

Social Networks and Unraveling in Labor Markets*

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Abstract

This paper develops a model of local unraveling (or early hiring) in entry-level labor markets. Information about workers' productivity is revealed over time and transmitted credibly via a two-sided network connecting firms and workers. While employment starts only after workers finish their formal training, firms and workers can sign an enforceable future employment contract at any time. By considering network based information flows and more than one period of unraveling, we modify results from less general models that predict that increasing efficiency in the post-graduation market leads to a decrease in unraveling and show that it might lead to an increase in unraveling. A more thorough investigation finds that unraveling increases in the span of network, in network concentration around a subset of workers, and in early information accuracy. Unraveling decreases in network concentration around a subset of firms. Network density increases unraveling when the network is sparse, but decreases unraveling in dense networks. The model also predicts earlier unraveling by workers that are connected to firms of heterogeneous qualities and by firms that are connected to less workers. Finally, we analyze the effects of unraveling on market outcomes and welfare and evaluate different policies with respect to their effect on unraveling. (JEL: A14, D40, D85, C78, J44, L14)

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1 Introduction

The timing of transactions is an important part of a market's behavior. In various contexts, early contracting has been shown to affect the market outcomes with respect to both distribution and welfare.¹ These effects are expected to be large in markets that involve high stakes and uncertainty that is resolved over time, as in many entry-level professional job markets such as the undergraduate college admissions, the market for positions in residency programs for young medical doctors, and the market for judicial clerks (see Avery et.al. 2001, 2007, Niederle and Roth 2005, and Roth 1984, 1990, 1996, and 2003). In these markets the decision involves a choice of geographic location, income, and specialty that will accompany the workers years into the future. It is also the case that the quality of the workers in these markets is revealed gradually, where the most complete information is available only towards their graduation.

Nevertheless, many of these markets have suffered from extremely early hiring of workers, long before students graduated and were ready to work (i.e. in the medical residencies market, residents were hired as early as the fall of their junior year). The phenomena of early contracting in the presence of uncertainty that will be resolved in the future is commonly called *unraveling*. Traditionally, unraveling is thought of as a dynamic phenomena, where early contracting by some agents may lead other agents to contract early as well, and sometimes to contract even earlier.²

In this paper, we aim to shed light on the inherently local nature of unraveling - firms hire workers who they know. This creates an adverse selection problem for firms that did not plan on using their connections and increases firms' tendency to hire through connections. We evaluate the impact that the network of connections has on the market: what network structure will better facilitate / prevent unraveling; who are the winners and the losers when unraveling occurs and what are the aggregate welfare implications when markets unravel. The analysis also suggests policy intervention.

In addition to introducing the network structure, we find the timing of unraveling to be an important factor. Unlike several previous papers in which unraveling happens in one period, we allow unraveling to happen over time as more information is being revealed.³ In our model,

¹See Niederle and Roth (2003 and 2005) and Fréchette, Roth, and Ünver (2007).

²See also Roth and Xing 1994.

³Ostrovsky and Schwarz (2007) allow information to be revealed dynamically and study the equilibrium measure of information revealed and its relation to unraveling. Damiano et al. (2005) study a dynamic model of unraveling, but do not allow for gradual revelation of information.

two types of unraveling can coexist: At an early stage in their training, workers face a large uncertainty regarding their future productivity and employment prospects. Consequently, all workers (even the best ones) will accept any early job offer from (even less preferred) firms, but some of the firms will prefer to wait with their offer to learn more about the workers. This results in *worker driven unraveling* as it is driven by workers' demand for insurance. Later on, workers have more information about their desirability in the post-graduation marketplace and good workers will refuse some less preferred offers. At this time firms are seeking for insurance against not filling their positions and all firms make early offers to workers of high enough potential. This late *firm driven unraveling* is driven by firms' demand for insurance.⁴ The division into two types of unraveling by their timing allows us to focus on a three period model that grasps the dynamic nature of unraveling, resulting in more realistic results and new policy implications. The results are summarized in table 1.

Notably, the impact of increasing connectivity or concentration in the network depends on the details of this increase. An increase in the number of connected workers and firms (network span) increases unraveling, while increasing the connectivity by adding more links between already connected agents (network density) has a non monotonic effect on unraveling and depends on the initial density. Similarly, increasing the concentration of connections around workers and around firms affects unraveling in qualitatively different ways.

There is a common line of reasoning connecting our results. On one hand, an increase in unraveling opportunities reduces the attractiveness of the post-graduation period and increases the incentives for unraveling. On the other hand, increasing late unraveling opportunities increases the option value of a connection at earlier periods and reduces firms' incentives for early unraveling. Our analysis consists of a careful evaluation of these countervailing effects.

Our results are a first step in understanding the relationship between network structure and the incentives for unraveling and suggest a possible explanation to the interesting unraveling patterns observed in labor markets. In particular, the market for Gastroenterology fellowship seems to have a sparse network of connections as each department is connected to a small number of internal medicine departments, yet the span of the network is quite large (many departments have at least one connection). In the market for judicial clerks the network looks quite different; it appears that there is high concentration around a small number of law schools

⁴The terms worker and firm driven unraveling have been suggested by Li and Suen (2000).

that are the focus of unraveling and each judge seems to be connected only to one or two schools.⁵ Nevertheless, both of these markets exhibited extensive unraveling at times.

In addition to exploring the effects of the various changes in network structure, the paper allows us to evaluate claims often used by market participants, such as (1) in the presence of extensive unraveling, some highly connected firms wait and do not hire until more information is revealed; and (2) competition between firms of different quality intensifies unraveling.⁶ Both claims are supported by our model.

Finally, the introduction of dynamics and local competition considerations modifies some of the results that were discussed in the literature in a way that has significant policy implications. Most notably, improving the efficiency in the post-graduation market has a qualitatively different effect on unraveling depending on its timing. Surprisingly, early worker driven unraveling increases in the efficiency of the post-graduation marketplace. Intuitively, there are conflicting effects from improving efficiency. On one hand, the post-graduation market becomes more appealing, so when close enough to the market, firms might not be able to hire on a local basis as high quality workers refuse many job offers. On the other hand, reducing this ability to contract late through the network creates an incentive for the firms to use the network early when the value of their connections is still high. For the earlier part of the unraveling process it turns out that the second indirect effect overcomes the direct one, indicating that an increase in the efficiency of a global market results in more worker driven unraveling.

Our measure of efficiency captures exogenous changes in IT and search costs. To that extent, this paper takes a first step towards an investigation of the impact of IT and communication devices such as the phone, E-mail, and the internet on labor markets outcomes.

In the following section, we present a general framework to analyze unraveling. In section 3, we analyze a two period model that sheds light on the role of the social network as defining infor-

⁵The definition of a connection between judges and law schools can take many forms; a judge can be connected to a school she graduated from, or might have been a classmate of one of the school's professors. While no data on judges hiring is available, informal observations suggest that judges are quite consistent with their hiring from a subset of schools.

⁶In an ongoing debate around the design of the market for judicial clerks, the California circuit has often claimed that the New York circuit can afford waiting for later periods before hiring as it is well connected to more high quality law schools. The New York circuit on the other hand holds the opinion that the reason that the California circuit hires earlier on average is due to difference in the demand for the positions, and that this creates pressure for unraveling on more prestigious circuits.

mation frictions in the market. We characterize the equilibria and analyze the effect of changing various parameters of the network structure, information accuracy, and post-graduation market efficiency. In section 4, we extend the model to include more than one period of unraveling and investigate the effects of intertemporal considerations and local competition. The results from sections 3 and 4 are summarized in table 1.

Section 5 suggests policy implications arising from the analysis. In particular, we analyze different policies banning 'exploding offers' in labor market, and policies regarding the standardization and improvements in national examination systems.

In Section 6 we discuss the relation between unraveling and market outcome. We show that unraveling reduces mobility, and early (worker driven) unraveling reduces mobility more than late (firm driven) contracting. We also find that connected workers gain from unraveling while the effect on connected firms is ambiguous (unconnected firms strictly lose from an increase in unraveling). While aggregate welfare is hard to analyze in matching markets, we are able to measure the welfare loss from unraveling in some markets. Finally, we derive the cost of lack of coordination in labor markets and suggest a reason why some markets are able to coordinate on preventing unraveling better than others.

In section 7 we extend the model and investigate the role of local competition between firms of different qualities in hiring in pre-graduation periods. We find that in early periods, competition over workers between firms of different qualities exacerbates unraveling in the market. We offer some concluding remarks in sections 8.

Table 1: Results summary and cross model comparison

	A static model of unraveling	A model with dynamic considerations	
		Worker driven unraveling	Firm driven unraveling
Interior equilibrium characterization	Only unstable	Stable or Unstable (claim 1)	
Network density	Increases unraveling for low densities and decrease for high densities	Increases unraveling for low densities and decrease for high densities (with exceptions – claim 2)	
The span of the network	Increases unraveling	Increases unraveling	
Concentration around firms	Decreases unraveling	Decreases unraveling	
Concentration around workers	Decreases unraveling	Increases Unraveling (with exceptions – claim 3)	
Information accuracy	Increases and then decreases unraveling	Increases unraveling	Decreases Unraveling
Efficiency	Decreases unraveling (weakly)	Increases unraveling	Decreases unraveling

2 The model

There is a continuum of firms and a continuum of workers in the economy, both are of measure one. Workers have T training periods before they can be employed by firms. In the training periods, workers spend time in training institutions (i.e. law school, medical school, residency programs, etc.). Workers need to formally finish their training before they can be employed. Each worker can work for at most one firm and each firm can employ at most one worker.

Production takes place at the post-graduation period and is a function of workers' productivity; worker i 's productivity is $q_i \in \{H, L\}$ and she receives a wage w_i . For the rest of the paper, we suppress wages and make them a part of the job description ($w_i = 0$). Hence, wages are independent of the time of contracting. This assumption is consistent with the markets motivating this paper.^{7,8} Moreover, this assumption can be substituted (with some technical burden) for some mild restrictions on preferences and/or wages.⁹ Firm j 's profit from employing worker i is:¹⁰

⁷Judicial clerks' wages are determined by federal law. In another market, medical residents' wages seems to be constructed in a very discrete way and limited to a small number of wage steps that can each be analyzed within our framework.

⁸The discussion of the role of wage in the analysis of unraveling is not new. In particular, models of matching markets can be analyzed using the assignment model (Koopmans and Beckmann 1957, Shapley and Shubik 1971) where wage is a part of the clearing mechanism, or using the marriage model (Gale and Shapley 1962) where wages are assumed out. It is interesting to note that even papers that use the assignment model such as Li and Suen (2000), that analyze unraveling in the context of college admissions, admit that "*the marriage model of Gale and Shapley (1962) might seem more appropriate than the assignment model to study college admission*" and that "*our [assignment model] analysis applies with a greater force to assignment markets in which payments transfers are explicitly negotiated*".

⁹One simple way of incorporating wages into the model is by making workers' preferences be based solely on wages, and assign the firms idiosyncratic preferences in a form similar to the workers preferences we have in the present model. Under a minimum wage assumption (specifically, let $w \geq 1 - \varepsilon$), wage competition will lead to dynamics that are very similar to the ones we get using our framework.

¹⁰Firms' preferences over workers that are used throughout the paper exhibit perfect correlation. As Halaburda (2007) shows some correlation across firms' preferences is necessary for unraveling to occur. To introduce some heterogeneity in firms' preferences, we have explored a model in which $\pi_j(q_i) = \begin{cases} 1 + \lambda\varepsilon_{ij} & \text{if } q_i = H \\ -1 + \lambda\varepsilon_{ij} & \text{if } q_i = L \end{cases}$. All of the results from our paper seem to hold for every $\lambda < \bar{\lambda}$ for some $\bar{\lambda} > 0$. As we are interested in the case of similar preferences that are considered to better facilitate unraveling, we omit the more complicated analysis.

$$\pi_j(q_i) = \begin{cases} 1 & \text{if } q_i = H \\ -1 & \text{if } q_i = L \end{cases} \quad (1)$$

Firms are a priori identical (In section 7 we extend our model and discuss firms' heterogeneity and systematic workers' preferences over firms). Nevertheless, workers have idiosyncratic preferences about firms. Specifically, let worker i 's utility function be:

$$u_i = \begin{cases} 1 + \varepsilon_{ij} & \text{if hired by firm } j \\ 0 & \text{if not hired} \end{cases} \quad (2)$$

Where ε_{ij} 's are i.i.d. and drawn from a distribution G that has mean 0 and positive density in any point in the support $[-\varepsilon, \varepsilon]$.

Let period 0 be the post-graduation period, when hiring decisions must be finalized and employment starts.¹¹ And let periods $-T, -T+1, \dots, -1$ be the training periods. In the training periods ($t < 0$), information about own productivity is gradually revealed to the workers (exam grades, reinforcements from teachers, etc.) in the following way: At a given period $t < 0$ worker i observes a noisy signal $s_i(t) \in \{h(t), l(t)\}$ that summarizes all the information available about the worker up to period t (ex-ante with equal probabilities, but conditional on the signals from previous periods the distribution might change). An $s_i(t) = h(t)$ worker has probability $\alpha(t) \in [\frac{1}{2}, 1]$ of being $q_i = H$ and probability $1 - \alpha(t)$ of becoming $q_i = L$. An $s_i(t) = l(t)$ worker has the reversed probabilities. $\alpha(t)$ is assumed to be an increasing function of t and at graduation q_i is revealed accurately to the worker so $\alpha(0) = 1$.

After graduation, information about workers productivity can be revealed credibly to all firms. However, before graduation, the training institution does not produce official transcripts or other reports. Therefore, a worker can credibly convey his early information only through personal connections, i.e. a worker's teacher might know a HR person in one (or more) of the firms. Formally, let $f \in [0, 1]$ be the proportion of firms that are connected to at least one worker (so $(1 - f)$ firms are not connected to any worker). Let each connected firm be connected to r_f workers and each connected worker be connected to r_w firms. Therefore, the proportion of connected workers is $w = (f \cdot r_f) / r_w$; we call $f + w$ the *span of the network* and r_θ the *rank* of a connected agent of type $\theta \in \{w, f\}$.¹²

¹¹During the training periods both firms' profits and workers' utilities are normalized to zero, so the only payoffs in the model are received in period 0.

¹²For tractability, we assume that all connected firms (workers) have the same rank r_f (r_w). This is a

Early contracts and the post-graduation marketplace. During the training periods, any worker and firm (irrespective of being connected or not) can sign employment contracts. A contract specifies that worker i will be employed by firm j in the post-graduation period, independent of any future realization. This limited contracting can result from incomplete contracts and from lack of verifiability of workers' quality.¹³ More important, it seems to be the format of contracts in the markets motivating this paper. Motivated by recent discussions on the the design of a new match for the Gastroenterology fellowship market, further discussion of the nature of the early contracts and their implications for unraveling can be found in section 5.

Workers and firms that did not sign an early contract reach the post-graduation period (period 0) unmatched. Without restricting the analysis to either a centralized or decentralized market we focus on the market's outcome rather than on the specific mechanism. In particular, information technologies (IT), search costs, and market culture affect the ability of firms and workers to find their most preferred partner. In a perfect market, workers are likely to find their most preferred firm, and if the worker is of high productivity, she will be hired by that firm. This guarantees a high quality worker with an expected utility arbitrarily close to $1 + \varepsilon$ in the post graduation period. In a virtually random market, a high quality worker will be matched with an arbitrary firm and will have an expected utility of 1. More generally, let $\phi \in [0, 1]$ be *the efficiency in the post-graduation marketplace* and the expected utility of a high quality worker in the post-graduation period be $1 + \phi \cdot \varepsilon$. The following examples provide two different possible stories that are consistent with our parameterization of the market's outcome and highlight its intuitive meaning.

Example 1 (*Random assignment order with learning of preferences*) Consider workers who do not know their firm specific preferences until they communicate with the firm (interview / fly-out etc.). Workers arrive in the market sequentially, each worker knows which firms have openings, but does not know her preferences over these firms. Firms do not know the worker's productivity. The worker can communicate with up to n firms. During this communication the firms find out the worker's productivity, and the worker finds out her firm specific preferences with respect

generalization of a regular network to a two sided network framework.

¹³These simple contract can also rise in equilibrium of a more complicated game. As the motivation for workers to sign early is the insurance against bad realizations. it is to be expected that they will not accept a contract that conditions on future outcomes.

to those firms (preferences might include wage, location, etc.). Following this communication process, firms can make a job offer to the worker and the worker can accept at most one job before the next worker arrives to the market.

Example 2 (*Decentralized marketplace with deadlines*) Consider a market in which the high quality workers are assigned sequentially. Worker i arrives to the market and becomes observable to all firms, the worker does not know which firms have hired in previous periods or earlier in that period and cannot take a job without examining an explicit job offer from a firm. All unmatched firms try to offer a job to worker i simultaneously. The technology is such that only the n first offers get through before the worker has to make a decision. The worker then accepts her most preferred job among the ones offered.¹⁴

In both of these examples consider $\phi = \phi(n)$ to be an increasing function of the quality of the information technology in the market as captured by the number of job offers a worker can consider.¹⁵

Our focus differs from that of a large part of the market design literature; the latter focuses on the efficiency generated by the mechanism, *given the information available in the market and (maybe) the reported preferences*, and traditionally full information is assumed. Several recent papers analyzed labor markets at the pre-interview stage, when firms choose which of a pool of candidates to interview, and possibly make an offer to.¹⁶ Consistent with the focus of this new literature we interpret ϕ in the following way. Small ϕ implies that it is hard to find your preferred partner - in example 1, small ϕ (or small n) implies that it is hard for a firm (worker) to learn about the qualities of the workers (firms); in example 2, the same implies that it is hard for a worker to find out who is interested in hiring her. This could result from high

¹⁴In the market for Clinical Psychologists offers were made by phone and the market was open for a limited period of time so hospitals often had their offer held by a worker that was still waiting for another offer until close to the market's closing time. This made the market stagnant and a lot of offers and acceptances were made around the market's closing time and on an exploding offer basis. As a result, each worker received a small number of offers before having to accept one of them. For more details see Roth and Xing (1997).

¹⁵In both examples, let w' high quality workers and f' firms reach the post-graduation marketplace unmatched. Given our i.i.d. assumption on ε_{ij} , as long as $f' > w' + n$, workers' expected utility is independent of f' and w' , rises in n and is bounded between 1 and $1 + \varepsilon$. It will become clear later that $f' > w' + n$ for every finite n in our model.

¹⁶See Lee and Schwarz (2007), and Coles and Niederle (2007).

search costs (like high interviewing costs), limited search time, etc. In fact, many well designed centralized markets are likely to have $\phi \ll 1$ due to inevitable information friction.

2.1 Unraveling

Unraveling has static and dynamic interpretations. In a static framework, unraveling is simply early contracting, i.e. contracting before some relevant information is available. In a dynamic context, unraveling is defined as escalation towards earlier contracting derived by the early contracting of others. As the signal of workers' productivity is noisy in early periods we refer to any employment contract signed prior to graduation as unraveling.

We focus in our analysis on sequential equilibria in undominated strategies. An immediate result is that in the post-graduation period, the market is global, high productivity workers are hired and low productivity ones are not. Also, in earlier periods, the local nature of information diffusion leads to a local market, namely, *in equilibrium*, we get that before graduation hiring is done only through connections.¹⁷

In any early period $t < 0$, let $\sigma_j(t) \in [0, 1]$ be the probability that firm j makes an offer in period t conditional on being connected to a worker with a positive signal ($s_i(t) = h(t)$). Throughout most of the analysis we focus on symmetric equilibria so $\sigma_j(t) = \sigma(t)$ for all j . Clearly, no firm contracts with a worker that has a negative signal ($s_i(t) = l(t)$) as such a worker provides a firm with negative expected profits, lower than the outside option of the firm. Similarly, let $\mu(t) \in [0, 1]$ be the probability that a worker with a positive signal refuses a job offer from a randomly drawn firm. Since the ε_{ij} 's are ex-ante unknown, this is the ex-ante probability that a worker refuses a job offer from any single firm. It is easy to see that $\mu(t)$ can be characterized by a threshold strategy by the workers. In period t , worker i will accept an offer from firm j if and only if $\varepsilon_{ij} \geq \bar{\varepsilon}(t)$. We defer the characterization of $\bar{\varepsilon}(t)$ and $\mu(t)$ to section 3.

Let $F(t, \sigma, \mu)$ be the excess expected profit for a firm in period t from *not offering* a job to a connected $h(t)$ worker. Namely, the difference between the expected profit from waiting and the expected profit from making an offer to the worker. We will characterize $F(t, \sigma, \mu)$ and show a straightforward way to derive μ for a range of interesting cases in the following section.

¹⁷In early periods information is transferred only through connections. Moreover, the expected profit from employing a random worker is at most 0.

For these cases, a symmetric sequential equilibrium can be characterized in the following way:

Corollary 1 *For every sequential equilibrium strategies $\sigma(t) = \sigma^*(t)$ and $\mu(t) = \mu^*(t)$, and for every period t , one of the following holds:*

1. $F(t, \sigma^*, \mu^*) = 0$ and $\sigma^*(t) \in [0, 1]$
2. $F(t, \sigma^*, \mu^*) > 0$ and $\sigma^*(t) = 0$
3. $F(t, \sigma^*, \mu^*) < 0$ and $\sigma^*(t) = 1$

We will be interested in changes in unraveling following changes in market fundamentals, such as the connectivity between firms and workers, the efficiency of the post graduation period marketplace, the accuracy of early information, etc. As economists oriented towards market design and policy it will be suitable to focus on changes in the incentives of both firms and workers to contract early.

Definition 1 *A change in market fundamentals leads to an **increase in unraveling** in period t if all of the following hold weakly, and at least one holds in a strict sense:*

1. *The change leads to an increase (decrease) in the equilibrium probability that a firm makes an offer to a worker with a high productivity signal ($\sigma^*(t)$) in every stable (unstable) equilibrium (σ^*, μ^*) .*
2. *The change leads to a decrease in the probability that a worker refuses a job offer ($\mu(t)$) in all equilibria.*

Where stable (unstable) equilibria are defined based on the standard notions of a fixed point stability and rise in our model when $\frac{dF(\cdot)}{d\sigma} > 0$ ($\frac{dF(\cdot)}{d\sigma} < 0$).

Our notion of an increase in unraveling is quite general, in particular, both the definition and the results to follow are consistent with a large variety of equilibrium selection rules (e.g. basin of attraction, selection of the most efficient equilibrium, etc.). We also suggest in the appendix a simple motivating dynamic process to demonstrate the intuitive nature of our definition.

2.1.1 Worker and firm driven unraveling

The relation between early information accuracy (α) and t is worth exploring a little as in can significantly influence the dynamic patterns of unraveling. Intuitively, when t is large, so is $\alpha(t)$. We argue (and show formally later on) that the incentives of firms to hire in period t increase in $\alpha(t)$ (i.e. $\frac{\partial F(\cdot)}{\partial \alpha} \leq 0$) and therefore increase in t . As a result, there exists $\bar{\alpha}(\bar{t})$ such that for every $\alpha > \bar{\alpha}(t > \bar{t})$, $F(\cdot) < 0$ independent of σ and all firms that are connected to $s_i = h$ workers make offers.

Also, the probability that a worker refuses a job offer in period t ($\mu(t)$) is increasing in α ($\frac{d\mu}{d\alpha} \geq 0$) and therefore in t ; a worker that is less concerned about being low productivity (and therefore undesirable later on) will be more selective with her early job offers. Hence, There exist $\underline{\alpha}(\underline{t})$ such that for every $\alpha < \underline{\alpha}(t < \underline{t})$ workers do not refuse early offers from firms. Furthermore, as long as ε is not too large, so that workers strictly prefer getting a job to taking a large risk of being unemployed for getting a more preferred job, $\bar{\alpha} \geq \underline{\alpha}(\bar{t} \geq \underline{t})$.

As in the introduction we say that earlier periods exhibit *worker driven unraveling*, as workers are always eager to contract while firms might contract or not. In the later (but still pre-graduation) periods all firms want to hire high potential workers, but high potential workers refuse less preferred offers. We refer to the unraveling in these periods as *firm driven unraveling* as $\sigma = 1$ while $\mu \in [0, 1]$. The distinction will become useful as some phenomena are inherently different between the different types of unraveling.

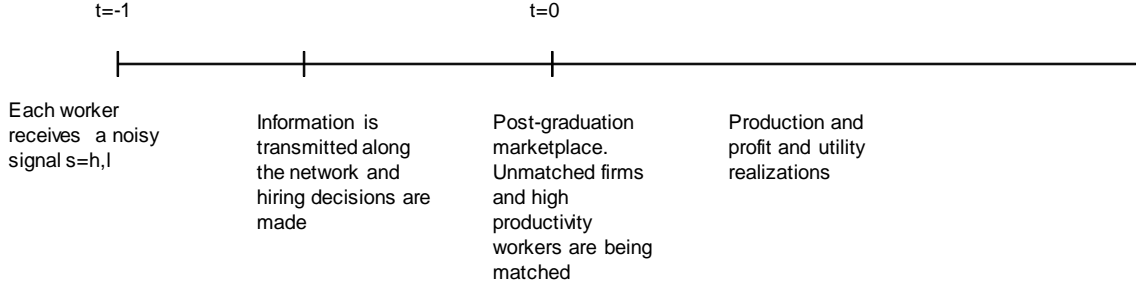
3 Social networks and information frictions

In this section, we start with a simple two period model ($T = 1$), in which firms and workers can sign early contracts in period $t = -1$ or wait for the post-graduation market. We characterize the unraveling in equilibrium and derive how unraveling is affected by changes in the network structure, the efficiency of the post-graduation marketplace and the accuracy of the early information. This model is instructive as it shows the role of the network structure in creating information frictions of a realistic nature.¹⁸

The two-period framework is also consistent with many previous models of unraveling (Li

¹⁸As there is only one period of unraveling, the model will allow for either worker or firm driven unraveling corresponding to low or high $\alpha(-1)$ respectively, but not both. Therefore, we delay any discussion on the different types of unraveling to the end of this section.

Figure 1: The timeline of the two-period model



and Rosen 1998, Li and Suen 2000,¹⁹ Suen 2000, Halaburda 2007) and highlights the value added by explicitly modeling the network. In section 4, we examine a more general model and the comparison of the results in the two models will allow us to discriminate between the role of the connections as defining (static) information frictions and more dynamic roles. See table 1 for a comparison between the two models with respect to results.

The timeline of the model is described in figure 1.

For notational simplicity we suppress the index -1 when it is clear from the context.

Denote $E_M[\pi] = E_M[\pi|\alpha, f, w, r_f, r_w, \phi, \sigma, \mu]$ as the expected profit for a firm that reaches the post-graduation marketplace unmatched; changes in $E_M[\pi]$ capture the externalities of early contracting. Let $\alpha = \alpha(-1)$, so firm j 's expected profit when connected to a $s_i = h$ worker is $\alpha - (1 - \alpha) = 2\alpha - 1$ if the firm and the worker contract in the first period.

Let $F(\cdot) = E_M[\pi] - 2\alpha + 1$. Furthermore, as worker i accepts a job offer from firm j in period -1 if and only if $1 + \varepsilon_{ij} > \alpha(1 + \phi \cdot \varepsilon)$, so $\mu = \mu(\alpha, \phi, G)$ is independent of σ .²⁰ Therefore, $\sigma^* = \sigma^*(-1) \in [0, 1]$ for which $F(\cdot) = 0$ are interior equilibria in which firms are indifferent between hiring a $s_i = h$ worker in the period -1 , and waiting for the post-graduation period. Possible corner solutions involve either full unraveling ($\sigma^* = 1$ and $F(\cdot) < 0$), or no unraveling ($\sigma^* = 0$ and $F(\cdot) > 0$).

To get an explicit expression for $E_M[\pi]$, note that the probability that a connected $s_i = h$ worker i receives a job offer from firm j can be expressed as

$$\tau = \sigma \sum_{m=0, \dots, r_f-1} \binom{r_f-1}{m} 0.5^m 0.5^{r_f-m-1} \cdot \frac{1}{m+1} \quad (3)$$

¹⁹Li and Suen (2000) suggest an extension of a multi-period model of unraveling.

²⁰In particular, μ is continuous and is increasing in α and in ϕ .

where σ is the probability that firm j makes an offer in period -1 , $\binom{r_f-1}{m}0.5^m0.5^{r_f-m-1}$ is the probability that there are m other $s_i = h$ workers that the firm is connected to, and $\frac{1}{m+1}$ is the conditional probability that the firm makes the offer to worker i . τ can be readily simplified to

$$\tau = \sigma (1 - 0.5^{r_f}) / (0.5 \cdot r_f) \quad (4)$$

Let P^w be the probability that a connected worker i who is of quality $s_i = h$ gets at least one job offer, and γ is the fraction of workers hired in period -1 . So,

$$\gamma = f \cdot (r_f/r_w) \cdot \frac{1}{2} \cdot P^w \quad (5)$$

where

$$P^w = 1 - [(1 - \tau) + \tau \cdot \mu]^{r_w} \quad (6)$$

Using γ the expected profit of a firm in the post-graduation marketplace can be expressed as:

$$E_M [\pi] = \frac{0.5 - \alpha\gamma}{1 - \gamma} \quad (7)$$

By continuity of $E_M [\pi]$ and $F(\cdot)$ and convexity of the support for σ an equilibrium always exists.²¹ Furthermore,

Proposition 1 *There is at most one interior equilibrium.*

1. *If an interior equilibrium exists, it is unstable. Moreover, there are two stable equilibria, one with all firms that are connected to $s_i = h$ workers make early offers ($\sigma = 1$), and one where no firm makes an early offer ($\sigma = 0$).*
2. *If an interior equilibrium does not exist, there is a unique stable equilibrium either at $\sigma = 1$ or at $\sigma = 0$.*

Proof. The proof is immediate as it is easy to verify that $\frac{dF(\cdot)}{d\sigma} < 0$ for every σ . ■

In light of proposition 1, our measure of unraveling can be interpreted as the tendency of a market to get to the full unraveling equilibrium. The claim also call attention to the

²¹If $F(\cdot) > 0$ ($F(\cdot) < 0$) for every σ , the unique equilibrium is no (full) unraveling. If there exist σ for which $F(\cdot) > 0$ and σ for which $F(\cdot) < 0$, we are guaranteed an interior solution by Weierstrass' fixed point theorem.

externalities involved in the unraveling process, as the incentives of a firm to unravel depend on the number of other firms that hired early.

The market for Gastroenterology fellowships seems to exhibit a pattern that is consistent with proposition 1. Before the establishment and after the collapse of the centralized match, unraveling seemed to be an extensive phenomena. In particular, in 1999, just before the match was formally abolished, only 14 out of more than 300 positions were filled in the post-graduation period. Nevertheless, during the most years that the match was in place, nearly all positions were filled through the match in the post-graduation period.

The importance of coordination in the Gastroenterology fellowships market can be seen from the following E-mail sent by Debbie Proctor, the gastroenterologist who took the lead in reorganizing the match, to the economists assisting in redesigning the Gastroenterology fellowships market:

“I’m answering 3-4 emails per day especially on this issue. ‘I want to make sure MY competition is in the match and that they don’t cheat.’ Well, this is another way of saying that if they cheat, then I will too!...Have you ever seen this before? The distrust amongst program directors? I find it hard to believe that we are unique. Maybe this is [a] social science phenomenon?”²²

It is also interesting to note that there are some markets that are not well characterized by proposition 1. One example is the market for judicial clerks; even in years of extensive unraveling, there has been a substantial number of judges that did not attempt to hire early (see Avery et. al. 2001, 2007). We will suggest in section 4 a resolution of this puzzle in the form of a more general model that allows for unraveling to happen in several points in time and characterize markets in which some of the firms will prefer to wait until close to the post-graduation period before hiring.

We now turn to evaluate the effect of changes in the market fundamentals on unraveling.

3.1 Network structure and unraveling

In this section we show how changes in the network structure affect unraveling in the two-period model. In particular, we are interested in network density, the span of the network, and network concentration around a subset of workers and / or firms. As network structure

²²See Niederle and Roth (forthcoming).

provides an explicit form of information frictions, the different effects show that different ways of adding information to the market in early periods affect in different ways. Intuitively, there are two main questions that can be asked about the network, who is connected, and how they are connected. Table 1 summarizes the finding in this section and all proofs are deferred to the appendix.

Network density and congestion. The market design literature emphasizes the importance of understanding *thickness* and *congestion* in the post-graduation marketplace and their effect on markets' efficiency (i.e. Niederle and Roth 2005 and 2007). A market can fail if it is not thick enough (if not enough participants reach the market). In this case it is beneficial for agents to seek ways of transacting outside the market - one way is unraveling. The flip side of thickness is congestion. If the market is very thick, but there is not sufficient time to explore the available options, agents will look for ways to extend the available search and bargaining time, sometimes by unraveling.

In a model of unraveling, a related (less studied) question rises. Can early periods suffer from congestion? The answer is yes. While no formal model has analyzed the relationship between congestion in pre-marketplace periods and unraveling, the observation that early congestion can prevent unraveling is not new. In particular, Kagel and Roth (2000) note that:

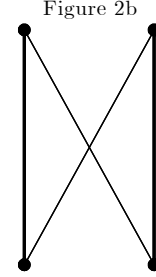
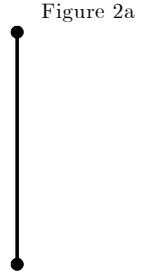
“when the centralized mechanisms are introduced, there is only a small rollback of the unraveling that developed when the market was decentralized. But because of the congestion and competition in the market, some firms and workers who intend to make early matches find themselves unable to do so, and these participate in the centralized mechanism.”²³

With respect to network structure, the density of the network is a measure of the thickness (and the resulting congestion when the number of offers made by firms in every period is limited) of the market in the early periods.

Definition 2 *For a fixed set of connected firms and worker (f and w), the **network density** is defined as $D = r_f + r_w$.*

For illustration, the networks in figure 2a and 2b have an identical set of connected firms and workers, and densities of 2 and 4 correspondingly.

²³Kagel and Roth (2000) paper examines the effect of the mechanism used in the post graduation market on the ability of the market to recover from unraveling. In their experiment the congestion in the early period is fixed and the variation comes from the matching mechanism.



Note that an increase in density does not change the aggregate information in the system, but rather increases the information that every firm is exposed to via an increase in the information overlap between firms.²⁴

Proposition 2 *For every fixed set of connected firms and workers (f and w) there exist upper and lower network density thresholds $\overline{D} = \overline{D}(f, w)$ and $\underline{D} = \underline{D}(f, w)$ such that for every low enough density ($D < \underline{D}$) an increase in network density leads to an **increase in unraveling**, and for every high enough density ($D > \overline{D}$) an increase in network density leads to a **decrease in unraveling**.*

Conjecture 1 $\underline{D} = \overline{D}$.²⁵

Proposition 2 focuses on the trade-off between two effects of an increase in network density: (1) an increase in *competition* for hiring each $s_i = h$ worker period -1 increases the number of $s_i = h$ workers hired early and reduces the attractiveness of the post-graduation marketplace; and (2) a weakening of the *coordination* of offers between the firms (provided by the network), increase the probability that some $s = h$ workers receive multiple offers while others receive non, and leads to a reduction in the number of workers that are hired early and an increase in the attractiveness of the post-graduation marketplace. In a sparse network, the first effect dominates, while in a thick network the second effect is more significant.²⁶

The following lemma, that is an important part of the proof, shows that even when actions are held fixed, increasing the number of connections does not necessarily increase contracting opportunities.

²⁴Later we will investigate the consequences of information infusion to the system in the form of an increase in the number of connected workers or in the early information accuracy.

²⁵Conjecture 1 is supported by extensive numerical analysis.

²⁶Clearly, in reality there might be an interesting process in which an increase in access for information increases coordination ability between the firms as well. Such a change will not be grasped by a change in network density, as firms coordinate by committing to not use some of their connections.

Lemma 1 *For every f , w , and σ there exist a threshold $D = \widehat{D}(f, w, \sigma)$ such that for every $D \leq \widehat{D}$, the number of workers hired in period -1 (γ) increases in D , and for every $D > \widehat{D}$, γ decreases in D .²⁷*

Lemma 1 grasps an inherent trade-off between coordination and competition in promoting efficiency in a static environment. While increasing overlap a little can raise competition as more firms are competing on every worker, it comes with a cost of the coordination that a sparse network provides.

For practical uses, it is instructive to focus on markets that unravel for which we are able to get an estimate of $\widehat{D}(f, w, \sigma)$ given a large σ . In particular, it is useful to know whether increasing density in a particular market can mitigate the unraveling problem. It turns out that if unraveling in the market is relatively high ($\sigma \geq 0.89$) then every increase in density reduces unraveling, even if we started with a very sparse network. To see why, look at the networks in figure 2 and assume full unraveling ($\sigma = 1$). In figure 2a, every $s_i = h$ worker will be hired early with probability one. In the more dense network in figure 2b, there is a probability $\frac{1}{8}$ that both workers are $s_i = h$ and only one of them is hired early.

Therefore, when unraveling cannot be pushed to an even earlier period, congestion in early periods can deter unraveling. We do not want to overstate the practical importance of this result. There are several market elements that should be considered before applying this result to a specific market: (1) a limited number of offers made at every period; and (2) a limited number of periods before the end market. In some markets these assumptions are realistic. In Harvard Business School, MBA students are not allowed to miss classes for interviews, and firms are not allowed to condition an interview on a student missing class time. This policy is enforced as firms that try to deviate are not invited to recruit in the school. In this case, recruiting periods are divided to very specific times and the number of interviews in every such time period is limited. To some extent all early interviews are done during breaks. In other entry-level market the structure is less rigid. However, it is not uncommon that recruiting schedule aligns with the academic calendar.

Despite its caveats, we find the result encouraging. Information technologies as the phone, E-mail, or the internet improve communication and reduce search cost and distance barriers.

²⁷Calvó-Armengol and Zenou (2005) prove a similar statement for a worker-only network and exogenously given σ in an employee referral model.

While it seems unlikely that market designers be able to reduce the number of communication channels between different agents, we might be able to influence the formation of different channels of communication, that facilitate credible information transmission without solving the congestion in early periods (like designing easy platforms to make professional webpages, and encouraging public exposure of students' work).

The span of the network is one of the most basic network descriptors as it describes the number of firms and workers that are connected by the network. As such, the span of the network is a basic measure of early information transmission in our model.

Proposition 3 *An increase in the span of the network (an increase of both f and w in identical proportions) leads to an **increase in unraveling**.*

As we saw, only connected agents will unravel. However, the following result is even stronger; an increase in the span of the network *will increase unraveling for firms and workers that were connected before the increase as well*. As the span of the network increases, the expected number of high quality workers in the post-graduation marketplace decreases for a fixed σ . This reduces the expected profit of a firm that reaches unmatched to the post-graduation marketplace and increase its incentive to hire early.

Network concentration around a subset of firms or workers is a measure of market power and of informational advantage of some firms or workers over the others. In addition, it is related to the span of the network. Namely, increasing network concentration around a subset of agents (either workers or firms) decreases the number of connected agents. Our next result shows that in a two period model this effect dominates and increasing the network's concentration decreases unraveling through a decrease in the span. In section 4 we introduce dynamic considerations and show that the effect of network concentration on unraveling is more complex and that there are inherent differences between increasing concentration around workers and around firms.

Proposition 4 *An increase in the concentration of connections around a subset of workers (an increase in r_w accompanied by a decrease in w) or firms (an increase in r_f accompanied by a decrease in f) leads to a **decrease in unraveling**.*

While tempting, it will not be correct to interpret proposition 4 as allowing comparison between firms with different levels of connectivity within the same equilibrium. The main force driving the result are the changes in the overall connectivity of workers and firms. In section 4, after we present the more general model of unraveling, we will be able to shed light on this question as well.

3.2 Information accuracy and efficiency

In this section, we go beyond the analysis of the network structure and analyze the effects of varying the information accuracy in early periods (α) and the efficiency of the post-graduation marketplace (ϕ).

The efficiency of the post-graduation marketplace can be affected by many market changes. For example, a reduction of search cost through lower interview costs or an improvement in evaluation standards can help firms and workers to improve their idiosyncratic match. In general, many changes in Information Technology (IT) can affect significantly the ability of firms and workers that are a better match to find each other.

Proposition 5 *An increase in the efficiency of the post-graduation marketplace (ϕ) leads to a decrease in unraveling if $\alpha > \frac{1-\varepsilon}{1+\varepsilon}$ and does not affect unraveling if $\alpha < \frac{1-\varepsilon}{1+\varepsilon}$.*

Intuitively, in the simple two period model, increasing efficiency affects only the workers' "pickiness" in the sense that it improves their post-graduation option and they can decline more firms. This becomes relevant when workers have a high enough α (relative to the strength of their preferences as described by ε) and decline job offers from their less preferred firms. When we introduce dynamic considerations in section 4 the result becomes more subtle as refusal of late offers by workers increases firms' incentives to make early offers.

Accuracy of early information as measured by α has a non-monotonic effect on unraveling.

Proposition 6 *There exist thresholds $\underline{\alpha}$ and $\bar{\alpha}$ such that:*

(1) *when $\alpha < \underline{\alpha}$ an increase in the accuracy of early information (α) leads to an **increase in unraveling**;²⁸ and,*

²⁸In particular, $\underline{\alpha} \geq \frac{1-\varepsilon}{1+\varepsilon}$.

(2) when $\alpha > \bar{\alpha}$ an increase in the accuracy of early information (α) leads to a **decrease in unraveling**.

Conjecture 2 $\underline{\alpha} = \bar{\alpha}$.²⁹

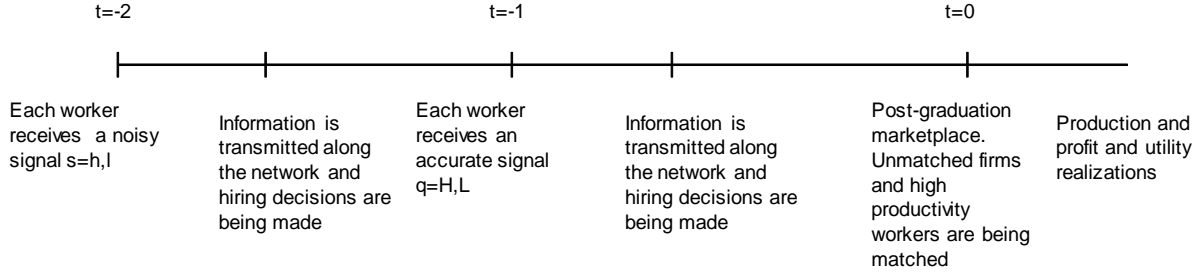
Increasing the accuracy of information in period -1 (by increasing α) can affect the incentives of both firms and workers to contract early. The effect on workers is straightforward, as the information accuracy grows, workers are more likely to decline early job offers. If α is high, unraveling is firm driven, and this effect dominates. If α is low, the effect of changes in information accuracy on firms' incentives becomes more important. This effect is driven by two forces. First, an increase in information accuracy increases the expected profit of a firm that hires a $s_i = h$ worker in period -1 ; we call this the direct effect. The major part of the proof is focused on the other, indirect, effect of increasing α on the expected profits of firms that do not unravel. This effect can be summarized by $\frac{\partial E_M[\pi]}{\partial \alpha} \leq 0$; for the same amount of unraveling, increasing α implies lower expected profit for firms that reach the post-graduation period unmatched. Intuitively, as α grows, there is a higher probability that a worker that was hired early, turns out to be of high quality and her absence from the post-graduation period marketplace will reduce the expected profit of firms that try to get matched in this period.

If we agree that in real markets information becomes more accurate over time, propositions 5 and 6 suggest that pre-graduation periods can be naturally divided into two time segments: early on, when α is small, workers will accept any job offer in order to be insured against unemployment. This *worker driven unraveling* can be influenced only by changes to firms' incentives. Closer to graduation, α is large, and the balance of power shifts. Firms are now ready to hire any high potential worker, while workers decline the less desirable job offers. This *firm driven unraveling* can be influenced by changing workers' incentives.

The partition suggests a tractable yet informative way of introducing dynamics. In the following sections we introduce a three period model that accommodate both worker and firm driven unraveling and allows us to internalize intertemporal consideration and local competition as affecting unraveling. This also allows us to refine our results from the above simplified model in a very realistic way.

²⁹Conjecture 2 is supported by extensive numerical analysis.

Figure 3: The timeline of the model



4 A model with worker and firm driven unraveling

In this section we extend our model and allow for intertemporal interaction in the unraveling process. We show how unraveling is affected by market structure and present an equilibrium characterization of a three period model that captures important differences between various labor market. The three period model also provides a more accurate description of the effects of changes in network concentration around workers and changes in efficiency of the post-graduation period’s marketplace. Table 1 summarizes the similarities and differences between the two and three period models with respect to results.

Consider workers and firms that live for three periods (periods -2 , -1 , and 0 , with 0 being the post graduation period). Motivated by our discussion at the end of the previous section, let $\alpha(-1) \rightarrow 1$ and $\alpha(-2) = \alpha < \frac{1-\varepsilon}{1+\varepsilon}$. This guarantees that in period -2 workers do not decline job offers and the unraveling is worker driven, while in period -1 firms make offers to high potential workers with probability 1 and the unraveling is firm driven. Therefore, in all equilibria $\sigma(-1) = 1$ and $\mu(-2) = 0$. For notational simplicity we say that in period -2 workers get a noisy signal $s_i \in \{h, l\}$ of their own productivity and in period -1 workers observe $q_i \in \{H, L\}$ which is their true productivity. Similarly, let $\sigma(-2) = \sigma$ and $\mu(-1) = \mu$. Other definitions follow through from the previous section. The timeline of the model is described in figure 3.

4.1 An example

The general three period model with a general network structure is very complex. Therefore, before turning to the analysis of the general model, we suggest an example with a simpler network structure that allow for a more tractable analysis and highlights some of the important

ways in which enriching the dynamics affects the results. In this example, each worker or firm has at most one connection ($r_w = r_f = 1$). The implied network is a set of firm-worker pairs.

Recall that $E_M[\pi] = E_M[\pi_j|\alpha, f, w, \phi, \sigma, \mu]$ is the expected profit for a firm that reaches the post-graduation marketplace unmatched. If γ_t is the number of workers hired in period t , then $E_M[\pi] = \frac{0.5 - \alpha\gamma_{-2} - \gamma_{-1}}{1 - \gamma_{-2} - \gamma_{-1}}$. Clearly, $\frac{\partial E_M[\pi]}{\partial \gamma_t} < 0$ for $t = -2, -1$, so the highest $E_M[\pi]$ is when there are no workers hired early ($\gamma_{-1} = \gamma_{-2} = 0$). This exercise highlights the potential for escalation in early contracting and captures the dynamic nature of unraveling.

A firm's expected profit when connected to a $s_i = h$ worker i is $2\alpha - 1$ if the firm and the worker contract in period -2 and

$$\begin{aligned} E_{-2}[\pi|\cdot] &= E_{-2}[\pi|\alpha, f, w, \phi, G, \sigma, \mu, s_i = h, \text{ does not hire } i \text{ in period } -2] = & (8) \\ &= \alpha(1 - \mu) + [1 - \alpha(1 - \mu)] \cdot E_M[\pi] \end{aligned}$$

otherwise. The first expression in (8) is the probability that the worker connected to the firm is revealed to be a H type, and that she accepts an offer from the firm in period -1 . The second expression is the probability that the firm reaches the post-graduation period unmatched multiplied by the expected profit in such case.

Let $F(\cdot) = F(-2, \sigma) = E_{-2}[\pi|\cdot] - 2\alpha + 1$. Therefore, $\sigma^* \in [0, 1]$ for which $F(\cdot) = 0$ are interior equilibria in which firms are indifferent between hiring a $s_i = h$ worker in period -2 , and waiting for period -1 . Possible corner solutions involve either full unraveling ($\sigma^* = 1$ and $F(\cdot) < 0$), or no unraveling ($\sigma^* = 0$ and $F(\cdot) > 0$). As in the previous section, an equilibrium is stable if $\frac{\partial F(\cdot)}{\partial \sigma} \geq 0$ and unstable otherwise. By continuity of $F(\cdot)$ and convexity of the support for σ an equilibrium always exists.

The following results are the counterparts of propositions 1, 3 and 6 respectively, and verify that the corresponding results from the two period model still hold.³⁰ The proofs are deferred to the appendix.

Proposition 7 *Consistent with the results from the two-period model, in the three period model:*

1. *(Equilibrium characterization) There exists an information accuracy threshold $\alpha = \bar{\alpha}$ such that for every higher information accuracy ($\alpha > \bar{\alpha}$) there is at most one interior*

³⁰As in this example we hold r_w and r_f constant, the analysis of the density and concentration results in the dynamic context are deferred to the next section.

equilibrium that is unstable. Otherwise, if there is no interior equilibrium, there is a unique stable equilibrium where either all firms that are connected to a $s_i = h$ worker make early offers ($\sigma = 1$) or no firm makes an early job offer ($\sigma = 0$). If ($\alpha < \bar{\alpha}$) the unique equilibrium involves no unraveling.

2. (The span of the network) An increase in the span of the network (η) leads to an **increase in unraveling**.
3. (Information accuracy) As long as $\alpha(-2) < \frac{1-\varepsilon}{1+\varepsilon}$ and $\alpha(-1) \rightarrow 1$, an increase in the accuracy of early information (at any pre-graduation period) leads to an **increase in worker driven (period -2) unraveling**.

The intuition for 1 and 2 is similar to the one in the two period model. We expand on 3 below.

The accuracy of early information is measured in this model by $\alpha(-1)$ and $\alpha(-2)$. While we are interested in a limited range of values for $\alpha(-1)$ (recall that $\alpha(-1) \rightarrow 1$), we can predict unambiguously the effect of changes in $\alpha(-1)$ when it is close to 1. Basically, for high $\alpha(-1)$ increasing $\alpha(-1)$ increases the refusal rate of workers in period -1 (firm driven unraveling) and therefore make it less profitable for firms to wait for period -1. As the proof of the following result is a derivative of the proof of proposition 8, it is omitted.

Lemma 2 *Increasing the accuracy of information in period -1 increases period -2 (worker driven) unraveling and decreases period -1 (firm driven) unraveling.*

It turns out that an increase in the accuracy of early information, no matter at which period has an unambiguous effect on early, worker driven (period -2) unraveling. This is important as the accuracy of information during a student's training can be affected by simple policies. We suggest an analysis of one such policy in section 5.

Lemma 2 and Proposition 7-(3) are consistent with the lack of information provided about workers by many training institutions. In particular, many schools cluster their grades around a very small support with only very few students graded at the extremes. In some schools this happens through grade inflation, in others, such as Harvard Business School, there are three possible grades, with 80% of the students receiving the same middle grade as dictated by school

policy. These schools provide students with insurance that is similar to the one provided by unraveling in the early (worker driven) contracting and prevent unraveling in the early stages when workers accept any job to insure themselves against being ranked low by the school.³¹

So far, the results from the two period model followed through. The following result is the first major modification to the limited two-period model and potentially carries important policy implications.

4.1.1 Market efficiency and the importance of incorporating intertemporal considerations in models of unraveling

The two period model produced a strong result: increasing the efficiency of the post-graduation match outcome can only reduce unraveling. Our next result suggests that the two period model is not sufficient for understanding the impact of match efficiency on unraveling, and emphasizes the inherent trade-off between static and dynamic efficiency.

Proposition 8 *Increasing the efficiency of the post-graduation marketplace (ϕ) leads to an increase in period -2 (worker driven) unraveling and to a decrease in period -1 (firm driven) unraveling.*

The main premise of the result is the understanding that a firm's connection is an asset that can be used in different times (and maybe for different uses), in the same way that other assets can be channeled to other uses. Intuitively, increasing the efficiency decreases the probability that a worker will accept a job offered to her in period -1 (μ). Hence, increasing ϕ reduces the probability that the firm could use a connection to a $q_i = H$ worker in period -1 and the future option value of the connection decreases. Therefore, the firm will prefer to capitalize the connection at an earlier stage.

The intuition goes beyond the model at hand. In many everyday applications, individuals and organizations tend to take early action or sign long term contracts when they believe that their strategic advantage might be jeopardized in the future. The intuition is also present in

³¹Our results are related to Ostrovsky and Schwarz (2006). They find that improving the information available to firms in the post graduation market might, under some conditions, triggers unraveling. Our results add to that and show adding information in other stages of training can also increase unraveling. Later, in proposition 8 we add another layer to this line of research and show that an improvement in the matching in the post-graduation marketplace can trigger unraveling.

search models and in the analysis of the classic optimal stopping problem. In a finite search an agent's threshold for stopping after the first draw will go down as we reduce the number of draws left. Similarly the intuition does not seem to depend on the number of pre-graduation periods in the model as any reduction of the opportunity cost from not keeping the connection will reduce the firm's incentives to wait further to realize the benefits of the connection.

In the unraveling case there is another force pulling in the other direction. Namely, the decrease in period -1 's hiring increases the expected profit of firms that reach the post-graduation period unmatched as the pool of workers in this period increases. Nevertheless, this second, indirect effect, is dominated by the direct effect of reduction in the option value of keeping the connection.

The proof consists of deriving $\frac{dF}{d\phi}$ and showing that it is strictly negative, so that firms' incentives to hire early increase with ϕ . Formally,

$$\frac{dF}{d\phi} = \left(\alpha \cdot (E_M[\pi] - 1) + [1 - \alpha(1 - \mu)] \cdot \frac{dE_M[\pi]}{d\mu} \right) \cdot \frac{d\mu}{d\phi} \quad (9)$$

Recall that $\frac{d\mu}{d\phi} > 0$ so we can focus on the expression in the parenthesis. An increase in μ reduces a firm's probability of hiring a worker that is connected to it in period -1 as the worker is more likely to refuse an offer. This direct effect reduces a firm's expected profit in periods later than -2 . Clearly, this affects the firms negatively only if the worker is revealed to be of high quality (which happens with probability α) and is captured by the (negative) first element in the summation $\alpha \cdot (E_M[\pi] - 1)$. On the other hand, reducing the hiring in period -1 done by other firms increases the expected profit for a firm that reach the post-graduation period unmatched. This change is grasped by the positive second element of the summation, $[1 - \alpha(1 - \mu)] \cdot \frac{\partial E_M[\pi]}{\partial \mu}$. It turns out that the first (direct) effect is greater than the externality on the post-graduation period.

Proposition 8 is different in spirit than many results in the market design literature and highlights the contrast between our notion of efficiency and the notions of stability or even mechanism efficiency. The market design literature usually assumes that information is publicly available and costless. This assumption is reasonable in this literature as it takes agents' preferences, including the number of their acceptable matches, as given and not affected by market parameters. However, to understand unraveling in a broader context we allow for changes in information technologies and search costs in the market by varying ϕ .

4.2 The general model

In this section we investigate the three period model with a general network structure. We show that although the equilibrium characterization is richer, the comparative statics from the example carry through. We claim that the extended equilibrium characterization that allows for the existence of stable equilibria, corresponds to market with more complex network structures. With respect to network structure, we verify that the results regarding the span of the network and the concentration around firms hold, and modify the results regarding the network density and the concentration around workers. The modified result demonstrate the considerations coming from intertemporal and local competition.

Allowing r_f and r_w to accept any positive integer value poses several technical difficulties. Therefore, in this section we characterize the equilibrium equation analytically, but use numerical methods to solve for equilibrium and comparative statics.³² Moreover, in order to be able to write down the equilibrium equation we are required to put more structure on the hiring process in period -1 , and introduce the following two simplifying assumptions:

Assumption 1: *In period -1 a worker chooses a firm to apply to (rather than a firm choosing a worker to propose to).*

It will become clear in the analysis that assumption 1 should not change our result qualitatively.³³ As before, in every period, an agent (firm or worker) can make at most one offer to any of the other agents.

Assumption 2: *Firms and workers draw new connections at every period, these connections substitute one-to-one their connections in the previous period.*

This assumption is a good approximation if firms cannot learn much from their own experience in period -2 . In our setup, a firm that was connected to at least one $s_i = h$ worker in period -2 and reaches period -1 unmatched only knows if it made an offer and was rejected or if it didn't make an offer, and does not have any information on the status of its other linked workers. This information is expected to be significant only when firms have a small number

³²The analysis was conducted using Matlab. The code is available upon request from the author.

³³The assumption can also be justified using a simple argument of market power; in period -2 , the unraveling is worker driven. Workers will accept any offer and do not have bargaining power as some firms might not want to make offers. In period -1 , the unraveling is firm driven. High productivity workers know that any firm will be willing to hire them, and it does not longer make sense to assume that they will be restricted to firms that make them an offer.

of connections. However, the qualitative results that we get for this case will be identical to the ones from the example above, that does not employ assumption 2, and allows each firm to have at most one connection.^{34,35}

The analysis of period -2 is almost identical to the analysis of the early period in the two period model given in equations (3) - (6), with the only change that the probability that a worker refuses an offer is $\mu(-2) = 0$.

In period -2 , let τ_{-2} be the probability that worker i that is of quality $s_i = h$ is offered a job by firm j that is connected to her, P_{-2}^w be the probability that a connected worker is offered any job, and γ_{-2} be the fraction of workers hired. The implied probability that a connected firm hires in period -2 is $\frac{\gamma_{-2}}{f}$.

In period -1 , the probability that firm j receives an application from a worker i of productivity $q_i = H$ is,

$$\tau_{-1} = \left[\frac{1}{2}\alpha (1 - P_{-2}^w) + \frac{1}{2}(1 - \alpha) \right] \cdot \Psi \quad (10)$$

where $\frac{1}{2}\alpha (1 - P_{-2}^w) + \frac{1}{2}(1 - \alpha)$ is the probability that the worker is of high productivity and was not hired in period -2 , and

$$\Psi = \sum_{m=0, \dots, r_w-1} \binom{r_w-1}{m} \left(1 - \frac{\gamma_{-2}}{f}\right)^m \left(\frac{\gamma_{-2}}{f}\right)^{r_w-m-1} \cdot \frac{1}{m+1} \cdot (1 - \mu^{m+1}) \quad (11)$$

is the probability that the worker applies for a position in firm j taking in consideration the probability distribution over the number of firms that did not hire in period -2 and are connected to the worker, and the probability that the firm is not acceptable by the worker. With some algebra this is reduced to

$$\tau_{-1} = \frac{1}{2} [1 - \alpha P_{-2}^w] \cdot \left\{ 1 - \left[\left(1 - \frac{\gamma_{-2}}{f}\right) \mu + \frac{\gamma_{-2}}{f} \right]^{r_w} \right\} / \left[\left(1 - \frac{\gamma_{-2}}{f}\right) r_w \right] \quad (12)$$

The probability that the firm gets at least one application in period -1 can be written as,

³⁴In an older version of the paper we investigated numerically in which links were not re-drawn periodically. We assumed instead that a firm can participate either in period -2 or in period -1 , but not in both. The results of this exercise were qualitatively the same.

³⁵In some markets assumption 2 might be a reasonable assumption, especially if a firm / worker is well connected and cannot communicate with all connections in every period. In that case, the connections counted for in the model represent a subset of the actual connections that are activated randomly to transmit or receive information.

$$P_{-1}^f = 1 - (1 - \tau_{-1})^{r_f} \quad (13)$$

and the number of workers hired in period -1 is,

$$\gamma_{-1} = f \cdot \left(1 - \frac{\gamma_{-2}}{f}\right) P_{-1}^f \quad (14)$$

As in the example $E_M[\pi] = \frac{0.5 - \alpha\gamma_{-2} - \gamma_{-1}}{1 - \gamma_{-2} - \gamma_{-1}}$ so that the expected profit of a firm that did not hire in period -2 is,

$$E_{-1}[\pi] = P_{-1}^f + (1 - P_{-1}^f) E_M[\pi] \quad (15)$$

The firm has an option to try and hire a worker in period -2 and get an expected profit of $2\alpha - 1$ if it succeeds and $E_{-1}[\pi]$ otherwise, or wait for period -1 and expect a profit of $E_{-1}[\pi]$. The implied equilibrium equation is therefore,

$$F(\cdot) = E_{-1}[\pi] - 2\alpha + 1 \quad (16)$$

Again, in period -2 there will be partial (worker driven) unraveling (an interior equilibrium) if and only if $F(\cdot) = 0$ while full or no unraveling can be achieved when $F(\cdot) < 0$ or $F(\cdot) > 0$ respectively.

To characterize the set of equilibria and derive its properties we constructed the grid presented in table 2:³⁶

Table 2: Parameters for numerical analysis

	lower bound	upper bound	step size
f	0.01	0.99	0.02
r_f	1	20	2
r_w	1	20	2
α	0.51	0.99	0.02
μ	0.51	0.99	0.02

In search of a solution for $F(\cdot) = 0$, for every set of parameters, we conducted a line search over $\sigma \in [0.01, 0.99]$ in steps of 0.01 to find a change in sign. For every change of sign we have

³⁶As μ is a trivial function of G and ϕ and we assumed G to be fixed, we use μ instead of deriving it from ϕ in the analysis.

conducted a binary search to find the root with tolerance 0.0001.³⁷

The results are summarized in table 1. The results regarding the span of the network, network concentration around firms and the accuracy of early information remain the same as in the two period model and in the example above and are omitted.

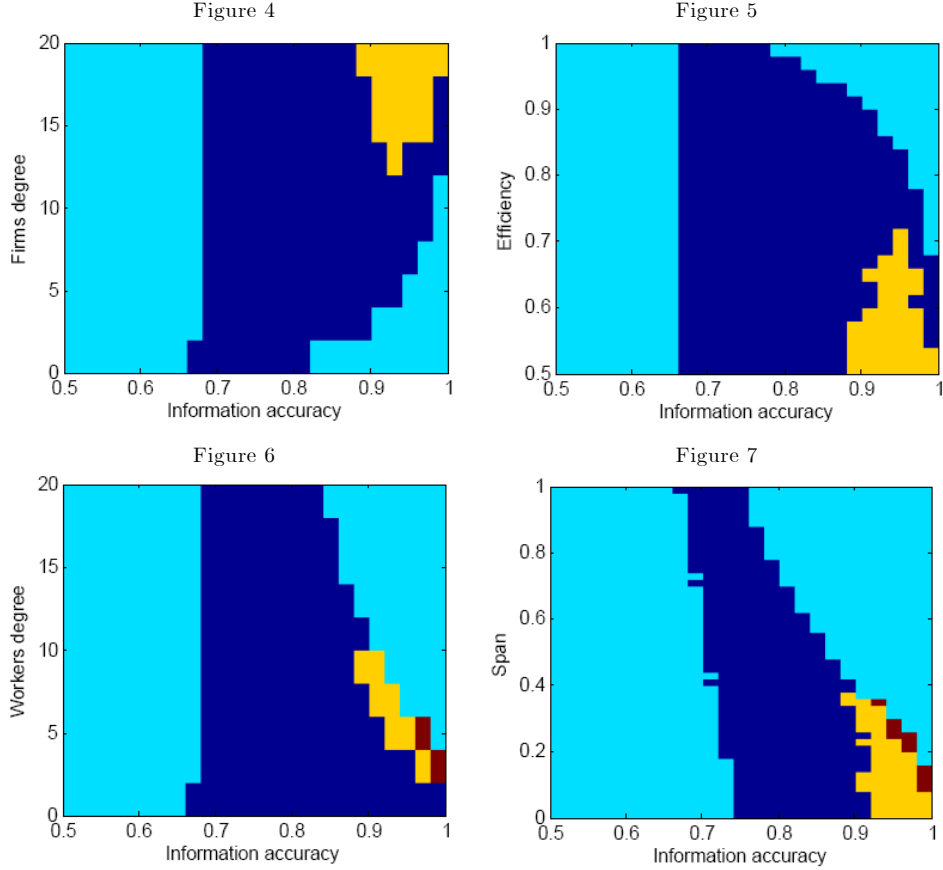
Equilibrium characterization

Claim 1 *For most parameters, there is either a unique pure strategy equilibrium where all firms take the same action in period -2 , or a unique interior (mixed strategy) equilibrium that is unstable and two pure equilibria (full and no unraveling in period -2 are equilibria). Stable partial unraveling equilibria arise when:*

- 1. The accuracy of early information and the network concentration around firms are high;*
and
- 2. The efficiency in the post-graduation marketplace, the network span, and the network concentration around workers is low.*

Claim 1 is summarized graphically in figures 4 - 7.

³⁷We have chosen this method after plotting $F(\sigma|\cdot)$ for a large set of specifications and verifying that multiplicity of equilibria is very rare and in any case, equilibria are at distance (in terms of the difference between the σ^* s) of more than 0.1 from each other.



Figures 4-7; *Equilibrium Characterization:* Each graph above divides the relevant plain into up to four regions according to the period -2 equilibrium structure attainable for the relevant pairs of parameters in the given region. The light blue regions have a unique pure strategy equilibrium in which either every firm that is connected to a $s_i = h$ worker makes an offer, or no firm makes an offer in period -2 (in each graph, in the left light blue region there is no unraveling in period -2 and in the right there is full unraveling in period -2). The dark blue regions have a unique interior (mixed strategy) unstable equilibrium and two pure strategy equilibria (a tipping point equilibria - if enough firms make early offers, every firm that is connected to a $s_i = h$ worker makes an offer). The brown regions have a unique interior stable equilibrium where a firm that is connected to a $s_i = h$ worker makes an offer with probability $\sigma \in (0, 1)$, and in the yellow region we have either stable or unstable interior equilibria (depending on the other parameters that are not on described by the plain)

Intuitively, when r_f is large, a firm has a relatively high probability of being connected

to an unemployed high productivity worker in period -1 even if many firms hired in period -2 , and when r_w is small there is a high probability that this worker will apply to that firm; this probability increases with the tendency of other firms to hire early. Low values for the span of the network and the efficiency parameters increase the sensitivity of the local (period -1) market, and decrease the sensitivity of the post-graduation market to the hiring in period -2 . Finally, The requirement for large α is somewhat less intuitive, however, for the regions of parameters that allow for a stable mixed strategy equilibrium, a lower α leads to a no unraveling equilibrium.

This result has practical implications. In an ongoing debate around the design of the market for judicial clerks, officials from the California circuit claimed that judges in the New York circuit can wait for later periods before hiring as they are well connected to more high quality law schools. Claim 1 suggests that a high number of connections can indeed allow for stability of an equilibrium in which some highly connected firms wait and do not unravel early. In fact, it is a somewhat trivial exercise to introduce a small number of firms with $r'_f > r_f$ and show that these firms will be the ones to wait.

Network density. The result that for low enough densities an increase in density leads to an increase in unraveling is trivial. We therefore investigate the effect of density on unraveling for high density levels. For this end, we have explored networks as dense as $r_w = r_f = 100$ and base our result on these networks.³⁸

Claim 2 *For most parameters and for $r_f = r_w = 100$, an increase in network density leads to a decrease in unraveling. In particular, unraveling increases with density when:*

1. *The accuracy of early information is high;*

and

2. *The efficiency in the post-graduation marketplace is not too high.*

Claim 2 is summarized graphically in figure 8.

³⁸While it is possible that higher densities will somewhat modify the results it is unlikely, as the effect of density on unraveling is already infinitesimally small at this point and seem to decrease rapidly.

Figure 8

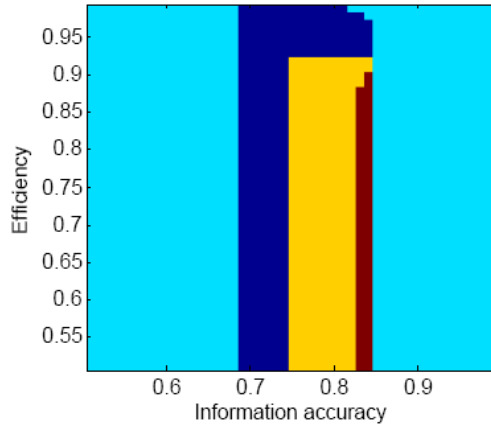


Figure 8; The effect of changes in the network density on unraveling in dense networks: The graph divides the relevant plain into four regions according to the effect of an increase in network density (in a dense network) for the relevant pairs of parameters in the given region. The light blue regions have a unique pure strategy equilibrium and therefore in these regions unraveling is not affected by changes in network density. For parameters in the dark blue region unraveling decreases with density. In the the brown region unraveling increases with density, while in the yellow region unraveling either increase or decrease with density (depending on the other parameters that are not in the grid).

Intuitively, high efficiency reduces the activity in period -1 and get us closer to the two period model analyzed in section 3 as there are almost no contracts signed in period -1 . To understand the relation between α and the effect of density on unraveling, recall that for most parameters we analyze tipping point equilibria, in which the interior equilibrium describe the minimal fraction of unraveling needed to induce full unraveling. High α suggests unstable equilibria with low tendency of firms to make offers in period -2 (low σ). When σ is low, the following effect dominates any other effect of congestion: an increase in the density increases the probability that a firm that is interested in making an offer is connected to a high potential worker in period -2 . Therefore, high α is likely to lead to a positive effect of density on unraveling even for relatively high density.

So far, the numeric results from the general model had relatively small modifications to the results from the two period model. The following two results show that adding intertemporal

and local considerations can have some significant implications, and shed light on important policy and design issues.

Network concentration around workers (r_w)

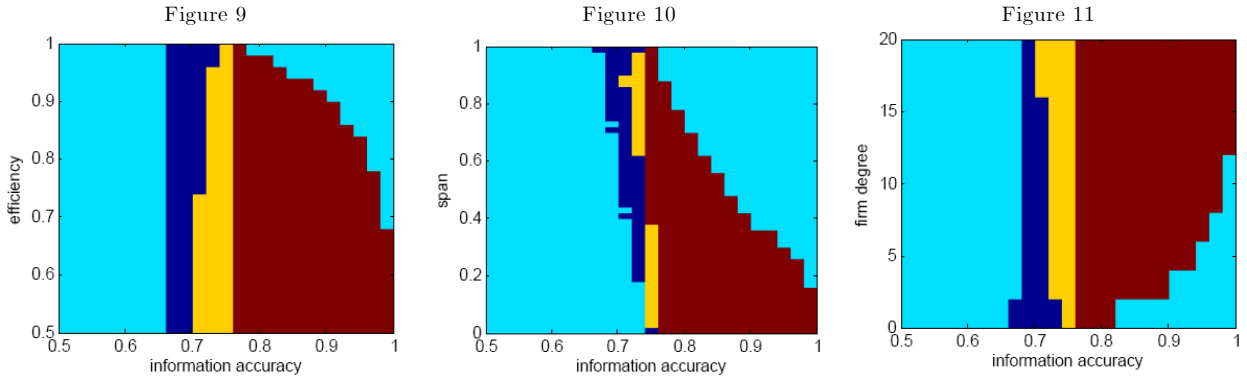
Claim 3 *For most parameters, an increase in network concentration around workers leads to an increase in unraveling. In particular, unraveling decreases with concentration around workers only when:*

1. *The accuracy of early information and the concentration around firms are low;*

and

2. *The efficiency in the post-graduation marketplace and the span of the network are high.*

Figures 9 - 11 present the main findings of claim 3.



Figures 9-11; The effect of changes in the network concentration around workers on unraveling: The graphs divide the relevant plane into four regions according to the effect of changes in the network concentration around workers on unraveling. Parameters in the light blue regions exhibit a unique pure strategy equilibrium and therefore are not affected by changes in the network. In the dark blue regions unraveling decreases with concentration around workers. In the brown regions there are only equilibria in which unraveling increases with concentration around workers, while in the yellow region unraveling either increases or decreases with network concentration around workers (depending on the other parameters that are not specified by the plane).³⁹

³⁹The disconnect between the two yellow regions in figure 10 is due to the coarseness of the grid used to plot the figures and is not of qualitative importance.

Increasing concentration around workers increases the competition that firms expect in period -1 on the remaining workers and therefore increases firms' incentives to make offers in period -2 . When the span of the network is large or when the efficiency of the post-graduation marketplace is high, the effect on the post-graduation period becomes strong relative to the effects on competition in period -1 . On the other hand, when α and r_f are large, firms tend to value their local position more and local competition becomes more significant.

The efficiency in the post-graduation marketplace (ϕ). We had different results with respect to efficiency in our two period model and in our three period example. The following claim verifies that proposition 8 derived from our simple three period example is robust to complex network structures. The intuition is the same as in the example.

Claim 4 *An increase in the efficiency of the post-graduation marketplace leads to an increase in worker driven unraveling (period -2) and to a decrease in firm driven unraveling (period -1).*

5 Labor market policies and unraveling

In this section we analyze two policies that were in affect or under discussion in various markets and in different contexts. The first, involving standardization of the grade system shows that more information might be bad news in the context of unraveling, and shows how dynamic models improve our ability to evaluate suggested policies. The second follows a strand of the market design literature and some practical design issues around the existence of exploding offers in a market and refines previous observations.

5.1 National grade standards and uniform graduation exams

Consider a market in which workers take exams towards graduation and these exam grades can be credibly revealed in the post-graduation marketplace. These can be either local school exams or nation-wide exams (SAT, GMAT, etc.).

A national exam allows all firms an identical interpretation of the grades. Therefore, an extensive national exam reduces information frictions and improves hiring in the post-graduation market. With verifiable information that is easy to transmit, a firm's interview cost can be

reduced and each firm can consider many candidates. In such a market, a high productivity worker is very likely to be considered by its most preferred firm and more likely to be employed by it. Consequently the efficiency of the post-graduation marketplace may increase.

The general model shows that such a policy will increase early (worker driven) unraveling and can reduce only late (firm driven) unraveling, while a limited two period model of unraveling will predict that such a policy will decrease unraveling as a whole.

The observation that finer information towards the end of a market's activity can increase unraveling is consistent with the analysis in Ostrovsky and Schwarz (2007) that focus on information disclosure and unraveling in matching markets.

5.2 Exploding offers

We assume throughout the paper that early contracts at any time t are enforceable and allow firms to give period specific offers commonly referred to in the literature as 'exploding offers'. This is consistent with observations from many entry level markets in which renegeing on early offers is rare. In particular, there is much evidence that markets in which renegeing is likely do not exhibit unraveling. Niederle and Roth (2004,2007) find experimental evidence that corroborates this evidence. The following corollary suggest that 'exploding offers' are a necessary condition for worker driven unraveling to occur.

Corollary 2 *If workers can costlessly renege on their contracts before the post-graduation period, no firms makes an offer to a worker in period -2 .*

Intuitively, in the presence of renegeing, the firm takes an unnecessary downside risk and insures the worker without gaining any of the upside.

The observation that allowing workers to renege before the post-graduation period (as opposed to allowing workers to go on the market and renege only if they found a better job) is sufficient to prevent worker driven unraveling, is relevant for practical reasons. In recent work on the design of a new match for the Gastroenterology fellowships market there has been much resistance to a policy that would allow residents who accepted an early offer to participate in the match and renege only if they got a better job. Based on intuition similar to the one presented here, an alternative option was suggested; allowing residents to renege on an early accepted offer before entering the match. Such a policy is likely to create less resistance as

Gastroenterology departments will get an opportunity to participate in the match in case they suffered from a cancellation. Corollary 2 suggests that the two policies are equivalent with respect to early (worker driven) unraveling. However, it is still the case that allowing for workers to renege *after* observing the post graduation marketplace outcome will prevent firm driven unraveling, while the more moderate option will not.

6 Unraveling and market outcomes

In this section we discuss briefly some ways that unraveling affects market outcomes as rise from our analysis above.

Mobility of the labor force corresponds to the scope of the labor market. As mobility goes down, the market becomes more fragmented as workers do not tend to move across regions. Niederle and Roth (2003) show empirically that unraveling reduced aggregate mobility in Gastroenterology fellowships. In particular, while the centralized match was in place and hospitals waited until the match to hire their residents, mobility was at its peak; residents were likely to move out of their first residency hospital, city, and state. Before the match was in place and after the match broke the market observed increased unraveling coupled with reduced mobility in the hospital, city, and state levels.⁴⁰ The following result shows that our model captures this relationship between the date of hiring and the scope of the market.

Corollary 3 *The number of contracts that are signed through the network is increasing in the tendency of firms to hire in period -2 .*

In equilibrium, early contracts are signed only through the network. The only thing left to note is that not every worker that is $s_i = h$ and not hired in period -2 , will be hired in period -1 . Hence, the two periods are not fully substitutable.

Welfare analysis in the context of a two-sided matching market is always subtle. There is an inherent trade-off between the gain of one and the loss of the other. We start by analyzing welfare changes for different groups of interest within the market. Later, we suggest ways to evaluate the total welfare change in the economy due to changes in unraveling.

⁴⁰The reasons for the collapse of the match are beyond the scope of this paper. See Niederle and Roth 2004 and McKinney, Niederle and Roth 2005 for further discussion.

There are four groups of interest in our model: connected workers, unconnected workers, connected firms and unconnected firms. Without further assumptions, the following observations are immediate.

Corollary 4 (1) *The utility of connected workers strictly increases in σ^* .*

(2) *The aggregate profit of all firms in the market is strictly decreasing in σ^* .*

The first claim grasps the insurance that unraveling provides to connected workers. The second points out that worker driven unraveling leads to hiring of non-productive workers and reduces aggregate firms' profits. For connected firms profits can change either way.

To evaluate total welfare changes in the economy one need to assume something about the outside option of both workers and firms. One approach is to assume that firms do not leave a job unattended, but rather hire some worker from the general pool of unemployed workers in the population with an expected profit of 0. In this case, there is no change in employment, and the loss of the firms from unraveling represents an upper bound on the total welfare loss in the economy as a result of unraveling. However, exploring the right approach for evaluating the total welfare outcomes of unraveling remains a challenge.⁴¹

The cost of (lack of) coordination For a large range of parameters, we found the equilibrium to have a tipping point structure. In such environments, strong coordination enables firms to choose between full unraveling and no unraveling equilibria. A simple back-of-the-envelope calculation shows that while the aggregate net-profit for firms is $\frac{1}{2}$ in the no-unraveling equilibrium, it is only

$$\begin{aligned} \sum \pi_j &= \frac{1}{2} - (1 - \alpha) (\gamma_{-2} | \sigma = 1) = \\ &= \frac{1}{2} - w \cdot \frac{1 - \alpha}{2} \left\{ 1 - \left[1 - \frac{1 - 0.5r_f}{0.5 \cdot r_f} \right]^{r_w} \right\} \end{aligned}$$

in the full unraveling equilibrium.

Let $c = w \cdot \frac{1 - \alpha}{2} \left\{ 1 - \left[1 - \frac{1 - 0.5r_f}{0.5 \cdot r_f} \right]^{r_w} \right\}$ be the cost of (lack of) coordination, and note that it has a tight upper bound of $\frac{1}{4}$ which is half of the maximal aggregate profit that could be achieved by the firms. c increases in the span of the network and decreases in the quality of the early information available on workers' productivity. Therefore we expect to see established

⁴¹Fr chet te, Roth, and  nver (2007) suggest using viewers ratings as a way of evaluating the welfare loss from unraveling in the context of NCAA football bowl games.

mechanisms that coordinate on no-unraveling equilibrium rise in unraveling markets with a large span and noisy early information.

Apart from its size, the market for medical residencies is characterized by a large span, as the teachers of the medicine students are themselves doctors that can potentially hire residents. It is also a market in which a struggle of the hospitals against unraveling has documented evidence since the 1920s (for an extensive review see Roth 2003). When by the beginning of the 1940s hiring became as early as the beginning of the students' junior year, it is reasonable to assume that not much information was available on students' future ability. It is at this stage that hospitals started coordinating. After several failed attempts, the National Resident Match Program (NRMP) was created and became one of the earliest matching programs established in the US. The market was also quick to react when the match broke and signs of unraveling were noticed due to graduation of couples, or to the introduction of specialties that require more than one residency. While each modification of the matching algorithm has an interesting story by itself, it is beyond the scope of this paper.⁴² Although only suggestive, the dynamics of the coordination of firms and the creation of the match in this market suggest that the model presented in this paper provides a consistent explanation.

7 Extension

7.1 Firms' heterogeneity

Heterogeneity of agents raised interest, but have not yet been fully understood in the unraveling literature. Li and Rosen (1998) allow for one quality of firms, but for different early signals of workers quality and get as a result that unraveling increases in the heterogeneity of the applicant pool. Halaburda (2007) allows for a specific kind of difference in firms' quality, where "*all workers agree on which firm is the best firm, the next-to-best or the worst firm*".

We now allow for similarity of workers preferences in a way that will divide firms into an arbitrary number of quality tiers. We then explore the effect of local competition between heterogenous firms and show that firms that compete 'out-of-their-class' in their local (network) environment, have a stronger incentive to hire early. We also allow for two components

⁴²The interested reader is referred to Roth 1984, 1990, 1991, 1996, 2003 and Roth and Peranson 1997, and 1999.

in workers' preferences over firms; a known perfectly correlated component, and a random, idiosyncratic one.

Consider an economy with $\beta_Q \in [0, 1]$ firms of quality $Q \in \{1, 2, \dots, Q_{max}\}$ ⁴³, and let $\sum_{Q \in \{1, 2, \dots, Q_{max}\}} \beta_Q = 1$. All workers prefer to be matched with a higher quality firm than with a lower quality one. Formally, worker i has a utility of $1 + Q \cdot g + \varepsilon_{ij}$ (let $g > \varepsilon$) from being matched with firm j that is of quality Q .⁴⁴

We modify the post-graduation marketplace by requiring that a firm of quality \widehat{Q} will not be matched with a high productivity worker unless all firms of quality $Q > \widehat{Q}$ are matched with high quality workers and the allocation of high quality workers to quality tiers of firms is random. High productivity worker that is assigned to a firm from quality tier Q achieves an expected utility of $1 + Q \cdot g + \phi \cdot \varepsilon$. To prevent all firms with $Q < Q_{max}$ from becoming obsolete in the market, assume that $g < \frac{1 - \alpha - (1 + \alpha)\varepsilon}{\alpha}$. This condition assures that some medium quality firms are attractive to $s_i = h$ workers in period -2 .⁴⁵

The following claim (while obvious) is key in simplifying the analysis.

Claim 5 *For every $0 < \alpha < 1$ there exist g, Q_{max} and a set of densities $\{\beta_Q | \beta_Q > 0\}_{Q \in \{1, 2, \dots, Q_{max}\}}$, such that in every equilibrium there are $\underline{Q}, \overline{Q} \in \{1, 2, \dots, Q_{max}\}$ for which:*

1. *Firms that are of quality $Q \leq \underline{Q}$ are not acceptable for a $q_i = H$ worker in period -1 .*
2. *Firms that are of quality $Q \geq \overline{Q}$ are assigned to a $q_i = H$ worker with probability 1 if they reached the post-graduation period unmatched.*

and

3. *There exist $\widehat{Q} < \underline{Q}$ such that offers from firms of quality $Q > \widehat{Q}$ are never rejected in period -2 .*

⁴³There are many dimensions along which a firms' quality can be described, i.e. wage level, facilities, working and social environment, boss reputation, prestige, etc. We use a reduced form and describe firms' qualities by a combination of aggregate and idiosyncratic workers preferences.

⁴⁴In the judicial clerks market, some circuits have more prestige then others. As a rough approximation, most clerks prefer working for judges from the more prestigious circuit. Within circuit, clerks might prefer to work for different judges (i.e. some clerks prefer to work for a more liberal judge while others prefer working for a more conservative one).

⁴⁵We focus on firms with close enough qualities, so they appeal to the same type of workers.

Proof. We construct a simple example (\widehat{Q} , \underline{Q} , and \overline{Q}) that is consistent with the claim for a given $0 < \alpha < 1$. Let $Q_{max} \rightarrow \infty$ and $\beta_Q \rightarrow 0$ for every Q . Let $\underline{Q} = \max \{Q \mid \text{A firm of quality } Q' \leq Q \text{ is rejected}\}$. It is easy to see that firms of quality \underline{Q} are never rejected in period -2 , so we can define $\widehat{Q} = \underline{Q} - 1$. Finally, we define $\overline{Q} = Q_{max}$ and the proof is complete. ■

In particular we will be interested in competition between firms such that according to some \widehat{Q} , \underline{Q} , and \overline{Q} that are consistent with the claim, are divided as follows: either one firm is of quality $\widehat{Q} < Q_L \leq \underline{Q}$ and the other is of quality $\underline{Q} < Q_H$, or one firm is of quality $\widehat{Q} < Q_L \leq \overline{Q}$ and the other is of quality $\overline{Q} < Q_H$. As a result, claim 5 allows us to focus on two qualities of firms $Q \in \{L, H\}$ without loss of generality. In particular, claim 5 and the way we have chosen the two qualities assures us that the equilibrium will be of (at least) one of the two following cases:

Case 1: In equilibrium, low quality firms are always rejected in the period -1 .

Case 2: In equilibrium there is no period -2 unraveling of high quality firms (as they are guaranteed to be matched to a $q_i = H$ worker in the post-graduation marketplace).

Finally, the expected profit of firms that do not unravel is strictly monotonic in their qualities. Consequently, there is no equilibrium in which more than one type of firm has a mixed strategy.

7.1.1 Local competition, firm heterogeneity, and unraveling

In this section, we investigate directly the local interaction between firms of different qualities. The simplest setup in which we can look at local effects of firms' heterogeneity is where $r_f = 1$ and $r_w = 2$. This allows us to separate connected components, into three types: (1) Uniform High (HH) - two high quality firms connected to a worker; (2) Uniform Low (LL) - two low quality firms connected to a worker; and (3) Mixed (HL) - one high quality firm and one low quality firm connected to the worker. We assume that firms and workers know what component that they are in. With some abuse of notation, let $C_k \in \{HH, LL, HL\}$ be the component type of agent (firm or worker) k .

We extend our assumption of symmetric equilibrium in an intuitive way, namely, all firms that are indifferent about offering a job in period -2 mix with the same probability. Still, the set of strategy profiles is richer than in the homogeneous case. Denote the probability with

which a firm of quality $Q \in \{L, H\}$ from a component C make an offer to a connected $s_i = h$ worker in period -2 by $\sigma_{Q,C}$.

The following result states that a worker that is connected to one H firm and one L firm is more likely to get an early offer from an H firm than a worker that is connected to two H firms, and that the same is true for offers from L firms. A revealed preference type argument suggests that in many cases a worker is better off having a diverse 'portfolio' of connections with firms of different qualities. Which component are preferred by H firms is ambiguous. If H firms are likely to make early offers in both a uniform and a mixed component, every firm will prefer to be in a component with a L firm. Nevertheless, if a H firm is significantly less likely to make an early offer when it is a part of a uniform component, H firms might strictly prefer to have another H firm in its component.

The following result that is proved in the appendix makes our statement more precise.

Proposition 9 *Let $\delta_{Q,C}$ be the probability that a worker that received a high signal at an early stage ($s_i = h$) and is a part of a $C \in \{HH, LL, HL\}$ component is offered a job by a $Q \in \{H, L\}$ quality firm. In every equilibrium, $\delta_{H,HH} \leq \delta_{H,HL}$ and $\delta_{L,LL} \leq \delta_{L,HL}$.*

To better understand the scope of the result, recall that L firms are of quality greater than \hat{Q} as defined in claim 5. Therefore, our focus is on firms that are relevant in competing for high quality workers, rather than very low quality firms. In a sense, although H firms are of higher quality, the jobs that H and L firms offer are comparable.

Intuitively, when a L firm is a part of a mixed component, it cannot defer making an offer to the worker to period -1 as it will then compete with a H firm. The L firm is then more likely to make an earlier offer to the worker. Since in period -2 the worker is likely to accept an offer even from a L firm, a H firm that is part of a mixed component does not expect to be able to utilize a link in the period -1 and is more likely to make an early offer itself.

Proposition 9 suggests that within equilibrium, regions of the network in which firms of different qualities compete, will exhibit more unraveling.

An additional question of interest is whether a market that facilitates more early competition between firms of different qualities is more likely to unravel as a whole. A full equilibrium analysis of the heterogeneous case is beyond the scope of this paper. Nevertheless, we are able to suggest a partial analysis comparing a network in which there are only mixed components to a network that contains only pure components.

It is interesting to see that while some sets of parameters allow for equilibria of both types to exist, when $\beta_H > \frac{1}{2}$, all equilibria are of case 1 (L firms are always rejected in the period -1) and when $\beta_H < \frac{1}{2}(1 - \alpha)$ equilibria exclusively belong to case 2 (with no period -2 unraveling of H firms). This implies that for a large region of parameters we can analyze each of the cases separately in our equilibrium analysis.

Corollary 5 *When $\beta_H > \frac{1}{2}$ (case 1 equilibrium) the mixed components network exhibit worker driven unraveling by more workers than the pure components network in every equilibrium.*

In equilibrium of case 1, low quality firms always unravel. Given that in the mixed components network low quality firms have access to all connected workers, it induces the highest level of unraveling possible with respect to the number of workers hired in period -2 .

Case 2 equilibrium is less obvious. On one hand only L firms unravel anyway, so when they have more access we expect more workers to unravel. On the other hand, low quality firms' level of unraveling might be different between the networks.

The notion that mixed qualities of firms connected to the same workers increase unraveling has some support in the evidence from the judicial clerks market. In particular, all circuits are connected and hire early from a small number of law schools (i.e. Chicago, Harvard, Michigan, Stanford, and Yale). When new norms imposing a starting date for hiring were introduced, it has been claimed that the more prestigious circuits were quick to obey, as long as the less prestigious ones did. Arguably (as none of the circuits admit to be less popular), the unraveling of the dates came along as the less prestigious ones tried to hire a little before the others, this lead a large number of firms (even more prestigious ones) to hire early.

The heterogeneous case also presents the opportunity to compare the effect of unraveling on the welfare of low and high quality firms. In general, low quality firms that are connected to workers that are in turn not connected to high quality firms, are most likely to gain from unraveling as they have significant probability of hiring the $s_i = h$ worker early, and had the least expected profit in the no-unraveling case. This might suggest the source of resistance that some market designer experience when trying to put in place regulations that are directed at preventing unraveling. i.e. in the Gastroenterology fellowships match, hospital with a less prestigious Gastroenterology department and above average internal medicine department tend to object to the introduction of a centralized match or any regulations that are aimed at preventing unraveling.

8 Conclusion

Labor economists have long recognized that many workers find their jobs through friends and relatives. Business people have adopted this way of thinking and are constantly on the look for new connections in key places, professional social networks on the internet (e.g. LinkedIn) thrive. It is only natural that social networks affect an inherently connection based phenomena as early contracting in the presence of incomplete information.⁴⁶

This paper is a first attempt to tackle the phenomena of early contracting in entry-level labor markets from the perspective of social networks. We propose a model of local interaction in which information flows via connections in the network, and show that unraveling increases with the span of the network, the effect of network concentration depends whether concentration around firms or workers is exhibited, and the density of the network has a non-monotonic effect. In particular, when the network is dense, an increase in network density reduces unraveling. Furthermore, we show that within a network, a more diverse region with firms of different qualities is more likely to exhibit unraveling. These results have clear and testable implications and seem to be consistent with suggestive evidence from the market design literature.

An analysis of the timing of the interaction leads to the observation that worker driven unraveling increases with an improvement in the efficiency in the post-graduation market. The relation between the efficiency and information technologies (i.e. internet, E-mail, telephone, etc.) or search costs in the market, suggest several policies to reduce unraveling.

Finally, the model sheds light on the understudied relationship between unraveling and market outcomes. In particular we are able to show that in line with empirical observations in the literature, unraveling reduces mobility in matching markets. We derive the aggregate welfare loss and the price of (lack of) coordination in the market, and detect the "winners" and "losers" from unraveling in a market. Again, there is evidence from the market design literature that is consistent with this analysis.

While the idea that social networks are used as a mean of transferring information (and in particular information related to job search) is widely accepted in the social network literature, it has not been used to analyze many labor market phenomena or in the context of market design, including unraveling.⁴⁷ Our results, supported by suggestive evidence, suggest that it

⁴⁶See also Montgomery (1991).

⁴⁷Montgomery (1991), Calvo-Armengol and Jackson (2004), Calvo-Armengol and Zenou (2005).

is useful to incorporate this idea into labor markets and market design analysis.

9 References

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10 Appendix

10.1 Dynamic unraveling and comparative statics

We present here a simple repeated game type framework of unraveling that demonstrates the intuitive nature of our definition of an increase in unraveling (definition 1).

Let new workers graduate every year. Let firms' demand be independent across years, so hiring in one year has no implication on hiring in the next. Firms have no information about the current year's market and choose their strategy (their function $\sigma^*(t)$ in the current year) as a 'naive' best response to last year's market.

Assume that in a certain year, the market was in an interior (mixed strategy) equilibrium in which firms' actions were defined by σ^* . After the year has ended (and before the subsequent year) market fundamentals have changed and the new (interior) equilibrium is defined by the function σ^{**} moved the equilibrium such that for in period t' , $\sigma^{**}(t') = \sigma^*(t') + \zeta$ (ζ is positive and infinitesimally small) and for every $t \neq t'$, $\sigma^*(t) = \sigma^{**}(t)$. In the year after the changes, firms choose best response to the previous year and act according to σ^* . Hence firms make less offers in period t' than the new equilibrium requires. This implies that $F(t', \sigma^{**}) \neq 0$ and that firms' best response (that they will carry the following year) changes. If unraveling has a

negative externality on firms' expected future in periods $t > t'$, $\frac{dF(\cdot)}{d\sigma} < 0$, and $F(t', \sigma^*) > 0$ so the equilibrium is unstable so firms will unravel less as years pass until a new equilibrium (with a lower level of unraveling) is reached.

On the other hand, if unraveling has a positive externality on firms' expected future in periods $t > t'$, $\frac{dF(\cdot)}{d\sigma} > 0$, and $F(t', \sigma^*) < 0$, so the equilibrium is stable and firms will eventually converge to $\sigma^{**}(t')$ with more unraveling in period t' than before the change in fundamentals.

10.2 Proofs

Lemma 1 - Proof. The lemma can be rephrased as follows: Hold fixed the number of connected firms and workers. There exist $r' \geq 1$ such that for $r_f < r'$ ($r_w < r'$), γ is increasing in r_f (r_w) and for $r_f > r'$ ($r_w > r'$), γ is decreasing in r_f (r_w). Namely that there exist $r' \geq 1$ such that $\partial \left(\gamma \mid \frac{r_w}{r_f} = c \right) / \partial r_f \geq 0$ for every $r_f < r'$ and $\partial \left(\gamma \mid \frac{r_w}{r_f} = c \right) / \partial r_f \leq 0$ for every $r_f > r'$.

For the rest of the proof, let $\frac{r_w}{r_f} = c$ and hold c fixed. Therefore,

$$\gamma = f \cdot (1/c) \cdot \frac{1}{2} \cdot \{1 - [1 - \sigma(1 - \mu)(1 - 0.5^{r_f}) / (0.5 \cdot r_f)]^{c \cdot r_f}\}$$

As $(1 - \mu) \in [0, 1]$ is independent of r_f , r_w , and σ is held constant throughout the proof, we abuse notation and let $\sigma = \sigma(1 - \mu)$ from this point on.

Note that,

$$\text{sign} \{ \partial \gamma / \partial r_f \} = -\text{sign} \{ \partial \{ \ln([1 - \sigma(1 - 0.5^{r_f}) / (0.5 \cdot r_f)]^{c \cdot r_f}) \} / \partial r_f \}$$

and

$$\ln([1 - \sigma(1 - 0.5^{r_f}) / (0.5 \cdot r_f)]^{c \cdot r_f}) = c \cdot r_f \cdot \ln[1 - \sigma(1 - 0.5^{r_f}) / (0.5 \cdot r_f)]$$

\Rightarrow

$$\text{sign} \left\{ \partial \left(\gamma \mid \frac{r_w}{r_f} = c \right) / \partial r_f \right\} = -\text{sign} \left\{ \partial \{ r_f \cdot \ln[1 - \sigma(1 - 0.5^{r_f}) / (0.5 \cdot r_f)] \} / \partial r_f \right\}$$

We can now focus on $\partial \{ r_f \cdot \ln[1 - \sigma(1 - 0.5^{r_f}) / (0.5 \cdot r_f)] \} / \partial r_f$.

We first show that $r_f \cdot \ln[1 - \sigma(1 - 0.5^{r_f}) / (0.5 \cdot r_f)]$ is convex in r_f and then show that it is decreasing for $r_f = 1$ and increasing when $r_f \rightarrow \infty$.

$$\begin{aligned} & \partial \{ r_f \cdot \ln[1 - \sigma(1 - 0.5^{r_f}) / (0.5 \cdot r_f)] \} / \partial r_f = \\ & = \left\{ \begin{array}{l} \ln[1 - \sigma(1 - 0.5^{r_f}) / (0.5 \cdot r_f)] + \\ + \sigma \left(\ln(0.5) \cdot 0.5^{r_f} \cdot r_f - 0.5^{r_f} \right) / [1 - \sigma(1 - \mu)(1 - 0.5^{r_f})] \end{array} \right\} + \sigma / [1 - \sigma(1 - 0.5^{r_f})] \end{aligned}$$

This can be positive or negative. However, note that,

$$\partial [(1 - 0.5^{r_f}) / (0.5 \cdot r_f)] / \partial r_f < 0$$

\Rightarrow

$$\partial [\ln[1 - \sigma(1 - 0.5^{r_f}) / (0.5 \cdot r_f)]] / \partial r_f > 0$$

Also,

$$\partial [\sigma (\ln (0.5) 0.5^{r_f} \cdot r_f - 0.5^{r_f}) / [1 - \sigma (1 - 0.5^{r_f})]] / \partial r_f > 0$$

and,

$$\partial [\sigma / [1 - \sigma (1 - 0.5^{r_f})]] / \partial r_f > 0$$

\Rightarrow

$$\partial^2 \{r_f \cdot \ln [1 - \sigma (1 - 0.5^{r_f}) / (0.5 \cdot r_f)]\} / (\partial r_f)^2 > 0$$

So that $r_f \cdot \ln [1 - \sigma (1 - 0.5^{r_f}) / (0.5 \cdot r_f)]$ is convex in r_f .

Moreover,

$$(\partial \{r_f \cdot \ln [1 - \sigma (1 - 0.5^{r_f}) / (0.5 \cdot r_f)]\} / \partial r_f |_{r_f = 1}) = \ln [1 - \sigma] + 0.5 \{ \ln (0.5) + 1 \} \sigma / [1 - 0.5\sigma]$$

and,

$$\begin{aligned} (\partial^2 \{r_f \cdot \ln [1 - \sigma (1 - 0.5^{r_f}) / (0.5 \cdot r_f)]\} / \partial r_f \partial \sigma |_{r_f = 1}) = \\ = \left\{ -\frac{[1-0.5\sigma]^2}{1-\sigma} + 0.5 \{ \ln (0.5) + 1 \} \right\} / [1 - 0.5\sigma]^2 < 0 \end{aligned}$$

This is because, $\frac{[1-0.5\sigma]^2}{1-\sigma} > \frac{[1-0.5]^2}{1} = 0.25$ and $0.5 \{ \ln (0.5) + 1 \} < 0.5 \{-0.65 + 1\} = 0.175$.

Hence,

$$(\partial \{r_f \cdot \ln [1 - \sigma (1 - 0.5^{r_f}) / (0.5 \cdot r_f)]\} / \partial r_f |_{r_f = 1}) \leq \ln [1 - 0] + 0.5 \{ \ln (0.5) + 1 \} \cdot 0 / [1 - 0.5 \cdot 0] =$$

0

On the other hand,

$$\lim_{r_f \rightarrow \infty} \partial \{r_f \cdot \ln [1 - \sigma (1 - 0.5^{r_f}) / (0.5 \cdot r_f)]\} / \partial r_f = \ln [1] + \sigma / [1 - \sigma] \geq 0$$

So there exist $r' \geq 1$ such that for every $r_f > r'$ we have that,

$$\partial \{r_f \cdot \ln [1 - \sigma (1 - 0.5^{r_f}) / (0.5 \cdot r_f)]\} / \partial r_f \geq 0$$

and for every $r_f < r'$ we have that

$$\partial \{r_f \cdot \ln [1 - \sigma (1 - 0.5^{r_f}) / (0.5 \cdot r_f)]\} / \partial r_f \leq 0$$

If we recall from the beginning of the proof that

$$\text{sign} \{ \partial \gamma / \partial r_f \} = -\text{sign} [\partial \{r_f \cdot \ln [1 - \sigma (1 - 0.5^{r_f}) / (0.5 \cdot r_f)]\} / \partial r_f]$$

We get that

$$\partial \left(\gamma \Big|_{\frac{r_w}{r_f} = c} \right) / \partial r_f \geq 0 \text{ for every } r_f < r'$$

and

$$\partial \left(\gamma \Big|_{\frac{r_w}{r_f} = c} \right) / \partial r_f \leq 0 \text{ for every } r_f > r' \quad \blacksquare$$

proposition 2 - Proof. Let $c = \frac{r_w}{r_f}$ be a constant.

Simple algebra shows that $\frac{dE_M[\pi]}{d\gamma} < 0$. Substituting in lemma 1, let $\bar{D}(f, w) = \text{argmax} \{ \hat{D}(f, w, \sigma) | \sigma \in [0, 1] \}$ and $\underline{D}(f, w) = \text{argmin} \{ \hat{D}(f, w, \sigma) | \sigma \in [0, 1] \}$, so that $\partial E_M[\pi] / \partial D \leq 0$ for $D < \underline{D}$ and

$\partial E_M[\pi]/\partial D \geq 0$ for $D > \bar{D}$. Consequently, $\partial F(\cdot)/\partial D \leq 0$ for $D < \underline{D}$ and $\partial F(\cdot)/\partial D \geq 0$ for $D > \bar{D}$. As $\frac{dF(\cdot)}{d\sigma} < 0$ we have that $\frac{d\sigma}{dD} \leq 0$ for $D < \underline{D}$ and $\frac{d\sigma}{dD} \geq 0$ for $D > \bar{D}$ which in turn implies the result for the interior unstable equilibrium. It is then enough to note that μ is not affected by D . ■

Proposition 3 - Proof. $\frac{dF(\cdot)}{df} = \frac{dF(\cdot)}{d\gamma} \cdot \frac{d\gamma}{df}$. Clearly, $\frac{d\gamma}{df} > 0$. Also, $\frac{dF(\cdot)}{d\gamma} = [-\alpha + 0.5]/(\cdot)^2 < 0$

so, holding r_f and r_w fixed, $\frac{dF(\cdot)}{df} < 0$. The result is then obtained directly using the implicit function theorem. ■

Proposition 4 (part 1) - Proof. $\frac{dF(\cdot)}{dr_w} = \frac{dF(\cdot)}{d\gamma} \cdot \frac{d\gamma}{dr_w}$. In the previous proof we showed that $\frac{dF(\cdot)}{d\gamma} < 0$, so we are left to demonstrate that $\frac{d\gamma}{dr_w} < 0$ so that $\frac{dF(\cdot)}{dr_w} > 0$ and the implicit function theorem gives us the result.

$$\frac{d\gamma}{dr_w} = \frac{f}{2} (r_f/r_w) \left\{ - (1/r_w) + (1/r_w) [1 - \tau + \tau \cdot \mu]^{r_w} - (1 - \tau + \mu\tau)^{r_w} \log(1 - \tau + \mu\tau) \right\}$$

denote $y = 1 - \tau + \tau \cdot \mu$ so $0 \leq y \leq 1$ and $\frac{d\gamma}{dr_w} = \frac{f}{2} (r_f/r_w) \left\{ - (1/r_w) + (1/r_w) y^{r_w} - y^{r_w} \log(y) \right\}$

\Rightarrow

$$\text{sign} \left\{ \frac{d\gamma}{dr_w} \right\} = \text{sign} \left\{ - (1/r_w) + (1/r_w) y^{r_w} - y^{r_w} \log(y) \right\}$$

Note that $d \left\{ - (1/r_w) + (1/r_w) y^{r_w} - y^{r_w} \log(y) \right\} / dy = -r_w \cdot y^{r_w-1} \log(y) > 0$ and that $- (1/r_w) + (1/r_w) y^{r_w} - y^{r_w} \log(y) = 0$ when $y = 1$. Hence $- (1/r_w) + (1/r_w) y^{r_w} - y^{r_w} \log(y) \leq 0$ for every $0 \leq y \leq 1$. ■

Proposition 4 (part 2) - Proof. Recall that $f = w \cdot (r_w/r_f)$ where w is a constant smaller than 1.

\Rightarrow

$$\gamma = f \cdot (r_f/r_w) \cdot \frac{1}{2} \cdot P^w = \frac{w}{2} \cdot P^w$$

and,

$$\frac{dF(\cdot)}{dr_f} = \frac{dF(\cdot)}{d\gamma} \cdot \frac{d\gamma}{dP^w} \cdot \frac{dP^w}{d\tau} \cdot \frac{d\tau}{dr_f} > 0 \text{ (as } \frac{dF(\cdot)}{d\gamma} < 0; \frac{d\gamma}{dP^w} > 0; \frac{dP^w}{d\tau} > 0; \frac{d\tau}{dr_f} < 0 \text{)}$$

and the result is reached by using the implicit function theorem. ■

Proposition 5 - Proof. $\frac{dF(\cdot)}{d\phi} = \frac{dF(\cdot)}{d\gamma} \cdot \frac{d\gamma}{d\mu} \cdot \frac{d\mu}{d\phi}$. Evaluating each expression separately, we get that,

$$\frac{dF(\cdot)}{d\gamma} = [-\alpha + 0.5]/(\cdot)^2 < 0;$$

$$\frac{d\gamma}{d\mu} < 0;$$

and

$$\frac{d\mu}{d\phi} = 0 \text{ if } \alpha < \frac{1-\varepsilon}{1+\varepsilon} \text{ and } \frac{d\mu}{d\phi} \geq 0 \text{ otherwise.}$$

Therefore, $\frac{dF(\cdot)}{d\phi} \geq 0$ if $\alpha > \frac{1-\varepsilon}{1+\varepsilon}$ and $\frac{dF(\cdot)}{d\phi} = 0$ otherwise. Using the implicit function theorem to get $\frac{d\sigma}{d\phi}$ the result is immediate. ■

Proposition 6 - Proof. For $\alpha < \frac{1-\varepsilon}{1+\varepsilon}$ a worker with a high signal does not refuse any job offer ($\mu = 0, \frac{d\mu}{d\alpha} = 0$) and $\frac{dF(\cdot)}{d\alpha} = \frac{-\gamma}{1-\gamma} - 2 < 0$ implies that $\frac{d\sigma}{d\alpha} \geq 0$ for stable equilibria and $\frac{d\sigma}{d\alpha} \leq 0$ for unstable equilibria. For $\alpha \rightarrow 1$ firms always make offers to workers that are connected to them ($\sigma = 1, \frac{d\sigma}{d\alpha} = 0$), and $\frac{d\mu}{d\alpha} > 0$. ■

Corollary 2 - Proof. As a worker can renege on any offer, a worker who accepts an offer in period -2 will renege on it with probability μ in period -1 . Hence, the firms expected profit from making an offer in period -2 to a $s_i = h$ worker is

$$\alpha \cdot E(\pi | q_i = H) + (1 - \alpha) \cdot (-1)$$

Which is strictly lower than the expected profit from making an offer in period -1 to a $q_i = H$ worker. ■

Proposition 7 (1) - Proof. As defined in the text, the (interior) equilibrium condition is $E_{-2}[\pi|\cdot] - 2\alpha + 1 = 0$. Where, $E_{-2}[\pi|\cdot] = E_{-2}[\pi | s_i = h, \text{ does not offer early}] = \alpha(1 - \mu) + [1 - \alpha(1 - \mu)] \cdot E_M[\pi]$

\Rightarrow

$$\frac{dE_{-2}[\pi|\cdot]}{d\sigma} = [1 - \alpha(1 - \mu)] \cdot \frac{dE_M[\pi]}{d\sigma}$$

As $\gamma_{-2} = \frac{f}{2} \cdot \sigma$ and $\gamma_{-1} = [1 - \mu] \cdot [\frac{\eta}{2} - \gamma_{-2} \cdot \alpha]$,

$$\frac{dE_M[\pi]}{d\sigma} = \frac{\partial E_M[\pi]}{\partial \gamma_{-2}} \cdot \frac{d\gamma_{-2}}{d\sigma} + \frac{\partial E_M[\pi]}{\partial \gamma_{-1}} \cdot \frac{d\gamma_{-1}}{d\sigma} = \frac{-\alpha + (\alpha - 1)\gamma_{-1} + 0.5}{(1 - \gamma_{-2} - \gamma_{-1})^2} \cdot \frac{d\gamma_{-2}}{d\sigma} + \frac{-0.5 + (1 - \alpha)\gamma_{-2}}{(1 - \gamma_{-2} - \gamma_{-1})^2} \cdot \frac{d\gamma_{-1}}{d\sigma}$$

and

$$\frac{d\gamma_{-1}}{d\sigma} = -\alpha [1 - \mu] \cdot \frac{d\gamma_{-2}}{d\sigma}$$

\Rightarrow

$$\begin{aligned} \frac{dE_M[\pi]}{d\sigma} &= \frac{-\alpha + (\alpha - 1)\gamma_{-1} + 0.5}{(1 - \gamma_{-2} - \gamma_{-1})^2} \cdot \frac{d\gamma_{-2}}{d\sigma} - \frac{-0.5 + (1 - \alpha)\gamma_{-2}}{(1 - \gamma_{-2} - \gamma_{-1})^2} \cdot \alpha [1 - \mu] \frac{d\gamma_{-2}}{d\sigma} = \\ &= \frac{B}{(1 - \gamma_{-2} - \gamma_{-1})^2} \cdot \frac{d\gamma_{-2}}{d\sigma} \end{aligned}$$

Where $B = -\alpha + 0.5 - (1 - \alpha) [1 - \mu] \cdot \frac{f}{2r} + 0.5 \cdot \alpha [1 - \mu]$

Given that

$$\frac{\partial B}{\partial \alpha} = -1 + [1 - \mu] \cdot \frac{f}{2} + 0.5 [1 - \mu] < 0$$

Let $\bar{\alpha} = \{0.5 - [1 - \mu] \cdot \frac{f}{2}\} / \{1 - 0.5 \cdot [1 - \mu] - [1 - \mu] \cdot \frac{f}{2}\}$, we have that:

1. For $\alpha < \bar{\alpha}$ we have that $\frac{dE_M[\pi]}{d\sigma} > 0$ and therefore $\frac{dE_{-2}[\pi|\cdot]}{d\sigma} > 0$ and we have a unique stable equilibrium.
2. For $\alpha > \bar{\alpha}$ we have that $\frac{dE_M[\pi]}{d\sigma} < 0$ and therefore $\frac{dE_{-2}[\pi|\cdot]}{d\sigma} < 0$ and we have a unique unstable interior equilibrium, and two stable equilibria in $\sigma = 0$ and $\sigma = 1$.

We are left to show that for $\alpha < \bar{\alpha}$ the unique equilibrium is no unraveling. Indeed, a simple calculation shows that $\frac{d\bar{\alpha}}{d\mu} < 0$ and $\frac{d\bar{\alpha}}{df} < 0$ substituting $\mu = 0.5$ and $f = 0$ we get an upper bound on $\bar{\alpha}$ of $\frac{2}{3}$. With $\alpha < \frac{2}{3}$, no firm makes an early offer, independent of the actions of other firms as $2\alpha - 1 = \frac{1}{3} = \min(E_M[\pi]) < E_{-2}[\pi|\cdot]$. ■

Proposition 7 (2) - Proof. It is straightforward to see that $\frac{\partial E_M[\pi]}{\partial f} < 0$. Since we know that $E_{-1}[\pi|q_i = H] = (1 - \mu) + \mu \cdot E_M[\pi]$, we have also that $\frac{\partial E_{-1}[\pi|q_i = H]}{\partial f} < 0$. Combining these two observations we get that $\frac{\partial E_{-2}[\pi|s_i = h, \text{ does not offer early}]}{\partial f} < 0$ and $\frac{\partial F(\cdot)}{\partial f} < 0$. Hence, $\frac{\partial \sigma}{\partial f} = -\left(\frac{\partial F(\cdot)}{\partial f} / \frac{\partial F(\cdot)}{\partial \sigma}\right) \leq 0$ for an unstable equilibrium, and $\frac{\partial \sigma}{\partial \eta} \geq 0$ for a stable equilibrium. So, as required, unraveling increases in f when keeping r_f and r_w fixed. ■

Proposition 7 (3) - Proof. An increase in the accuracy of information in period -1 is covered by lemma 2. We show below that $\frac{\partial F(\cdot)}{\partial \alpha} < 0$. Hence, $\frac{\partial \sigma}{\partial \alpha} = -\left(\frac{\partial F(\cdot)}{\partial \alpha} / \frac{\partial F(\cdot)}{\partial \sigma}\right) \leq 0$ for an unstable equilibrium, and $\frac{\partial \sigma}{\partial \alpha} \geq 0$ for a stable equilibrium. So unraveling increases in α .

We start by deriving $F(\cdot)$ by α : $\frac{\partial F(\cdot)}{\partial \alpha} = (1 - \mu) - (1 - \mu) \cdot E_M[\pi] + [1 - \alpha(1 - \mu)] \cdot \frac{\partial E_M[\pi]}{\partial \alpha} - 2$

Recall that γ_t is the number of workers hired in period t , and that $\frac{\partial \gamma_{-1}}{\partial \alpha} = -\gamma_{-2} \cdot \alpha [1 - \mu]$.

Plugging that into $E_M[\pi]$ we get that,

$$\frac{\partial E_M[\pi]}{\partial \alpha} = \frac{\gamma_{-2} \cdot K}{(1 - \gamma_{-2} - \gamma_{-1})^2}$$

where

$$K = -1 + \gamma_{-2} + [1 - \mu] \frac{f}{2} + \alpha [1 - \mu] (0.5 - 2\gamma_{-2} + \alpha\gamma_{-2})$$

so that,

$$\frac{\partial K}{\partial \gamma_{-2}} = 1 + \alpha [1 - \mu] (-2 + \alpha) \geq 0$$

\Rightarrow

$$K \leq (K|\gamma_{-2} = 0.5 \cdot \eta) = -1 + 0.5 \cdot \eta + [1 - \mu] \frac{\eta}{2} + \alpha [1 - \mu] (0.5 - \eta + 0.5 \cdot \alpha\eta)$$

Also,

$$\frac{\partial(K|\gamma_{-2}=0.5\cdot\eta)}{\partial f} = 0.5 + [1 - \mu] \left[\frac{1}{2} + \alpha(-1 + 0.5\alpha) \right] > 0$$

\Rightarrow

As $f \leq 1$,

$$K \leq (K|\gamma_1 = 0.5) = -0.5 + 0.5[1 - \mu] + \alpha[1 - \mu](0.5 - 1 + 0.5\alpha) \leq 0$$

\Rightarrow

$$\frac{\partial E_M[\pi]}{\partial \alpha} \leq 0$$

\Rightarrow

$$\frac{\partial F(\cdot)}{\partial \alpha} = (1 - \mu) - (1 - \mu) \cdot E_M[\pi] + [1 - \alpha(1 - \mu)] \cdot \frac{\partial E_M[\pi]}{\partial \alpha} - 2 < 0 \quad \blacksquare$$

Proposition 8 - Proof. First, we derive the value of $\frac{dF}{d\mu}$ and show that it is strictly negative. Given that $\frac{d\mu}{d\phi} \geq 0$, and that $\frac{\partial F(\cdot)}{\partial \sigma} \leq 0$ for unstable equilibria and $\frac{\partial F(\cdot)}{\partial \sigma} \geq 0$ for stable equilibria, this implies that $\frac{\partial \sigma}{\partial \mu} = -\left(\frac{\partial F(\cdot)}{\partial \mu} / \frac{\partial F(\cdot)}{\partial \sigma}\right) \leq 0$ for an unstable equilibrium, and $\frac{\partial \sigma}{\partial \mu} \geq 0$ for a stable equilibrium, and that unraveling increases in μ and hence in ϕ .

First note that $\frac{d\gamma_{-2}}{d\mu} = 0$; and $\frac{d\gamma_{-1}}{d\mu} = -\left[\frac{f}{2} - \gamma_{-2} \cdot \alpha\right]$.

$$\text{Also, } \frac{dE_M[\pi]}{d\gamma_{-1}} = \frac{(1-\alpha)\gamma_{-2}-0.5}{(1-\gamma_{-2}-\gamma_{-1})^2}, \text{ therefore, } \frac{dE_M[\pi]}{d\mu} = \frac{(\gamma_{-2})^2 \cdot \alpha - (\alpha)^2 (\gamma_{-2})^2 - 0.5 \cdot \gamma_{-2} \cdot \alpha - \frac{f}{2} \gamma_{-2} + \frac{f}{2} \alpha \gamma_{-2} + \frac{f}{2} 0.5}{(1-\gamma_{-2}-\gamma_{-1})^2}$$

\Rightarrow

$$\frac{dF}{d\mu} = -\alpha + \alpha \cdot E_M[\pi] + [1 - \alpha(1 - \mu)] \cdot \frac{dE_M[\pi]}{d\mu} < 0$$

Note that the last inequality is not straightforward, as $[1 - \alpha(1 - \mu)] \cdot \frac{\partial E_M[\pi]}{\partial \mu} > 0$. We have used an analytic function in "Wolfram Mathematica 6.0" to verify this result. \blacksquare

Proposition 9 - Proof. Some additional notation is needed to proceed with the proof; denote $E_{-2}[\pi^Q|s_i = h, C, \text{ does not offer}]$ as the expected profit of a firm of quality Q that belongs to a C component, connected to a $s_i = h$ worker in period -2 and does not offer the worker a job in the period -2 .

For brevity, we denote $E_{-2}[\pi^Q|s_i = h, C, \text{ does not offer}]$ as $E_{-2}[\pi^Q|C]$ throughout the rest of the analysis. Let p_H and p_L be the period -1 probability that a $q_i = H$ worker accepts a job offer from a high and low quality firms respectively (if this is her only offer in that period) and let $E_{-1}[\pi^Q|C]$ be the expected utility of a firm of quality Q that is connected to a $q_i = H$ worker (that is not yet hired) and belongs to a C component at the beginning of period -1 . Finally, Q firm's expected profit if not matched before the post-graduation marketplace is $E_M[\pi^Q]$.

By claim 5, and the analysis that follows, we can focus on the two following cases:

- Case 1 - $p_L = 0$ and $E_{-1} [\pi^L] = E_M [\pi^L] = 0$ so, $E_{-2} [\pi^H|\cdot] > E_{-2} [\pi^L|\cdot]$ and $E_{-2} [\pi^L|LL] = E_{-2} [\pi^L|HL] = 0$. Clearly, a L firm will make an early job offer in period -2 , so $\sigma_{L,LL} = \sigma_{L,HL} = 1$.

Looking at the incentives of H firms, either $E_M [\pi^H] = 1$ (and $\sigma_{H,HH} = \sigma_{H,HL} = 0$), or $E_M [\pi^H] < 1$, so $E_{-2} [\pi^H|HH] \geq E_{-2} [\pi^H|HL]$ with equality only when $\sigma_{H,HH} = 1$.

\Rightarrow

One of the following hold: (1) $\sigma_{H,HH} = \sigma_{H,HL} \in \{0, 1\}$; (2) $\sigma_{H,HH} < 1$ and $\sigma_{H,HL} = 1$; or (3) $\sigma_{H,HH} = 0$ and $\sigma_{H,HL} > 0$.

- Case 2 - $\sigma_{H,HL} = \sigma_{H,HH} = 0$ and $E_M [\pi^H] = 1$. It is immediate that one of the following holds: (1) $\sigma_{L,HL} = \sigma_{L,LL} \in \{0, 1\}$; (2) $\sigma_{L,HL} = 1$ and $\sigma_{L,LL} < 1$; or (3) $\sigma_{L,HL} > 0$ and $\sigma_{L,LL} = 0$.

\Rightarrow

$$\delta_{H,HH} \leq \delta_{H,HL}$$

$$\delta_{L,LL} \leq \delta_{L,HL} \quad \blacksquare$$