The Effect of Better Information on Growth and Welfare

by

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June 2001

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*Financial support from the Stifterverband für die Deutsche Wissenschaft and the Commerzbank is gratefully acknowledged.

Abstract

We develop an OLG economy with accumulation in human capital. Heterogeneity among individuals in each generation results mainly from the (random) innate ability assigned to each individual. When young, all agents observe a private signal which reveals some information about a person's ability. Private investment in education and training is based on this signal and therefore varies from one young person to the other. We analyze how *better information*, which allows better 'screening' during the 'youth' period when the individual human capital is being formed, affects welfare and the accumulation process of human capital in each generation. Our analysis highlights the critical role played by the degree of risk aversion in the economy and by the availability of a risk sharing market.

1 Introduction

The role of human capital in enhancing economic growth has been analyzed extensively in the literature of the last two decades. Following the seminal contributions of Becker emphasizing the link between education and productivity [see, for example, Becker (1964)], the role of human capital became central in endogenous growth models [see, for example, Razin (1972), Lucas (1988) and Azariadis and Drazen (1990)]. The assumptions regarding the process of human capital formation became significant in the evolution of these dynamic models. The production function of human capital is clearly a complex one since it is affected by many factors including the home and the social environment, provision of education, motivation etc. [see, for example, Jovanovic and Nyarko (1995), Laitner (1997) and Orazem and Tesfatsion (1997)]. This aspect of the human capital formation process is a central point in our work. We integrate two strands in the literature: endogenous growth with human capital accumulation and the role of information. In our framework, information affects the process of human capital formation and, hence, economic growth.

Since the seminal contributions by Blackwell (1951, 1953) on the positive welfare implications of 'more information' for an individual decision maker, this topic has attracted substantial attention among economists. The decision maker observes a signal, correlated to the state of nature, and updates his/her probability distribution before taking an action. However, Blackwell's result holds in an economic environment where the signals are private information. If information is public signals affect the opportunity sets of decision makers and more information may result in a lower welfare. The negative value of public information in equilibrium has been established for certain types of exchange economies by Hirshleifer (1971,1975), Green (1981), Orosel (1996), Schlee (2001) and others. When the model includes production the welfare implications of better information are more gratifying. Eckwert and Zilcha (2001) demonstrate that whether signals reveal information about 'uninsurable risks' or 'insurable risks' is important in determining the value of information.

The aforementioned papers dealing with the value of information consider static models and therefore ignore the implications of information for economic growth. Economic growth affects the welfare of future generations, hence the welfare analysis should be extended. Our paper takes into account the impact of information on the formation process of human capital and thereby on the evolution of the economy over time. It is widely recognized that investment in human capital is subject to considerable risk. In addition, the possibilities for diversification are quite limited because human capital cannot be traded on markets and cannot be separated from individuals [see Levhari and Weiss (1974), Stiglitz (1975)]. In some cases, insurance contracts which are contingent on the human capital of an individual may be tradable, thereby allowing the agents to share part of their idiosyncratic risks. Our study compares the welfare effects of information under two different scenarios. The first scenario is characterized by the absence of any risk sharing arrangements; and under the second scenario agents are able to obtain partial insurance for the risky returns of investments in human capital.

The framework we use for our analysis is an overlapping generations economy with production [see Diamond (1965)], a continuum of households in each generation, and no population growth. Individuals in the same generation differ in their (random) innate abilities. We assume that the human capital of an individual depends upon his/her innate ability as well as the 'environment', represented by the average human capital level of the older generation (the generation of the teachers and parents).[1] When ability is still unknown each individual decides how much 'effort' to invest in his/her education and training. The return to this investment, in term of wages during the working period, is random since it depends on the realization of the ability. However, prior to making the decision about investment in education (i.e., effort in our case), each agent observes a signal which reveals in a Bayesian manner some information about his/her personal ability. Thus, the 'screening' process, and hence the accumulation of human capital, depends on the

[1] Endogenous growth models in which human capital operates as the engine of growth have been widely used in the literature to analyze various economic issues related to economic policy [see, e.g., Lucas (1988), Azariadis and Drazen (1990), Eckstein and Zilcha (1994), Galor and Tsiddon (1997), and Orazem and Tesfatsion (1997). We use a similar framework for our study of the dynamic effects of better information.

informativeness of the signals. In our model better information means better screening with respect to individual ability when education and training are being formed. In the extreme case where signals are uninformative private investment in education is uncorrelated to ability.

Better information affects economic welfare in two ways. First, as signals become more reliable the agents are exposed to less uncertainty when they make their decisions. This reduction in uncertainty has an impact on welfare which is called the *direct* effect. Second, better information creates an externality through its impact on the accumulation of human capital: future generations benefit from a higher accumulation rate because they inherit part of their human capital from the previous generation. This mechanism is called the *indirect* welfare effect. We show that if no risk sharing is available then (a) the direct effect on welfare is always positive; (b) the indirect effect, i.e., the effect via growth, is positive in economies with moderately risk averse agents, and negative in highly risk averse economies. We also demonstrate that the operation of a risk sharing market can potentially interfere with the informative structure that the economy displays and, hence, with the ability of the screening process to enhance welfare. More precisely, if part of the human capital risk can be insured, both the direct and the indirect welfare effects are negative in economies with highly risk averse agents, and positive in economies with moderately risk averse agents. Thus, if the consumers are highly risk averse, under certain conditions better screening during the youth period is harmful and will reduce the welfare of all generations. Our dynamic model demonstrates that, in equilibrium, the value of information depends heavily on the risk sharing arrangements that exist in the market. In particular, it matters whether the information relates to risks which can be insured or to risks that are uninsurable

The rest of the paper is organized as follows: We present the model in Section 2. In Section 3 we consider the case where the human capital risk cannot be insured. The analysis in Section 4 analyzes the role of information when part of the human capital risk is insurable. Section 4 concludes the paper. All proofs are gathered in a separate Appendix.

2 The Model

Consider an overlapping generations economy with a single commodity and a continuum of individuals in each generation (but no population growth). The commodity can be either consumed or used as an input (physical capital) in a production process. Individuals live for three periods: 'youth' where they obtain education (while still supported by parents), 'middle-age' where they work and consume, and 'retirement' where they only consume. We denote generation t by $G_t, t = 0, 1, \cdots .G_t$ consists of all individuals born at date $t-1$.

One of the main features of our economy is the heterogeneity of individuals with regard to their human capital generated by a random innate ability. When individual i is born his ability is yet unknown. The uncertainty about the agent's ability is described by some random variable $\tilde{A}^i$ which realizes at the beginning of the next period and takes values in some interval $\mathcal{A} \subset \mathbb{R}_+$. We assume that the random variables $\tilde{A}^i, i \in G_t, t = 0, 1, \cdots$, are i.i.d; thus, in particular, the ex ante distribution of ability is the same for all agents and does not depend on time or on the history of the economy.

Human capital of individual $i \in G_t$ depends on ability $\tilde{A}^i$ (which is random), effort $e^i \in \mathbb{R}_+$ invested in education by this individual, and the 'environment', represented here by the average human capital of agents in the previous generation (who are currently active economically). Thus we write,

$$\tilde{h}^i = \tilde{A}^i g(H_{t-1}, e^i) \tag{1}$$

where i belongs to generation t, and H_{t-1} is the average human capital of G_{t-1}, (see the role of H_{t-1} in generating human capital of G_t, for example, in Lucas (1988), Azariadis and Drazen (1990)).

Assumption 1 *The function $g(H, e)$ is strictly increasing and $g_{12} \geq 0, g_{22} < 0$.*

A priori the distribution of random ability $\tilde{A}^i$ is the same for all agents i both within the same generation and across generations.[2] However, before choosing

[2]In the sequel we will therefore suppress the index i and write $\tilde{A}$ instead of $\tilde{A}^i$. Note, however,

optimal effort in the youth period each individual observes a signal which contains information about his own ability. We model the informational structure of the economy as follows: let $\tilde{y}$ be a real-valued random variable which takes values in $Y \subset \mathbb{R}$ and is correlated to $\tilde{A}$. Each agent $i \in G_t, t = 0, 1, \cdots$, with ability A observes an individual signal y^i which is drawn randomly from the distribution of the random variable $(\tilde{y}|A)$.[3] By construction, this individual signal is correlated to i's ability. Therefore, when agent i makes his decision about how much effort e^i to invest in education, the relevant c.d.f. for random ability is the posterior distribution of $\tilde{A}$ given the individual signal y^i.

Let $\nu : \mathcal{A} \to \mathbb{R}_+$ be a (Lebesgue)-density function where $\nu(A)$ is the density of agents with ability A. We assume

$$\int_{\mathcal{A}} \nu(A)\mathrm{d}A = 1,$$

i.e., the measure of agents is normalized to 1. The distribution of individual signals received by agents in generation t has the density[4]

$$\mu(y) = \int_{\mathcal{A}} \nu_A(y)\nu(A)\mathrm{d}A, \tag{2}$$

where ν_A denotes the density of the random variable $(\tilde{y}|A)$. Average ability of all agents who have received the individual signal y is

$$\bar{A}(\nu_y) := \int_{\mathcal{A}} A\nu_y(A)\mathrm{d}A. \tag{3}$$

ν_y denotes the density of the random variable $(\tilde{A}|y)$.

We assume that signals are public information while the effort employed by the individual is private information. This assumption will be relaxed later on.

Individuals derive negative utility from 'effort' while they are young. Denote their consumption in the working period by c_1, and in retirement period by c_2. For

that in general the random variables $\tilde{A}^i$ and $\tilde{A}^j$ differ for $i \neq j$; only their distributions are the same.

[3]Throughout the paper we shall refer to the realizations of $\tilde{y}$ as signals, and to the realizations of the $\tilde{y}^i$'s as *individual* signals.

[4]Note that, by the law of large numbers, μ does not depend on t.

each agent, the lifetime utility function is given by

$$U(e, c_1, c_2) = v(e) + u_1(c_1) + u_2(c_2). \tag{4}$$

We assume that,

Assumption 2 *The utility functions v and u_j, $j = 1, 2$, have the following properties:*

(i) $v : \mathbb{R}_+ \longrightarrow \mathbb{R}_-$ *is decreasing and strictly concave,*

(ii) $u_j : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ *is increasing and strictly concave, $j = 1, 2$.*

In each period, production in our economy, is carried out by competitive firms who use two production factors: physical capital K and human capital H. The process is described by an aggregate production function $F(K, H)$, which exhibits constant returns to scale. If individual i supplies l^i units of labor in his 'working period', his supply of human capital equals $l^i h^i$. We assume that l^i is a constant and it is equal 1 for all i.

Assumption 3 *$F(K, H)$ is concave, homogeneous of degree 1, and satisfies $F_K > 0$, $F_H > 0, F_{KK} < 0, F_{HH} < 0$.*

We assume throughout this paper full international capital mobility, while human capital is assumed to be immobile. Thus the interest rate $\bar{r}_t$ is exogenously given at each date t. This implies that marginal productivity of aggregate physical capital K_t must be equal to $1 + \bar{r}_t$ (assuming full depreciation of capital in each period). On the other hand, given the aggregate stock of human capital at date t, H_t, the stock K_t must adjust such that

$$1 + \bar{r}_t = F_K(K_t, H_t) \qquad t = 1, 2, 3, \cdots \tag{5}$$

holds. But this implies, by Assumption 3, that $\frac{K_t}{H_t}$ is determined by the international rate of interest $\bar{r}_t$. Hence the wage rate w_t (price of one unit of human capital), given in equilibrium by the marginal product of aggregate human capital, is also determined once $\bar{r}_t$ is given. Thus we may write

$$w_t = F_L\Big(\frac{K_t}{H_t}, 1\Big) =: \zeta(\bar{r}_t) \qquad t = 1, 2, 3, \cdots. \tag{6}$$

Now let us consider the optimization problem that each $i \in G_t$ faces, given $\bar{r}_t, w_t$, and H_{t-1}. At date $t-1$, when 'young', this individual chooses the optimal level of effort employed in obtaining education. This decision is made under random ability $\tilde{A}$, but after the individual signal y^i has been observed. The decision about saving, s^i, to be used for consumption when 'old' is taken in the second period, after the realization of $\tilde{A}$, and hence when the human capital h^i is known. Thus s^i will depend on h^i via the wage earnings $w_t h^i$.

For given levels of h^i, w_t and $\bar{r}_t$, the optimal saving decision of individual $i \in G_t$ is determined by

$$\begin{aligned} \max_{s^i} \ & u_1(c_1^i) + u_2(c_2^i) \\ \text{s.t. } & c_1^i = w_t h^i - s^i \\ & c_2^i = (1+\bar{r}_t)s^i \end{aligned} \tag{7}$$

and satisfies the necessary and sufficient first order condition

$$-u_1'(w_t h^i - s^i) + (1+\bar{r}_t)u_2'((1+\bar{r}_t)s^i) = 0 \tag{8}$$

for all h^i. From equation (8) we find optimal saving as a function of each realized h^i, i.e., $s^i = s_t(h^i)$. The optimal level of effort invested in education, e^i, is determined by

$$\begin{aligned} \max_{e^i} \ & E[v(e^i) + u_1(\tilde{c}_1^i) + u_2(\tilde{c}_2^i)|y^i] \\ \text{s.t. } & \tilde{c}_1^i = w_t \tilde{h}^i - \tilde{s}^i \\ & \tilde{c}_2^i = (1+\bar{r}_t)\tilde{s}^i, \end{aligned} \tag{9}$$

where $\tilde{h}^i$ is given by equation (1) and $\tilde{s}^i$ satisfies equation (8). Due to the Envelope theorem and the strict concavity of the utility functions, problem (9) has a unique solution determined by the first order condition

$$v'(e^i) + w_t g_2(H_{t-1}, e^i)E[\tilde{A}u_1'(w_t\tilde{h}^i - \tilde{s}^i)|y^i] = 0. \tag{10}$$

Since u_1' is a decreasing function we also conclude from (8) that $s_t(h^i)$ and $w_t h^i - s_t(h^i)$ are both increasing in h^i. This implies, in particular, that the LHS in (10) is strictly decreasing in e^i. Similarly, from equation (10) we obtain the optimal level of effort as a function of the conditional distribution ν_{yi}, i.e., $e^i = e_t(\nu_{yi})$. Note that any two agents in generation t who receive the same individual signal will choose the same effort level.

Using (2) and (3) the aggregate stock of human capital at date t can be expressed as

$$H_t = E_y[\bar{h}_t(\nu_y)] = \int_Y \bar{h}_t(\nu_y)\mu(y)\mathrm{d}y, \tag{11}$$

where

$$\bar{h}_t(\nu_y) := \bar{A}(\nu_y)g(H_{t-1}, e_t(\nu_y)) \tag{12}$$

is the average human capital of agents in G_t who have received the signal y.

Definition 1 *Given the international interest rates $(\bar{r}_t)$ and the initial stock human capital H_0, a competitive equilibrium consists of a sequence $\{(e^i, s^i)_{i\in G_t}\}_{t=1}^{\infty}$, and a sequence of wages $(w_t)_{t=1}^{\infty}$, such that:*

(i) *At each date t, given $\bar{r}_t$, H_{t-1}, and w_t, the optimum for each $i \in G_t$ in problems (9) and (7) is given by (e^i, s^i).*

(ii) *The aggregate stocks of human capital, $H_t, t = 1, 2, \cdots$, satisfy (11).*

(iii) *Wage rates $w_t, t = 1, 2, \cdots$, are determined by (6).*

2.1 Information Systems

The ability of each individual is a random variable $\tilde{A}^i$. Let $\tilde{A}$ be an arbitrarily chosen element of the family $(\tilde{A}^i)_i$. Since the random variables $\tilde{A}^i$ are i.i.d. they all have the same distribution as $\tilde{A}$. We shall refer to the realizations of $\tilde{A}$ as the states of nature. Before a young agent with ability A chooses an optimal effort level he observes an individual signal which is drawn randomly from the distribution of the random variable $(\tilde{y}|\tilde{A}^i = A) = (\tilde{y}|\tilde{A} = A) =: (\tilde{y}|A)$. Thus, ex ante the conditional distributions of the individual signals are identical. For convenience, we shall refer to the realizations of $\tilde{y}$ simply as signals.

An information system, denoted ν_A, specifies for each state of nature A a conditional probability function over the set of signals. The positive real number $\nu_A(y)$ defines the conditional probability (density) that if the state of nature is A, then the signal y will be sent.

By the law of large numbers, the prior distribution over $\mathcal{A}$ coincides with the ex post distribution of ability across agents. Also the prior distribution over Y coincides with the ex post distribution of individual signals across agents and, hence, is given by equation (2). Finally, the density function for the updated posterior distribution over $\mathcal{A}$ is

$$\nu_y(A) = \nu_A(y)\nu(A)/\mu(y). \tag{13}$$

Following Blackwell (1953) a criterion can be defined that compares different information systems by their informational contents.[5] Suppose $\bar{\nu}_A$ and $\hat{\nu}_A$ are two information systems with associated density functions $\bar{\nu}_y$, $\hat{\nu}_y$, $\bar{\mu}$, $\hat{\mu}$. The informativeness of an information system can be defined as follows:

Definition 2 *(informativeness:) Let $\bar{\nu}_A$ and $\hat{\nu}_A$ be two information systems. $\bar{\nu}_A$ is said to be more informative than $\hat{\nu}_A$(expressed by $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$), if there exists an integrable function $\lambda : Y^2 \to \mathbb{R}_+$ such that*

$$\int_Y \lambda(y', y)\mathrm{d}y' = 1 \tag{14}$$

holds for all y, and

$$\hat{\nu}_A(y') = \int_Y \bar{\nu}_A(y)\lambda(y', y)\mathrm{d}y \tag{15}$$

holds for all $A \in \mathcal{A}$.

[5]The Blackwell-criterion is quite demanding. It does not allow a comparison of *any* two information structures and, therefore, induces an incomplete ordering on the set of information systems. For a generalization of this concept see Athey and Levine (1998).

The concept of informativeness is based on a simple intuitive idea: consider a stochastic mechanism, compatible with equation (14), that transforms a signal y into another signal y' according to the probability density $\lambda(y', y)$. If the y'-values are generated in this way, the information system $\hat{\nu}_A$ can be interpreted as being obtained from the information system $\bar{\nu}_A$ by adding some random noise. The following criterion turns out to be a useful tool for the analysis of our model:

Lemma 1 *Information system $\bar{\nu}_A$ is more informative than information system $\hat{\nu}_A$, if and only if*

$$\int_Y F(\bar{\nu}_y)\bar{\mu}(y)\mathrm{d}y \geq \int_Y F(\hat{\nu}_y)\hat{\mu}(y)\mathrm{d}y$$

holds for every convex function F on the set of density functions over $\mathcal{A}$.

A proof of Lemma 1 can be found in Kihlstrom (1984). Note that $\bar{\nu}_y$ and $\hat{\nu}_y$ are the posterior beliefs under the two information systems. Thus, Lemma 1 implies that a more informative structure (weakly) raises the expectation of any convex function of posterior beliefs. For concave functions, F, the inequality is reversed, and for linear functions it holds with equality.

3 Information in the Absence of Risk Sharing

Let us analyze first the effect of better information on the welfare of the first generation G_1 and on the welfare of future generations. Consider the optimization in (7) and (9) under some given information system ν_A, and denote by $e_t(\nu_{yi})$ and $s_t(h^i)$ the decision rules for agents in generation t. The value function, V_t, of generation t associates to any realization of an individual signal, $y^i, i \in G_t$, the level of i's expected utility,[6]

$$V_t(\nu_{yi}) = v\big(e_t(\nu_{yi})\big) + E_{\tilde{A}^i}\Big[u_1\big(w_t\tilde{h}^i - s_t(\tilde{h}^i)\big) + u_2\big((1+\bar{r}_t)s_t(\tilde{h}^i)\big)|y^i\Big], \tag{16}$$

[6]Note that all agents in generation t have the same value function because the random variables $(\tilde{A}_i)$ are i.i.d.

where $\tilde{h}^i = \tilde{A}^i g(H_{t-1}, e_t(\nu_{yi}))$. Economic welfare, W_t, of an individual in generation t is defined as the ex ante expected utility at the outset of his lifetime:

$$\begin{aligned} W_t(\nu_A) &= E_y[V_t(\nu_y)] \\ &= E_y\Big\{v\big(e_t(\nu_y)\big) + E_{\tilde{A}}\Big[u_1\big(w_t\tilde{h}_t - s_t(\tilde{h}_t)\big) + u_2\big((1+\bar{r}_t)s_t(\tilde{h}_t)\big)|y\Big]\Big\}, \end{aligned} \tag{17}$$

where $\tilde{h}_t = \tilde{A}g(H_{t-1}, e_t(\nu_y))$. Observe that W_t does not depend on the particular agent i chosen from G_t, i.e., all individuals within the same generation attain the same level of welfare.

We say that the value of information is positive for G_t, if $W_t(\bar{\nu}_A) \geq W_t(\hat{\nu}_A)$, whenever $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$.

Proposition 1 *Let $\bar{\nu}_A$ and $\hat{\nu}_A$ be two information systems satisfying $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$. Given any initial conditions, all members of G_1 are better-off (or at least nobody is worse-off) under $\bar{\nu}_A$ than under $\hat{\nu}_A$.*

Proof: See Appendix.

Thus, for all agents in G_1 information has positive value, i.e., these agents will benefit from a more informative system. *Future* generations G_t, $t > 1$, differ from G_1 only by their inherited stock of human capital, H_{t-1}. The welfare of future generations therefore depends on two, possibly conflicting, factors. The first factor represents the mechanism characterized in Proposition 1. This factor which, in the absence of risk sharing, has a positive impact on the welfare of all generations will be called the *direct* welfare effect. The second factor is the aggregate stock of human capital, H_{t-1}, which affects human capital, and hence welfare, of agents in G_t. This factor will be called the *indirect* welfare effect. Future generations unambiguously benefit from a better information system only if these two factors work in the same direction, i.e., if under a more informative system the aggregate stock of human capital is higher at all dates $t > 1$.

To facilitate the analysis of this issue we shall restrict the utility functions $u_1(\cdot), u_2(\cdot)$, and $v(\cdot)$ to be in the family of CRRA; namely we assume in the sequel that

$$u_1(c_1) = \frac{c_1^{1-\gamma_u}}{1-\gamma_u}; \quad u_2(c_2) = \beta\frac{c_2^{1-\gamma_u}}{1-\gamma_u}; \quad v(e) = -\frac{e^{\gamma_v+1}}{\gamma_v+1}, \tag{18}$$

with strictly positive measures of relative risk aversion; i.e., $\gamma_u > 0,\ \gamma_v > 0$. We shall also assume occasionally that the function g in (1) has the form

$$g(H,e) = \hat{g}(H)e^{\alpha}, \tag{19}$$

where $\hat{g}$ is strictly increasing in H, and $\alpha \in (0,1)$.

Using the functional forms of u_j, $j = 1,2$, in (18), it follows from equation (8) that, given $\bar{r}_t$ and w_t, the saving s^i is proportional to the human capital level h^i. In other words, for each t and for each $i \in G_t$ we have:

$$s^i = m_t h^i, \quad 0 < m_t < w_t, \quad t = 1, 2, \cdots \tag{20}$$

Our next proposition assesses the impact of better information on economic growth. This assessment depends on the property of co-monotonicity, resp. anti-monotonicity, in the signal y between the random variables $\bar{A}(\nu_y)$ and $\bar{A}^{1-\gamma_u}(\nu_y) := \int_{\mathcal{A}} A^{1-\gamma_u}\nu_y(A)\mathrm{d}A$. These two random variables are said to be co-monotone (anti-monotone) in y, if

$$[\bar{A}(\nu_y) - \bar{A}(\nu_{y'})][\bar{A}^{1-\gamma_u}(\nu_y) - \bar{A}^{1-\gamma_u}(\nu_{y'})] \overset{(\leq)}{\geq} 0$$

holds for all $y, y' \in Y$. As $\bar{A}(\nu_y)$ and $\bar{A}^{1-\gamma_u}(\nu_y)$ are expectations, in general $\gamma_u \leq 1$ does not imply co-monotonicity; nor does $\gamma_u \geq 1$ imply anti-monotonicity. If, however, the information system ν_y is fully informative, then the signal y reveals the true state A. Thus, we may write $A(\nu_y)$ and $\big(A(\nu_y)\big)^{1-\gamma_u}$ instead of $\bar{A}(\nu_y)$ and $\bar{A}^{1-\gamma_u}(\nu_y)$. In this special case co-monotonicity obviously prevails for $\gamma_u \leq 1$, and anti-monotonicity prevails for $\gamma_u \geq 1$.

Proposition 2 *Let $\bar{\nu}_A$ and $\hat{\nu}_A$ be two information systems satisfying $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$ and denote by $H_t(\bar{\nu}_A)$ and $H_t(\hat{\nu}_A)$, $t = 0, 1, \cdots$, the corresponding stocks of human capital. Assume that the functions v, u_1, u_2, and g satisfy the specifications in (18) and (19).*

(i) *If* $\bar{A}(\bar{\nu}_y)$ *and* $\bar{A}^{1-\gamma_u}(\bar{\nu}_y)$ *are co-monotone in the signal* y *and* $\rho := \alpha/[\gamma_v + \alpha(\gamma_u - 1) + 1]$ *is larger than or equal to 1, then better information (weakly) enhances growth, i.e.,* $H_t(\bar{\nu}_A) \geq H_t(\hat{\nu}_A)$ *for all* $t \geq 0$.

(ii) *If* $\bar{A}(\bar{\nu}_y)$ *and* $\bar{A}^{1-\gamma_u}(\bar{\nu}_y)$ *are anti-monotone in the signal* y *and* ρ *is less than or equal to 1, then better information (weakly) reduces growth, i.e.,* $H_t(\bar{\nu}_A) \leq H_t(\hat{\nu}_A)$ *for all* $t \geq 0$.

Proof: See Appendix.

The co-monotonicity assumption in part (i) of Proposition 2 is obviously satisfied for $\gamma_u = 0$, i.e., when agents are risk neutral with regard to future consumption risks. For $\gamma_u = 1$, human capital $\bar{h}_t(\nu_y)$ in equation (28) is linear in the posterior belief ν_y and, hence, Lemma 1 implies that better information does not affect growth.

In order to gain some intuition for the implications of Proposition 2, let us discuss the special case where random ability $\tilde{A}$ takes on only two values, say A_H and A_L with $A_H \geq A_L$: denoting the corresponding posterior belief by $\nu = (\nu_H, \nu_L) = (\nu_H, 1 - \nu_H)$, we get

$$\begin{aligned} \bar{A}(\nu_H, 1-\nu_H) &= \nu_H A_H + (1-\nu_H) A_L \\ \bar{A}^{1-\gamma_u}(\nu_H, 1-\nu_H) &= \nu_H A_H^{1-\gamma_u} + (1-\nu_H) A_L^{1-\gamma_u}. \end{aligned}$$

Obviously, for $\gamma_u \leq 1$ co-monotonicity prevails since $\bar{A}(\nu)$ and $\bar{A}^{1-\gamma_u}(\nu)$ are both monotone increasing in ν_H. By contrast, anti-monotonicity prevails in case $\gamma_u \geq 1$. Observe that $\gamma_u \leq 1$ is implied by $\rho \geq 1$. $\rho \geq 1$ therefore ensures that better information enhances growth.

Consider now the case where $\bar{\nu}_A$ is fully informative. As mentioned earlier $\bar{A}(\bar{\nu}_y)$ and $\bar{A}^{1-\gamma_u}(\bar{\nu}_y)$ are co-monotone in the signal y, if $\gamma_u \leq 1$, and anti-monotone, if $\gamma_u \geq 1$. Noting that $\rho \geq 1$ implies $\gamma_u \leq 1$ and, hence, $\gamma_u \geq 1$ implies $\rho \leq 1$, from Proposition 2 we obtain the following

Corollary 1 *Assume that the functions* v, u_1, u_2, *and* g *satisfy the specifications in (18) and (19). Let* $\bar{\nu}_A$ *and* $\hat{\nu}_A$ *be two information systems and assume that* $\bar{\nu}_A$ *is fully informative.*[7]

[7]Observe that $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$ is automatically satisfied if $\bar{\nu}_A$ is fully informative.

(i) *If $\rho \geq 1$, then economic growth is (weakly) higher under $\bar{\nu}_A$ than under $\hat{\nu}_A$.*

(ii) *If $\gamma_u \geq 1$, then economic growth is (weakly) lower under $\bar{\nu}_A$ than under $\hat{\nu}_A$.*

The future generations G_t, $t \geq 2$, differ from G_1 only by their inherited stocks of human capital. Since the factor prices $\bar{r}_t$ and w_t do not depend on H_t, all future generations will benefit from higher growth. Propositions 1 and 2 therefore imply:

Corollary 2 *Let $\bar{\nu}_A$ and $\hat{\nu}_A$ be two information systems satisfying $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$ and assume that the conditions in Proposition 2(i) are satisfied. In any competitive equilibrium information has positive value in the sense that all generations are (weakly) better-off under $\bar{\nu}_A$ than under $\hat{\nu}_A$.*

Combining corollaries 1 and 2, better information benefits all generations, if $\rho \geq 1$ and $\bar{\nu}_A$ is fully informative. However, the restriction $\rho \geq 1$ imposes a strong condition on the parameters of our model. This conditions will be violated unless α is sufficiently large and the measures of relative risk aversion, γ_v and γ_u, are sufficiently small. In the special case where $\gamma_v = \gamma_u = \gamma$, the restriction $\rho \geq 1$ is violated whenever $\gamma > 1/2$.

4 Information with Risk Sharing

This section proceeds on the assumption that part of the uncertainty of an agent's ability is insurable. Let $\tilde{A} = \tilde{A}_1 \cdot \tilde{A}_2$, where $\tilde{A}_1$ and $\tilde{A}_2$ are stochastically independent random variables which take values in $\mathcal{A}_1$ and $\mathcal{A}_2$. Before agents make decisions about effort they can insure the risk which is associated with the $\tilde{A}_1$- component of their (unknown) ability. The random variable $\tilde{A}$ has the same properties as in the previous section. In particular, since individual ability is identically and independently distributed across the members of each generation, there exists no aggregate risk in our economy. As a consequence, the insurance market for the $\tilde{A}_1$-risk will be unbiased, i.e., the agents can share this risk on fair terms. While in Section 3 the signals affected only uninsurable risks, we will assume in the sequel that the signals contain only information about the insurable risk factor $\tilde{A}_1$.[8]

In order to introduce the risk sharing market we need to assume that the $\tilde{A}_1$-component of individual ability is verifiable by the insurers. The random future income of each individual will then have an insurable component as well as an uninsurable component. Denote by $\bar{A}_1(\nu_y)$ the expected value of $\tilde{A}_1$ if the signal y has been observed,

$$\bar{A}_1(\nu_y) := \int_{\mathcal{A}} \tilde{A}_1 \nu_y(A)\, \mathrm{d}A. \tag{21}$$

Since the insurance market is unbiased, all agents find it optimal to completely eliminate the $\tilde{A}_1$- risk from income in their second period of life. Thus the optimal saving and effort decisions of individual $i \in G_t$ satisfy the following first order conditions

$$(1+\bar{r}_t)u_2'\big((1+\bar{r}_t)s^i\big) - u_1'\big(w_t \bar{A}_1(\nu_{y^i})A_2 g(H_{t-1}, e^i) - s^i\big) = 0 \quad (A_2 \in \mathcal{A}_2) \tag{22}$$

$$v'(e^i) + w_t g_2(H_{t-1}, e^i) E\big[\tilde{\bar{A}}(\nu_{y^i}) u_1'\big(w_t \tilde{\bar{A}}(\nu_{y^i}) g(H_{t-1}, e^i) - s^i\big) | y^i\big] = 0 \quad (y^i \in Y), \tag{23}$$

where

$$\tilde{\bar{A}}(\nu_{y^i}) := \bar{A}_1(\nu_{y^i}) \cdot \tilde{A}_2. \tag{24}$$

It is our aim to analyze the impact of information on growth and welfare if agents are able to share part of the uncertainty about their random ability. We are also interested in studying the role of risk sharing for economic growth for a *given* information system. Our next proposition deals with the former issue and demonstrates that better information about the insurable risk increases the aggregate stocks of human capital if the relative measure of risk aversion γ_u is less than 1, but does the opposite when γ_u exceeds 1.

[8] In fact, the analysis in Section 3 can be understood as being conducted in the same stochastic framework as here with the signals containing only information about the uninsurable risk factor $\tilde{A}_2$.

Proposition 3 *Assume that the functions v, u_1, u_2, and g satisfy the specifications in (18) and (19). In the presence of an insurance market for the $\tilde{A}_1$-risk, better information enhances growth for all $t \geq 1$, if $\gamma_u < 1$. Better information reduces growth for all $t \geq 1$, if $\gamma_u > 1$. When $\gamma_u = 1$ it has no effect.*

Proof: See Appendix.

Proposition 3 contains a similar same message as Corollary 1: loosely speaking, better information enhances growth in moderately risk averse economies, and depresses growth in highly risk averse economies. In this general form the statement is valid both in the presence and in the absence of a risk sharing market for the $\tilde{A}_1$-risk, although the conditions which guarantee positive growth effects are somewhat less restrictive if a risk sharing market exists.

In equilibrium there are two channels through which the precision of information signals affects economic growth. Consider the case $\rho \geq 1$ (which implies $\gamma_u < 1$, i.e., the economy is moderately risk averse). First, under a more informative system private investment in education will be better in line with the distribution of talent across agents: when signals are more reliable, it is less likely that an agent with low ability receives a signal which suggests high talent, and which induces him to invest heavily in education; or that an agent with high ability receives a signal which suggests low talent, thereby inducing him to invest too little. This allocative effect has a positive impact on growth.

Second, according to (30) the conditional expectation $\tilde{\bar{A}}^{1-\gamma_u}(\nu_y) = E\left[\tilde{A}^{1-\gamma_u}|y\right]$ aggregates all relevant information conveyed by the signal y. The higher is $\tilde{\bar{A}}^{1-\gamma_u}(\nu_y)$, the more favorable is the signal y. When signals become more informative agents with good signals invest more in education, and agents with bad signals invest less. However, $\rho \geq 1$ implies that $\bar{h}_t$ in (29) is a convex function of $\tilde{\bar{A}}^{1-\gamma_u}(\nu_y)$. Thus, the additional effort of agents with good signals adds more to the stock of human capital than is detracted from it through the reduced effort of agents with bad signals. The strength of this positive effect on aggregate human capital is inversely related to risk aversion because ρ is decreasing in γ_v and γ_u.

The overall impact of better information on growth combines these two effects:

on the one hand the allocation of investment in education becomes more efficient; and on the other hand the distribution of individual effort levels becomes more dispersed[9] hence, since $\rho \geq 1$, it contributes positively to the stock of human capital. If the economy is moderately risk averse, these two effects work in the same direction and stimulate economic growth.

Consider now the case where $\gamma_u \geq 1$ (which implies $\rho \leq 1$). Regarding the second effect discussed above observe that, again, the dispersion of individual effort levels increases with better information. However, now the resulting effect on the stock of human capital is negative, because $\rho \leq 1$ implies that $\bar{h}_t$ in (29) is concave as a function of the aggregated information $\tilde{\tilde{A}}^{1-\gamma_u}(\nu_y)$. The first effect which is due to a more efficient allocation of effort also works in the opposite direction as before: according to (30) agents who have received a good signal (and will probably be highly talented) invest less in education than agents with bad signals (and low talent). By responding to low expected talent with higher investment in education agents attempt to achieve a satisfactory level of human capital in their second period of life. When the signals become more reliable, agents who have received bad signals will step up their effort and invest more in education. By contrast, agents who have received good signals will cut back on their spending for education. While this kind of behavior is efficient from the decision makers' point of view it is, of course, detrimental to economic growth. Thus, again, the two effects work in the same direction. However, in a highly risk averse economy they depress economic growth.

According to Proposition 3, better information stimulates economic growth in the presence of an insurance market for the $\tilde{A}_1$-risk, if relative risk aversion γ_u is sufficiently small. In deriving this result we kept the market structure fixed while switching to a more informative system. In our next proposition we keep the information structure fixed and ask under what conditions the introduction of an insurance market for the $\tilde{A}_1$-risk will enhance economic growth. It turns out that, again, the growth effects depend critically on the measure of relative risk aversion in the second and third periods of the agents lives.

Proposition 4 *Consider our economy for a given information system ν_A and as-*

[9]Note that all agents choose the same effort level if the signals are uninformative.

sume that the functions v, u_1, and u_2 satisfy the specifications in (18).

(i) *If the relative measure of risk aversion, γ_u, is less than 1, then the introduction of an insurance market for the $\tilde{A}_1$-risk results in higher stocks of human capital at all dates and thus, in this sense, enhances growth.*

(ii) *If the relative measure of risk aversion, γ_u, is larger than 1, then such insurance will lower human capital at all dates and, hence, is detrimental to growth.*

Proof: See Appendix.

Comparing Proposition 3 and Proposition 4 shows that the introduction of a risk sharing market has the same qualitative impact on economic growth as the introduction of a better information system. The symmetry of the results in these two propositions is not surprising because both risk sharing and better information affect individual behavior in a similar way, namely by eliminating part of the risks to which agents are exposed.

Let us now turn to the question whether better information enhances economic welfare when a risk sharing market for the $\tilde{A}_1$-risk is available. Schlee (1999) showed that under certain conditions in exchange economies with efficient risk sharing arrangements better information will always be harmful. Eckwert und Zilcha (2001) demonstrate for a class of production economies that the welfare effects of information critically depend on the degree of risk aversion of the consumers. However, neither of these papers takes into account the externality created by private investment into the human capital stock.

Denote by $e(\cdot)$ and $s(\cdot)$ the optimal effort and saving decision of an agent in generation 1 (omitting the indices i and t); and let $R_j(c_j) := -u_j''(c_j)c_j/u_j'(c_j)$, $j = 1, 2$, be the relative measures of risk aversion in the agent's working period and retirement period. Recall that the signal affects only the insurable risk $\tilde{A}_1$. Thus, according to (23), $e(\cdot)$ depends on the posterior belief ν_y only via $\bar{A}_1(\nu_y)$. Similarly, in view of (22), $s(\cdot)$ depends on ν_y only via $\tilde{\bar{h}} := \tilde{\bar{A}}(\nu_y)g(H_0, e(\cdot))$, where $\tilde{\bar{A}}(\nu_y)$ has been defined in (24). Thus we may write the value function as

$$V\big(\bar{A}_1(\nu_y)\big) = v\big(e(\cdot)\big) + E\Big[u_1\Big(w_1\tilde{\bar{h}}(\cdot) - s\big(\tilde{\bar{h}}(\cdot)\big)\Big)\Big] + E\Big[u_2\Big((1+\bar{r})s\big(\tilde{\bar{h}}(\cdot)\big)\Big)\Big], \quad (25)$$

where $e(\cdot)$ and $\tilde{\bar{h}}(\cdot)$ are functions of $\bar{A}_1(\nu_y)$.

Proposition 5 *Let $\bar{\nu}_A$ and $\hat{\nu}_A$ be two information systems satisfying $\bar{\nu}_A \succ_{\inf} \hat{\nu}_A$, and assume that all agents have access to the insurance market. Given any initial conditions, all members of G_1 are worse-off (or at least nobody is better-off) under $\bar{\nu}_A$ than under $\hat{\nu}_A$, if*

(i) $R_1(c_1) \geq \frac{1}{2}$, *for all* $c_1 \geq 0$

(ii) $R_2(c_2) \geq R_1(c_1)$, *for all* $c_1, c_2 \geq 0$

is satisfied.

Proof: See Appendix.

If u_1 and u_2 satisfy the specifications in (18), the conditions (i) and (ii) in Proposition 5 boil down to the restriction $\gamma_u \geq 1/2$. Thus better information may stimulate growth and, at the same time, reduce welfare of the agents in G_1. Under the assumptions of Proposition 3 this happens if $1/2 \leq \gamma_u \leq 1$. If γ_u exceeds 1, better information depresses growth according to Proposition 3. In this case the direct and the indirect welfare effects are both negative and, hence, all generations are worse-off under a more informative system. Future generations are hit harder than the current generation: they suffer not only from the negative direct welfare effect but also from the indirect welfare effect induced by lower growth. We summarize this observation in

Corollary 3 *Assume that the functions v, u_1, u_2, and g satisfy the specifications in (18) and (19) with $\gamma_u \geq 1$. Assume further that agents have access to an insurance market for the $\tilde{A}_1$-risk. In any competitive equilibrium, better information about the $\tilde{A}_1$-risk has negative value in the sense that all generations are (weakly) worse-off under a more informative system.*

Proposition 1 and Proposition 5 suggest that the *direct* welfare effect, i.e., the impact of better information on the welfare of G_1, is less favorable (or even harmful) when agents are able to hedge against the risk on which information is revealed. This result can be interpreted in terms of two opposing mechanisms which affect economic welfare. The first mechanism was pointed out by Blackwell (1953): when agents receive more reliable information they are able to improve the quality of their effort and saving decisions. And better individual decisions result in higher welfare.

The second mechanism captures the so-called Hirshleifer-effect (Hirshleifer (1971, 1975)). The Hirshleifer-effect rests on a deterioration of the risk allocation due to better information: more reliable information signals typically restrict the risk sharing opportunities in an economy, which leads to lower welfare. In our model the risk sharing market opens after the signals have been observed. Thus, on this market the agents can only insure that part of the $\tilde{A}_1$-risk which has not yet been resolved through the signals. Accordingly, with more informative signals the insurable part of the $\tilde{A}_1$-risk will be smaller and, hence, economic welfare will be lower. The welfare loss caused by the uninsured risks is small, if the economy is only slightly risk averse, but may assume significant proportions in highly risk averse economies.

In economies where no risk sharing arrangements are operative, the Hirshleifer-effect is nil and, hence, better information increases welfare (Proposition 1). If, by contrast, the $\tilde{A}_1$-risk can be insured, then the *direct* impact of better information on economic welfare depends on a subtle interaction between the positive Blackwell-effect and the negative Hirshleifer-effect: in weakly risk averse economies welfare will rise; and in strongly risk averse economies, where the Hirshleifer-effect dominates the Blackwell-effect, welfare will decline. According to Proposition 5, the critical value of relative risk aversion, beyond which the Hirshleifer-effect outweighs the Blackwell-effect, is 1/2.

Better information raises economic welfare (in the Pareto sense) only if the direct and the indirect effects are both positive. Under both market structures considered in sections 3 and 4 this requirement will be violated unless relative risk aversion, γ_u, is sufficiently small.[10] In the more realistic case considered in Corollary 3, where the $\tilde{A}_1$-risk is insurable and γ_u exceeds 1, information has negative value in the sense that the economy is better off under a less informative system. This result may explain why in some countries (like Israel) the use of aptitude tests as a screening device for entrance to high education has recently been subjected to a critical reevaluation.

5 Concluding Remarks

There is an extensive literature which analyzes the role of information in the operation of risk sharing markets. Most of these studies are conducted either in partial equilibrium framework, or within a static theoretical set up. By construction, partial equilibrium models are of limited value for an analysis of the welfare implications of better information. We argue in this paper that a similar caveat applies to static models: these models do not explain economic growth which, however, contributes to the welfare of future generations. Our paper therefore proposes such analysis in a dynamic framework in which the role of information in enhancing growth and economic welfare can be studied.

Better information creates a *direct* and an *indirect* welfare effect. The direct welfare effect arises because, under a more informative system, agents are able to anticipate the uncertain future economic environment in a more reliable way. Better information also has implications for economic growth via more efficient investment in human capital (indirect effect) thereby affecting the welfare of future generations. We have shown the the direct and the indirect welfare effects can be in conflict with each other. This happens if no risk sharing takes place and relative risk aversion, γ_u, is higher than 1 (direct effect positive, indirect effect negative); and, in the presence of a risk sharing market, if γ_u exceeds 1/2 but is less than 1 (direct effect negative, indirect effect positive).

In either case, i.e., with and without risk sharing, both effects are positive, if

[10]Observe from eq. (37) that, in the presence of an insurance market, $\gamma_u = 0$ implies the convexity of V. Thus, by Lemma 1, the direct welfare effect is positive.

γ_u is sufficiently small. Thus, in 'slightly risk averse economies' better information enhances welfare. By contrast, if a risk sharing market exists and γ_u exceeds 1, both effects are negative which means that all generations are worse-off under a more informative system. As a rule of thumb we may paraphrase these findings as follows: The impact of better information on welfare is less favorable (or even harmful), when risk sharing arrangements are more effective and/or when risk aversion is 'high'.

Finally we would like to stress that the welfare implications of better information cannot be properly studied in models of pure exchange. Nevertheless, due to their analytic simplicity, these models dominate the literature on the value of information. By construction, models of pure exchange are unable to capture the growth-induced indirect welfare effect. In addition, these models also entail an unsatisfactory representation of the direct welfare effect. In general, the direct effect is the result of an interaction between the positive Blackwell-effect and the negative Hirshleifer-effect. In models of pure exchange, however, the Blackwell-effect is absent and, hence, the direct welfare effect is non-positive in any efficient equilibrium allocation of risk (Schlee 1998).

Appendix

In this appendix we prove propositions 1-5.

Proof of Proposition 1: Denote by $e(\nu_y)$ and $s(h)$ the optimal decision of an agent in G_1 (omitting the indices i and t), and define $U\big(\tilde{h}, s(\tilde{h})\big) := u_1\big(w\tilde{h} - s(\tilde{h})\big) + u_2\big((1+\bar{r})s(\tilde{h})\big)$. With this notation we may state the value function as

$$V(\nu_y) = v\big(e(\nu_y)\big) + \int\limits_{\mathcal{A}} U\big(\tilde{h}, s(\tilde{h})\big)\nu_y(A)\mathrm{d}A.$$

We show that the value function is convex in the posterior belief ν_y. Assume $\nu_y = \alpha\bar{\nu}_y + (1-\alpha)\hat{\nu}_y, \alpha \in [0,1]$, and denote by $(e(\bar{\nu}_y), \bar{s}(h))$ and $(e(\hat{\nu}_y), \hat{s}(h))$ the optimal decisions under the posterior beliefs $\bar{\nu}_y$ and $\hat{\nu}_y$. We obtain

$$V(\nu_y) = v\big(e(\nu_y)\big) + \int\limits_{\mathcal{A}} U\big(\tilde{h}, s(\tilde{h})\big)[\alpha\bar{\nu}_y(A) + (1-\alpha)\hat{\nu}_y(A)]\mathrm{d}A$$
$$= \alpha\Big[v\big(e(\nu_y)\big) + \int\limits_{\mathcal{A}} U\big(\tilde{h}, s(\tilde{h})\big)\bar{\nu}_y(A)\mathrm{d}A\Big] + (1-\alpha)\Big[v\big(e(\nu_y)\big) + \int\limits_{\mathcal{A}} U\big(\tilde{h}, s(\tilde{h})\big)\hat{\nu}_y(A)\mathrm{d}A\Big]$$
$$\leq \alpha\Big[v\big(e(\bar{\nu}_y)\big) + \int\limits_{\mathcal{A}} U\big(\tilde{h}, \bar{s}(\tilde{h})\big)\bar{\nu}_y(A)\mathrm{d}A\Big] + (1-\alpha)\Big[v\big(e(\hat{\nu}_y)\big) + \int\limits_{\mathcal{A}} U\big(\tilde{h}, \hat{s}(\tilde{h})\big)\hat{\nu}_y(A)\mathrm{d}A\Big]$$
$$= \alpha V(\bar{\nu}_y) + (1-\alpha)V(\hat{\nu}_y).$$

The inequality holds because $(e(\bar{\nu}_y), \bar{s}(h))$ and $(e(\hat{\nu}_y), \hat{s}(h))$ maximize expected utility, if the posterior belief is given by $\bar{\nu}_y$ and $\hat{\nu}_y$, respectively.

We have shown that the value function is convex in the posterior beliefs. Now the claim in Proposition 1 follows from Lemma 1. □

The proof of Proposition 2 requires some preparatory work. Define

$$\phi(y, y') := \lambda(y', y)\bar{\mu}(y)/\hat{\mu}(y').$$

Note that for any $y' \in Y$, the function $\phi(\cdot, y')$ constitutes a probability density over Y, i.e., $\int_Y \phi(y, y')\mathrm{d}y = 1$.[11] For any integrable function $\vartheta : Y \to \mathbb{R}$, let $\Gamma\big(\vartheta(y); y'\big)$ be its expectation with respect to the probability density $\phi(\cdot, y')$, i.e.,

$$\Gamma\big(\vartheta(y); y'\big) := \int_Y \vartheta(y)\phi(y, y')\,\mathrm{d}y.$$

Direct computation yields

$$\hat{\nu}_{y'}(A) = \Gamma\big(\bar{\nu}_y(A); y'\big) \tag{26}$$
$$\bar{A}(\hat{\nu}_{y'}) = \Gamma\big(\bar{A}(\bar{\nu}_y); y'\big). \tag{27}$$

[11]The interpretation of $\phi : Y \times Y \to \mathbb{R}_+$ is the following: If the signal y has realized under the information system $\bar{\nu}_A$, then $\phi(y, y')$ is the probability (density) that y' would have been observed under the information system $\hat{\nu}_A$.

Proof of Proposition 2: Since the initial stock of human capital, H_0, is fixed, it suffices to show that for any given $H_{t-1},\ t \geq 1$, the aggregate stock of human capital at date t,

$$H_t = \int\limits_Y \bar{h}_t(\nu_y)\mu(y)\mathrm{d}y, \tag{28}$$

is (weakly) higher under the more informative system. Using (19) in (12) we get

$$\bar{h}_t(\nu_y) = \bar{A}(\nu_y)\hat{g}(H_{t-1})\big(e_t(\nu_y)\big)^{\alpha}, \tag{29}$$

where $e_t(\nu_y)$ is given by[12]

$$e_t(\nu_y) = \delta_t\bigg(E[\tilde{A}^{1-\gamma_u}|y]\bigg)^{\frac{1}{\gamma_v+\alpha\gamma_u+1-\alpha}} \tag{30}$$

with

$$\delta_t := \left[\frac{\alpha w_t(\hat{g}(H_{t-1}))^{1-\gamma_u}}{(w_t - m_t)^{\gamma_u}}\right]^{\frac{1}{\gamma_v+\alpha\gamma_u+1-\alpha}}.$$

Combining (29) and (30) with (28) we arrive at

$$H_t(\nu_A) = \delta_t^{\alpha}\hat{g}(H_{t-1})\int_Y \bar{A}(\nu_y)\left[\bar{A}^{1-\gamma_u}(\nu_y)\right]^{\rho}\mu(y)\mathrm{d}y. \tag{31}$$

(i) The representation in (31) implies the following assessment with regard to the information systems $\bar{\nu}_A$ and $\hat{\nu}_A$:

$$\begin{aligned}
H_t(\bar{\nu}_A)/\delta_t^{\alpha}\hat{g}(H_{t-1}) &= \int_Y \bar{A}(\bar{\nu}_y)\big(\bar{A}^{1-\gamma_u}(\bar{\nu}_y)\big)^{\rho}\,\mathrm{d}\bar{\mu}(y) \\
&= \int_Y \Gamma\Big(\bar{A}(\bar{\nu}_y)\big(\bar{A}^{1-\gamma_u}(\bar{\nu}_y)\big)^{\rho}, y'\Big)\,\mathrm{d}\hat{\mu}(y') \\
&\geq \int_Y \Gamma\big(\bar{A}(\bar{\nu}_y), y'\big)\Gamma\Big(\big(\bar{A}^{1-\gamma_u}(\bar{\nu}_y)\big)^{\rho}, y'\Big)\,\mathrm{d}\hat{\mu}(y') \\
&\geq \int_Y \bar{A}(\hat{\nu}_{y'})\big(\bar{A}^{1-\gamma_u}(\hat{\nu}_{y'})\big)^{\rho}\,\mathrm{d}\hat{\mu}(y') = H_t(\hat{\nu}_A)/\delta_t^{\alpha}\hat{g}(H_{t-1}) \qquad (32)
\end{aligned}$$

In (32) the first inequality follows from the co-monotonicity assumption and the second inequality follows from $\rho \geq 1$.

[12]Equation (30) can be derived from (10) using (18), (19) and (20).

(ii) Under these assumptions both inequalities in (32) are reversed. Thus the same reasoning as in (i) implies the claim. □

Proof of Proposition 3: Proceeding along the same line as in the proof of Proposition 2 we obtain

$$\bar{h}_t(\nu_y) = \delta_t^{\alpha} \hat{g}(H_{t-1}) \bar{A}_2 \big(\bar{A}_1(\nu_y)\big)^{1+\varrho(1-\gamma_u)} \Big(E\big[\tilde{A}_2^{1-\gamma_u}\big]\Big)^{\varrho}, \tag{33}$$

where $\varrho > 0$ has been defined in Proposition 2. $\bar{h}_t$ depends on the posterior belief ν_y only via $\bar{A}_1(\nu_y)$. Since $\bar{A}_1(\cdot)$ is linear in ν_y, $\bar{h}_t$ is convex (concave) in ν_y, if and only if $\hat{h} : \mathbb{R}_+ \to \mathbb{R}_+$,

$$\hat{h}(\bar{A}_1) := \bar{A}_2 \bar{A}_1^{1+\varrho(1-\gamma_u)} \Big(E\big[\tilde{A}_2^{1-\gamma_u}\big]\Big)^{\varrho}, \tag{34}$$

is a convex (concave) function. Obviously $\hat{h}$ is convex, if $\gamma_u \leq 1$, and concave, if $\gamma_u \geq 1$ holds. Lemma 1 therefore implies the claim in the proposition. □

Proof of Proposition 4: Denote by $(\bar{e}^i, \bar{s}^i(\bar{h}^i))$ and $(\hat{e}^i, \hat{s}^i(\hat{h}^i))$, respectively, the optimal decisions of $i \in G_1$ with signal y^i in the presence and in the absence of an insurance market for the $\tilde{A}_1$-risk. $(\hat{e}^i, \hat{s}^i(\hat{h}^i))$ satisfies (8), (10), and $(\bar{e}^i, \bar{s}^i(\bar{h}))$ satisfies (22), (23) for $t = 1$.

(i) In this case $xu_1'(x)$ is a strictly concave function, so that

$$v'(\hat{e}^i) + w_1 g_2(H_0, \hat{e}^i) E\big[\tilde{\bar{A}}(\nu_{yi}) u_1'\big((w_1 - m_1)\tilde{\bar{A}}(\nu_{yi}) g(H_0, \hat{e}^i)\big)\big] > 0 \tag{35}$$

follows from (10). We have noted earlier that, by the strict concavity of the decision problem (9), the function

$$v'(e) + w_t g_2(H_{t-1}, e) E\big[\tilde{\bar{A}}(\nu_y) u_1'\big((w_t - m_t)\tilde{\bar{A}}(\nu_y) g(H_{t-1}, e)\big)\big]$$

is strictly decreasing in e for all $t \geq 1, y \in Y$. Therefore, comparing (35) with (23), we obtain $\hat{e}^i < \bar{e}^i$ for all $y^i \in Y, i \in G_1$. Thus all individual effort levels are higher in the presence of the insurance market. This implies that the aggregate stock of human capital, and hence total output, is higher if a risk sharing market exists. Obviously this process can be continued for all periods $t > 1$.

(ii) In this case the inequality in (35) is reversed since $xu_1'(x)$ is a strictly convex function. Similar reasoning as above shows that $\hat{e}^i > \bar{e}^i$ for all $y^i \in Y, i \in G_1$. Continuing the argument for all $t > 1$, we conclude that the introduction of an insurance market for the $\tilde{A}_1$-risk lowers total output and the aggregate stock of human capital at all dates. □

Proof of Proposition 5: In view of Lemma 1 we have to show that under the conditions of the proposition the value function in (25) is concave in the posterior belief ν_y. Since $\bar{A}_1(\nu_y)$ is linear in ν_y, the value function will be concave in ν_y if it is concave in $\bar{A}_1$. Making use of the Envelope theorem, differentiation of (25) with respect to $\bar{A}_1$ yields

$$V'(\bar{A}_1) = E\Big[u_1'\Big(w_1\tilde{\bar{A}}g\big(H_0, e(\bar{A}_1)\big) - s(\cdot)\Big)w_1\tilde{A}_2 g\big(H_0, e(\bar{A}_1)\big)\Big] \tag{36}$$

and (omitting the arguments of all functions)

$$V'' = E\Big[w_1\tilde{A}_2 g_2 u_1' e' + w_1\tilde{A}_2 g u_1''\{w_1\big(\tilde{A}_2 g + \tilde{\bar{A}}g_2 e'\big) - s'\}\Big]. \tag{37}$$

e' and s' denote, respectively, the derivatives of $e(\cdot)$ and $s(\cdot)$ with respect to $\bar{A}_1$. Differentiate equation (23) with respect to $\bar{A}_1$ and multiply by e' to obtain

$$0 = v''e'^2 + E\Big[\big(\tilde{A}_2 w_1 g_2 e' + \tilde{\bar{A}}w_1 g_{22}e'^2\big)u_1'\Big] + E\Big[\{w_1\big(\tilde{A}_2 g + \tilde{\bar{A}}g_2 e'\big) - s'\}w_1 g_2\tilde{\bar{A}}e'u_1''\Big]. \tag{38}$$

Adding (37) and (38), and rearranging, yields

$$\begin{aligned} V'' &= e'^2 v'' + w_1 g_{22} e'^2 E\Big[\tilde{\bar{A}}u_1'\Big] \\ &+ E\Big[\big(w_1\tilde{A}_2 g + w_1\tilde{\bar{A}}g_2 e'\big)^2 u_1'' - s'u_1'' w_1\big(\tilde{A}_2 g + g_2\tilde{\bar{A}}e'\big) + 2\tilde{A}_2 w_1 g_2 e' u_1'\Big]. \end{aligned} \tag{39}$$

The two terms in the first line on the RHS of (39) are negative. We show that the sign of the term in the second line of (39) is negative as well. From (22) we get

$$s' = \hat{m}\big(\tilde{\bar{A}}\big)\big(\tilde{A}_2 g + g_2 e'\tilde{\bar{A}}\big), \tag{40}$$

where

$$\hat{m}(\tilde{\tilde{A}}) := \frac{u_1'' w_1}{(1+\bar{r}_1)^2 u_2'' + u_1''} \leq w_1.$$

Using (40) we rewrite the term in the second line of (39) as

$$E\Big[\big(w_1 - \hat{m}(\tilde{\tilde{A}})\big) w_1 \big(\tilde{A}_2 g + \tilde{\tilde{A}} g_2 e'\big)^2 u_1'' + 2\tilde{A}_2 w_1 g_2 e' u_1'\Big]. \tag{41}$$

Noting that $\tilde{c}_1 = w_1 \tilde{\tilde{A}} g - s$, the expression in (41) can be transformed into

$$E\left[\frac{u_1' w_1 g(\tilde{A}_2 + B)^2}{\tilde{\tilde{A}}} \left\{\frac{2B\tilde{A}_2}{(\tilde{A}_2 + B)^2} - R_1(\tilde{c}_1) + \frac{u_1''}{u_1'}\big[s - \hat{m}(\tilde{\tilde{A}})\tilde{\tilde{A}} g\big]\right\}\right], \tag{42}$$

where $B := \tilde{\tilde{A}} g_2 e'/g$. Since $2B\tilde{A}_2/(\tilde{A}_2 + B)^2 = 2(B/\tilde{A}_2)/[1 + (B/\tilde{A}_2)]^2$ is bounded from above by 1/2, $[2B\tilde{A}_2/(\tilde{A}_2 + B)^2] - R_1(\tilde{c}_1)$ is negative under the assumptions of the proposition. Thus the proof is complete if we can show that

$$s \geq \hat{m}(\tilde{\tilde{A}})\tilde{\tilde{A}} g \tag{43}$$

is satisfied.

By assumption, $R_2(\tilde{c}_2) \geq R_1(\tilde{c}_1)$ holds for all $\tilde{c}_1, \tilde{c}_2$. From this we conclude, using (22),

$$\begin{aligned} R_2(\tilde{c}_2) \geq R_1(\tilde{c}_1) \quad &\Longleftrightarrow \quad \frac{u_1'}{u_2'} u_2''(1+\bar{r}_1) \leq u_1''\left[\frac{w_1\tilde{\tilde{A}} g}{s} - 1\right] \\ &\Longleftrightarrow \quad (1+\bar{r}_1)^2 u_2'' \leq u_1''\left[\frac{w_1\tilde{\tilde{A}} g}{s} - 1\right]. \end{aligned} \tag{44}$$

If $\tilde{\tilde{A}} g$ is locally increasing (decreasing) in $\bar{A}_1$, then (44) in combination with (22) implies that $w_1\tilde{\tilde{A}} g/s$ is locally increasing (decreasing) in $\bar{A}_1$. Hence we obtain

$$\begin{aligned} 0 \leq \frac{d}{d\bar{A}_1}(\tilde{\tilde{A}} g)\frac{d}{d\bar{A}_1}\left(\frac{w_1\tilde{\tilde{A}} g}{s}\right) &= \big(\tilde{A}_2 g + \tilde{\tilde{A}} g_2 e'\big)\frac{w_1}{s^2}\big[\big(\tilde{A}_2 g + \tilde{\tilde{A}} g_2 e'\big)s - s'\tilde{\tilde{A}} g\big] \\ &= w_1\left(\frac{\tilde{A}_2 g + \tilde{\tilde{A}} g_2 e'}{s}\right)^2 \big[s - \hat{m}(\tilde{\tilde{A}}) g \tilde{\tilde{A}}\big], \end{aligned} \tag{45}$$

where in the last equality we have made use of (40). Obviously, (45) implies the inequality in (43). □

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