

# Labor Market Experience and the Gender Gap<sup>\*</sup>

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## Abstract

We present a model in which the gender gap in wages displays non-monotonic dynamics of the type observed in the US during the twentieth century. We show that the dynamics of the gender gap depend on the number of women that work at home in the early stage of their life and join the labor force late in life with low skills and little labor market experience. Consistent with empirical findings, we conclude that the gender gap increases when this dynamic labor profile is sufficiently widespread, and vice versa. We argue that this profile abounds when wages grow sufficiently rapidly.

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# 1. Introduction

U.S. labor force participation (LFP) of women has been lower than that of men, and gradually increasing since 1890.<sup>1</sup> Since Mincer's 1962 seminal work, the explanation for these phenomena focuses on the observed gradual increase in women wages. Along this line, Galor and Weil (1996) argue that due to a difference in physical strength between men and women, the accumulation of physical capital in the process of development have complemented female's comparative advantage in mental (rather than physical) labor input, raised their relative wages, reduced their fertility rates and permitted an increase in their labor force participation. Greenwood, Seshardi and Yorukoglu (2001) follow via a dynamic general equilibrium model, Gronau (1977) who studies the choice between leisure, home production and LFP. They focus on the rise in overall real wages, in conjunction with the introduction of labor-saving household appliances, as the engine behind the rise in female LFP.

Seemingly contradicting with this approach is the empirical finding regarding the dynamic pattern of the ratio of the mean wage of working women to the mean wage of working men, often referred to as the "gender gap in wages". The data presented in Smith and Ward (1989) and O'Neil and Polachek (1993) show that this ratio has presented a non-monotonic dynamics during the 20<sup>th</sup> century.

Our purpose in this paper is to present a model that generates non-monotonic dynamics of the gender gap of the type observed by the above-mentioned empirical studies. In our model, the wage per unit of labor is continuously increasing due to capital accumulation. This entails a continuing growth in women's LFP, as in Galor and Weil (1996). The non-monotonic dynamics of the means ratio springs from the evolution of the compositions of the populations of working men and women.

Our explanation is related to the empirical works of Smith and Ward (1989), Goldin (1990) and O'Neil and Polachek (1993). These studies distinguish between two kinds of periods. In periods of the first type, a sufficiently large number of new entrants were married women older than about 35 years with low skills and little labor market experience. The new entrants have lowered the average market skills of working women,

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<sup>1</sup> See The Handbook of Labor Economics (1986), chapter 2.

slowing down the growth in average women earnings, and making it grow slower than men's. In periods of the second type the dominant factor in the expansion in female LFP was the rise in the participation rates of younger women. In those periods, due to the early participation, women's experience has approached that of men and the main reason for the decrease in the ratio of mean wages was removed.

The main contribution of our paper is the analysis of the individuals' considerations underlying their observed labor choices. In particular we conclude that a massive entry of women in a relatively late stage of their lives into the labor force occurs in periods in which wages grow sufficiently rapidly. In such periods the wage growth makes the difference between the wages a woman faces when she is young and when she is older sufficiently large to induce a difference in her LFP choices over time. In periods in which wages grow sufficiently slow, the similarity in wages during the different periods of the woman's life leads to similarity in her LFP decisions in those periods.

We present an overlapping generations model in which individuals live and work for two periods. The economy comprises three production sectors: home production, a physical sector and a modern sector. At home and in the physical sector labor is the sole input, while the modern sector production utilizes both labor and capital. We refer to employment in the physical and the modern sectors as labor market participation. We assume that individuals differ in their ability and that the individual's ability, together with the individual's labor market experience, determines the amount of efficiency units of labor that she or he can supply to the modern sector. The distribution of abilities that are relevant to the modern sector among women is identical to that among men. The only difference we assume between women and men's abilities is that men have higher productivity in the physical sector. A key assumption in the model is that by entering the labor force early in life, individuals acquire experience that increases their amount of efficiency units later in life. Several simplifying assumptions regarding productivity at home and in the physical sector ensure that women never choose to work in the physical sector and men never choose home production. We further assume that men working in the physical sector make more than women working at home. Since women's alternative to working in the modern sector is lesser than men's, the average ability and the average income of women in the modern sector are lower than those of men.

Due to these assumptions, in the initial stage of the economy's growth, women labor can follow three alternative dynamic labor profiles: The least able women work at home in both their lives' periods; abler women work at home in the first period of their life and in the modern sector in the second period; the ablest women work in the modern sector in both periods. The middle group exists because in this initial stage wages grow rapidly, attracting men and women in their second period of life to the modern sector. Since these men have labor market experience while these women do not, the mean income of men rises more than the mean income of women.

As the wages growth decelerates, the middle group disappears and with it the differences in experience between men and women of the same generation in the modern sector. The growth of wages in the modern sector affects women average income more than it affects men's, since female LFP consists only of work in the modern sector while men also work in the physical sector. Thus, in this stage the gender gap narrows.

The paper is organized as follows. In section 2 we present the basic structure of the model. In section 3 we analyze the individuals' labor supply decisions. In section 4 we analyze the equilibrium and the dynamics of the economy portrayed by the model. In section 5 we compare the dynamics of the gender gap implied by the model with the observed dynamics of the gender gap. Combining the data in Smith and Ward (1989), Goldin (1990) and O'Neil and Polachek (1993) shows that the ratio of female to male earnings has presented a W-shaped dynamics for the US during the twentieth century with a local maximum in the 1950s. Goldin (1991) interprets World War II (WWII) as a period of a temporary decrease in the value of home production. Her and Acemoglu, Autor and Lyle (2002) show that WWII induced a temporary shock with a positive effect on female LFP. Inserting a temporary decline in the value of home production in our model indeed generates W-shaped mean earnings ratio dynamics, where the local maximum comes later than the shock, due to the effect of the accumulated labor market experience. In section 6 we provide concluding remarks.

## 2. The structure of the economy

Consider a closed overlapping generations economy that operates in a perfectly competitive environment. Time is discrete and infinite. In every period the economy

produces a single good that can be used for consumption or investment.

## 2.1 Production

Production can take place at home or in the market. There are two production sectors in the market: the physical sector and the modern sector. Working in the market, in either sector, in life's first period rewards the individual with labor market experience that increases her or his productivity in the market production in life's second period.

The marginal productivity of labor at home is the constant  $H$ , regardless of gender and experience. In contrast, the marginal productivity of labor in the physical sector differs across the genders. The productivity of man  $j$  in the physical sector equals  $e^j P$ , and the productivity of woman  $j$  in the physical equals  $e^j P'$ , where  $P$  and  $P'$  are constants satisfying  $P > P'$  and  $e^j$  is a function of  $j$ 's labor market experience.<sup>2</sup>

The production function in the modern sector is:

$$Q_t = K_t^{0.5} L_t^{0.5} \quad (1)$$

Where  $Q_t$  is output,  $K_t$  is the amount of capital and  $L_t$  is the amount of efficiency units of labor in this sector in period  $t$ .<sup>3</sup>

Markets are assumed to be competitive. Hence the wage of one unit of efficiency labor in period  $t$  and the return to one unit of capital in period  $t$  are respectively:

$$w_t = 0.5 K_t^{0.5} L_t^{-0.5} \quad (2)$$

$$R_t = 0.5 K_t^{-0.5} L_t^{0.5} \quad (3)$$

Thus:

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<sup>2</sup> The function  $e^j$  is defined in the next sub-section.

<sup>3</sup> The specific value of 0.5 in the exponent of the production function is chosen to enable closed form solution for the variables of the model.

$$R_t = \frac{1}{4w_t} \tag{4}$$

## 2.2 Individuals

In each period, a generation of measure 2 joins the economy where the measures of the women and the men in each generation are normalized to 1. All individuals live and work for two periods. For simplicity, we assume that individuals' preferences are defined only over consumption in their second period of life. Due to this assumption, individuals maximize the net present value of their earnings.

Individuals differ in their amount of efficiency units. Let  $a^j$  be the amount of efficiency units that individual  $j$  has in his or her first life period. We assume  $a^j \sim U[0,1]$  regardless of  $j$ 's gender. In life's second period,  $j$ 's amount of efficiency units is  $e^j a^j$  where  $e^j$  is a function of the labor market experience  $j$  acquired in  $j$ 's life's first period. The function  $e^j$  takes three values: 1,  $\theta_1$  and  $\theta_2$ . It equals 1 if  $j$  has no market experience due to working at home in  $j$ 's life's first period,  $\theta_1$  if  $j$  works in the same market sector in both periods of  $j$ 's life or  $\theta_2$  if  $j$  moves from one market sector to the other, where  $\theta_1 \geq \theta_2 > 1$ . In order to focus efficiently on the dynamics of the women's choice between working at home and working in the market sectors we simplify the analysis of the choice between the two market sectors by assuming from now on that  $\theta_1 = \theta_2 \equiv \theta$ .<sup>4</sup> Finally, we assume that  $P > H$ , an assumption which assures that men do not work at home.

## 3. Labor supply

In this section we analyze the labor supply of each of the four groups of individuals that exist in the economy. We denote the number of group  $z$  members that work in the modern sector in period  $t$  by  $x_t^z$ , where  $z$  is a group index satisfying  $z \in \{A, B, C, D\}$  where  $A, B, C$  and  $D$  respectively represent: women in their life's first period; women in their life's second period; men in their life's first period; and men in their life's second period. In a similar way we denote the amount of efficiency units of labor supplied to the modern

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<sup>4</sup> Angrist (1990) shows a strong impact of labor market experience on lifetime earnings.

sector by members of group  $z$  by  $L_t^z$ .

### 3.1 Men's Labor choice

In each period  $t$  man  $j$  chooses to work in the modern sector if:

$$a^j w_t \geq P \quad (5)$$

Note that if (5) holds then  $a^j \theta w_t \geq \theta P$ , and thus,  $j$  prefers the modern sector regardless of his experience. We can define therefore an ability threshold for men denoted by  $a_t^Y$ , where if  $a^j \geq a_t^Y$  then  $j$  works in the modern sector, and if  $a^j < a_t^Y$  he works in the physical sector. The threshold  $a_t^Y$  satisfies:

$$a_t^Y = \begin{cases} \frac{P}{w_t} & \text{if } \frac{P}{w_t} < 1 \\ 1 & \text{otherwise} \end{cases} \quad (6)$$

Due to the uniform distribution of  $a^j$ :

$$x_t^C = x_t^D = 1 - a_t^Y = \begin{cases} 1 - \frac{P}{w_t} & \text{if } \frac{P}{w_t} < 1 \\ 0 & \text{if } \frac{P}{w_t} \geq 1 \end{cases} \quad (7)$$

The total amount of men in the modern sector is therefore  $x_t^C + x_t^D = 2x_t^C = 2x_t^D$ . Given  $w_t$ , the amount of efficiency units supplied to the modern sector in period  $t$  by men of generation  $t$  is:

$$L_t^C(w_t) = \int_{a_t^Y}^1 ada = \frac{1 - (a_t^Y)^2}{2} = \begin{cases} \frac{1 - \left(\frac{P}{w_t}\right)^2}{2} & \text{if } \frac{P}{w_t} < 1 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

and the amount of efficiency units supplied to the modern sector in period  $t$  by men of generation  $t-1$  is:

$$L_t^D(w_t) = \theta \int_{a_t^Y}^1 ada = \theta \frac{1 - (a_t^Y)^2}{2} = \begin{cases} \theta \frac{1 - \left(\frac{P}{w_t}\right)^2}{2} & \text{if } \frac{P}{w_t} < 1 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

### 3.2 The labor supply of women in their life's first period

In contrast to men, the period  $t$  occupational choice of a woman born in period  $t$  depends not only on current wage,  $w_t$ , but also on  $w_{t+1}$ . The connection between the occupational choices in period  $t$  and period  $t+1$  springs from the different effect that experience has on productivity in modern sector and on productivity at home.

As discussed in the introduction, the main results of our paper depend on the dynamics of the number of women who choose to work at home in their life's first period and join the modern sector in their second's. To focus more efficiently on these dynamics we take simplifying assumptions that ensure that women either work at home or in the modern sector, but not in the physical sector. Specifically, we assume for that purpose that  $P'=0$ .<sup>5</sup> In the appendix we show conditions which ensure that there will be women who work at home in their life's first period and join the modern sector in the second even if  $P' > 0$  and some women do work in the physical sector.

Given this assumption, each woman born in period  $t$  has to choose in that period one

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<sup>5</sup> If  $P' = 0$ , then working in the modern sector, which yields a positive income and an experience premium, is always better than working in the physical sector for each woman in each period of her life.

of the following dynamic labor profiles:

Profile 1: Home production in period  $t$  and in period  $t+1$ .

Profile 2: Home production in period  $t$  and work in the modern sector in period  $t+1$ .

Profile 3: Work in the modern sector in period  $t$  and home production in period  $t+1$ .

Profile 4: Work in the modern sector in period  $t$  and in period  $t+1$ .

We define  $V^i$  as the present value of earnings under each of the profiles  $i \in \{1,2,3,4\}$ .

Given  $w_t$ ,  $w_{t+1}$  and  $R_{t+1}$ ,  $V^i$  satisfies:

$$\begin{aligned} V^1 &= H + \frac{H}{R_{t+1}} & V^2 &= H + \frac{aw_{t+1}}{R_{t+1}} \\ V^3 &= aw_t + \frac{H}{R_{t+1}} & V^4 &= aw_t + \frac{a\theta w_{t+1}}{R_{t+1}} \end{aligned}$$

We define  $a_{mn,t}$  as follows: all the women to whom  $a > a_{mn,t}$  prefer profile  $m$  to profile  $n$ , where  $m > n$  and  $m, n \in \{1,2,3,4\}$ . For each of the possible  $(m,n)$  combinations,  $a_{mn,t}$  is the value of  $a$  for which  $V^m(a, w_t, w_{t+1}) = V^n(a, w_t, w_{t+1})$ . Thus,  $a_{mn,t}$  is a function of  $w_t$  and  $w_{t+1}$ :

$$\begin{aligned} a_{21,t} &= \frac{H}{w_{t+1}} & a_{31,t} &= \frac{H}{w_t} & a_{41,t} &= \frac{H + 4Hw_{t+1}}{w_t + 4\theta w_{t+1}^2} \\ a_{43,t} &= \frac{H}{\theta w_{t+1}} & a_{42,t} &= \frac{H}{w_t + (\theta - 1)4w_{t+1}^2} \end{aligned} \tag{10}$$

We start analyzing the labor profile choice of women born in period  $t$  with the case where  $w_t < w_{t+1}$ , then we turn to the case where  $w_t \geq w_{t+1}$ .

The following two propositions show that when wages are increasing over time there are always women who choose profiles 1 and 4 and no woman chooses profile 3. In addition, some women may choose profile 2, but this occurs only if the increase in wages is sufficiently rapid. The women who choose profile 4 are more able than those who

choose profile 2, and the women who choose profile 2 are more able than those who choose profile 1.

**Proposition 1**

If  $w_{t+1} > w_t$ , working in the first period in the market and in the second period at home (profile 3) cannot be optimal for any woman.

**Proof**

For profile 3 to be optimal for some women,  $V^3$  must exceed  $V^1$ ,  $V^2$  and  $V^4$  for some  $a$ . However, if  $V^3 > V^1$  for a certain  $a$  then  $aw_t > H$  which means that  $a\theta w_{t+1} > H$  and therefore that  $V^4 > V^3$  for that  $a$ . Thus, profile 3 cannot be optimal for any woman. ■

**Proposition 2:**

If  $w_t$  is sufficiently smaller than  $w_{t+1}$  then  $a_{21,t} < a_{41,t} < a_{42,t}$ . If  $w_t$  is smaller than, but sufficiently close to,  $w_{t+1}$  then  $a_{42,t} < a_{41,t} < a_{21,t}$ .

**Proof**

If  $w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2$ , i.e.,  $w_t$  is sufficiently below  $w_{t+1}$ , then  $a_{21,t} < a_{41,t} < a_{42,t}$ , as follows directly from (10). If  $w_t < w_{t+1}$  and  $w_t > w_{t+1} - 4(\theta - 1)w_{t+1}^2$ , i.e.,  $w_t$  is below  $w_{t+1}$  but sufficiently close to it, then  $a_{21,t} < a_{41,t} < a_{42,t}$ , as also follows directly from (10). ■

Figure 1.a shows the distribution of labor profiles according to abilities as follows from proposition 2 for the case when  $w_t$  is sufficiently smaller than  $w_{t+1}$ .<sup>6</sup> Figure 1.b shows the distribution of labor profiles according to abilities as follows from proposition 2 for the case in which  $w_t$  is not sufficiently below  $w_{t+1}$ .

The rationale behind these results is as follows: When  $w_t$  is sufficiently smaller than  $w_{t+1}$  the growth rate of wages is high. We therefore observe the ‘middle’ group: women who find that the loss in income in the first period is greater than the gain in income resulting from acquiring experience. However, since next period wage will rise significantly, it would be better to work next period in the market than at home. When  $w_t$

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<sup>6</sup> Specifically the relevant range is  $w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2$ .

is not sufficiently below  $w_{t+1}$  wages change relatively little over time. Thus if staying at home in the first period is optimal then staying at home in the second period is also optimal. From the analysis above we can obtain the proportion of women born in period  $t$  who work in the modern sector in that period for the case where  $w_{t+1} \geq w_t$ :

$$x_t^A = \begin{cases} 1 - a_{42,t} & \text{if } w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2 \\ 1 - a_{41,t} & \text{if } w_{t+1} - 4(\theta - 1)w_{t+1}^2 < w_t < w_{t+1} \end{cases} \quad (11)$$

A similar analysis to the one taken for the case where wages are increasing over time yields that when wages are decreasing there are always women who choose profiles 1 and 4 and no woman who chooses profile 2. Some women choose profile 3 but this occurs only if the decrease in wages is sufficiently rapid. The women who choose profile 4 are more able than those who choose profile 3 and the women who choose profile 3 are more able than those who choose profile 1. For this case in which  $w_t \geq w_{t+1}$  the proportion of women born in period  $t$  who work in the modern sector in that period:

$$x_t^A = \begin{cases} 1 - a_{31,t} & \text{if } w_t > \theta w_{t+1} \\ 1 - a_{41,t} & \text{if } w_{t+1} < w_t < \theta w_{t+1} \end{cases} \quad (12)$$

Combining both cases,  $w_t < w_{t+1}$  and  $w_t \geq w_{t+1}$ , we get:<sup>7</sup>

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<sup>7</sup> Equation A.3 in the appendix shows  $x_t^A(w_t, w_{t+1})$  explicitly.

$$x_t^A(w_t, w_{t+1}) = \begin{cases} 1 - a_{42,t}(w_t, w_{t+1}) & \text{if } w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2 \\ 1 - a_{41,t}(w_t, w_{t+1}) & \text{if } w_{t+1} - 4(\theta - 1)w_{t+1}^2 < w_t < \theta w_{t+1} \\ 1 - a_{31,t}(w_t, w_{t+1}) & \text{if } \theta w_{t+1} < w_t \end{cases} \quad (13)$$

It follows from the analysis in this sub-section that the women who are born in period  $t$  and choose profile 4 are those whose abilities satisfy  $1 - x_t^A \leq a \leq 1$ . Based on the  $[0,1]$  uniform distribution of abilities, the amount of efficiency units of labor supplied to the modern sector in period  $t$  by this group is therefore:

$$L_t^A(w_t, w_{t+1}) = \int_{1-x_t^A}^1 ada = \frac{x_t^A(2-x_t^A)}{2}. \quad (14)$$

Note that  $L_t^A$  is a function of  $w_t$  and  $w_{t+1}$  since, by (13),  $x_t^A$  is.

### 3.3 The labor supply of women in their life's second period

As was established in section 3.2 there exists an ability threshold, which we denote in this sub-section by  $a_{t-1}^X$ , such that in period  $t-1$  the young women with  $0 < a < a_{t-1}^X$  work at home while the young women with  $a_{t-1}^X < a < 1$  work in the modern sector.<sup>8</sup>

There are three possible cases. In the first case,  $a_{t-1}^X < \frac{H}{\theta w_t} < \frac{H}{w_t}$ . In this case the women who did not acquire experience in period  $t-1$ , those with  $0 < a < a_{t-1}^X$ , do not work in the market in period  $t$  since for them  $a < \frac{H}{w_t}$  implying that  $aw_t < H$ . Women who did acquire experience in period  $t-1$  and their amount of efficiency units is in the range

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<sup>8</sup> If  $w_t > w_{t-1}$ ,  $a_{t-1}^X = a_{41,t-1}$ , otherwise  $a_{t-1}^X = a_{31,t-1}$ .

$a_{t-1}^x < a < \frac{H}{\theta w_t}$  also do not work in the market since for them  $a\theta w_t < H$ . The only women

born in period  $t-1$  who work in period  $t$  are those with  $\frac{H}{\theta w_t} < a < 1$ . Thus,  $x_t^B = 1 - \frac{H}{\theta w_t}$ , due

to the uniform distribution of  $a$ . The period  $t$  labor supply of these women in this case is  $L_t^B = \int_{\frac{H}{\theta w_t}}^1 ada$ .

In the second case,  $\frac{H}{\theta w_t} < a_{t-1}^x < \frac{H}{w_t}$ . As in the previous case, the women who did not acquire experience in period  $t-1$ , do not work in the market in period  $t$  since for them  $a < \frac{H}{w_t}$ . On the other hand, all the women who did acquire experience in period  $t-1$  work

in the market in period  $t$ , since for them  $a > a_{t-1}^x$  and therefore  $a > \frac{H}{\theta w_t}$ . Thus, in this case

$x_t^B = x_{t-1}^A$ . The period  $t$  labor supply of women born in period  $t-1$  in this case is  $L_t^B = \theta \int_{a_{t-1}^x}^1 ada$ .

In the third case,  $\frac{H}{\theta w_t} < \frac{H}{w_t} < a_{t-1}^x$ . As in the second case, all the women who did acquire experience in period  $t-1$  work in the market in period  $t$ , for the same reason as in that case. In addition, some women born in period  $t-1$  who worked at home in period  $t-1$  will work in the market in period  $t$  too. These women are the ones with  $\frac{H}{w_t} < a < a_{t-1}^x$ . The

rest of the women who were born in period  $t-1$  and worked at home in that period are the ones to whom  $0 < a < \frac{H}{w_t}$  and therefore they work at home in period  $t$  too. The period  $t$

labor supply of women born in period  $t-1$  in this case is  $L_t^B = \theta \int_{a_{t-1}^x}^1 ada + \int_{\frac{H}{w_t}}^{a_{t-1}^x} ada$ . Equation

(15) summarizes the analysis of this section.

$$L_t^B(x_{t-1}^A, w_t) = \begin{cases} \frac{1 - \left(\frac{H}{\theta w_t}\right)^2}{2} & \text{if } a_{t-1}^x < \frac{H}{\theta w_t} \\ \theta \frac{1 - (1 - x_{t-1}^A)^2}{2} & \text{if } \frac{H}{\theta w_t} < a_{t-1}^x < \frac{H}{w_t} \\ \frac{\theta - (\theta - 1)(1 - x_{t-1}^A)^2 - \left(\frac{H}{w_t}\right)^2}{2} & \text{if } \frac{H}{w_t} < a_{t-1}^x \end{cases} \quad (15)$$

## 4. Equilibrium and Dynamics

In each period  $t$  two stocks are created and transferred to period  $t+1$ : the stock of physical capital,  $K_{t+1}$ , and a stock that measures the proportion of women in their life's second period with modern sector work experience,  $x_t^A$ . In this section we show that given the initial values of these stocks, denoted by  $K_0$  and  $x_{-1}^A$ , a unique Perfect Foresight Equilibrium (PFE) exists. We first define the PFE and then turn to its determination and properties.

**Definition 1:** A PFE is a set of allocations  $\{(L_t^A, L_t^B, L_t^C, L_t^D, K_t)\}_{t=0}^\infty$  and a set of prices  $\{(w_t, R_t)\}_{t=0}^\infty$  that satisfy (2), (3), (8), (9), (14), (15), the law of motion of capital given in (16) and  $L_t = \sum_z L_t^z$ ,  $z \in \{A, B, C, D\}$  for all  $t$ ,  $t = 0, \dots, \infty$ .

### 4.1 Determination of the Equilibrium

Unlike some dynamical macroeconomic models, it is impossible in our model to obtain the allocations and prices of the period  $t$  equilibrium by solving a finite set of equations which depend on period  $t$  variables alone, given the stocks created in period  $t-1$ . Instead, given  $K_t$  and  $x_{t-1}^A$ , period  $t$  variables must be found jointly with the variables of periods  $t+1, t+2, \dots, \infty$ . This is because the period  $t$  labor supply of women born in that depends not only on current wage,  $w_t$ , but also on future wage,  $w_{t+1}$ .

We develop this section in the following four steps: First, we argue that if a pair of  $(x_t^A, w_t)$ , is an element of the PFE then the full set of the period  $t$  equilibrium can be obtained. Second, we show that  $(x_{t+1}^A, w_{t+1})$  is uniquely determined by  $(x_t^A, w_t)$ . These first two steps imply that this two-dimensional dynamic system fully describes the evolution of our economy. In the third step, we show the existence and the uniqueness of a steady state and in the fourth step, we characterize the steady state and show that it is a saddle. The last two steps imply that given the initial stocks,  $K_0$  and  $x_{-1}^A$ , only one pair of  $(x_0^A, w_0)$  is consistent with definition 1.<sup>9,10</sup>

**Step 1:** If  $(x_t^A, w_t)$  is an element of the PFE then the full set of the period  $t$  equilibrium can be obtained.

Given  $w_t$  we can obtain  $L_t^C$  and  $L_t^D$  using (8) and (9). Given  $w_t$  and  $x_{t-1}^A$  we can obtain  $L_t^B$  using (15), and given  $x_t^A$  we can obtain  $L_t^A$  using (14). Then, since by definition,  $L_t = \sum_z L_t^z$ , we can obtain  $K_t$  from (2) and then we can have  $R_t$  from (3).

**Step 2:**  $(x_{t+1}^A, w_{t+1})$  is uniquely determined by  $(x_t^A, w_t)$ .

From (13) it follows that in the relevant range  $w_{t+1}$  can be shown as a function of  $x_t^A$  and  $w_t$ , we denote this function by  $w_{t+1}(x_t^A, w_t)$  and present it explicitly in equation (A.4) in the appendix.

The stock of capital available in period  $t+1$  is also a function of  $x_t^A$  and  $w_t$ . The stock of capital available in period  $t+1$  equals the period  $t$  savings. These savings are the incomes of the generation born in period  $t$ :

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<sup>9</sup> As will become apparent in the analysis below, a pair of  $(x_0^A, w_0)$  which is not on the saddle path can be an equilibrium in period  $t$  only if individuals' expectations for some period  $s, s > t$  are not rational.

<sup>10</sup> Note that although in definition 1 the set of allocations contains the amount of efficiency unit of labor,  $L_t^z$ , the dynamical system is expressed in terms of the number (and due to the normalization the proportion) of young women who supply their labor to the modern sector. Equation (14) shows the unique mapping between these two variables.

$$K_{t+1}(x_t^A, w_t) = L_t^A w_t + L_t^C w_t + (1 - x_t^A)H + (1 - x_t^C)P \quad (16)$$

Note that  $x_t^C$  and  $L_t^C$  are functions of  $w_t$ , as follows from (7) and (8).  $L_t^A$  is a function of  $x_t^A$ , as follows from (14). Manipulating (2) yields  $L_{t+1}$  as a function of  $w_{t+1}$  and  $K_{t+1}$  and therefore as the following function of  $x_t^A$  and  $w_t$ :

$$L_{t+1}(x_t^A, w_t) = \frac{K_{t+1}(x_t^A, w_t)}{4w_{t+1}(x_t^A, w_t)} \quad (17)$$

By definition of  $L_{t+1}^z$ ,  $L_{t+1} = \sum_z L_{t+1}^z$ . Note from (8) and (9) that  $L_{t+1}^C$  and  $L_{t+1}^D$  are functions of  $w_{t+1}$  and therefore of  $x_t^A$  and  $w_t$ .  $L_{t+1}^B$ , the period  $t+1$  labor supply of women born in period  $t$ , is a function of  $x_t^A$  and  $w_{t+1}$ , as follows from section 3.3. Thus, in labor market equilibrium  $L_{t+1}^A$  is the following function of  $x_t^A$  and  $w_t$ :

$$L_{t+1}^A(x_t^A, w_t) = \quad (18)$$

$$L_{t+1}(x_t^A, w_t) - L_{t+1}^B[x_t^A, w_{t+1}(x_t^A, w_t)] - L_{t+1}^C[w_{t+1}(x_t^A, w_t)] - L_{t+1}^D[w_{t+1}(x_t^A, w_t)]$$

Finally, applying (18) into (14) yields  $x_{t+1}^A$  as a function of  $x_t^A$  and  $w_t$ :

$$x_{t+1}^A(x_t^A, w_t) = 1 - \sqrt{1 - 2L_{t+1}^A(x_t^A, w_t)} \quad (19)$$

Thus, the model generates a dynamical system where  $x_{t+1}^A$  and  $w_{t+1}$  are uniquely determined by  $x_t^A$  and  $w_t$ . Equations (A.4) and (A.5) - (A.8) in the appendix show this system explicitly.

**Step 3:** Existence and Uniqueness of the Steady State.

**Proposition 3:** The dynamical system  $x_{t+1}^A(x_t^A, w_t)$ ,  $w_{t+1}(x_t^A, w_t)$  has a unique steady state.

**Proof:** See part E in the appendix.

**Step 4:** Stability of the Steady State.

**Proposition 4:** The Steady State of the dynamical system  $x_{t+1}^A(x_t^A, w_t)$ ,  $w_{t+1}(x_t^A, w_t)$  is a saddle.

**Proof:** See part F in the appendix.

Steps 3 and 4 as established in propositions 3 and 4, imply that only one pair of  $(x_0^A, w_0)$  is consistent with definition 1. Thus the economy converges to the steady state along the saddle path.

Figure 2 shows the phase diagram of this system. The  $ww$  curve in figure 2 is the set of pairs of  $(x_t^A, w_t)$  for which  $w_{t+1}=w_t$ . Likewise, the  $xx$  curve in figure 2 is the set of pairs of  $(x_t^A, w_t)$  for which  $x_{t+1}^A=x_t^A$ . Consistent with proposition 3, the two curves intersect once and only once.

Differentiating  $w_{t+1}(x_t^A, w_t)$  with respect to  $x_t$  yields  $\frac{\partial w_{t+1}}{\partial x_t} > 0$ , which implies that

above the  $ww$  curve  $w$  increases and vice versa. Similarly, differentiating  $x_{t+1}^A(x_t^A, w_t)$

with respect to  $w_t$  yields  $\frac{\partial x_{t+1}^A}{\partial w_t} > 0$  which implies that above the  $xx$  curve  $x$  decreases and

vice versa. The proof of proposition 4 implies the existence of a saddle path that leads to the steady state. This path is presented in figure 2 by the increasing dotted line, which crosses the steady state point.

## 4.2 An Application

In this subsection we show the possibility of non-monotonic dynamics of the ratio of the mean wage of working women to that of men, which we denote by  $rw$ . This possibility is presented using a numerical example.

Given the following parameter values,  $P=0.025$ ,  $H=0.02$  and  $\theta=1.5$  we show that if the initial conditions,  $K_0$  and  $x_{-1}$ , are sufficiently low, the dynamics of the model starts in a range where each of the dynamic labor profiles 1, 2, and 4, are optimal for some young women. The least able women choose profile 1, which means that they stay at home for their two life's period; the abler women choose profile 2 which means that they stay at home in their life's first period and work in the modern sector in their life's second period; the ablest women choose profile 4, which means that they work in the modern sector in both their life's period. Thus, since in that range profile 2 is optimal for some women, there are men and women in their life's second period who are newcomers to the modern sector. In the previous period, these men acquired market skills by working in the physical sector, whereas these women worked at home and therefore have no market skills. This causes the relative market skills of women (to men) in the modern sector to decrease and thus the ratio of the mean wage of working women to the mean wage of working men decreases. As the economy develops the growth in wages slows down until the rate of growth of wages is sufficiently low to make profile 2 no longer optimal for any woman. Thus, at this stage the reason for the decrease in  $rw$  is removed. From then on, the increase in wages in the modern sector affects women average income more than it affects men's, since female LFP consists only of work in the modern sector while men also work in the physical sector. As a result, in this stage  $rw$  increases. Figure 3a shows this non-monotonic dynamics of  $rw$ , given the above values of  $P$ ,  $H$ ,  $\theta$ , and the following values for the initial stocks:  $K_0=0.0001$  and  $x_{-1}^A = 0.02$ . Figure 3b shows the reason why profile 2 existed in the early stage of the development of the economy but vanished later - the concavity of the increase in wages in the modern sector.

## 5. Gender Gap Data and the Dynamics in the Model

In this section we compare the dynamics of the gender gap implied by the model with the observed dynamics of this gap during the 20<sup>th</sup> century in the US. Smith and Ward (1989) show that the gender gap in 1980 was larger than in 1920. They also show that between 1968 and 1986 the gender gap has been gradually decreasing. Such non-monotonic dynamics were also documented by O'Neil and Polachek (1993). While Smith and Ward

(1989) estimate hourly wages of working women as a percent of those of working men, O’Neil and Polachek (1993) look at ratio of median earnings for full-time year-round workers. However, in years that appear in both studies the difference between the two ratios is negligible.<sup>11</sup> Therefore we combine the data from both sources and present it in figure 4. The figure shows that the ratio of mean incomes has presented a W-shaped dynamics with a local maximum in the 1950’s.

It is possible to interpret the local peak in the 1950’s as a result of a temporary decrease in the value of home production during World War II. Goldin (1991, p.741) states that: “A husband’s absence often meant that his wife has less to do in the home”. Obviously, this decline in the value of home production can be viewed as a temporary shock.<sup>12</sup> Table 4 in Goldin (1991) shows that 25 percent of the working women in the group age of 27-51 in January 1951 did not work in December 1941 but worked in March 1944. According to the composition effect described in our model the substantial growth in female LFP during the 1940’s would lead to a similarly sharp decline in the experience differences between men and women during the 1950’s.

Following this logic, inserting an exogenous temporary decrease to the value of home production in our model ( $H$ ) would lead to a W-shaped dynamics instead of the U-shaped dynamics derived in the previous sections.

## 6. Concluding Remarks

In this paper we have developed a model that generates non-monotonic dynamics of the ratio of the mean wages of women to that of men, as observed in the US during the 20<sup>th</sup> century. Our approach follows the empirical studies that have shown that when this ratio was decreasing many women were entering the labor force in a late phase of their life

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<sup>11</sup> Hourly wage rates and earnings of full-time year-round workers are comparable, if female (and male) labor force participation is “heterogeneous”. Heterogeneous labor force means that if the participation rate is 20 percent, then 20 percent of the women are in the labor force all year and 80 percent are not in at all. Goldin (1989) shows that the female labor force was relatively heterogeneous from the 1920s to 1950s and Heckman and Willis (1977) show that heterogeneity continued into the 1970s. In fact, the maximal difference between the two ratios appears in 1980. Smith and Ward (1989) report a ratio of 0.605 and O’Neil and Polachek (1993) report a ratio of 0.602.

<sup>12</sup> Assuming that the temporary decline in the value of home production during the war time was followed by a temporary increase (relative to the value before the war) would only strengthen this argument. Such an increase could be attributed to the “baby boom” after the war.

with low skills and little labor market experience, and that this ratio started increasing together with the disappearance of this behavior of women. In our model entering the labor market in a relatively late phase of their lives is optimal for some women when wages grow sufficiently rapidly but later disappears when wages growth slows down.

In the model we assume that individuals have perfect foresight. Assuming alternatively that individuals are myopic in the sense that their choice whether to work at home or in the market in a certain period depends only on the wages in that period, would not change the main qualitative results of the paper. This occurs because these results rely on the existence of a group of women who choose a labor profile in which they work at home in that period and work in the market in the second period, if wages grow sufficiently fast. Each woman that chooses such profile taking into account the growth in wages and the role of market experience would necessarily take the same decision if in each period she were merely comparing the earnings from working at home to her earnings in the market in that period. In fact, this group is even larger under myopic behavior since under perfect foresight there are women who work in the market in the first period of their lives although they earn in that period less than they could earn at home. If these women were myopic they would not work in the market in the first period of their lives. Technically, the model under the assumption of myopic behavior would be much simpler since equilibrium in each period would not depend on the future equilibrium.

Another assumption in our model, taken for simplicity sake, is that productivity at home is constant. Greenwood, Seshardi and Yorukoglu (2001) have shown that time-saving technological improvements in home production play an important role in explaining female LFP dynamics in the past century in the US. Incorporating such progress in home production in our model requires that the assumption that individuals either works at home or in the market in each period should be replaced by the assumption that in each period individuals optimally allocate part of their time to home production and the rest to LFP. Such technological improvements in home production should have the same positive effect on individuals LFP decision as an increase in wages. Thus our conclusion that the gender gap decreases when wages grow sufficiently fast and vice versa, would be modified and become that the gender gap decreases when the

growth in wages, or the time saving technological progress in home production, are sufficiently fast, and vice versa.

## Appendix

A.

Here we show conditions which ensure that there will be women who work at home in their life's first period and join the modern sector in the second even if  $P' > 0$  and some women work in the physical sector.

The additional possible labor profiles:

Profile 5: Work in the physical sector in period  $t$  and in the modern sector in  $t+1$ .

Profile 6: Work in the physical sector in periods  $t$  and  $t+1$ .

Profile 7: Work in the modern sector in period  $t$  and in the physical sector in  $t+1$ .

Profile 8: Work in the physical sector in period  $t$  and at home in  $t+1$ .

Profile 9: Work at home in period  $t$  and in the physical sector in  $t+1$ .

Assuming  $\theta P' < H$  rules out labor profiles 6, 7, 8 and 9. Profiles 6, 8 and 9 are dominated by profile 1 for all  $a$ , and profile 7 is dominated by profile 3 for all  $a$ . Following the

procedure developed in section 3.2,  $V^5 = P' + \frac{a\theta w_{t+1}}{R_{t+1}}$  and  $a_{52,t} = \frac{H - P'}{(\theta - 1)4w_{t+1}^2}$ . First, note

that for some women to prefer profile 2 over profiles 1 and 4,  $w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2$  should hold. This implies that  $4(\theta - 1)w_{t+1} < 1$ . Using the values of  $a_{52,t}$  and  $a_{42,t}$ , the

assumption  $\theta P' < H$  and the last inequality provide us with sufficient condition for  $a_{42,t} >$

$a_{52,t}$ :  $w_t < \frac{\theta - 1}{\theta}$ . If this condition holds profile 5 is optimal for some women if  $a_{52,t} > a_{21,t}$ .

Using the values of  $a_{52,t}$  and  $a_{21,t}$  and the assumption  $\theta P' < H$  provides us with the

following sufficient condition:  $w_{t+1} < \frac{1}{4\theta}$ . Note that this condition does not contradict the

condition  $4(\theta - 1)w_{t+1} < 1$  that is necessary for some women to prefer profile 2 over profiles 1 and 4.

### B. The function $x_t^A(w_t, w_{t+1})$

Substituting (10) into (13) yields:

$$x_t^A = \begin{cases} 1 - \frac{H}{w_t + (\theta - 1)4w_{t+1}^2} & \text{if } w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2 \\ 1 - \frac{H(1 + 4w_{t+1})}{w_t + 4\theta w_{t+1}^2} & \text{if } w_{t+1} - 4(\theta - 1)w_{t+1}^2 < w_t < \theta w_{t+1} \\ 1 - \frac{H}{w_t} & \text{if } \theta w_{t+1} < w_t \end{cases} \quad (\text{A.3})$$

Note that despite the division to three ranges,  $x_t^A$  is continuous in  $w_t$  and  $w_{t+1}$ .

Differentiation of the first two lines of (A.3) shows that  $\frac{\partial x_t^A}{\partial w_{t+1}} > 0$  and therefore that

$$x_t^A \geq 1 - \frac{H}{w_t} \text{ for all } w_{t+1}.$$

### C. The function $w_{t+1}(x_t^A, w_t)$

For convenience of notation we define  $h_t(x_t^A) \equiv h_t \equiv \frac{H}{1 - x_t^A}$ . Since  $x_t^A \geq 1 - \frac{H}{w_t}$  for all

$w_{t+1}$  it follows that  $h_t \geq w_t$ . Manipulating the first line in (A.3) in order to isolate  $w_{t+1}$  yields

$$w_{t+1} = \sqrt{\frac{h_t - w_t}{4(\theta - 1)}}. \text{ Applying this in } w_t < w_{t+1} - 4(\theta - 1)w_{t+1}^2 \text{ shows that this case is}$$

relevant in the range  $E \equiv \{(w_t, x_t^A) : w_t < h_t - 4(\theta - 1)h_t^2\}$ . Likewise, isolating  $w_{t+1}$  in the

second line of (A.3) yields  $w_{t+1} = \frac{h_t + \sqrt{h_t^2 - \theta(w_t - h_t)}}{2\theta}$  and the relevant range becomes

$F \equiv \{(w_t, x_t^A) : h_t - 4(\theta - 1)h_t^2 < w_t < h_t\}$ . In the third line of (A.3)  $w_t > \theta w_{t+1}$  implying a

rapid decline in wages over time. The economy will therefore be in this range only if the initial stock of physical capital is sufficiently large. Note that in this range there is a set of

values of  $w_{t+1}$ , rather than a single value, that corresponds to a given pair of  $(w_t, x_t^A)$ . Since the case of rapidly declining wages is not in the focus of this paper, we assume that the initial stock of physical capital is not that large. (A.4) summarizes this analysis:

$$w_{t+1}(w_t, x_t^A) = \begin{cases} \sqrt{\frac{h_t - w_t}{4(\theta - 1)}} & \text{if } (w_t, x_t^A) \in E \\ \frac{h_t + \sqrt{h_t^2 - \theta(w_t - h_t)}}{2\theta} & \text{if } (w_t, x_t^A) \in F \end{cases} \quad (\text{A.4})$$

Note that  $x_t^A$  appears on the RHS of (A.4) through  $h_t$ .

**D. The function  $x_{t+1}^A = (x_t^A, w_t)$**

Following the procedure presented in detail in section 4.1 the explicit form of the function  $x_{t+1}^A = (x_t^A, w_t)$  is:

*In the range where  $(w_t, x_t^A) \in E$  and  $w_t \geq P$*

$$x_{t+1}^A = 1 - \sqrt{2(\theta + 1) - 2(\theta - 1) \frac{w_t - \left(\frac{H}{h_t}\right)^2 \frac{w_t}{2} + \frac{P^2}{2w_t} + \frac{H^2}{h_t} + 2(1 + \theta)P^2 + 2H^2}{h_t - w_t} - (\theta - 1) \left(\frac{H}{h_t}\right)^2} \quad (\text{A.5})$$

*In the range where  $(w_t, x_t^A) \in E$  and  $w_t < P$*

$$x_{t+1}^A = 1 - \sqrt{-2(\theta - 1) \frac{\left[1 - \left(\frac{H}{h_t}\right)^2\right] \frac{w_t}{2} + \frac{H^2}{h_t} + P + 2H^2}{h_t - w_t} + (\theta + 1) - (\theta - 1) \left(\frac{H}{h_t}\right)^2} \quad (\text{A.6})$$

*In the range where  $(w_t, x_t^A) \in F$  and  $w_t \geq P$*

$$x_{t+1}^A = 1 - \sqrt{2(\theta + 1) - 2\theta^2 \frac{w_t - \frac{w_t}{2} \left(\frac{H}{h_t}\right)^2 + \frac{P^2}{2w_t} + \frac{H^2}{h_t} + 2(\theta + 1)P^2}{2h_t^2 + \theta(h_t - w_t) + 2h_t \sqrt{h_t^2 + \theta(h_t - w_t)}} - \theta \left(\frac{H}{h_t}\right)^2} \quad (\text{A.7})$$

In the range where  $(w_t, x_t^A) \in F$  and  $w_t < P$

$$x_{t+1}^A = 1 - \sqrt{-2\theta^2 \frac{w_t - \frac{w_t}{2} \left(\frac{H}{h_t}\right)^2 + \frac{P^2}{2w_t} + \frac{H^2}{h_t}}{2h_t^2 - \theta(w_t - h_t) + 2\sqrt{h_t^2 - \theta(w_t - h_t)}} - \theta \left(\frac{H}{h_t}\right)^2 + \theta + 1} \quad (\text{A.8})$$

### E. Proof of Proposition 3 – Existence and Uniqueness of the Steady State

The proof is given in two steps. In step 1 we obtain an equation in  $\bar{w}$  which is the steady state level of  $w$ . In step 2 we show that it has a unique positive solution.

**Step 1:** Since in the steady state  $w = \bar{w}$  is constant over time, only profiles 1 and 4 are optimal for women. Thus, in the steady state the number of women in the life's first period and the number of women in the life's second period who work in the modern sector is given by:  $x = 1 - a_{42}$  for each group. The number of men in the life's first period and men in the life's second period is given by:  $1 - \frac{P}{\bar{w}}$ . Using (14), (15), (8) and (9) we can calculate the steady state amount of labor supplied to the modern sector:

$$\bar{L} = (1 + \theta) \left[ 1 - \frac{P^2 + H^2 \left(\frac{1 + 4\bar{w}}{1 + 4\theta\bar{w}}\right)^2}{2\bar{w}^2} \right]$$

Using (14), (8) and (16) we can calculate the stock of physical capital in the steady state:

$$\bar{K} = \frac{2\bar{w}^2 + P^2 + \frac{2H^2(1 + 4\bar{w})}{(1 + 4\theta\bar{w})} - \frac{H^2(1 + 4\bar{w})^2}{(1 + 4\theta\bar{w})^2}}{2\bar{w}}$$

Equation (2) implies that in the steady state:  $\bar{w} = 0.5\bar{K}^{0.5}\bar{L}^{0.5}$ , or equivalently,  $\bar{K} = 4\bar{w}^2\bar{L}$ .

Using  $\bar{K}$  and  $\bar{L}$  from above in this equation and manipulating yields:

$$128(1+\theta)\theta^2\bar{w}^5 + 32\theta[2+\theta]\bar{w}^4 + 8[1-\theta-8\theta^2P^2(1+\theta)-8H^2]\bar{w}^3 - 2[8\theta P^2(2+3\theta)+1+8H^2(1+7\theta)]\bar{w}^2 - 4[P^2(1+3\theta)+H^2(7\theta+1)]\bar{w} - P^2 - H^2 = 0$$

**Step 2:** define  $F(w)$  as follows:

$$F(w) \equiv 128(1+\theta)\theta^2w^5 + 32\theta[2+\theta]w^4 + 8[1-\theta-8\theta^2P^2(1+\theta)-8H^2]w^3 - 2[8\theta P^2(2+3\theta)+1+8H^2(1+7\theta)]w^2 - 4[P^2(1+3\theta)+H^2(7\theta+1)]w - P^2 - H^2$$

Note that  $F$  is a polynomial of fifth degree in  $w$  where  $a_5, a_4 > 0$  and  $a_3, a_2, a_1, a_0 < 0$ . We denote by  $F^{(i)}$  the  $i$ -th derivative of  $F$ . Therefore:

(a)  $F^{(5)}(w) > 0$  for all  $w$ .

(b)  $F^{(4)}(0) > 0$ .

Due to (a) and (b):

(c)  $F^{(4)}(w) > 0$  for all  $w > 0$ .

Also:

(d)  $F^{(3)}(0) < 0$

(e)  $\lim_{w \rightarrow \infty} F^{(3)}(w) = \infty$

Due to (c), (d) and (e):

(f) In the range  $w > 0$  there is a unique  $w^* > 0$  for which  $F^{(3)}(w^*) = 0$ .

Also:

$$(g) \quad F^{(2)}(0) < 0$$

$$(h) \quad \lim_{w \rightarrow \infty} F^{(2)}(w) = \infty$$

Due to (f), (g) and (h):

$$(i) \quad w^* \text{ is the unique minimum of } F^{(2)}(w) \text{ in the range } w > 0.$$

$$(j) \quad F^{(2)}(w^*) < 0$$

$$(k) \quad \text{There is a unique } w^{**} > w^* > 0 \text{ for which } F^{(2)}(w^{**}) = 0.$$

Also:

$$(l) \quad F^{(1)}(0) < 0$$

$$(m) \quad \lim_{w \rightarrow \infty} F^{(1)}(w) = \infty$$

Due to (k), (l) and (m):

$$(n) \quad w^{**} \text{ is the unique minimum of } F^{(1)}(w) \text{ in the range } w > 0.$$

$$(o) \quad F^{(1)}(w^{**}) < 0$$

$$(p) \quad \text{There is a unique } w^{***} > w^{**} > w^* > 0 \text{ for which } F^{(1)}(w^{***}) = 0.$$

Finally,

$$(q) \quad F(0) < 0.$$

$$(r) \quad \lim_{w \rightarrow \infty} F(w) = \infty.$$

Due to (p), (q) and (r):

$$(s) \quad w^{***} \text{ is the minimum of } F(w) \text{ in the range } w > 0.$$

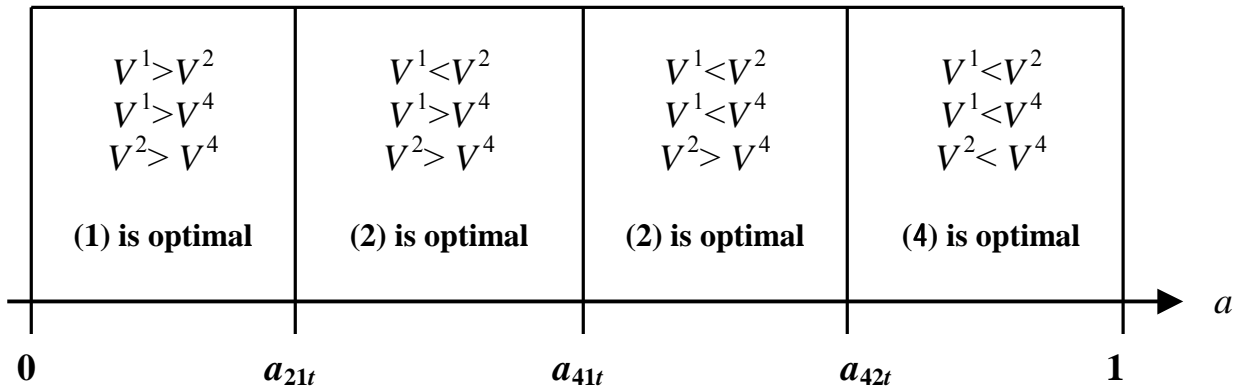
$$(t) \quad F(w^{***}) < 0$$

There is a unique  $\bar{w} > w^{***} > w^{**} > w^* > 0$  for which  $F(\bar{w}) = 0$ . *QED*.

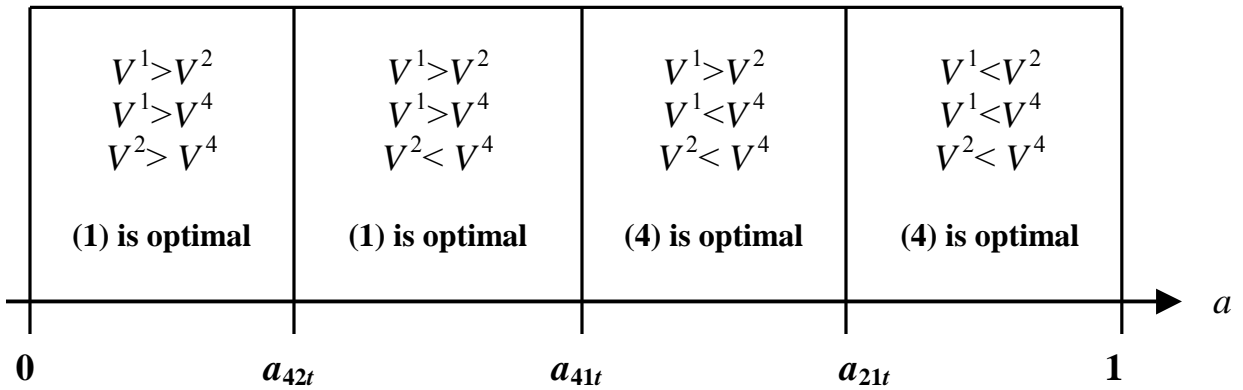
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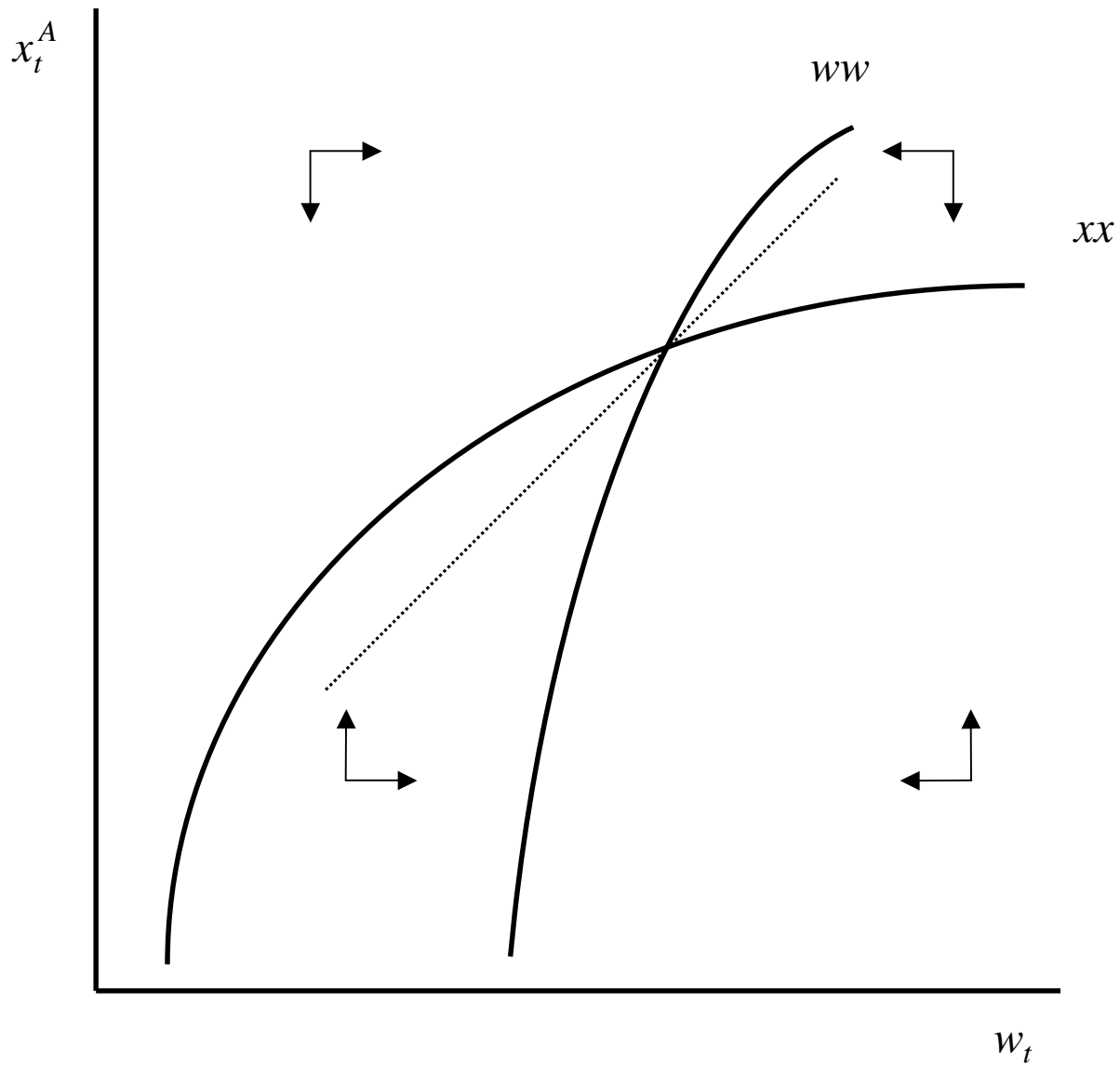
**Figure 1- the distribution of women's labor profiles over ability**



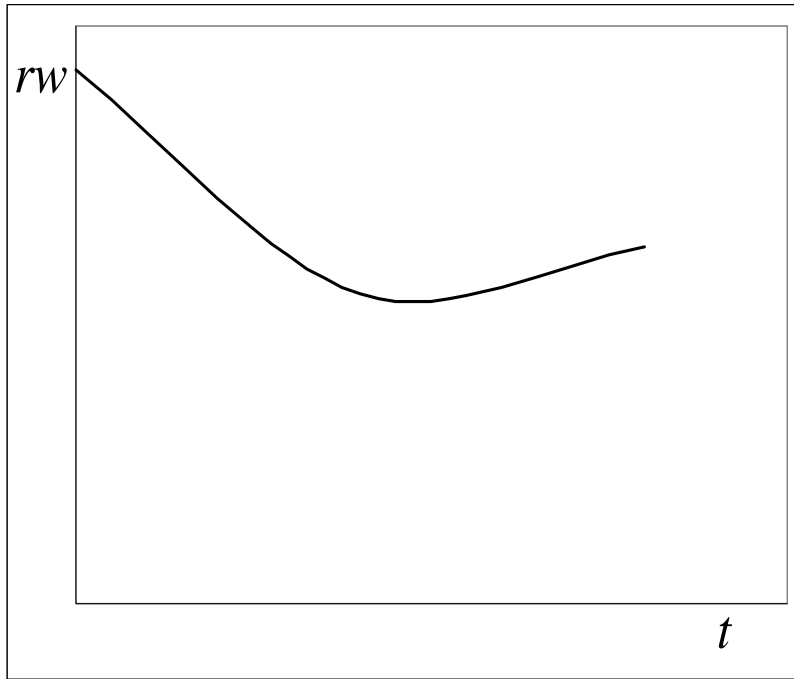
**Figure 1.a: When  $w_t$  is sufficiently smaller than  $w_{t+1}$**



**Figure 1.b: When  $w_t$  is not sufficiently smaller than  $w_{t+1}$**

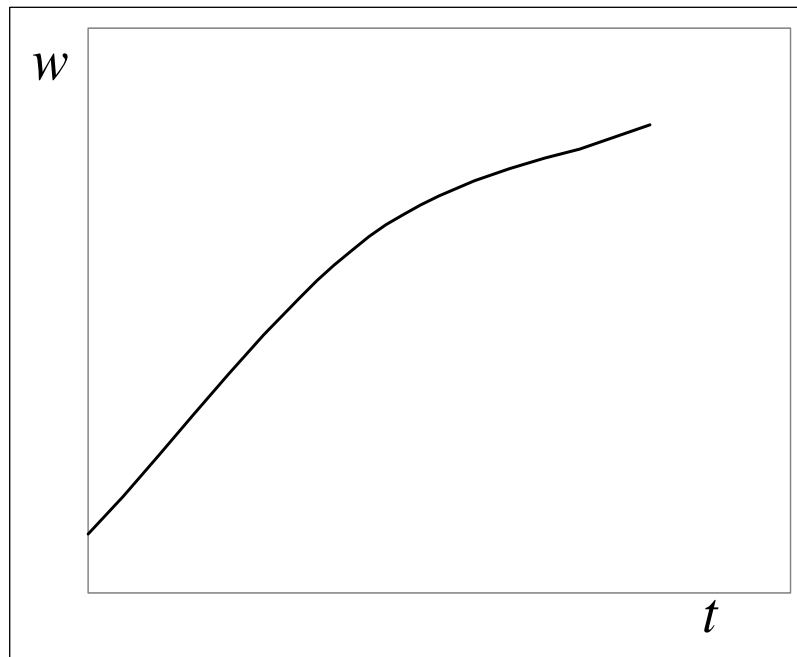


**Figure 2 – the Phase Diagram**



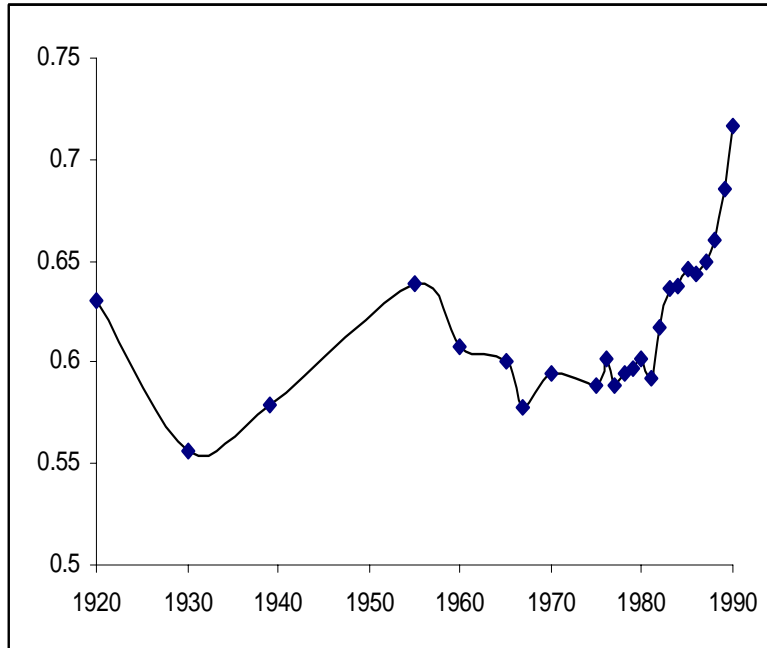
### 3.a - The ratio of means wages over time

Parameter values:  $H=0.02$ ,  $P=0.025$ ,  $\theta=1.5$ ,  $K_0=0.0001$ ,  $x_{-1}^A=0.02$



### 3.b - The wage for one unit of efficiency labor over time

Parameter values:  $H=0.02$ ,  $P=0.025$ ,  $\theta=1.5$ ,  $K_0=0.0001$ ,  $x_{-1}^A=0.02$



**Figure 4: The ratio of mean earnings in the US** (sources - Smith and Ward (1989) and O'Neil and Polachek (1993). See text for details)