*$*
2. *No potential buyer can do better by offering to buy the asset at a price $r > r^*$*
3. *There is perfect information regarding prices and frictionless search for trading partners.*

Note that when the market is stable at a price r there could not be agents with the same ownership status and realized income that make different buy sell decisions unless they are just indifferent between buying and selling at this price. Therefore, if at the market price a high income agent prefers to buy the asset, it must be that the number of sellers is just equal to the number of high income agents without the asset, and similarly for the buyers. Therefore, at any price for which both sides of the market are not indifferent, the share of agents owning the asset, p, must be such that these two sides of the market are of equal size.

An equilibrium is a triplet (p, r, s) where p is a steady-state fraction of agents who own the asset, r is the steady-state price of the asset and s is the exchange structure, i.e., the exchange decision of each agent condition on his group affiliation and income.

Definition 2 *The triplet (p, r, s) is an equilibrium if:*

1. *The exchange structure s is stable given r and p — no agent can do better by changing his buy-sell decision following s given the asset price r and the assumption that future generations will follow the equilibrium.*

2. *The market is stable at the price r given the market structure implied by s and p.*

It may be illustrative to consider the following extreme example of a social asset. Suppose that each agent is endowed with the property rights to a unit of housing identified by its location on the interval $[0, 1]$. Agents can trade the property rights for their housing units, and property rights transfer to offspring. An endogenous social asset can be a "good" house where a good house is any unit with a number higher that some $\tau \in [0, 1]$. In particular, while it may be that all units are intrinsically the same, the equilibrium we construct in the following section gives rise to a price differential between houses with number lower than τ and those with a number higher than τ, creating a valuable social asset.

3 A two-tier equilibrium

We look for an equilibrium where a social asset arises endogenously, i.e., a certain asset or attribute with possibly no productive value, is traded at a positive price implying that owners of the asset enjoy a higher expected stream of utilities. Both the quantity and the price of the social asset are determined *endogenously* in equilibrium so that the market for the asset clears. The equilibrium involves the following transactions. Once income is realized, a rich agent without the asset purchases the asset at market price from some poor agent who owns it. All agents consume all income remained after market transactions. An agent who owns an asset leaves it as bequest to his child.[6]

3.1 The case of an asset with no intrinsic value

Proposition 3 *For $\beta > u'(H)/(qu'(H) + (1 - q)u'(L))$, there exists an equilibrium (p, r, s) with:*

(a) $p = q$.

[6] We assume that agents cannot bequeath money to their children. Alternatively, if there is a risk-free asset that gives a zero (real) interest rate, agents are indifferent between saving this amount of money and sotring it through acquiring the asset.

(b) s, the exchange structure, is such that: H agents who do not own the asset purchase it at the price r from L agents who own

the asset.

(c) $r \in [0, \bar{r}]$*, where* $0 < \bar{r} < H - L$ *is the solution of the equation*

$$u(L+\bar{r}) - u(L) = \frac{1/\beta - q}{1-q}\left(u(H) - u(H-\bar{r})\right)$$

Next we prove the Proposition. The equilibrium gives rise to the following value equations:

$$\begin{aligned} V^e &= q((1-\beta)u(H) + \beta V^e) + (1-q)((1-\beta)u(L+r) + \beta V^{ne}) \\ V^{ne} &= q\left((1-\beta)u(H-r) + \beta V^e\right) + (1-q)((1-\beta)u(H) + \beta V^{ne})). \end{aligned}$$

For exchange to be stable it must be that:

$$\begin{aligned} (condL) &: \quad (1-\beta)u(L+r) + \beta V^{ne} \geq (1-\beta)u(L) + \beta V^e \\ (condH) &: \quad (1-\beta)u(H-r) + \beta V^e \geq (1-\beta)u(H) + \beta V^{ne} \end{aligned}$$

Solving for V^e and V^{ne} gives

$$\begin{aligned} V^e - V^{ne} &= (1-\beta)\left(q\left(u(H) - u(H-r)\right) + (1-q)\left(u(L+r) - u(L)\right)\right) \\ &\text{and} \\ V^e &= qu(H) + (1-q)u(L+r) - \beta(1-q)\left(q\left(u(H) - u(H-r)\right) + (1-q)\left(u(L+r) - u(L)\right)\right) \\ V^{ne} &= qu(H-r) + (1-q)u(L) + \beta q\left(q\left(u(H) - u(H-r)\right) + (1-q)\left(u(L+r) - u(L)\right)\right) \end{aligned}$$

Suppose that $V^e - V^{ne}$ was some exogenous amount, then concavity of the utility function implies that *condL* must be satisfied whenever *condH* is, i.e., poor agents are willing to trade off this amount of utility at any price that the rich are willing to pay for it. However, values are endogenous, hence substituting the difference into the constraints yields

$$u(L+r) - u(L) \geq \beta\left(q\left(u(H) - u(H-r)\right) + (1-q)\left(u(L+r) - u(L)\right)\right) \geq u(H) - u(H-r)$$

For the two constraints to be satisfied simultaneously it must be that $r \leq H - L$. In this domain, the right-hand side, corresponding to the constraint for the poor, is implied by concavity for the reason provided above (and since $\beta < 1$). The left-hand side, corresponding to the constraint for the rich agents, imposes an upper bound on the price r, denoted by $\bar{r}$, where the inequality binds. The condition in the Proposition, requiring β to be sufficiently large, is necessary and sufficient for $\bar{r}$ to be strictly positive. Note that if we assume an Inada condition, i.e., that $\lim_{\varepsilon \to 0} u'(L+\varepsilon) = \infty$, then an equilibrium exists for any positive value of β.

Naturally, in equilibrium, whenever $r > 0$ it implies that $V^e > V^{ne}$. Also, define $V = qV^e + (1-q)V^{ne}$ then,

$$V - (qu(H) + (1-q)u(L)) = q(1-q)\left((u(L+r) - u(L)) - (u(H) - u(H-r))\right) > 0$$

since utility is concave and the equilibrium price is necessarily below $H - L$. Therefore, the expected value of a randomly drawn agent is higher than in the absence of coordination on the social asset. What about the price that maximizes this average value?

$$\frac{\partial V}{\partial r} = -q(1-q)(u'(H-r) - u'(L+r))$$

The first and second order conditions imply that the average value is maximized at the price $r^* = (H-L)/2$. At this price the net income levels of those buying the asset and those selling are equal. We can interpret this as "full insurance" in the two states of the world in which a transaction occurs. Depending on the size of β and the utility function $u()$, the maximal price $\bar{r}$ in equilibrium can be below or above r^*. In the event that $r^* < \bar{r}$ there could be equilibria with too much insurance, i.e., with a transaction price that is higher than the price that maximizes average value.

Moreover,

$$\begin{aligned} V^{ne} &\geq qu(H) + (1-q)u(L) \text{ with equality at } \bar{r} \\ V^{e} &\geq V^{ne} \text{ with equality at } r = 0 \end{aligned}$$

Whenever the equilibrium is stable it is welfare improving for both types of agents. This is the case since those born without the asset today appropriate part of the value it creates in the event that they are rich and acquire the asset. To establish some additional comparative statics results we differentiate the values

$$\begin{aligned} \frac{\partial V^{e}}{\partial r} &= (-1+q)\left(\beta q u'(H-r) - (1-\beta+\beta q)u'(L+r)\right) \\ \frac{\partial V^{ne}}{\partial r} &= q\left(-(1-\beta q)u'(H-r) + \beta(1-q)u'(L+r)\right) \end{aligned}$$

Second order conditions imply that the solution to these conditions characterizes the maximum. Denote the optimal price for V^e (V^{ne}) by $\bar{r}$ ($\underline{r}$). Simple algebra shows that $\underline{r}(\beta) \leq r^* \leq \bar{r}(\beta)$ for any $\beta \in [0,1]$ with equality at $\beta = 1$.[7]

The difference in welfare $V^e - V^{ne} = (1-\beta)\Delta_{(q,q)}(r)$ is increasing in the equilibrium price r.

This analysis of this case illustrates how values is created for a non-productive asset through the inter-generational insurance it provides. In the next two sections we consider equilibria of this kind for assets that have a productive value. There are two ways in which the asset can be fundamentally valuable. First, we assume that the asset provides access to a superior technology of producing income. In particular, this implies that all gains from acquiring the asset are enjoyed by future generations. Second, we assume that the asset is a source of direct utility to its owner though it does not alter one's prospects for future income.

[7] Also, $\underline{r}(\beta)$ is increasing in β, and $\underline{r}(\beta) = 0$ for $\beta < u'(H)/(qu'(H)+(1-q)u'(L))$. On the other hand, $\bar{r}(\beta)$ is decreasing in β, and $\bar{r}(\beta) > H - L$ (which is the above any equilibrium price) for $\beta < u'(H)/((1-q)u'(H) + qu'(L))$.

3.2 The case of a productive asset

We could generalize the case of a non-productive asset that was analyzed in the previous section. Suppose that ownership of the asset determines the technology that produces income. In particular, an agent who owns the asset faces stochastic income drawn from $(H, q; L, 1-q)$, and an agent without the asset faces $(H, t; L, 1-t)$. Define the following functions,

$$\Delta_{(q,t)}(r) = (qu(H) + (1-q)u(L+r) - (tu(H-r) + (1-t)u(L)) \quad (1)$$

$$\Delta_H(r) = u(H) - u(H-r) \quad (2)$$

$$\Delta_L(r) = u(L+r) - u(L) \quad (3)$$

With an exchange structure as defined above, where rich agents acquire the asset from poor agents owning it, the expression for the values is:

$$V^e = qu(H) + (1-q)u(L+r) - \frac{\beta(1-q)}{1+\beta(t-q)}\Delta_{(q,t)}(r) \quad (4)$$

$$V^{ne} = tu(H-r) + (1-t)u(L) + \frac{\beta t}{1+\beta(t-q)}\Delta_{(q,t)}(r) \quad (5)$$

Conditions for stability of the transaction scheme require that:

$$u(H) - u(H-r) \leq \frac{\beta}{1+\beta(t-q)}\Delta_{(q,t)}(r) \leq u(L+r) - u(L) \quad (6)$$

This condition implies, in particular that

$$V^e \geq V^{ne} \quad \Leftrightarrow \quad \Delta_{(q,t)}(r) \geq 0 \; .$$

As before, condition 6 requires that $r < H - L$.

Consider first the case where $q > t$, i.e., owning the asset makes the agent more productive. A two-tier equilibrium with the asset traded at a positive price is stable when conditions 6 are satisfied. The left-hand side of condition 6 defines a strictly positive lower bound $\underline{r}$ and the right-hand side defines an upper bound $\bar{r}$. Refer to Figure 3. The equilibrium is stable for any price $r \in [\underline{r}, \bar{r}]$.

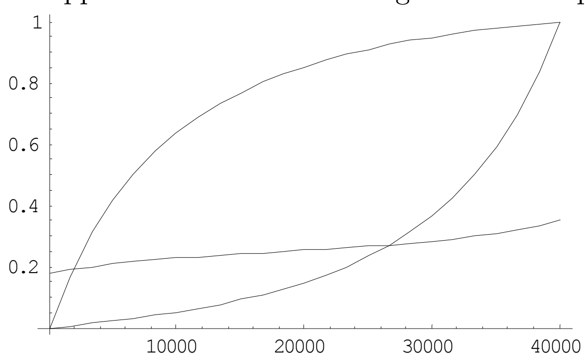

Figure 3. $\beta = 0.3, q = 0.9, t = 0.3$.

Proposition 4 *When* $\beta/(1+\beta(t-q))<1$ *there exist* $0<\underline{r}<\bar{r}<H-L$ *, where*[8]

(a) $\underline{r}$ *is the solution to the equation*

$$u(L+\underline{r})-u(L)=\frac{\beta}{1+\beta(t-q)}\left(qu(H)+(1-q)u(L+\underline{r})-tu(H-\underline{r})-(1-t)u(L)\right) \tag{7}$$

(b) $\bar{r}$ *is the solution to the equation*

$$u(H)-u(H-\bar{r})=\frac{\beta}{1+\beta(t-q)}\left(qu(H)+(1-q)u(L+\bar{r})-tu(H-\bar{r})-(1-t)u(L)\right) \tag{8}$$

such that (p,r,s) *with:*

(a) $p=t/(1-q+t)$.

(b) s*, the exchange structure, is such that:* H *agents who do not own the asset buy it from* L *agents who own it at the price* r.

(c) $r\in[\underline{r},\bar{r}]$.

Note that for an equilibrium to exist, β needs to be below $1/(1+q-t)$ and so cannot be too high. Otherwise poor agents with the asset will not be willing to sell it at a price that rich agents are willing to pay. Alternatively, we can view the condition as requiring that $q-t<(1-\beta)/\beta$ where the interpretation is the the larger the difference between the technologies with and without the asset, the lower has to be the discount factor for the poor agents with the asset to be willing to depart from it and hence worsen their offspring prospects.

The equilibrium price of the asset incorporates its productive value and its social value which arises from the insurance created by the possibility of trade. In particular, Eq. 7 implies that the minimal equilibrium price satisfies that

$$\Delta_L(r)=\frac{\beta}{1+\beta(t-q)}\Delta_{(q,t)}(r)>\beta\left(qu(H)+(1-q)u(L)-tu(H)-(1-t)u(L)\right)=\beta(q-t)(u(H)-u(L))$$

that is, the utility gain today (for a poor person) compensates for the future loss in the expected stream of utility from income. It is easy to see that the difference in welfare is increasing in the equilibrium price r since

$$\frac{\partial(V^e(r)-V^{ne}(r))}{\partial r}=\frac{1-\beta}{1-\beta(q-t)}\frac{\partial\Delta_{(q,t)}(r)}{\partial r}=\frac{1-\beta}{1-\beta(q-t)}\left(qu'(H-r)+(1-q)u'(L+r)\right)>0.$$

Let $E_q(u)=qu(H)+(1-q)u(L)$ denote the welfare in autarchy when the probability of high income is q. Then,

$$\begin{aligned} V^e(\underline{r}) &= E_q(u)>V^{ne}(\underline{r})>E_t(u)\Rightarrow V^e-V^{ne}<E_q(u)-E_t(u)=(q-t)(u(H)-u(L)) \\ V^e(\bar{r}) &> E_q(u)>E_t(u)=V^{ne}(\bar{r})\Rightarrow V^e-V^{ne}>E_q(u)-E_t(u)=(q-t)(u(H)-u(L)) \end{aligned}$$

Therefore, while the role of the productive asset as a social asset is welfare increasing, it can, depending on the equilibrium price, increase or decrease the inequality relative to the difference in welfare that is due to the objective difference in technologies (and endowments).

[8] If there exists more that one solution to each equation, then $\underline{r}$ and $\bar{r}$ are the smallest positive ones respectively.

3.3 The case of a counter-productive asset

Before analyzing a variation of the model where the asset is a source of direct utility, we want to complete our understanding of the social value of an asset in equilibrium when it is counter productive. Suppose ownership of the asset implies an inferior technology of generating income, that is $t > q$.Agents can freely dispose of the asset and use the technology $(H, t; L, 1-t)$. Could the asset be valuable in equilibrium, i.e., provide a higher stream of utility? The answer is yes, where the source of value is the social value that is generated by the possibility of transferring money across generations.

The values and stability constraints are given by equations 4 and 6. Since $t > q$, the right-hand side inequality is implied by concavity and the fact that $\beta/(1+\beta(t-q)) < 1$. The left-hand side inequality imposes a (strictly) lower bound and an upper bound on r. Depending on the parameters q, t, β and the utility function u, the lower bound may or may not be within the required range, i.e., below $H - L$, in which case an equilibrium does not exist. See Figures 1 and 2 for numerical illustrations.

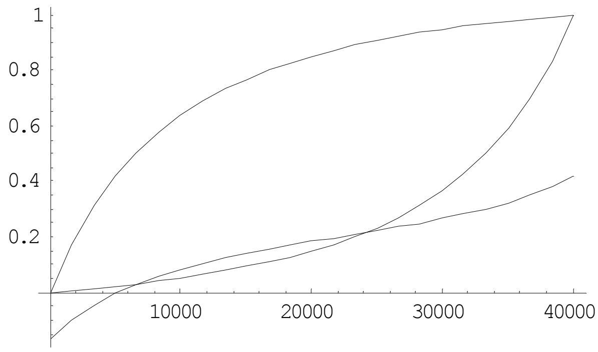

Figure 1. $\beta = 0.5, q = 0.2, t = 0.5$.

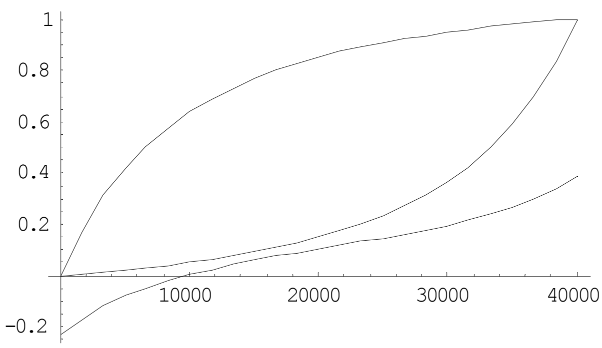

Figure 2. $\beta = 0.5, q = 0.1, t = 0.7$.

When an equilibrium exists, the asset is traded for a positive price, and no agent exercises free disposal. Note that although the asset, which is counter productive, is owned by a fraction of the population in equilibrium, $V^e > V^{ne} \geq tu(H) + (1-t)u(L) = u(CE(H, t; L, 1-t))$. Therefore, this social arrangement is welfare improving relative to the alternative where all agents choose not to own the asset and be (individually) more productive. Formally,

Proposition 5 *If* $\underline{r} < \min\{\bar{r}, H - L\}$, *where* $\underline{r}$ *and* $\bar{r}$ *are the solutions (if exist) to the equation*

$$u(H) - u(H-r) = \frac{\beta}{1+\beta(t-q)} \left(qu(H) + (1-q)u(L+r) - tu(H-r) - (1-t)u(L)\right),$$

then there exists an equilibrium (p, r, s) *with:*

(a) $p = t/(1-q+t)$.

(b) s, *the exchange structure, is such that:* H *agents who do not own the asset trade it with* L *agents who own it for*

the price r.

(c) $r \in [\underline{r}, \min\{\bar{r}, H-L\}]$.

3.4 An asset generating consumption value

Let us start by returning to the motivating example where the asset was assumed to be a housing unit. While in the non-productive case we assumed all housing units were identical, here we assume that certain housing units are intrinsically better than others, providing their owners an additional utility equal to U relative to inferior housing units. We assume that this utility differential U is additively separable, i.e., an agent with an income level x owning the asset enjoys a total utility of $u(x) + U$. However, ownership of the asset is assume to have no effect on future income distribution.

The time line is as follows. Agents inherit the asset from their parents. Individual incomes are realized, followed by transactions over the asset. The "consumption" value of the asset, i.e., the utility differential is enjoyed by the post-transaction owners of the asset along with consumption of the remaining income. The value equations are therefore,

$$\begin{aligned} V^e &= q\left(u(H) + U\right) + (1-q)u(L+r) - \beta(1-q)\widetilde{\Delta}_{(q,q)}(r) \\ V^{ne} &= q\left(u(H-r) + U\right) + (1-q)u(L) + \beta q\widetilde{\Delta}_{(q,q)}(r) \\ \text{with } \widetilde{\Delta}_{(q,q)}(r) &= q\left(u(H) + U\right) + (1-q)u(L+r) - q\left(u(H-r) + U\right) - (1-q)u(L) = \Delta_{(q,q)}(r) \end{aligned}$$

Stability requires that

$$u(H) - u(H-r) - U \leq \beta\Delta_{(q,q)}(r) \leq u(L+r) - u(L) - U$$

Proposition 6 *When $\beta\left(u(H) - u(L)\right) < u(H) - u(L) - U$ there exist $0 < \underline{r} < \bar{r} < H - L$, where*

(a) $\underline{r}$ is the solution to the equation

$$u(L+\underline{r}) - u(L) - U = \beta\left(qu(H) + (1-q)u(L+\underline{r}) - qu(H-\underline{r}) - (1-q)u(L)\right) \tag{9}$$

(b) $\bar{r}$ is the solution to the equation

$$u(H) - u(H-\bar{r}) - U = \beta\left(qu(H) + (1-q)u(L+\bar{r}) - qu(H-\bar{r}) - (1-q)u(L)\right) \tag{10}$$

such that (p, r, s) with:

(a) $p = q$

(b) s, the exchange structure, is such that: H agents who do not own the asset buy it from L agents who own it at the price r.

(c) $r \in [\underline{r}, \bar{r}]$.

Proof. Mimicking the proof of Proposition 4 with $q = t$ and D_L and D_H shifted downwards by U, it is easy to see that the condition of the proposition implies that $F(r)$ is below the shifted $D_H(r)$ at price $r = H - L$. This implies that there exist $0 < \underline{r} < \bar{r} < H - L$ as needed. ■

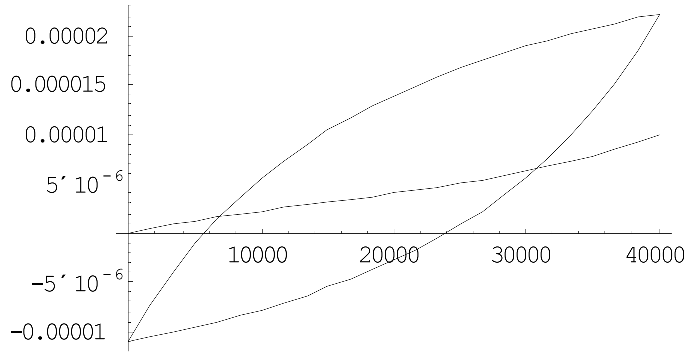

Figure 1:

Similarly to the case of a productive asset, the difference in welfare is increasing in the equilibrium price. It is strictly smaller than U for the minimal equilibrium price, $\underline{r}$, and it is strictly higher than U for the maximal equilibrium price $\overline{r}$. Therefore the effect in inequality depends on the equilibrium price.

One can easily combine the analysis performed in this section and the previous section to characterize equilibria for an asset which is both productive, in terms of the income generating technology, and is a source of direct utility.

4 Inducing risk taking through social assets

Next we want to introduce choice among actions and investigate the effect of the mechanism of coordinating on social assets on the incentives to take risky more productive actions. We consider the following extension to the model explored in the previous section. Assume that each agent can choose between an action (technology) which yields a lottery over income $(H, q; L, 1-q)$ and an action (technology) leading to a sure income M, where

$$CE(H, q; L, 1-q) < M < qH + (1-q)L$$

By construction, without a mechanism of transferring wealth to future generations, agents in equilibrium will choose the safe action over the more productive though risky action. The wealth of each agent in any given generation is M while it would have been $qH + (1-q)L$ had agents chose the risky more productive action. The gap between these two values increases in the degree of risk aversion. Therefore, it is in the interest of members in the group to introduce a mechanism to insure against individual risk and induce the choice of the more productive action. In the absence of financial risk-free instruments to transfer wealth across generations, the group may create mechanisms of insurance by coordinating on (possibly non-productive) social assets. Alternatively, since agents care about their offspring, institutions that transfer money from the rich to the poor in each generation in return for the promise to do so in future generations may arise. To elicit contributions from the rich, payments to future offspring need to be conditioned on a person's record of contributions. Enforcing this mechanism requires monitoring and punishment of offspring of those who fail to contribute. Kocherlakota **??** investigates the formal connections between subgame perfect equilibria under no commitment (with society memory) and monetary equilibria. The following propositions investigate how the mechanism of creating means of exchange — through social assets or other forms of fiat money — alters the incentives to take actions.

Proposition 7 *There does not exist an equilibrium with agents in both groups choosing the safe action and a social asset traded for a positive price.*

Intuitively, in the absence of risk, transaction over the asset is reduced to a transaction between two agents with the same income level and so it is not feasible. Formally,

$$\begin{aligned} V^e &= (1-\beta)u(M+r) + \beta V^{ne} \\ V^{ne} &= (1-\beta)u(M-r) + \beta V^e \\ & \quad \textit{and} \\ (condL) &: (1-\beta)u(M+r) + \beta V^{ne} \geq (1-\beta)u(M) + \beta V^e \\ (condH) &: (1-\beta)u(M-r) + \beta V^e \geq (1-\beta)u(M) + \beta V^{ne} \end{aligned}$$

This system of equations does not have a solution. While the non-existence of this equilibrium seems to depend heavily on the symmetry of choices, the following result sharpens the role of risk. In particular, it shows that there could not be an asymmetric equilibrium where the group owning the asset elects the safe action over the more-productive riskier action while those without the asset prefer the risky action. Since ownership of the asset provides an insurance against a realization of low income, it could not be that agents without this insurance prefer taking the risk while those insured choose the opposite.

Proposition 8 *There does not exist an asymmetric equilibrium* (p, r, s) *such that*

(a) $p, r > 0$

(b) s is such that: (i) agents not inheriting the asset choose the risky action; (ii) agents inheriting the asset choose the safe

action;

(iii) rich agents without the asset transact with (all or part) of the asset owners.

Quite naturally, there exists a symmetric equilibrium where agents in both groups choose the risky action. The equilibrium structure is similar to the one stated in Proposition 3 discussed in the previous section. However, the presence of the safe action imposes additional constraints on stability. In particular, for this to be an equilibrium, it must be that agents in each group prefer the risky action over the safe one given the market price and the possibility to trade at this price. Consequently, in addition to the stability constraints listed in Proposition 3, which put an upper bound on the price, there are three additional constraints:

$$\begin{aligned}
(ne,1) \quad &: \quad V^{ne} \geq u(M) \\
&\Leftrightarrow \quad (qu(H-r)+(1-q)u(L)) + \beta q \Delta_{(q,q)}(r) \geq u(M) \\
(ne,2) \quad &: \quad V^{ne} \geq (1-\beta)u(M-r) + \beta V^{e} \\
&\Leftrightarrow \quad qu(H-r)+(1-q)u(L) + \beta q \Delta_{(q,q)}(r) > u(M-r) + \beta \Delta_{(q,q)}(r) \\
(e,2) \quad &: \quad V^{e} \geq (1-\beta)u(M+r) + \beta V^{ne} \\
&\Leftrightarrow \quad (qu(H)+(1-q)u(L+r)) - \beta(1-q)\Delta_{(q,q)}(r) \geq u(M+r) - \beta \Delta_{(q,q)}(r)
\end{aligned}$$

These constraints, depending on the value of M, β and $u()$, define upper and lower bounds on r. In particular, given a concave utility function, there is a range of values for M above the certainty equivalent and β sufficiently close to one, such that there is a non-trivial range of prices $r > 0$ for which the equilibrium is stable.

Proposition 9 *There exist $\bar{\beta} < 1$ such that for any $\beta > \bar{\beta}$ there is $\bar{M} > M = CE(H,q;L,1-q)$ so that for all $M' \in [M, \bar{M}]$ there are $0 \leq \underline{r}(M,\beta) < \bar{r}(M,\beta) < H - L$ for which (p,s) as defined in Proposition 3 and $r \in [\underline{r}(M,\beta), \bar{r}(M,\beta)]$ is an equilibrium.*

Proof. Set $M = CE(H,q;L,1-q)$. Start with the set of equilibria identified in Proposition 3.Let

$$\begin{aligned}
\Delta_L(r) &= u(L+r) - u(L);\ \Delta_H(r) = u(H) - u(H-r) \\
\Delta_M^{+}(r) &= u(M+r) - u(M);\ \Delta_M^{-}(r) = u(M) - u(M-r)
\end{aligned}$$

The Proof of Proposition 3 established that $V^{ne} > u(M)$ in any equilibrium with a strictly positive price and so $(ne,1)$ is satisfied. In particular for $\overline{r}$ which is the solution to the equation

$$\beta \Delta_{(q,q)}(r) = \Delta_H(r) \tag{11}$$

where there is a minimal $\underline{\beta}$ for which such a solution exists. Note that as β approaches one, $\overline{r}$ approaches $H - L$.

For $M = CE(H,q;L,1-q)$, condition $(ne,2)$ and $(e,2)$ simplify to

$$\begin{aligned}
(ne,2) \quad &: \quad \Delta_M^{-} - q\Delta_H(r) \geq \beta(1-q)\Delta_{(q,q)}(r) \\
(e,2) \quad &: \quad \beta q \Delta_{(q,q)}(r) + (1-q)\Delta_L(r) \geq \Delta_M^{+}(r)
\end{aligned}$$

For $\overline{r}$ we can substitute Eq. 11 into the conditions

$$\begin{aligned} (ne, 2) &\quad : \quad \Delta^{-}_{M}(\overline{r}) - \Delta_H(\overline{r}) \geq 0 \\ (e, 2) &\quad : \quad q\Delta_H(\overline{r}) + (1-q)\Delta_L(\overline{r}) = \Delta_{(q,q)}(\overline{r}) \geq \Delta^{+}_{M}(\overline{r}) \end{aligned}$$

Condition $(ne, 2)$ is implied by concavity with a strict inequality, and so holds for a neighborhood of $r \leq \overline{r}$.

Concavity implies that condition $(e, 2)$ is satisfied with strict inequality for any r greater than some $r' < H - M$. Therefore, let $\overline{\beta}$ be the value for which

$$\overline{r}(\beta) = r'.$$

Since $\overline{r}(\beta)$ is increasing with β, for any $\beta > Max\{\overline{\beta}, \underline{\beta}\}$ condition $(e, 2)$ is satisfied for $r' \leq r \leq \overline{r}$. Since inequalities are strict, there exists an $\underline{r} < \overline{r}$ such that the equilibrium exists for any intermediate r.

Since all the inequalities are strict, for any $\beta > Max\{\overline{\beta}, \underline{\beta}\}$ there is $\overline{M} \geq CE(H, q; L, 1-q)$ such that equilibria with $r \in (\underline{r}(M), \overline{r}(M))$ exist for $M \in [CE(H, g; L, 1-q), \overline{M})$. ■

Next we investigate the existence of asymmetric equilibria in which one group chooses the risky action while the other group chooses the safe action. Note that in an asymmetric equilibrium, although groups face the same set of actions, they are ex-post different in the observed choices of actions. These difference manifest in different expected income distributions, as well as in a positive price for the social asset.

First we identify an equilibrium where those owning the social asset take the risky more productive action while among those who do not own the asset only a fraction chooses the risky action while the others take the safe action. Trade occurs between owners who get a low-income draw and non-owners who chose the risky action and got a high-income draw. Interpreted as social mobility, only those who choose the risky action and succeed get to move into the elite on the expense of those in the elite who got a bad income shock. Since non-owners choose both the safe action and the risky action it must be that they are indifferent among the two. This condition pins down the equilibrium price, which is thus unique. However, the equilibrium amount of the social asset is not unique — as long as the share non-owners taking the risky action is determined so that demand and supply for the asset are equated the equilibrium looks the same. Formally,

Proposition 10 *There exist $\bar{\beta} < 1$ such that for any $\beta > \bar{\beta}$ there is $\bar{M} > M = CE(H, q; L, 1-q)$ so that for all $M' \in [M, \bar{M}]$ there is a range of equilibria (p, r, s) such that:*

(a) $p \leq q$.

(b) s, the exchange structure, is such that: (i) agents who own the asset choose the risky action. If their realized income is L they sell the asset; (ii) α of the agents who do not own the asset choose the risky action. If their realized income is H is purchase the asset; (iii) $1-\alpha$ of those who do not own the asset choose the safe action.

(c) $\alpha = \frac{p(1-q)}{(1-p)q}$

(d) r *solves the equation*

$$u(M) = qu(H-r) + (1-q)u(L) + \beta q(q(u(H) - u(H-r)) + (1-q)(u(L+r) - u(L)))$$

Proof. Since transactions over the asset take place among agents who choose the risky action, the same as in the equilibrium of Proposition 3 the value equations for owners and non-owners are the same, i.e.,

$$V^e = qu(H) + (1-q)u(L+r) - \beta(1-q)\Delta_{(q,q)}(r) \quad (12)$$

$$V^{ne} = qu(H-r) + (1-q)u(L) + \beta q\Delta_{(q,q)}(r) \quad (13)$$

In addition, since non-owners are indifferent between this and choosing the safe action (and not acquiring the asset)

$$V^{ne} = u(M)$$

Stability conditions require

$$(e,1) \quad : \quad \Delta_L(r) \geq \beta\Delta_{(q,q)}(r)$$

$$(e,2) : \quad V^e \geq (1-\beta)u(M+r) + \beta V^{ne}$$

$$(ne,M) \quad : \quad u(M) \geq (1-\beta)u(M-r) + \beta V^e$$

$$(ne,q,1) \quad : \quad \Delta_H(r) \leq \beta\Delta_{(q,q)}(r)$$

When $M = CE(H,q;L,1-q)$, then the equilibrium price is $\overline{r}$ which is the solution to the equation

$$qu(H-r) + (1-q)u(L) + \beta q\Delta_{(q,q)}(r) = u(M) = qu(H) + (1-q)u(L)$$

and so it simplifies to

$$\beta\Delta_{(q,q)}(r) = \Delta_H(r) \quad (14)$$

Therefore, condition $(ne,q,1)$ is satisfied with equality and condition $(e,1)$ is implied by concavity. Condition $(e,2)$ is satisfied for the same reasons established in the proof of Proposition 9 for $\beta > \overline{\beta}$ where $\overline{\beta}$ is the same as in that proof. We are left to show that condition (ne,M) is satisfied. Remembering that

$$V^e - V^{ne} = (1-\beta)\Delta_{(q,q)}(r),$$

we can re-write condition (ne,M) as

$$u(M) \geq (1-\beta)u(M-r) + \beta\left(u(M) + (1-\beta)\Delta_{(q,q)}(r)\right)$$

and so for M set at the certainty equivalent and $r = \overline{r}$ it simplifies to

$$\Delta_M^-(\overline{r}) \geq \beta\Delta_{(q,q)}(\overline{r}) = \Delta_H(\overline{r})$$

which is implied by concavity. By continuity, for any $\beta > \overline{\beta}$ there is a range of M for which this equilibrium exists for sufficiently high β. ■

Proposition 11 *For a given utility function u, parameters M, β, q and a price r, the symmetric equilibrium characterized in Proposition 9 and a (strict) asymmetric equilibrium cannot coexist.*

The intuition for the proof is the following. Consider the equilibrium where the elite, i.e., those inheriting the asset, choose the risky action and the non-elite is chooses the safe action. The values associated with each group member depend on the action chosen by the group members and the action chosen by members of the other group. Consequently, the value to the elite group from choosing the risky action is different depending on whether the non-elite is doing the same, as in the symmetric equilibrium, or choosing the safe action, as in the asymmetric equilibrium. Naturally, the value to the non-elite is different in the symmetric equilibrium where it takes the risky action and in the asymmetric equilibrium where it chooses the safe action. Notwithstanding, since the elite in both cases is behaving in the same manner, i.e., choosing the risky action, the stability condition for the non-elite in the two equilibria coincides. That is, requiring that an agent prefers the safe action over the risky action in the asymmetric equilibrium is exactly in contrast to requiring the agent to prefer the risky action over the safe one in the symmetric equilibrium. We provide the proof in the Appendix.

We illustrate the existence of a strict asymmetric equilibrium using numerical simulations. Since this equilibrium cannot coexist with the symmetric two-tier equilibrium, and the latter exists for a wide range of the parameters, this strict asymmetric equilibrium lives on a relatively small domain. We denote this equilibrium the outcast equilibrium, as it is the case that if one happens to have a bad income shock, he leaves the group and for one period he takes the safe action and buys his way back into the group. The edge of this set of equilibria is an equilibrium of the type characterized in Proposition 10.

Proposition 12 *There exists a range of M (strictly above the certainty equivalent) and β (sufficiently large) such that there exists $0 < \underline{r}(M, \beta) < \bar{r}(M, \beta) < H - L$ for which the following (p, s, r) is an equilibrium:*

(a) $p = 1/(2 - q)$

(b) s, the exchange structure, is such that: agents who do not inherit the asset choose the safe action; agents who inherit the

asset choose the risky action. All agents without the asset transact with poor agents who own the asset for the price r.

(c) $r \in [\underline{r}(M, \beta), \bar{r}(M, \beta)]$.

5 Comparison with heritable social assets

This section explores a generalization of the mechanism discussed Section 2 where rather than trading the asset at the market place, agents can trade a lottery over the asset. Introducing this instrument allows us to include a version of the mixed matching equilibrium constructed in M&P [8] in our framework. In M&P, a certain (non productive) heritable attribute becomes a social asset in equilibrium when mixed matching is stable in the population — rich agents without the attribute are willing to forego current consumption by matching with poor agents who possess the attribute. Therefore, they are suffering a reduction in today's consumption, by sharing a smaller joint income, for the prospect of their offspring inheriting the attribute. As a result, having the attribute implies better prospect of marriage and higher expected utility in equilibrium. Similarly to this paper, the source of value is risk, and so mixed matching (in the absence of other risk-free storable assets) serves as an instrument of transferring wealth across generations. Since the asset at stake is a heritable attribute, transactions are done through matching and sharing joint income. In M&P's basic model, genetic transmission is such that the matching structure does not affect the share of agents with the attribute in the population, i.e., the two offspring of parents both with (without) the attribute possess (do not possess) the attribute, while among the two children of a mixed couple (one with the attribute and one without) one possesses the attribute and one does not. Therefore, when a rich agent without the attribute matches with a poor agent with the attribute, the average expected utility of his two offspring is equivalent to the expected utility of each of his children if the attribute transfers with a probability 1/2 given that one parent carries it.

Consider the following generalization of the basic model presented in Section 2. Suppose that agents can trade lottery tickets over the asset, where exchange (of the asset or its property rights) occurs with probability τ. That is, when a person without the asset acquires the ticket, he receives the asset with probability τ, in which case the owner of the asset looses its ownership. The basic model of Section 3 corresponds to $\tau = 1$. Moreover, the mixed matching in M&P described above corresponds to $\tau = 0.5$.[9] Given any fixed probability τ, the amount of the asset required for the market to clear is determined by the mass of rich agents in the population which is equal to q. The value equations are:

$$\begin{aligned} V^e_\tau &= q((1-\beta)u(H) + \beta V^e) + (1-q)((1-\beta)u(L+r) + \beta((1-\tau)V^e + \tau V^{ne}) \\ V^{ne}_\tau &= q\left((1-\beta)u(H-r) + \beta((1-\tau)V^{ne} + \tau V^e\right) + (1-q)((1-\beta)u(H) + \beta V^{ne})) \end{aligned}$$

Solving for the values, we get

$$V^e_\tau - V^{ne}_\tau = \frac{(1-\beta)}{(1-\beta(1-\tau))}\Delta_{(q,q)}(r).$$

[9] Note that assuming joint income is a public good corresponds to restricting the price of the asset to be $r = H - L$ in our model. Alternatively, if we allow any division of joint income and assume private consumption we get a mechansim analogous to our market mechanism with $\tau = 0.5$.

A sufficient condition for the existence of an equilibrium is

$$\beta > \frac{u'(H)}{(1-\tau(1-q))u'(H)+\tau(1-q)u'(L)}$$

Stability requires the equilibrium price to be in the interval $[0, \overline{r}]$, where $\overline{r}$ is the solution to the equation

$$\Delta_H(r) = \frac{\beta\tau}{1-\beta(1-\tau)}\Delta_{(q,q)}(r).$$

Given a fixed equilibrium price r, the average expected utility in the population is independent of τ, i.e.,

$$qV^e(r) + (1-q)V^{ne}(r) = q(qu(H) + (1-q)u(L+r)) + (1-q)(qu(H-r) + (1-q)u(L))$$

Moreover, differentiating the values yields the following,

Proposition 13 *Given an equilibrium price r, $V^e(\tau)$ is decreasing in τ and $V^{ne}(\tau)$ is increasing in τ, while the average expected utility is independent of τ.*

Proof. Solving for the values and differentiating yields

$$\begin{aligned}
\frac{\partial V^{ne}(\tau)}{\partial \tau} &= \frac{(1-\beta)\beta q(q\Delta_H(r) + (1-q)\Delta_L(r))}{(1-\beta(1-\tau))^2} > 0 \\
\frac{\partial V^{e}(\tau)}{\partial \tau} &= \frac{(1-\beta)\beta(-1+q)\,(q\Delta_H(r) + (1-q)\Delta_L(r))}{(1-\beta(1-\tau))^2} < 0
\end{aligned}$$

■

Therefore, we see that the mechanism of transacting over social assets can be viewed as a reduced form for a variety of social arrangements which in equilibrium involve coordination on the value of some asset, widely defined, which can be traded physically (and let as bequest) or can be acquired for one's offspring through matching. The effect of introducing lotteries on the nature of equilibria, in particular when agents can bargain over the pair of (τ, r) is an interesting one, still left to be explored. A recent paper by Berentsen, Molico and Wright [2], introduces lotteries into a search-theoretic model of fiat money. In contrast to the framework suggested above, they show that when the good is divisible good, a nonrandom quantity is traded with probability one while money trades with some probability smaller than one if buyers have sufficient bargaining power. It is interesting to apply their framework to our model where risk is the source of value for money.

6 Appendix

Proof of Proposition 4. Define

$$\begin{aligned}
F(r) &= \frac{\beta}{1+\beta(t-q)}\left(qu(H) + (1-q)u(L+r) - tu(H-r) - (1-t)u(L)\right) \\
D_L(r) &= u(L+r) - u(L) \\
D_H(r) &= u(H) - u(H-r)
\end{aligned}$$

$F(0) > 0$ since $q > t$, while $F(H-L) < u(H) - u(L)$ since $\beta/(1+\beta(t-q)) < 1$. Note that

$$\begin{aligned} D_L(r) &\geq D_H(r) \text{ for } 0 \leq r \leq H-L \\ D_L(0) &= D_H(0) = 0, \quad D_L(H-L) = D_H(H-L) = u(H) - u(L) \end{aligned}$$

Therefore, $F(r)$ must cross $D_L(r)$ at some $0 < \underline{r}$. Moreover,

$$\begin{aligned} F'(r) &= \frac{\beta}{1+\beta(t-q)} \big((1-q)u'(L+r) + tu'(H-r)\big) > 0 \\ F''(r) &= \frac{\beta}{1+\beta(t-q)} \big((1-q)u''(L+r) - tu''(H-r)\big) \end{aligned}$$

and so the function $F(r)$ can at most change, as r increases, from being concave to convex. Since $D_L(r) \geq D_H(r)$ it implies that F must cross D_H from above before it crosses D_L again. Also, since $F(H-L) < D_H(H-L)$, this crossing point, denoted by $\bar{r}$ must be below $H-L$. ■

Proof of Proposition 8. Consider the values:

$$\begin{aligned} V^e &= (1-\beta)u(M+r) + \beta V^{ne} \\ V^{ne} &= q\left((1-\beta)u(H-r) + \beta V^e\right) + (1-q)\left((1-\beta)u(L) + \beta V^{ne}\right) \end{aligned}$$

Stability requires that, on the one hand, the elite prefers the safe action over taking the lottery and cashing out if poor, while, on the other hand, the non-elite prefers the risky action over the value of the safe action:

$$\begin{aligned} V^e &\geq q\left((1-\beta)u(H) + \beta V^e\right) + (1-q)\left((1-\beta)u(L+r) + \beta V^{ne}\right) \\ V^{ne} &\geq u(M) \end{aligned}$$

Substituting the values into the stability condition of the elite one gets:

$$\frac{-(1-\beta)}{(1+\beta q)} \big(\beta q^2 \left(D_H(r) + D_L(r)\right) + q\left(u(H) - u(L+r) + \beta D_L(r)\right) + u(L+r) - u(M+r)\big) \geq 0$$

This requires the parabola in q to be non-negative. Note that the parabola has a negative coefficient on q^2. It intersects zero in (x_1, x_2) where

$$\begin{aligned} x_{1,2} &= \frac{-u(H) + \beta u(L) + (1-\beta)u(L+r) \mp \sqrt{\Delta}}{2\beta\left(D_H(r) + D_L(r)\right)} \\ \text{with } \Delta &= \left(u(H) - \beta u(L) - (1-\beta)u(L+r)\right)^2 - 4\beta\left(D_H(r) + D_L(r)\right)\left(u(L+r) - u(M+r)\right) \end{aligned}$$

Algebra calculations show that $x_1 > 0, x_2 > 1$. The stability condition for the non-elite is a linear function in q with a positive slope coefficient,

$$\frac{q\left(u(H-r) - u(L) + \beta\left(u(M+r) - u(M)\right)\right) + u(L) - u(M)}{(1+\beta q)}$$

It intersects the parabola at $(-1/\beta, x_3)$ where $0 < x_3 < 1$. Graphing the two functions gives that, for values greater than $-1/\beta$, since the parabola is above the line it cannot be that the parabola is negative while the line is positive until x_3 where they cross. But then the parabola only becomes negative again at a value of q greater than 1. Therefore, these two conditions cannot be satisfied together for $0 < q < 1$ and an the proposed equilibrium does not exist. ■

Proof of Proposition 11. The values in the symmetric equilibrium are given by

$$\begin{aligned} V^{e,qq} &= q\left((1-\beta)u(H) + \beta V^{e,qq}\right) + (1-q)\left((1-\beta)u(L+r) + \beta V^{ne,qq}\right) \\ V^{ne,qq} &= q\left((1-\beta)u(H-r) + \beta V^{e,qq}\right) + (1-q)\left((1-\beta)u(L) + \beta V^{ne,qq}\right) \end{aligned}$$

with the stability condition for the non-elite requiring that

$$V^{ne,qq} \geq (1-\beta)u(M-r) + \beta V^{e,qq}$$

Solving for the values and substituting into the constraint gives:

$$(1-\beta)\left(qu(H-r) + (1-q)u(L) - u(M-r) - \beta q \Delta_{(q,q,)}(r)\right) \geq 0$$

The values in the only candidate asymmetric equilibrium, given Proposition 8, is where the elite is taking the risk action while the non-elite members are choosing the safe action resulting in

$$\begin{aligned} V^{e,qM} &= q\left((1-\beta)u(H) + \beta V^{e,qM}\right) + (1-q)\left((1-\beta)u(L+r) + \beta V^{ne,qM}\right) \\ V^{ne,qM} &= (1-\beta)u(M-r) + \beta V^{e,qM} \end{aligned}$$

with the stability condition for the non-elite requiring that

$$V^{ne,qM} \geq q\left((1-\beta)u(H-r) + \beta V^{e,qM}\right) + (1-q)\left((1-\beta)u(L) + \beta V^{ne,qM}\right)$$

Solving for the values and substituting into the constraint gives

$$\frac{(1-\beta)}{-(1+\beta(1-q))}\left(qu(H-r) + (1-q)u(L) - u(M-r) - \beta q \Delta_{(q,q,)}(r)\right) \geq 0$$

Hence the two constraint could not be satisfied simultaneously. ■

References

[1] S. Ball, C. Eckel, P. Grossman and W. R. Zame, Status in Markets, *Quarterly Journal of Economics,* 116 (2001), 161-188.

[2] A. Berentsen, M. Molico and R. Wright, Indivisibilities, Lotteries and Monetary Exchange, mimeo 2000.

[3] T. Bewely, A Difficulty with the Optimal Quantity of Money, *Econometrica*, 51 (1983), 1485-1504.

[4] G. Cozzi, Culture as a Bubble, *Journal of Political Economy*, 106 (1998), 376-394.

[5] N. Kiyotaki, and R. Wright, On Money as a Medium of Exchange, *Journal of Political Economy*, 97 (1989), 927-954.

[6] N. Kocherlakota, Money is Memory, *J. Econ Theory*, 81 (1998), 232-251.

[7] D. Levine and W. Zame, Does Market Incompleteness Matter?, *Econometrica*, 70 (2001), 1805-1840.

[8] G. Mailath and A. Postlewaite, Social Assets, University of Pennsylvania Working Paper, 2002.