The Effect of Better Information on Income Inequality

by

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August 2002

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Abstract

We consider an OLG economy with endogenous investment in human capital. Heterogeneity in individual human capital levels is generated by random innate ability. The production of human capital depends on each individual's investment in education. This investment decision is taken only after observing a signal which is correlated to his/her true ability, and which is used for updating beliefs. Thus, a better information system affects the distribution of human capital in each generation. Assuming separable and identical preferences for all individuals, we derive the following results in equilibrium: (a) If the relative measure of risk aversion is less (more) than 1 then more information raises (reduces) income inequality. (b) When a risk sharing market is available better information results in higher inequality regardless of the measure risk aversion.

1 Introduction

In recent years we have witnessed growing interest of economists in the determinants of income inequality and its evolution in dynamic models. Many papers in this field focus on the role of education systems for the distribution of income. These papers have produced mixed results, both on the theoretical and empirical level, suggesting that better public education systems do not necessarily lead to less income inequality [e.g., Glomm and Ravikumar (1992), Sylwester (2002a,2002b)]. Another main issue relates to the effects of various institutional settings and macroeconomic policies on the distribution of income in equilibrium [see, for example, Loury (1981), Galor and Zeira (1993), Benabou (1996), Orazem and Tesfatsion (1997), Aghion (2002)].

The endogenous growth literature has investigated the causes of inequality in income distribution, concentrating on three main transmission channels. Firstly, differences in unobservable individual talent may generate income inequality [e.g., Juhn et al. (1993)]. Secondly, based on subjective assessments of individual talent agents may choose different levels of investment in education [e.g., Galor and Tsiddon (1997), Viaene and Zilcha (2002)]. Thirdly, the stock of human capital of parents may affect their children's learning. If this linkage is specific to the household it will contribute to income inequality [e.g., Hassler and Mora (2000)]. Our paper focuses on the first two channels. We develop an approach which embeds the social mechanism of selecting individual education levels in an endogenous growth model where agents differ with respect to ability. When young, each agent is screened by an information system which provides him with imperfect information about his talent. The central issue of our study is how better information (i.e., more efficient screening in the education period) affects the intragenerational income distribution.

Over the last decades the literature in the field of the economics of information has put much emphasis on the issue how welfare effects of information are related to the market structure of an economy. Many studies have examined the value of information in various partial equilibrium models [e.g.,Blackwell (1951,1953), Green (1981)] and in general equilibrium frameworks [e.g., Hirshleifer (1971,1975), Orosel (1996), Schlee (2001), Eckwert and Zilcha (2001a)]. However, to the best of our knowledge, this paper contains the first attempt to study the impact of information systems on the distribution of income in an endogenous growth model.

Our analytical framework is an OLG economy where investment in education is done under uncertainty. Individuals in the same generation differ in their (random) innate abilities. When ability is still unknown each individual decides how much 'effort' to invest in his/her education and training. The return to this investment, in term of wages during the working period, is random since it depends on the realization of the ability. The effort decision is made after observing a signal which contains information about the agent's random ability. We analyze the effect of better information system, i.e., more efficient screening, upon the distribution of income in each generation. Of course, better information has a significant impact on the accumulation of human capital as well and, hence, on economic growth. This aspect has been studied in a separate paper [see, Eckwert and Zilcha (2001b)] to which we will refer occasionally.

Our analysis concentrates on the intragenerational distribution of average income across groups of individuals with a given, but unknown, ability. We demonstrate that income inequality may either increase or decrease with a better information system. More precisely, assuming constant relative risk aversion utility functions, we show that better information increases (decreases) income inequality if the relative measure of risk aversion is smaller (larger) than 1. Risk aversion plays such a crucial role because it affects the behavior of the optimal effort level: the effort level increases (decreases) as the agent receives a more favorable information signal if relative risk aversion is below 1 (above 1). Thus, better information may either increase or decrease the dispersion in the distribution of investments in education – a fact which critically contributes to our result about the consequences of better information for income inequality.

We also study the role of risk sharing arrangements for the link between information and the distribution of income. Assuming that the signals convey information related to an insurable part of the random ability (or, the rate of return to investment in human capital) we show that better information always enhances income inequality. By contrast, if the information system is fixed, risk sharing reduces income inequality if relative risk aversion exceeds 1.

The paper is organized as follows. In section 2 we describe the OLG economy and define our concept of informativeness. In section 3 we study the effect of better information on income inequality. Section 4 deals with the same issue in the presence of an insurance market. In section 5 we analyze the consequences of risk sharing on income inequality for a given information system. To facilitate reading all proofs are relegated to section 6, the Appendix.

2 The Model

Consider an overlapping generations economy with a single commodity which is traded each period $t = 0, 1, \cdots$. The commodity can either be consumed or used as an input (physical capital) in a production process (see, e.g., Azariadis and Drazen (1990)) . The generations reproduce identically over time. Each generation consists of a continuum of individuals who live for three periods. In their first period ('youth') agents obtain education while they are still supported by their parents. In their second period ('middle-age') they work and spend part of the labor income for consumption; and in their third period ('retirement') they consume their savings. We denote by $G_t, t = 0, 1, \cdots$ the generation of all agents born at date $t - 1$.

The agents within a given generation differ with regard to their human capital. Human capital, $\tilde{h}^i$, of agent $i \in G_t$ is determined by a random innate ability, $\tilde{A}^i$, the effort $e^i \in \mathbb{R}_+$ invested in education by this individual, and the 'environment' when education takes place represented by the average human capital of agents in the previous generation:

$$\tilde{h}^i = \tilde{A}^i g(H_{t-1}, e^i) \tag{1}$$

H_{t-1} denotes the average human capital of G_{t-1} (to be defined below), and $g : \mathbb{R}^2_+ \to \mathbb{R}_+$ is a twice differentiable function which is strictly increasing in both arguments.

When 'young' the ability of agent i is uncertain. The random variable $\tilde{A}^i$ realizes at the beginning of agent i's middle-age period and takes values in some interval $\mathcal{A} \subset \mathbb{R}_+$. We assume that the random variables $\tilde{A}^i, i \in G_t, t = 0, 1, \cdots$, are i.i.d., so that the ex ante distribution of ability is the same for all agents.[1]

[1] In the sequel we will therefore suppress the index i and write $\tilde{A}$ instead of $\tilde{A}^i$. Note, however, that in general the random variables $\tilde{A}^i$ and $\tilde{A}^j$ differ for $i \neq j$; only their distributions are the

Before an agent chooses optimal effort in the youth period he receives a signal which contains information about his unknown ability. We consider information systems in which signals are represented by a random variable $\tilde{y}$ taking values in $Y \subset \mathbb{R}$. Each agent $i \in G_t, t = 0, 1, \cdots$, with ability A observes an individual signal y^i which is drawn randomly from the distribution of the random variable $(\tilde{y}|A)$.[2] By construction, this individual signal is correlated to i's ability. Therefore, when agent i makes his decision about how much effort e^i to invest in education, the relevant c.d.f. for random ability is the posterior distribution of $\tilde{A}^i$ given the individual signal y^i. Since the random variables $\tilde{A}^i, \in G_t, t = 0, 1, \cdots$, are i.i.d, any two agents who receive the same signal will base their respective effort decisions on the same posterior distribution of ability.

For convenience we normalize the measure of agents in each generation to 1:

$$\int_{\mathcal{A}} \nu(A) \mathrm{d}A = 1,$$

where $\nu(A)$ is the (Lebesgue)-densitiy of agents with ability A. Denote by $f(\cdot|A)$ the density of the random variable $(\tilde{y}|A)$, and by $\nu_y(\cdot)$ the density of the random variable $(\tilde{A}|y)$. Using this notation, the distribution of signals received by agents in the same generation has the density[3]

$$\mu(y) = \int_{\mathcal{A}} f(\cdot|A)\nu(A)\mathrm{d}A. \tag{2}$$

And average ability of all agents who have received the signal y is

$$\bar{A}(\nu_y) := \int_{\mathcal{A}} A\nu_y(A)\mathrm{d}A. \tag{3}$$

The agents are expected utility maximizers with von-Neumann Morgenstern lifetime utility function

$$U(e, c_1, c_2) = v(e) + u_1(c_1) + u_2(c_2). \tag{4}$$

Individuals derive negative utility from ‘effort’ while they are young and positive utility from consumption in the working period, c_1, and from consumption in the retirement period, c_2.

same.

[2]Throughout the paper we shall refer to the realizations of $\tilde{y}$ as signals, and to the realizations of the $\tilde{y}^i$'s as *individual* signals.

[3]Note that, by the law of large numbers, μ does not depend on t.

Assumption 1 *The utility functions* v *and* u_j, $j = 1, 2$, *have the following properties:*

(i) $v : \mathbb{R}_+ \to \mathbb{R}_-$ *is decreasing and strictly concave,*

(ii) $u_j : \mathbb{R}_+ \to \mathbb{R}$ *is increasing and strictly concave,* $j = 1, 2$.

In each period, competitive firms transform physical capital K and human capital H into a consumption/investment good. The transformation process can be described by an aggregate production function $F(K, H)$ which exhibits constant returns to scale. If individual i supplies l^i units of labor in his 'working period', his supply of human capital equals $l^i h^i$. We assume inelastic labor supply, i.e., that l^i is a constant and it is equal to 1 for all i.

Assumption 2 $F(K, H)$ *is concave, homogeneous of degree 1, and satisfies* $F_K > 0$, $F_H > 0, F_{KK} < 0, F_{HH} < 0$.

We also assume throughout this paper full international capital mobility, while human capital is assumed to be immobile. As a consequence, at each date t the interest rate $\bar{r}_t$ is exogenously given and marginal productivity of aggregate physical capital K_t is equal to $1 + \bar{r}_t$ (assuming full depreciation of capital in each period). Thus, given the aggregate stock of human capital at date t, H_t, the stock K_t must adjust such that

$$1 + \bar{r}_t = F_K(K_t, H_t) \qquad t = 1, 2, 3, \cdots \tag{5}$$

holds. Equation (5) and Assumption 3 imply that $\frac{K_t}{H_t}$ is determined by the international rate of interest $\bar{r}_t$. Hence the wage rate w_t (price of one unit of human capital), is given in equilibrium by the marginal product of aggregate human capital, is also determined once $\bar{r}_t$ is given. Thus we may write

$$w_t = F_L\Big(\frac{K_t}{H_t}, 1\Big) =: \zeta(\bar{r}_t) \qquad t = 1, 2, 3, \cdots . \tag{6}$$

Now let us consider the optimization problem that each $i \in G_t$ faces, given $\bar{r}_t, w_t$, and H_{t-1}. At date $t-1$, when 'young', this individual chooses the optimal level of effort employed in obtaining education. This decision is made under random ability $\tilde{A}$, but after the individual signal y^i has been observed. The decision about saving, s^i, to be used for consumption when 'old' is taken in the second period, after the realization of $\tilde{A}$, and hence when the human capital h^i is known. Thus s^i will depend on h^i via the wage earnings $w_t h^i$.

For given levels of h^i, w_t and $\bar{r}_t$, the optimal saving decision of individual $i \in G_t$ is determined by

$$\begin{aligned} \max_{s^i} \ & u_1(c_1^i) + u_2(c_2^i) \\ \text{s.t. } & c_1^i = w_t h^i - s^i \\ & c_2^i = (1+\bar{r}_t)s^i \end{aligned} \tag{7}$$

and satisfies the necessary and sufficient first order condition

$$-u_1'(w_t h^i - s^i) + (1+\bar{r}_t)u_2'((1+\bar{r}_t)s^i) = 0 \tag{8}$$

for all h^i. From equation (8) we find optimal saving as a function of each realized h^i, i.e., $s^i = s_t(h^i)$. The optimal level of effort invested in education, e^i, is determined by

$$\begin{aligned} \max_{e^i} \ & E[v(e^i) + u_1(\tilde{c}_1^i) + u_2(\tilde{c}_2^i)|y^i] \\ \text{s.t. } & \tilde{c}_1^i = w_t \tilde{h}^i - \tilde{s}^i \\ & \tilde{c}_2^i = (1+\bar{r}_t)\tilde{s}^i, \end{aligned} \tag{9}$$

where $\tilde{h}^i$ is given by equation (1) and $\tilde{s}^i$ satisfies equation (8). Due to the Envelope theorem and the strict concavity of the utility functions, problem (9) has a unique solution determined by the first order condition

$$v'(e^i) + w_t g_2(H_{t-1}, e^i) E[\tilde{A} u_1'(w_t \tilde{h}^i - \tilde{s}^i)|y^i] = 0. \tag{10}$$

Since u_1' is a decreasing function we also conclude from (8) that $s_t(h^i)$ and $w_t h^i - s_t(h^i)$ are both increasing in h^i. This implies, in particular, that the LHS in (10) is strictly decreasing in e^i. Similarly, from equation (10) we obtain the optimal level of effort as a function of the conditional distribution ν_{yi}, i.e., $e^i = e_t(\nu_{yi})$. Note that any two agents in generation t who receive the same individual signal will choose the same effort level.

Using (2) and (3) the aggregate stock of human capital at date t can be expressed as

$$H_t = E_y[\bar{h}_t(\nu_y)] = \int_Y \bar{h}_t(\nu_y)\mu(y)\mathrm{d}y, \tag{11}$$

where

$$\bar{h}_t(\nu_y) := \bar{A}(\nu_y)g(H_{t-1}, e_t(\nu_y)) \tag{12}$$

is the average human capital of agents in G_t who have received the signal y.

Definition 1 *Given the international interest rates $(\bar{r}_t)$ and the initial stock of human capital H_0, a competitive equilibrium consists of a sequence $\{(e^i, s^i)_{i \in G_t}\}_{t=1}^{\infty}$, and a sequence of wages $(w_t)_{t=1}^{\infty}$, such that:*

(i) *At each date t, given $\bar{r}_t$, H_{t-1}, and w_t, the optimum for each $i \in G_t$ in problems (9) and (7) is given by (e^i, s^i).*

(ii) *The aggregate stocks of human capital, $H_t, t = 1, 2, \cdots$, satisfy (11).*

(iii) *Wage rates $w_t, t = 1, 2, \cdots$, are determined by (6).*

2.1 Information Systems

The ability of each individual i is a random variable $\tilde{A}^i$. We assume that the random variables $\tilde{A}^i$ are i.i.d. across individuals in G_t, $t = 0, 1, 2...$, and that they all have the same distribution as $\tilde{A}$. We shall refer to the realizations of $\tilde{A}$ as the states of nature. Before a young agent with ability A chooses an optimal effort level he observes an individual signal which is drawn randomly from the distribution of the random variable $(\tilde{y}|\tilde{A}^i = A) = (\tilde{y}|\tilde{A} = A) =: (\tilde{y}|A)$. Thus, ex ante the conditional distributions of the individual signals are identical. For convenience, we shall refer to the realizations of $\tilde{y}$ simply as signals.

An information system, which will be represented by $f : Y \times \mathcal{A} \to \mathbb{R}_+$ throughout the paper, specifies for each state of nature A a conditional probability function over the set of signals. The positive real number $f(y|A)$ defines the conditional probability (density) that if the state of nature is A, then the signal y will be sent. We assume that the densities $\{f(\cdot|A), A \in \mathcal{A}\}$ have the strict monotone likelihood ratio property (MLRP): $y' > y$ implies that for any given (nondegenerate) prior distribution for A, the posterior distribution conditional on y' dominates the posterior distribution conditional on y in the first-order stochastic dominance.[4] As a consequence, $\int_{\mathcal{A}} \varphi(A)\nu_{y'}(A)\mathrm{d}A > \int_{\mathcal{A}} \varphi(A)\nu_y(A)\mathrm{d}A$ holds for any strictly increasing function φ.

By the law of large numbers, the prior distribution over $\mathcal{A}$ coincides with the ex post distribution of ability across agents. Also the prior distribution over Y coincides with the ex post distribution of individual signals across agents and, hence, is given by equation (2). Finally, the density function for the updated posterior distribution over $\mathcal{A}$ is

$$\nu_y(A) = f(y|A)\nu(A)/\mu(y). \tag{13}$$

Blackwell (1953) proposed a criterion that compares different information systems by their informational contents. Suppose $\bar{f}$ and $\hat{f}$ are two information systems with associated density functions $\bar{\nu}_y$, $\hat{\nu}_y$, $\bar{\mu}$, $\hat{\mu}$. Blackwell defined the informativeness of an information system as follows:

Definition 2 *Let $\bar{f}$ and $\hat{f}$ be two information systems. $\bar{f}$ is said to be more informative in the Blackwell sense than $\hat{f}$(expressed by $\bar{f} \succ_{\text{inf}}^{B} \hat{f}$), if there exists an integrable function $\lambda : Y^2 \to \mathbb{R}_+$ such that*

$$\int_Y \lambda(y', y)\mathrm{d}y' = 1 \tag{14}$$

holds for all y, and

$$\hat{f}(y'|A) = \int_Y \bar{f}(y|A)\lambda(y', y)\mathrm{d}y \tag{15}$$

holds for all $A \in \mathcal{A}$.

According to this criterion $\bar{f} \succ_{\text{inf}}^{B} \hat{f}$ holds if $\hat{f}$ can be obtained from $\bar{f}$ through a process of randomization, i.e., by adding some random noise.

Given an information system f and an ability level A, define

$$L_f^A(z) := \mathrm{pr}\left[\frac{f_A}{f}(\tilde{y}|A) \leq z\right] = \int_{\frac{f_A}{f}(y|A)\leq z} f(y|A)\,\mathrm{d}y$$

[4]For details see Milgrom (1981).

where f_A denotes the partial derivative. $L_f^A(z)$ is called the *likelihood ratio distribution* of an agent with ability A under information system f.

Lemma 1 *$\frac{f_A}{f}(y|A)$ is monotone increasing in y and, hence, the likelihood ratio distribution function can be written as*

$$L_f^A(z) = F\left(\left(\frac{f_A}{f}\right)^{-1}(z)\Big|A\right), \tag{16}$$

where $F(y|A)$ is the c.d.f. for the random variable $(\tilde{y}|A)$.

Kim (1995) has shown that the likelihood ratio distribution under $\bar{f}$, $L_{\bar{f}}^A(z)$, is a mean preserving spread of that under $\hat{f}$, $L_{\hat{f}}^A(z)$, if $\bar{f}$ is more informative (in the Blackwell sense) than $\hat{f}$:

Lemma 2 *Let $\bar{f}$ and $\hat{f}$ be two information systems such that $\bar{f} \succ_{\text{inf}}^B \hat{f}$. For any $A \in \mathcal{A}$, $L_{\bar{f}}^A(z)$ is a mean preserving spread (MPS) of $L_{\hat{f}}^A(z)$. That is, both distribution functions have the same mean and*

$$\int^{z'} \bar{F}\left(\left(\frac{\bar{f}_A}{\bar{f}}\right)^{-1}(z)\Big|A\right) \mathrm{d}z \geq \int^{z'} \hat{F}\left(\left(\frac{\hat{f}_A}{\hat{f}}\right)^{-1}(z)\Big|A\right) \mathrm{d}z \quad \forall z' \in \mathbb{R} \tag{17}$$

with the strict inequality holding for some range of $z' \in \mathbb{R}$ with positive measure.

Proof: see Kim (1995).

Inequality (17) can be transformed into an integral condition that will turn out to be a useful tool for the analysis in this paper.

Lemma 3 *Inequality (17) is satisfied for all $z' \in \mathbb{R}$ if and only if the following integral condition holds for all $\vartheta \in [0,1]$:*

$$S(\vartheta|A) := \int_0^{\vartheta} \left[\frac{\bar{f}_A}{\bar{f}}\left(\bar{F}^{-1}(s|A)\Big|A\right) - \frac{\hat{f}_A}{\hat{f}}\left(\hat{F}^{-1}(s|A)\Big|A\right)\right] \mathrm{d}s \leq 0. \tag{18}$$

Proof: This lemma is a straightforward modification of Proposition 3 in Demougin and Fluet (2001). The proof is therefore omitted. □

$\bar{f} \succ_{\inf}^{B} \hat{f}$ implies that the expectation of ability $\tilde{A}$ conditional on $\underline{y}$ is higher under information system $\hat{f}$ than under information system $\bar{f}$. This observation follows from the following assessment (for arbitrary $\hat{A} \in \mathcal{A}$):[5]

$$
\begin{aligned}
E^{\bar{f}}[\tilde{A}|\bar{F}^{-1}(0|\hat{A})] &= E^{\bar{f}}[\tilde{A}|\underline{y}] = \int_{\mathcal{A}} A\bar{\nu}_{\underline{y}}(A)\,\mathrm{d}A \\
&< \frac{1}{\hat{\mu}(\underline{y})} \int_{Y} \bar{\mu}(y')\lambda(\underline{y}, y') \int_{\mathcal{A}} A\bar{\nu}_{y'}(A)\,\mathrm{d}A\mathrm{d}y' \\
&= \int_{\mathcal{A}} A\hat{\nu}_{\underline{y}}(A)\,\mathrm{d}A = E^{\hat{f}}[\tilde{A}|\underline{y}] = E^{\hat{f}}[\tilde{A}|\hat{F}^{-1}(0|\hat{A})]. \qquad (19)
\end{aligned}
$$

By a similar argument, the expectation of $\tilde{A}$ conditional on $\bar{y}$ is higher under information system $\bar{f}$ than under $\hat{f}$,

$$
E^{\bar{f}}[\tilde{A}|\bar{F}^{-1}(1|\hat{A})] > E^{\hat{f}}[\tilde{A}|\hat{F}^{-1}(1|\hat{A})], \quad \forall \hat{A} \in \mathcal{A}. \qquad (20)
$$

From (19) and (20) we conclude that the conditional expectations $E^{\bar{f}}[\tilde{A}|\bar{F}^{-1}(s|\hat{A})]$ and $E^{\hat{f}}[\tilde{A}|\hat{F}^{-1}(s|\hat{A})]$ cross at least once.

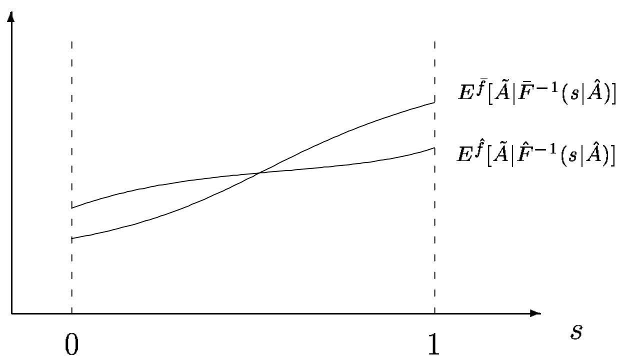

Figure 1: Conditional expectations under information systems f and $\hat{f}$.

In this paper we will focus on pairs of information systems for which the ratio

[5]The second equality makes use of Bayes' rule which implies

$$
\hat{\nu}_y(A) = \frac{1}{\hat{\mu}(y)} \int_Y \bar{\mu}(y')\bar{\nu}_{y'}(A)\lambda(y, y')\,\mathrm{d}y'.
$$

of the corresponding conditional expectations of ability is strictly monotone. This implies the single crossing property (as in the above figure). Since a signal reveals the same information about the random variable $\tilde{A}$ as about $h(\tilde{A})$ when $h : \mathcal{A} \to \mathbb{R}$ is strictly monotone, the ordering of information systems should be invariant with respect to strictly monotone transformations. We will therefore work with the following concept of informativeness:

Definition 3 (informativeness) *Let $\bar{f}$ and $\hat{f}$ be two information systems. $\bar{f}$ is more informative than $\hat{f}$ (expressed by $\bar{f} \succ_{\inf} \hat{f}$), if*

(i) *the integral condition (18) is satisfied,*

(ii) *for any $\hat{A} \in \mathcal{A}$ and any strictly increasing (decreasing) function $h : \mathcal{A} \to \mathbb{R}_+$ the ratio of the conditional expectations*

$$\frac{E^{\bar{f}}[h(\tilde{A})|\bar{F}^{-1}(s|\hat{A})]}{E^{\hat{f}}[h(\tilde{A})|\hat{F}^{-1}(s|\hat{A})]}$$

is strictly increasing (decreasing) in s.

Condition (ii) postulates that under a better information system the conditional expectation of any monotone transformation of $\tilde{A}$ reacts more sensitively, in percentage terms, to changes in the (transformed) signal s. In this sense an increase in the transformed signal s constitutes better news if the information system is more informative, i.e., if the signal is more reliable.

3 Comparison of Income Distributions

To facilitate the comparison of income distributions under different information systems we restrict the utility functions $u_1(\cdot)$, $u_2(\cdot)$, and $v(\cdot)$ to be in the family of CRRA:

$$u_1(c_1) = \frac{c_1^{1-\gamma_u}}{1-\gamma_u}; \quad u_2(c_2) = \beta \frac{c_2^{1-\gamma_u}}{1-\gamma_u}; \quad v(e) = -\frac{e^{\gamma_v+1}}{\gamma_v+1}. \tag{21}$$

γ_u and γ_v are strictly positive constants. γ_v parametrizes the curvature of the utility function in the youth period, v; and γ_u parametrizes the curvature of the utility functions in the middle age period and retirement period, u_i, $i = 1, 2$.

We also assume that the function g in (1) has the form

$$g(H,e) = \hat{g}(H)e^{\alpha}, \tag{22}$$

where $\hat{g}$ is strictly increasing in H, and $\alpha \in (0,1)$.

Using the functional forms of u_j, $j = 1, 2$, in (21), it follows from equation (8) that, given $\bar{r}_t$ and w_t, the saving s^i is proportional to the human capital level h^i. In other words, for each t and for each $i \in G_t$ we have:

$$s^i = m_t h^i, \quad 0 < m_t < w_t, \quad t = 1, 2, \cdots \tag{23}$$

The specifications in (21), (22) and (23) allow us to solve equation (10) for the optimal effort level as a function of the conditional distribution ν_y:[6]

$$e_t(\nu_y) = \delta_t \Big(E[\tilde{A}^{1-\gamma_u}|y] \Big)^{\rho/\alpha} \tag{24}$$

where

$$\delta_t := \left[\frac{\alpha w_t (\hat{g}(H_{t-1}))^{1-\gamma_u}}{(w_t - m_t)^{\gamma_u}} \right]^{\rho/\alpha}; \quad \rho = \frac{\alpha}{\gamma_v + \alpha(\gamma_u - 1) + 1}.$$

We will discuss the role of information for the distribution of expected individual incomes across agents of different types *before* ability and signals are revealed. For that purpose we focus on the average income of all agents with given ability A. The distribution of average income within this class of agents across different ability levels will serve as a measure of inequality.

3.1 Information and Inequality in the Absence of Risk Sharing

Let

$$\begin{aligned} I_t^f(A) &:= \int_Y w_t \tilde{h} f(y|A)\,\mathrm{d}y \\ &= w_t \hat{g}(H_{t-1}) \delta_t^{\alpha} A \int_Y \Big(E^f[\tilde{A}^{1-\gamma_u}|y] \Big)^{\rho} f(y|A)\,\mathrm{d}y \end{aligned} \tag{25}$$

be the expected income, as of date $t-1$ of an agent in generation t with ability A. We measure ex ante income inequality by the elasticity of expected income with respect to ability.

[6]Since all agents in generation t who receive the same signal will make the same savings and effort decisions we suppress the index i in the sequel.

Definition 4 (income inequality) *Income distribution under information system $\bar{f}$ is said to be more unequal than income distribution under information system $\hat{f}$, if*

$$\varepsilon[I_t^{\bar{f}}, A] := \frac{\partial I_t^{\bar{f}}(A)}{\partial A} \frac{A}{I_t^{\bar{f}}(A)} \geq \frac{\partial I_t^{\hat{f}}(A)}{\partial A} \frac{A}{I_t^{\hat{f}}(A)} =: \varepsilon[I_t^{\hat{f}}, A] \tag{26}$$

holds for all $t \geq 0$ and $A \in \mathcal{A}$.

We will show below (cf. part (i) in Theorem 1) that for $\gamma_u \leq 1$ and given $A \in \mathcal{A}$ the elasticity $\varepsilon[I_t^f, A]$ is bounded from below by $\varepsilon[I_t^{f^0}, A]$, where f^0 denotes the uninformative system. From (25) it is immediate that $\varepsilon[I_t^{f^0}, A] = 1$ for all A. Similarly, for $\gamma_u \geq 1$ the elasticity $\varepsilon[I_t^f, A]$ is bounded from below by $\varepsilon[I_t^{f^1}, A]$ where f^1 is the fully informative system. Since under f^1 the signal reveals an agent's talent, (25) reduces to

$$[w_t \hat{g}(H_{t-1}) \delta_t^\alpha]^{-1} I_t^{f^1}(A) = A^{1+(1-\gamma_u)\rho}$$

and, hence, $\varepsilon[I_t^{f^1}, A] = 1 + (1-\gamma_u)\rho = (1+\gamma_v)/[1+\gamma_v+\alpha(\gamma_u - 1)] > 0$. Thus the elasticities in (26) are strictly positive.

Remark: The above definition of income inequality implies the definition given by Atkinson (1970) in the sense that the normalized income under $\hat{f}$, $||I_t^{\hat{f}}(A)|| := I_t^{\hat{f}}(A)/E[I_t^{\hat{f}}(A)]$, dominates in second degree stochastic dominance the normalized income under $\bar{f}$, $||I_t^{\bar{f}}(A)|| := I_t^{\bar{f}}(A)/E[I_t^{\bar{f}}(A)]$. In other words, $||I_t^{\bar{f}}(A)||$ and $||I_t^{\hat{f}}(A)||$ differ by a MPS and, hence, the Lorenz curve for $||I_t^{\hat{f}}(A)||$ lies strictly above that for $||I_t^{\bar{f}}(A)||$.

Observe that

$$\operatorname{sign}\big(\varepsilon[I_t^{\bar{f}}, A] - \varepsilon[I_t^{\hat{f}}, A]\big) = \operatorname{sign}\big(\varepsilon[\bar{I}^{\bar{f}}, A] - \varepsilon[\bar{I}^{\hat{f}}, A]\big), \tag{27}$$

where

$$\bar{I}^f(A) := I_t^f(A)/w_t \hat{g}(H_{t-1}) \delta_t^\alpha A, \quad f = \bar{f}, \hat{f}.$$

$\bar{I}^f(A)$ can be rewritten as

$$\begin{aligned} \bar{I}^f(A) &= \int_0^1 \left(E^f[\tilde{A}^{1-\gamma_u} | F^{-1}(s|A)]\right)^\rho f\big(F^{-1}(s|A)|A\big)\big(F^{-1}\big)'(s)\,\mathrm{d}s \\ &= \int_0^1 \left(E^f[\tilde{A}^{1-\gamma_u} | F^{-1}(s|A)]\right)^\rho \mathrm{d}s, \quad (f, F) = (\bar{f}, \bar{F}), (\hat{f}, \hat{F}) \end{aligned} \tag{28}$$

Differentiating (28) we obtain

$$\frac{\partial \bar{I}^f(A)}{\partial A}\frac{1}{\bar{I}^f(A)} = \frac{1}{\bar{I}^f(A)}\int_Y \left(E^f[\tilde{A}^{1-\gamma_u}|y]\right)^\rho f_A(y|A)\,\mathrm{d}y$$

$$= \int_0^1 \frac{\left(E^f\left[\tilde{A}^{1-\gamma_u}|F^{-1}(s|A)\right]\right)^\rho}{\int_0^1 \left(E^f\left[\tilde{A}^{1-\gamma_u}|F^{-1}(\hat{s}|A)\right]\right)^\rho \mathrm{d}\hat{s}} \frac{f_A}{f}\left(F^{-1}(s|A)\Big|A\right)\mathrm{d}s. \quad (29)$$

Combining (27) and (29) yields a useful characterization of ex ante inequality. Define

$$\Gamma^F(s|A) := \frac{\left(E^f\left[\tilde{A}^{1-\gamma_u}|F^{-1}(s|A)\right]\right)^\rho}{\int_0^1 \left(E^f\left[\tilde{A}^{1-\gamma_u}|F^{-1}(\hat{s}|A)\right]\right)^\rho \mathrm{d}\hat{s}}; \quad (f,F) = (\bar{f},\bar{F}),(\hat{f},\hat{F}) \quad (30)$$

Proposition 1 *Income inequality in the sense of Definition 4 is higher under information system $\bar{f}$ than under information system $\hat{f}$, if and only if*

$$\int_0^1 \left[\Gamma^{\bar{F}}(s|A)\frac{\bar{f}_A}{\bar{f}}\left(\bar{F}^{-1}(s|A)\Big|A\right) - \Gamma^{\hat{F}}(s|A)\frac{\hat{f}_A}{\hat{f}}\left(\hat{F}^{-1}(s|A)\Big|A\right)\right]\mathrm{d}s \geq 0, \quad (31)$$

holds for all $A \in \mathcal{A}$.

If relative risk aversion, γ_u, is equal to 1, expected individual income is linear in ability and, hence, income inequality is not affected by better information. To verify this claim we first observe that

$$\int_0^1 \frac{f_A}{f}\left(F^{-1}(s|A)\Big|A\right)\mathrm{d}s = \int_Y f_A(y|A)\,\mathrm{d}y = 0. \quad (32)$$

Proposition 2 *If $\gamma_u = 1$, income inequality does not depend on the chosen information system.*

Proof: $\gamma_u = 1$ and equation (32) together imply that (31) is satisfied with equality for all $A \in \mathcal{A}$. Thus inequality of the income distribution is the same for any two information systems $\bar{f}$ and $\hat{f}$. □

For $\gamma_u = 1$ the individual effort level is not responsive to the information revealed by a signal (cf. (24)). As a consequence, the distribution of expected incomes across agents does not depend on the information system either. Yet, when γ_u differs from 1 a better information system may give rise to more or less income inequality.

Theorem 1 *Let $\bar{f}$ and $\hat{f}$ be two information systems with $\bar{f} \succ_{\text{inf}} \hat{f}$.*

(i) *If $\gamma_u \leq 1$, the income distribution under $\bar{f}$ is more unequal than the income distribution under $\hat{f}$.*

(ii) *If $\gamma_u \geq 1$, the income distribution under $\hat{f}$ is more unequal than the income distribution under $\bar{f}$.*

Information affects the distribution of efforts at each date. Under a more precise information system the agents' effort decisions respond more aggressively to signals. Also, on average highly talented individuals receive better signals than individuals with low talent. When $\gamma_u < 1$ individual effort is increasing in the signal. This implies more dispersion in the distribution of efforts (see equation (25)) and, as a result, we find that a more informative system enhances income inequality. For $\gamma_u > 1$ better talented individuals invest less in education (see (25) again), i.e., effort levels are less dispersed in this case and, hence, inequality is reduced.

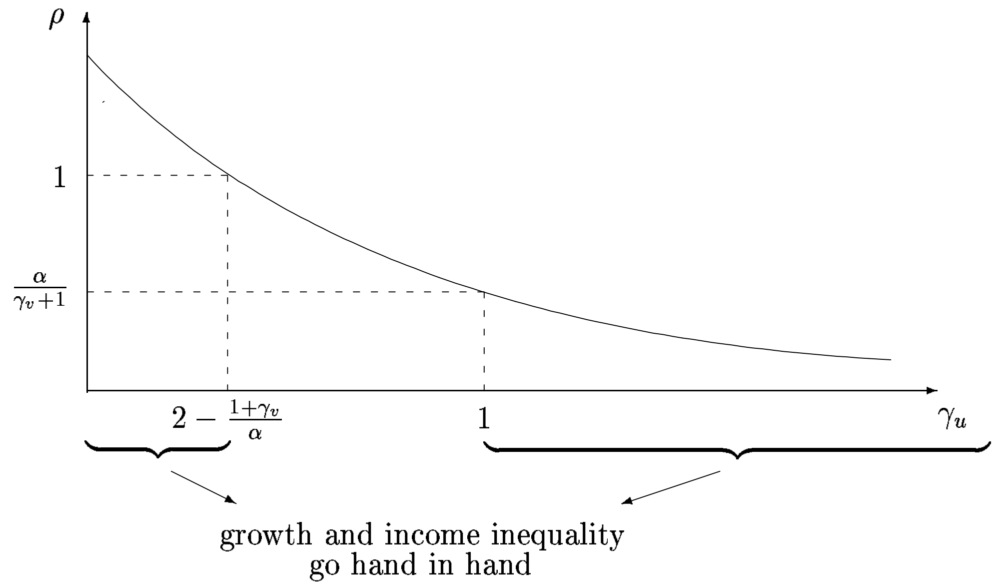

Figure 2.

The relationship between income inequality and growth is a controversial issue in the literature (see, e.g., Persson and Tabellini (1994), Forbes (2000)). Let us relate to this issue in our information context. We have shown in a separate work (see Eckwert and Zilcha (2001b)) that in this framework, when ρ is larger than 1 (hence γ_u is less than $2-(1+\gamma_v)/\alpha$) more information enhances growth; and if γ_u is larger than 1, more information reduces the stock of human capital in all dates, hence reduces growth. Thus, given our above Theorem, when γ_u is larger than 1 lower growth and less inequality occur simultaneously with more information. On the other hand, if γ_u is less than $2-(1+\gamma_v)/\alpha$, higher growth and higher inequality go hand in hand as the information system improves. Thus, whenever $\gamma_u \notin [2-(1-\gamma_v)/\alpha, 1]$, better information entails that growth and inequality move in the same direction.

4 Information and Inequality with Risk Sharing

This section proceeds on the assumption that part of the uncertainty of an agent's ability is insurable. Let $\tilde{A} = \tilde{A}_1 \cdot \tilde{A}_2$, where $\tilde{A}_1$ and $\tilde{A}_2$ are stochastically independent random variables which take values in $\mathcal{A}_1$ and $\mathcal{A}_2$. Before agents make decisions about effort they can insure the risk which is associated with the $\tilde{A}_1$- component of their (unknown) ability. The random variable $\tilde{A}$ has the same properties as in the previous section. In particular, since individual ability is identically and independently distributed across the members of each generation, there exists no aggregate risk in our economy. As a consequence, the insurance market for the $\tilde{A}_1$-risk will be unbiased, i.e., the agents can share this risk on fair terms. In Section 3.1 the signals affected only uninsurable risks. In this section we assume that the signals contain only information about the insurable risk factor $\tilde{A}_1$, i.e., $\tilde{A}_2$ and the signal are independent.[7]

In order to introduce the risk sharing market we need to assume that the $\tilde{A}_1$-component of individual ability is verifiable by the insurers. The random future income of each individual will then have an insurable component as well as an uninsurable component. Denote by $\bar{A}_1(\nu_y)$ the expected value of $\tilde{A}_1$ if the signal y

[7] In fact, the analysis in Section 3 can be understood as being conducted in the same stochastic framework as here with the signals containing only information about the uninsurable risk factor $\tilde{A}_2$.

has been observed,

$$\bar{A}_1(\nu_y) := \int_{\mathcal{A}} A_1 \nu_y(A)\,\mathrm{d}A. \tag{33}$$

Since the insurance market is unbiased, all agents find it optimal to completely eliminate the $\tilde{A}_1$- risk from income in their second period of life. Thus the optimal saving and effort decisions of individual $i \in G_t$ satisfy the following first order conditions

$$(1+\bar{r}_t)u_2'\big((1+\bar{r}_t)s^i\big) - u_1'\big(w_t\bar{A}_1(\nu_{y^i})A_2 g(H_{t-1},e^i) - s^i\big) = 0 \quad (A_2 \in \mathcal{A}_2) \tag{34}$$

$$v'(e^i) + w_t g_2(H_{t-1},e^i)E\big[\tilde{\bar{A}}(\nu_{y^i})u_1'\big(w_t\tilde{\bar{A}}(\nu_{y^i})g(H_{t-1},e^i) - s^i\big)|y^i\big] = 0 \quad (y^i \in Y), \tag{35}$$

where

$$\tilde{\bar{A}}(\nu_{y^i}) := \bar{A}_1(\nu_{y^i}) \cdot \tilde{A}_2. \tag{36}$$

It is our aim to analyze the impact of information on income inequality if agents are able to share part of the uncertainty about their random ability. We are also interested in studying the role of risk sharing for income inequality for a *given* information system. Our next theorem deals with the former issue. It investigates whether better information about the insurable risk increases or decreases income inequality.

Using the functional specifications (21)-(24) the average income of an agent with ability $A = A_1 \cdot A_2$ is

$$\begin{aligned} I_t^f(A_1,A_2) &= w_t\delta_t^\alpha\hat{g}(H_{t-1})\Big(E\big[\tilde{A}_2^{1-\gamma_u}\big]\Big)^\rho A_2 \int_Y \Big(E^f\big[\tilde{A}_1|y\big]\Big)^\tau f(y|A_1)\,\mathrm{d}y \\ &= w_t\delta_t^\alpha\hat{g}(H_{t-1})\Big(E\big[\tilde{A}_2^{1-\gamma_u}\big]\Big)^\rho A_2 \int_0^1 \Big(E^f\big[\tilde{A}_1|F^{-1}(s|A_1)\big]\Big)^\tau \mathrm{d}s, \end{aligned} \tag{37}$$

where

$$\tau := 1+\rho(1-\gamma_u) = \frac{1+\gamma_v}{\gamma_v+\alpha\gamma_u+(1-\alpha)} > 0. \tag{38}$$

Since $I_t^f(\cdot)$ is linear in A_2 regardless of the information system, the elasticity of expected income with respect to A_1 is now the relevant measure of inequality: the income distribution is more unequal under $\bar{f}$ than under $\hat{f}$ if $\varepsilon[I_t^{\bar{f}},A_1] \geq \varepsilon[I_t^{\hat{f}},A_1]$ holds for all $t \geq 0$ and $A_1 \in \mathcal{A}_1$.

Define

$$\Delta^F(s|A_1) := \frac{\left(E^f\left[\tilde{A}_1 \middle| F^{-1}(s|A_1)\right]\right)^{\tau}}{\int_0^1 \left(E^f\left[\tilde{A}_1 \middle| F^{-1}(\hat{s}|A_1)\right]\right)^{\tau} \mathrm{d}\hat{s}}. \tag{39}$$

The same procedure as in Section 3.1 yields

Proposition 3 *Income inequality is higher under information system $\bar{f}$ than under information system $\hat{f}$, if and only if*

$$\int_0^1 \left[\Delta^{\bar{F}}(s|A_1)\frac{\bar{f}_{A_1}}{\bar{f}}\left(\bar{F}^{-1}(s|A_1)\middle|A_1\right) - \Delta^{\hat{F}}(s|A_1)\frac{\hat{f}_{A_1}}{\hat{f}}\left(\hat{F}^{-1}(s|A_1)\middle|A_1\right)\right] \mathrm{d}s \geq 0, \tag{40}$$

holds for all $A_1 \in \mathcal{A}_1$.

In the absence of risk sharing opportunities the impact of better information on income inequality was critically dependent on the risk aversion parameter γ_u. By contrast, if those risks which are affected by the signals can be insured on fair terms better information will always increase income inequality:

Theorem 2 *Assume that an unbiased insurance market for the $\tilde{A}_1$-risk is available. Let $\bar{f}$ and $\hat{f}$ be two information systems with $\bar{f} \succ_{\inf} \hat{f}$. The income distribution under $\bar{f}$ is more unequal than the income distribution under $\hat{f}$.*

If the signals convey information about insurable risks more efficient screening (i.e., a better information system) results in more inequality regardless of the measure of relative risk aversion. Eckwert and Zilcha (2001b) found that in this framework better information enhances growth when γ_u is less than 1; and better information reduces growth when γ_u exceeds 1. Thus, in economies with low risk aversion ($\gamma_u < 1$) information induced growth always comes at the cost of higher income inequality. For strongly risk-averse economies ($\gamma_u > 1$), by contrast, our analysis suggests an inverse linkage between growth and inequality.

5 The Effect of Risk Sharing on Inequality

In this section we analyze the impact of risk sharing opportunities for the $\tilde{A}_1$-component of individual ability on income inequality. The comparison of the corresponding income distributions with and without risk sharing will be based on a given information system f. If no risk sharing opportunities are available the average income of an agent with ability $A = A_1 \cdot A_2$ is

$$\breve{I}_t^f(A_1, A_2) = w_t \delta_t^\alpha \hat{g}(H_{t-1}) A_1 A_2 \Big(E\big[\tilde{A}_2^{1-\gamma_u}\big]\Big)^\rho \int_0^1 \Big(E^f\big[\tilde{A}_1^{1-\gamma_u} \big| F^{-1}(s|A_1)\big]\Big)^\rho \mathrm{d}s. \tag{41}$$

If an insurance market for the $\tilde{A}_1$-risk exists average income is given by (37) which we restate here for convenience.

$$\hat{I}_t^f(A_1, A_2) = w_t \delta_t^\alpha \hat{g}(H_{t-1}) A_2 \Big(E\big[\tilde{A}_2^{1-\gamma_u}\big]\Big)^\rho \int_0^1 \Big(E^f\big[\tilde{A}_1 \big| F^{-1}(s|A_1)\big]\Big)^\tau \mathrm{d}s. \tag{42}$$

Let us consider the case where $\gamma_u \geq 1$ and, hence, $\tau \leq 1$ (cf. (38)). If the system f is uninformative, $f = f^0$, the integrals in (41) and (42) do not depend on A_1. Thus

$$\breve{\varepsilon}\big[I_t^{f^0}, A_1\big] = 1 \tag{43}$$
$$\hat{\varepsilon}\big[I_t^{f^0}, A_1\big] = 0, \tag{44}$$

where $\breve{\varepsilon}$ and $\hat{\varepsilon}$ denote the income elasticities in the absence and in the presence of risk sharing opportunities, respectively.

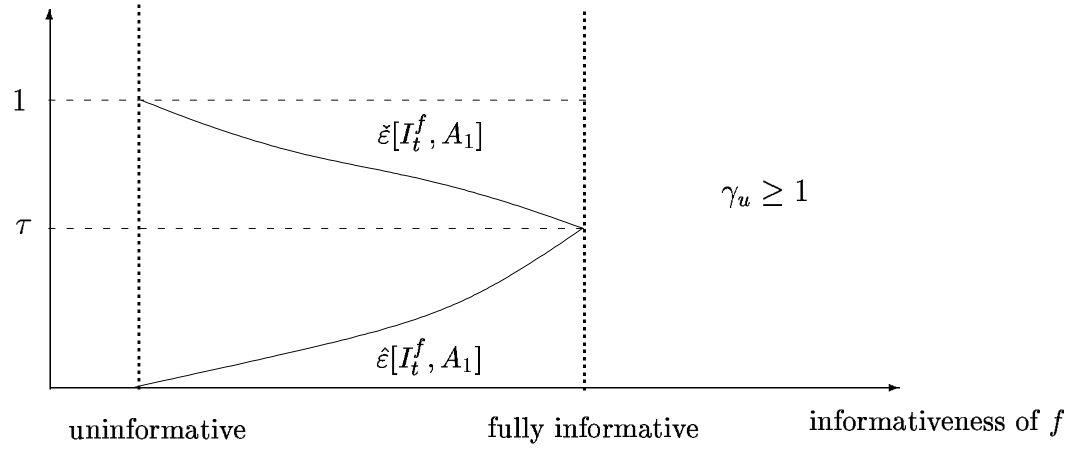

Figure 3.

Similarly, if the system f is fully informative, $f = f^1$, the signal $F^{-1}(s|A_1)$ reveals the agent's talent A_1 and, hence, equations (41) and (42) imply

$$\breve{\varepsilon}\big[I_t^{f^1}, A_1\big] = \hat{\varepsilon}\big[I_t^{f^0}, A_1\big] = \tau. \tag{45}$$

According to Theorem 1, $\check{\varepsilon}[I_t^f, A_1]$ declines as f becomes more informative; and according to Theorem 2, $\hat{\varepsilon}[I_t^f, A_1]$ increases with better information.

Our discussion has shown that the availability of risk sharing opportunities reduces income inequality if $\gamma_u \geq 1$; and that this effect is stronger in less well informed economies.

Theorem 3 *Assume $\gamma_u \geq 1$, i.e., the economy is strongly risk averse. Under a market structure which allows agents to insure (part of) their talent risk on fair terms the income distribution will be less unequal than in the absence of such risk sharing opportunities. The impact of risk sharing on income inequality is weaker in economies with better information systems.*

If $\gamma_u \leq 1$ the equations (43)-(45) continue to hold. In view of theorems 1 and 2 both $\check{\varepsilon}[I_t^f, A_1]$ and $\hat{\varepsilon}[I_t^f, A_1]$ increase as f becomes more informative. For some information systems, therefore, risk sharing may lead to higher income inequality (see Figure 4).

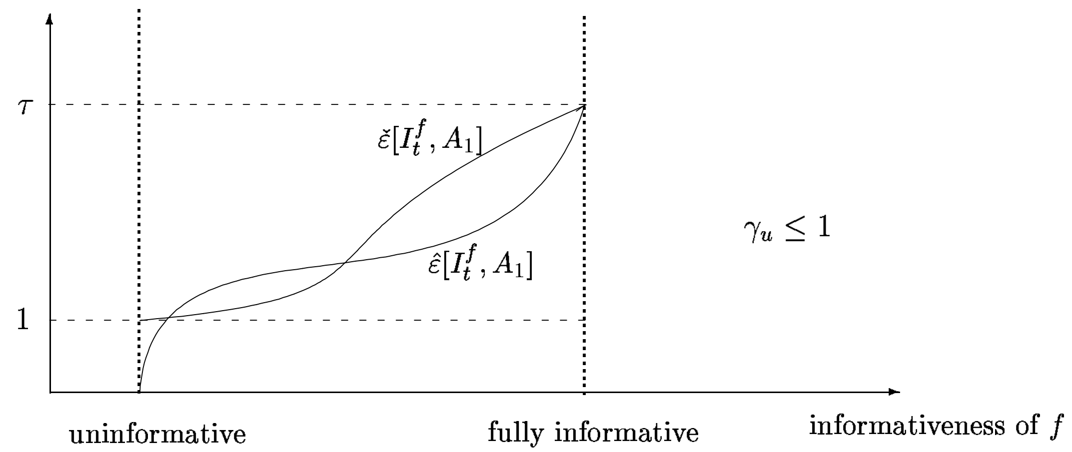

Figure 4.

6 Concluding Remarks

Education systems in all modern societies are based on screening mechanisms which generate information about the (unobservable) talent of young individuals. This paper is an attempt to analyze the implications of such mechanisms for the distribution of intragenerational income. A screening mechanism is modeled as an information system that allows the interpretation (in a Bayesian way) of signals which are specific to the agents.

Our analysis has shown that the implementation of a more efficient screening mechanism for individual talent will have effects on the inequality of the income distribution. These effects critically depend on the market structure of the economy. If the screening mechanism is applied to uninsurable risks, the income distribution will become more (less) unequal with more efficient screening if the economy is moderately (strongly) risk averse. Yet, if the screening mechanism applies to insurable risks, better screening always produces more income inequality. We have also addressed the controversial question about the linkage between inequality and economic growth. For most parameter constellations our analysis suggests a positive link between growth and inequality if the screening process provides information about non-insurable risks. By contrast, when these risks are insurable, higher growth implies more (less) income inequality if the economy is moderately (strongly) risk-averse.

In our framework individuals make their decision about investment in education after they observe a signal correlated to their ability, but prior to knowing the rate of return on this investment, i.e., before the realization of their 'type'. As a result 'income' can be defined at three points of time: (i) ex-ante income, i.e., expected income for each given level of ability A, (ii) expected income given the signal each individual observes, and (iii) ex-post income, i.e., the realized income when state of nature is known. We have considered here the ex-ante income distribution, but we are aware of the fact that our results may significantly change if we choose a different notion of income. Each type of inequality has a different conceptual meaning and it is not the place here to compare them due to the deep normative aspects it entails. We intend, however, to examine the issues discussed in this paper for the other notions of income as well.

To illustrate the significance of the underlying income concept for the properties of the equilibrium income distribution let us consider the subset D of all individuals with the same ability A. Under the ex ante specification (i) all individuals in this subset have the same income. Now consider the distribution of income after each person has observed an individual signal (specification (ii)). Obviously, agents in the subset D will almost surely have different incomes unless the information system is either fully informative (where the signal reveals the state) or uninformative (where the signals convey no information). A similar argument applies to the income specification (iii). Clearly we cannot expect that the results of our paper generalize to income concepts which correspond to the specifications (ii) or (iii).

Appendix

Let us prove some preliminary results before we proceed with the proofs of the theorems.

Proof of Lemma 1: Define $h(y|A,\hat{A}) := f(y|A)/f(y|\hat{A})$. Obviously, $h_y(y|A,\hat{A}) = 0$, $\forall y$, if $A = \hat{A}$. Also, by MLRP, $h_y(y|A,\hat{A}) > 0$, $\forall y$, if $A > \hat{A}$. From this observation we conclude

$$0 \leq \frac{\partial}{\partial A}\Big(h_y(y|A,\hat{A})\Big)\Big|_{A=\hat{A}} = \frac{\partial}{\partial y}\left(\frac{\partial h(y|A,\hat{A})}{\partial A}\bigg|_{A=\hat{A}}\right) = \frac{\partial}{\partial y}\left(\frac{f_A(y|A)}{f(y|A)}\right),$$

which proves the claim. □

Lemma 4 *Let $\theta : X := [\underline{x}, \bar{x}] \to \mathbb{R}$ be an integrable function satisfying*

$$\int_{\underline{x}}^{\hat{x}} \theta(x)\,\mathrm{d}x \leq 0; \qquad \int_X \theta(x)\,\mathrm{d}x = 0$$

for all $\hat{x} \in X$. Then

$$\int_X h(x)\theta(x)\,\mathrm{d}x \overset{(\leq)}{\geq} 0$$

holds for any differentiable monotone increasing (decreasing) function $h : X \to \mathbb{R}$.

Proof: Integration by parts gives

$$\begin{aligned}\int_X h(x)\theta(x)\,\mathrm{d}x &= h(x)\int_{\underline{x}}^{x}\theta(\hat{x})\,\mathrm{d}\hat{x}\bigg|_{\underline{x}}^{\bar{x}} - \int_X h'(x)\int_{\underline{x}}^{x}\theta(\hat{x})\,\mathrm{d}\hat{x}\,\mathrm{d}x \\ &= -\int_X h'(x)\int_{\underline{x}}^{x}\theta(\hat{x})\,\mathrm{d}\hat{x}\,\mathrm{d}x.\end{aligned}$$

The last term is non-negative if $h(x)$ is increasing; and it is non-positive if $h(x)$ is decreasing.

□

Lemma 5 *Let $\vartheta, z : [0,1] \to \mathbb{R}$ be two integrable functions satisfying*

$$\int_0^1 \big[\vartheta(s) - z(s)\big]\,\mathrm{d}s = 0.$$

If $\vartheta(s)/z(s)$ is monotone increasing (decreasing), then

$$\int_0^x \big[\vartheta(s) - z(s)\big]\,\mathrm{d}s \overset{(\geq)}{\leq} 0 \quad \forall x \in [0,1]. \tag{A1}$$

Proof: Suppose that $\vartheta(s)/z(s)$ is strictly monotone increasing (decreasing). Obviously, $\vartheta(s)$ and $z(s)$ have the single crossing property and, therefore, the integral in (A1) is non-positive (non-negative). By a continuity argument the inequality in (A1) remains valid if $\vartheta(s)/z(s)$ is merely monotone rather than strictly monotone. □

Proof of Theorem 1: We apply the characterization in Proposition 1. Consider the ratio

$$\frac{\Gamma^{\bar{F}}(s|A)}{\Gamma^{\hat{F}}(s|A)} = \left(\frac{E^{\bar{f}}\big[\tilde{A}^{1-\gamma_u}\big|\bar{F}^{-1}(s|A)\big]}{E^{\hat{f}}\big[\tilde{A}^{1-\gamma_u}\big|\hat{F}^{-1}(s|A)\big]}\right)^{\rho} \frac{\int_0^1 \Big(E^{\hat{f}}\big[\tilde{A}^{1-\gamma_u}\big|\hat{F}^{-1}(\hat{s}|A)\big]\Big)^{\rho}\,\mathrm{d}\hat{s}}{\int_0^1 \Big(E^{\bar{f}}\big[\tilde{A}^{1-\gamma_u}\big|\bar{F}^{-1}(\hat{s}|A)\big]\Big)^{\rho}\,\mathrm{d}\hat{s}}.$$

By (ii) in Definition 3, $\Gamma^{\bar{F}}(s|A)/\Gamma^{\hat{F}}(s|A)$ is monotone increasing (decreasing) in s for $\gamma_u \overset{(\geq)}{\leq} 1$. Applying Lemma 5 we obtain for $\gamma_u \overset{(\geq)}{\leq} 1$:

$$\int_0^{\vartheta} \big[\Gamma^{\bar{F}}(s|A) - \Gamma^{\hat{F}}(s|A)\big]\,\mathrm{d}s \overset{(\geq)}{\leq} 0 \quad \forall \vartheta \in [0,1]. \tag{A2}$$

(i) Consider the case $\gamma_u \leq 1$. The validity of the condition in (31) is immediate from the following assessment:

$$\begin{aligned}
\int_0^1 \Gamma^{\bar{F}}(s|A)\frac{\bar{f}_A}{\bar{f}}\Big(\bar{F}^{-1}(s|A)\Big|A\Big)\mathrm{d}s \;&\geq\; \int_0^1 \Gamma^{\bar{F}}(s|A)\frac{\hat{f}_A}{\hat{f}}\Big(\hat{F}^{-1}(s|A)\Big|A\Big)\mathrm{d}s \\
&\geq\; \int_0^1 \Gamma^{\hat{F}}(s|A)\frac{\hat{f}_A}{\hat{f}}\Big(\hat{F}^{-1}(s|A)\Big|A\Big)\mathrm{d}s
\end{aligned}$$

In the first of the above inequalities we have used Lemma 4 and the integral condition (18). The second inequality makes use of (A2), Lemma 1 and Lemma 4.

(ii) If $\gamma_u \geq 1$, the last two inequalities are reversed. Proposition 1 then implies the claim.

□

Proof of Theorem 2: We apply the characterization in Proposition 3. Consider the ratio

$$\frac{\Delta^{\bar{F}}(s|A)}{\Delta^{\hat{F}}(s|A)} = \left(\frac{E^{\bar{f}}\left[\tilde{A}_1|\bar{F}^{-1}(s|A)\right]}{E^{\hat{f}}\left[\tilde{A}_1|\hat{F}^{-1}(s|A)\right]}\right)^{\tau} \frac{\int_0^1 \left(E^{\hat{f}}\left[\tilde{A}_1|\hat{F}^{-1}(\hat{s}|A)\right]\right)^{\tau} \mathrm{d}\hat{s}}{\int_0^1 \left(E^{\bar{f}}\left[\tilde{A}_1|\bar{F}^{-1}(\hat{s}|A)\right]\right)^{\tau} \mathrm{d}\hat{s}}.$$

By (ii) in Definition 3, $\Delta^{\bar{F}}(s|A)/\Delta^{\hat{F}}(s|A)$ is monotone increasing in s. Now the proof proceeds as in part (i) of Theorem 1. □

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