# The Case for Discrimination*

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**Abstract**

We defend the use of asymmetric norms which grant greater privileges to minorities than to majorities. The norms we discuss include norms facilitating the establishment or prohibition of minority-only or majority-only institutions, neighborhoods, or associations.

While traditionally the primary arguments favoring minorities' privileges have been based on considerations of fairness or justice, we show that there are simple environments where asymmetric norms maximize aggregate sum of individual utilities. A utilitarian may therefore support the establishment of black colleges or Hassidic only neighborhoods while at the same time oppose exclusion of blacks or Jews from white or Christian neighborhoods.

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# 1 Introduction

This paper defends the use of asymmetric legal norms which grant greater privileges to members of minority groups than to members of majority groups. In contrast to previous attempts to justify minority privileges, this paper does it without making any assumptions concerning justice, rights, or public policy. The norms we discuss are norms facilitating or prohibiting the establishment of minority-only or majority-only institutions, neighborhoods, or associations. For instance, we address the question of why the legal system differentiates between norms governing black-only colleges and white-only colleges (and imposes much greater legal obstacles in establishing the latter than in establishing the former).[1] We argue that (under some plausible assumptions), to defend the privileges of minorities under asymmetric norms one need not deviate from the weak assumption that society should maximize total utility.[2]

The justifiability of asymmetric norms is debated in the literature and legislators, courts and theorists are ambivalent with respect to them. At the same time it is evident that even the opponents of asymmetric norms do not perceive these norms as pernicious and reprehensible as simple racist laws, i.e., laws that benefits members of majority groups or discriminate against minorities. It seems much more reprehensible to exclude blacks from a white college than to exclude whites from a black college. An evidence to this is the famous statement of Justice O'Connor that "we expect that twenty five years from now, the use of racial preferences will no longer be necessary to further the interest approved today."[3] Presumably the Justice would not have been equally tolerant towards racial prejudices *against* minorities. Her view presupposes that even if any racial preference is problematic, racial preferences favoring the minorities are not as pernicious as those that favor majorities.

The exclusion of blacks from a certain neighborhood is considered to be a blatant expression of racism while maintaining the black character of traditional black neighborhoods such as Harlem is regarded by many as dictated by principles of justice.[4] Similar attitudes are prevalent when the relevant

[1]For an analysis of the legality of black colleges, see Luti (12) and Bell (2)).

[2]The debate concerning asymmetrical norms is a specific instance of the more general debate concerning affirmative action, see Eastland (5).

[3]See Grutter v. Bollinger 539 U.S. 306, 343 (2003).

[4]See http://www.thedefendersonline.com/2010/01/26/harlem-going-going-gone-or-

minority is a religious rather than ethnic minority. Many seem to be willing to accept the resistance of the ultra-orthodox Jewish residents of Monsey, NY to having a Wal-Mart store in their neighborhoods, resistance that is based on the argument that this will expose local residents (and especially their children) to alien lifestyles. At the same time, few will be willing to tolerate communities that exclude members of Monsey's Hassidic sects from residing within them.[5]

## 1.1 Non-Utilitarian Arguments

Some theorists who justify the existence of asymmetrical norms believe that one can justify them simply on the grounds that some preferences (e.g., the exclusion of blacks) ought not to be respected because they are immoral and unjust, or because they are external preferences relating to the life style of others, or because such preferences are unstable and can be easily transformed and their transformation is socially desirable. We examine each of these claims.

One attempt to exclude preferences is based on the view that there are some preferences — for example, those of murderers and rapists — that are too hideous or pernicious. Filtering or censoring such preferences is legitimate and therefore such preferences ought not to count in the social calculus. This idea was supported by Harsanyi (10, p. 56), who claimed that society has no moral obligation supporting a person's desire to harm another person (see also Goodin (8)). This view under which certain preferences ought not to be aggregated has also been supported by the courts in the context of racial discrimination. Courts thought that the rules should be designed such that illegitimate racial preferences will be excluded from decision-making. "Indeed, the purpose of strict scrutiny is to 'smoke out' illegitimate uses of race by assuring that the legislative body is pursuing a goal important enough to warrant use of a highly suspect tool. The test also ensures that the means chosen 'fit' this compelling goal so closely that there is little or no possibility that the motive for the classification was illegitimate racial prejudice or stereotype." [6]

Moreover, there are cases in which individuals have second order preferences, i.e., preferences over what preferences they want to have (see Frank-

just-invisible/

[5]See nytimes.com/2008/03/12/nyregion/12monsey.html.

[6]City of Richmond v. J.A. Croson Co. 488 US 469, 493 (1989).

furt (7)). Thus, even without excluding preferences simply because of their content, it is possible to justify the exclusion of racist preferences to the extent that individuals wish they did not have such preferences.

It is easy to see how one can use these arguments to justify the asymmetric regulation of segregation, namely rules that differentiate between whites only and black only neighborhoods. Some may judge that preferences of whites for living in a white only community are morally inappropriate and therefore that these preferences ought not to count, but that the preferences of blacks to live in a black only community are not morally inappropriate and therefore should be counted. The first type of preferences is founded on hatred or prejudices that ought to be ignored while the second is founded on feelings of empathy and solidarity and therefore should be counted.

Another suggestion was raised by Ronald Dworkin (4, pp. 234–238). In his view preferences ought to be divided into external and internal, where the former are preferences about the extent to which the preferences of other people ought to be satisfied. In contrast, internal preferences are preferences relating exclusively to one's own tastes and inclinations.[7] In Dworkin's view only the second type of preferences ought to be calculated. For example, the fact that the majority favor a sports stadium rather than an opera house counts as an argument for subsidizing the stadium as these are presumably internal preferences. But the fact that the majority regards homosexuality as immoral does not count as an argument for legislating against homosexuality because the preferences here are external ones; they concern the assignment of liberties not to the individual who wishes to bar homosexuality but to other individuals. Dworkin argues that although individuals may in their personal lives act upon at least some of their external preferences, governments and legislators must base their decisions entirely on internal preferences. In the context of racially-related preferences, one can easily maintain that racial hatred is an external preference and therefore segregation grounded in racial hatred ought to be barred. A third approach to segregation is based on the view that racist preferences are manipulable and may be transformed by regulating segregation. The idea that the law has a role in changing preferences is common among legal scholars. For instance, Dau-Schmidt (3) maintains that criminal law is primarily designed to transform and change preferences. In our context the simplest and the most commonsensical conjecture is that by prohibiting segregation or forcing desegregation racial preferences for seg-

[7]For a discussion and a critique of Dworkin, see Ely (6).

regation disappear, or at least diminish in intensity. People who are forced to live in a multicultural community transform their preferences to suit their environment. Prohibiting segregation is therefore one way of manipulating preferences.

Each of these solutions raises problems of its own. Resorting to non-utilitarian considerations (as suggested by Harsanyi and Goodin), in particular resorting to the principle differentiating between “good” and “bad” preferences is founded on the conviction that utility alone cannot justify asymmetric norms; justice and public policy need to be brought in to salvage what utility alone cannot accomplish. But determining what the just preferences are is controversial and even dangerous. It may undermine the legitimacy of the legal system if it is required to examine the justness of preferences. The traditional view in the legal literature on cost benefit analysis is that it ought to rely on actual rather than informed or ideal preferences (see Adler and Posner (1)).

Furthermore, racial animosity often disguises itself as a cultural preference. Arguably, symmetric norms favoring members of the majority can be condemned as incompatible not only with the pursuit of utility but also with the autonomy and freedom of choice. Why should not a person who likes American food and American norms of behavior in the public sphere be able to exclude foreigners who may not speak his language, do not share his preferences in food or share his enthusiasm for football? It seems that the boundaries between pernicious racial preferences and benign non-racial preferences are not always as clear as most people tend to believe. Moreover, the need to resort to normative judgments as to what preferences are legitimate and what preferences are illegitimate seems to be incompatible with legal ideals of neutrality.[8] Why should judges (and even legislatures) be assigned with the power to make these judgments? Given these forceful considerations, it is not surprising that judges who are subjected to such hard questions are inclined to give up the use of asymmetric norms altogether and strike down affirmative action programs on the grounds that these programs are discriminatory. The powerful appeal of “color neutrality” provides an effective weapon in the hands of opponents of affirmative action.

Dworkin’s distinction between external and internal preferences is equally problematic. Regan (13, p. 1221 (note 18)) has argued forcefully that most preferences have external and internal aspects and that an endorsement of

[8] For neutrality as a central liberal ideal see Kymlicka (11).

Dworkin's proposals would exclude many of the most legitimate preferences. In fact, as Regan has pointed out, Dworkin concedes the relevance of external preferences in the appendix to the book in which he argued forcefully in favor of the distinction.[9]

The claim that racial preferences are unstable seems empirically dubious and more empirical work needs to be done before it can be substantiated. Also, the proponents of this view must rely on an asymmetric evaluation of the preferences of the different groups. They assume that minorities' preferences for separate accommodation are more stable than the majorities' as the former preferences are not sanctioned by the legal system.

## 1.2 Utilitarianism and Asymmetrical Norms

The present paper maintains that instead of resorting to controversial value judgments, or to the dubious distinction between internal and external preferences, or even to empirical conjectures concerning the stability of racial preferences, a simple utilitarian calculus can justify the use of asymmetric norms. Under certain reasonable assumptions, asymmetric norms would maximize aggregate sum of individual utilities and consequently justify the use of asymmetric norms. A utilitarian may then support the establishment of black colleges or Hassidic-only neighborhoods while, at the same time, opposing exclusion of blacks or Jews from white or Christian neighborhoods.

To simplify the discussion we analyze a stylized problem, where people of two types (eg. language, religion, nationality) share a country. Individuals belong either to a minority group, e.g., blacks or Hispanic or to the majority group, e.g., Caucasians. The country is divided into up to three (disjoint, but not necessarily non-empty) parts: $J$ and $N$, which are used exclusively by members of the majority and the minority groups, and $M$, which is shared by both types. The only difference between members of society is the majority/minority division. In particular, they all have the same preferences with respect to themselves and their respective groups in the sense that ceteris paribus they prefer to live with members of their own group and they would like to be able to reside in as large part of the country as possible. Furthermore the larger the percentage of individuals of one group residing in their neighborhood, the higher is the well-being of each such individual. Thus,

[9] In this appendix Dworkin says: "[People] will vote their external preferences; they will vote for legislators, for example, who share their own theories of political justice. How else should they decide for whom to vote?" See Dworkin (4, p. 358).

under our view, the utility of each person is increasing with the size of the part of the country that is is kept for the exclusive use of his type, with the size of the mixed area, and with his type's density in that region.

We show below that under some plausible assumptions regarding the shape of the utility function, the utilitarian solution is to favor the minority while restraining the majority. In the above example, the optimal division of the country is into two areas only, one which is for the exclusive use of the minority group and one which is open for all. No majority-only segment is allowed. According to the utilitarian calculation, the minority should "discriminate" against the majority, but the majority does not possess similar reciprocal rights.

The paper is organized as following: Section 2 presents the basic structure of social policies. Individual preferences are discussed in Section 3 and in Appendix 1. Social welfare of policies is calculated in Section 4 and the applications of these calculations to some specific utility functions are presented in Section 5. The main conclusion from these examples is that there is a strong utilitarian justification for policies that restrict the majority, but not the minority.

## 2 Preliminaries

Society is composed of two groups, majority ($j$) and minority ($n$), with proportions $\alpha > \frac{1}{2}$ and $1-\alpha$, respectively. Society needs to allocate the use of a certain source (land, schools, etc.) between the two groups. A social policy is a pair $(\beta, \gamma)$, meaning that the majority can use $\beta \geqslant \alpha$ part of the good and the minority can use $\gamma \geqslant 1-\alpha$ part of it. For example, when $\alpha = 0.8$, a possible social policy is to allow the majority to reside in 0.9 of the country and to allow the minority to reside in 0.4 of it. Such a policy divides the country into three parts: 0.6 $(= 1 - 0.4)$ will be used exclusively by the majority,[10] 0.1 $(= 1 - 0.9)$ exclusively by the minority, and the remaining 0.3 will be inhabited by both. Of these, 0.2 $(= 0.8 - 0.6)$ are members of the majority group and 0.1 $(= 0.2 - 0.1)$ of the minority, hence in the 30% of the land devoted to mixed use, the densities of the majority and the minority are two thirds and one third, respectively.

[10] We assume throughout the same population density everywhere. In other words, 60% of the population will reside in the 0.6 part of land that is allocated for the exclusive use of the majority. Hence three quarters of the majority will live in majority-only areas.

In general, we will get the following allocation. Denote by $s_j$ and $s_n$ the relative sizes of the parts of the land where only the majority and minority people can live, respectively, and denote by $t$ the size of the mixed area. Then

- Since the minority can reside only in $\gamma$ part of the country, the remaining $1-\gamma$ will have to be used solely by members of the majority. Hence $s_j = 1-\gamma \leqslant \alpha$. The rest of the majority, that is, $\alpha - s_j = \alpha + \gamma - 1$, live in the mixed area.
- $s_n = 1-\beta \leqslant 1-\alpha$. The rest of the minority, $\beta - \alpha$, live in the mixed area.
- $t = 1 - s_j - s_n = \beta + \gamma - 1$

Let $d_j$ and $d_n$ be the densities of the two types in the mixed area. Then

$$d_j = \frac{\alpha+\gamma-1}{\beta+\gamma-1}, \quad d_n = \frac{\beta-\alpha}{\beta+\gamma-1}$$

and $d_j + d_n = 1$. Finally, observe that

$$s_j + td_j = \alpha \text{ and } s_n + td_n = 1-\alpha \tag{1}$$

# 3 Individual Preferences

Individuals derive satisfaction from three variables: $s$, the size of the area that is assigned exclusively for their group; $t$, the size of the mixed area, and the density $d$ of their type in this mixed area. Of course, for a given group in a given society, it is impossible to increase all three variables and there is a trade-off between the possible triples each group will face. For example, increasing the size of the mixed area $t$ at the cost of reducing $s_n$, the exclusive area of the minority, will reduce $d_j$, the density of the majority in the mixed area, but will increase $d_n$, the density of the minority there. The preferences we discuss are abstract and related to hypothetical states, existing or not.

We assume that everyone has the same preferences over triples $(s, t, d)$. That is, the asymmetric results we obtain do not follow from asymmetric preferences. As the aim of the present analysis is to show the possibility that the optimal policy restrains the majority but not the minority (that is, $\beta < 1$

and $\gamma = 1$), the results will be more convincing if the individual preferences are as simple as possible. We will therefore analyze the case where these preferences can be represented by the function $u$, given by

$$u(s, t, d) = s + tg(d)$$

where $g$ is strictly increasing, $g(0) = 0$ and $g(1) = 1$. As we show in Appendix 1, these preferences can be derived from some simple axioms. We assume that this is the utility of a person independently of the location of his residence — in the segregated or in the mixed area. In other words, our analysis is ex ante, before anyone knows where exactly he'll live, rather than ex post (on the basis of the utility gained from living in either a mixed or a homogenous area). Under our model, people gain utility not from living in a specific area (which is either mixed or homogenous) but from living in a jurisdiction which has certain rules governing the allocation of rights of different groups.

We maintain that the ex ante perspective is indeed a better one than the ex post one. Recall that our analysis applies to all spheres of life. It applies to neighborhoods, schools, social clubs, restaurants, offices, etc. Given the broad scope of our analysis the rules governing the allocation of rights of different groups become primary in determining the utility of individuals during the course of their lives. Suppose that one's utility is the sum of the utilities derived from the composition of each one of the institutions to which a person belongs as well as the expectations concerning the institutions to which one will belong in the future. Any rule concerning segregation (a rule that prohibits segregation, an asymmetrical rule favoring minorities or a symmetrical rule) will have applications with respect to all spheres of life. Furthermore, it will have applications that extend to the expectations of individuals concerning the future as people may join new clubs and leave old ones. The overall utility is therefore a function of the ex ante perspective (based on the rules governing the permissibility of segregation) rather than a function of the ex post perspective.

Following the analysis of the previous section, the policy $(\beta, \gamma)$ leads members of the majority to the utility

$$\begin{aligned} u(s_j, t, d_j) &= u(1-\gamma, \beta+\gamma-1, \frac{\alpha+\gamma-1}{\beta+\gamma-1}) = \\ & 1-\gamma+(\beta+\gamma-1)\, g(\frac{\alpha+\gamma-1}{\beta+\gamma-1}) \end{aligned} \tag{2}$$

while the utility of members of the minority will be

$$\begin{aligned} u(s_n, t, d_n) &= u(1-\beta, \beta+\gamma-1, \frac{\alpha+\gamma-1}{\beta+\gamma-1}) = \\ & 1-\beta+(\beta+\gamma-1)\, g(\frac{\beta-\alpha}{\beta+\gamma-1}) \end{aligned} \quad (3)$$

Observe that if $g(d) = d$ for all $d$, then individuals don't care how the country is divided, as the utility of the majority is $\alpha$ and the utility of the minority is $1-\alpha$, regardless of the actual division.

**Remark:** The preferences discussed above are different from the expected ex post utilities restricted to the various areas. Formally, one can suggest a model where the utility of those who live in the segregated area equals the size of that area, and the utility of those who live in the mixed area is the size of the area multiplied by $g(d)$ where $d$ is the density of their type in this area. Finally, the ex ante utility will be the expected utility from the two areas, where the weights of the two utilities are proportional to the division of the type between them. According to this proposal, the policy $(\beta, \gamma)$, leading to the division $1-\gamma$ for the majority only, $1-\beta$ for the minority only, and $\beta+\gamma-1$ mixed area yields $1-\gamma$ members of the majority utility $1-\gamma$ and it yields the remaining $a+\gamma-1$ members of the majority utility $(\beta+\gamma-1)g(\frac{a+\gamma-1}{\beta+\gamma-1})$. The expected value of these utilities is

$$\frac{1-\gamma}{a} \times (1-\gamma) + \frac{a+\gamma-1}{a} \times (\beta+\gamma-1)g(\frac{a+\gamma-1}{\beta+\gamma-1})$$

When $g$ is linear, this expression reduces to $\frac{1}{a}[(1-\gamma)^2+(a+\gamma-1)^2]$, hence is sensitive to changes in $\gamma$. As we saw above with respect to our model, when $g$ is linear, individual utilities are not sensitive to social policies.

# 4 Social Welfare

If everyone prefers full segregation or if everyone prefers no segregated areas, the social optimum will agree with these preferences. This is the case when $g$ is either convex or concave.

If $g$ is convex (in fact, even if for every $0 < d < 1$, $g(d) \leqslant d$), then

$$u(s,t,d) = s + tg(d) \leqslant s + td = u(\alpha, 0, 0)$$

where the last equation-sign follows by eq. (1). In this case, everyone in society (weakly) prefers full segregation ($t = 0$), and therefore any social preferences satisfying unanimity will rank this policy best. On the other hand, if $g$ is concave, then having no exclusive areas is optimal (see proposition 2 below). We now turn to the construction of individual and social preferences that will lead society to restrict only the majority, but not the minority. We assume that society has preferences over policies, that is, over pairs $(\beta, \gamma)$. Following Harsanyi (9) (but assuming same weights to all), we assume that these preferences can be represented by the sum of individual utilities. This form of utilitarianism seems to us to serve our purpose — everyone counts, and no one has higher value than others.

As the proportion of the majority is $\alpha$, it follows by eqs. (2) and (3) that society is looking to maximize, with respect to $\beta$ and $\gamma$, the function $W$ given by

$$\begin{aligned} W(\beta,\gamma) &= \alpha u(s_j,t,d_j) + (1-\alpha)u(s_n,t,d_n) = \\ & \alpha[1-\gamma+(\beta+\gamma-1)\, g(\frac{\alpha+\gamma-1}{\beta+\gamma-1})] + \\ & \qquad (1-\alpha)[1-\beta+(\beta+\gamma-1)\, g(\frac{\beta-\alpha}{\beta+\gamma-1})] = \\ & 1-\alpha\gamma-(1-\alpha)\beta + \\ & \qquad (\beta+\gamma-1)[\alpha g(\frac{\alpha+\gamma-1}{\beta+\gamma-1}) + (1-\alpha) g(\frac{\beta-\alpha}{\beta+\gamma-1})] \end{aligned} \tag{4}$$

The next proposition shows that if the optimal policy is not full separation, then at least one of the two groups should not be restricted at all.

**Proposition 1** The maximum of the above function $W$ of eq. (4) is at one (or more) of the following points:

1. $\beta = \alpha$, $\gamma = 1-\alpha$,
2. $\beta = 1$,
3. $\gamma = 1$.

**Proof:** Obviously, if $\beta = \alpha$ [alt. $\gamma = 1-\alpha$] then effectively $\gamma = 1-\alpha$ [alt. $\beta = \alpha$]. The claim thus rules out the possibility that $\alpha < \beta < 1$ and

$1-\alpha < \gamma < 1$. Suppose that social welfare is maximized at $(\beta^*, \gamma^*) \in (\alpha, 1) \times (1-\alpha, 1)$. Define $\kappa = (\gamma^* + \alpha - 1)/(\beta^* - \alpha)$ and consider the function

$$\begin{aligned} F(\xi) &= W(\alpha+\xi, 1-\alpha+\kappa\xi) = \\ & 1-2\alpha(1-\alpha) - \alpha\kappa\xi - (1-\alpha)\xi + \\ & \xi(1+\kappa)[\alpha g(\frac{\kappa}{1+\kappa}) + (1-\alpha) g(\frac{1}{1+\kappa})] \end{aligned}$$

For $\xi = \beta^* - \alpha$, $F(\xi) = W(\beta^*, \gamma^*)$. The function $F$ is linear in $\xi$, hence is maximized either at $\xi = 0$ (case 1 of the claim) or when either $\alpha + \xi = 1$ (case 2) or $1 - \alpha + \kappa\xi = 1$ (case 3). ■

The following claim follows from the last proposition:

**Proposition 2** *If $g$ is concave then $(\beta, \gamma) = (1, 1)$ is optimal. In other words, having no segregated areas is not inferior to any other policy.*

**Proof:** By the concavity of $g$,

$$W(1,1) = \alpha g(\alpha) + (1-\alpha) g(1-\alpha) \geqslant \alpha^2 + (1-\alpha)^2 = W(\alpha, 1-\alpha)$$

Hence we should check cases 2 and 3 in proposition 1. Suppose $\beta = 1$ is optimal and compare it with the case $(\beta, \gamma) = (1, 1)$. Given policy $(1, \gamma)$, $1-\gamma$ members of the majority live in an exclusive area. The remaining $\alpha + \gamma - 1$ members of the majority together with all $1-\alpha$ members of the minority live in the mixed area. The size of the mixed area is $\gamma$, and the corresponding densities there are $\frac{\alpha+\gamma-1}{\gamma}$ and $\frac{1-\alpha}{\gamma}$. The utility of each member of the minority under policy $(1, \gamma)$ is therefore

$$\begin{aligned} \gamma g(\frac{1-\alpha}{\gamma}) &= \gamma g(\frac{1-\alpha}{\gamma}) + (1-\gamma) g(0) \leqslant \\ & g(\gamma \times \frac{1-\alpha}{\gamma} + (1-\gamma) \times 0) = g(1-\alpha) \end{aligned}$$

where the inequality follows by the concavity of $g$. Likewise, the utility of each member of the majority is

$$\begin{aligned} 1-\gamma+\gamma g(\frac{\alpha+\gamma-1}{\gamma}) &= (1-\gamma) g(1) + \gamma g(\frac{\alpha+\gamma-1}{\gamma}) \leqslant \\ & g((1-\gamma) \times 1 + \gamma \times \frac{\alpha+\gamma-1}{\gamma}) = g(\alpha) \end{aligned}$$

Hence $W(1,1) \geqslant W(1, \gamma)$. Similarly, $W(1,1) \geqslant W(\beta, 1)$. ■

# 5 Examples

In this section we show that all the cases of proposition 1 may happen. In particular, we show that there are functions $g$ such that the unique optimal social policy is to set $\gamma = 1$ and $\beta < 1$. Such policies restrict the majority without putting any restrictions on the minority.

According to proposition 1 we have to check for maximum points of $W(\beta, 1)$ with respect to $\beta$ and of $W(1, \gamma)$ with respect to $\gamma$, and compare them with $W(\alpha, 1-\alpha)$. Setting $\gamma = 1$ in eq. (4) we obtain that society should maximize, as a function of $\beta$,

$$(1-\alpha)(1-\beta) + \beta[\alpha g(\frac{\alpha}{\beta}) + (1-\alpha)g(1-\frac{\alpha}{\beta})] \tag{5}$$

First and second order conditions for maximization with respect to $\beta$ are

$$-(1-\alpha) + \alpha g(\frac{\alpha}{\beta}) + (1-\alpha)g(1-\frac{\alpha}{\beta}) \\ -\frac{\alpha^2}{\beta}g'(\frac{\alpha}{\beta}) + \frac{\alpha(1-\alpha)}{\beta}g'(1-\frac{\alpha}{\beta}) = 0 \tag{6}$$

$$\alpha g''(\frac{\alpha}{\beta}) + (1-\alpha)g''(1-\frac{\alpha}{\beta}) < 0 \tag{7}$$

Likewise, setting $\beta = 1$ in eq. (4) we obtain that society should maximize, as a function of $\gamma$,

$$\alpha(1-\gamma) + \gamma[\alpha g(1-\frac{1-\alpha}{\gamma}) + (1-\alpha)g(\frac{1-\alpha}{\gamma})]$$

First and second order conditions for maximization with respect to $\gamma$ are

$$-\alpha + \alpha g(1-\frac{1-\alpha}{\gamma}) + (1-\alpha)g(\frac{1-\alpha}{\gamma}) + \\ \frac{\alpha(1-\alpha)}{\gamma}g'(1-\frac{1-\alpha}{\gamma}) - \frac{(1-\alpha)^2}{\gamma}g'(\frac{1-\alpha}{\gamma}) = 0 \tag{8}$$

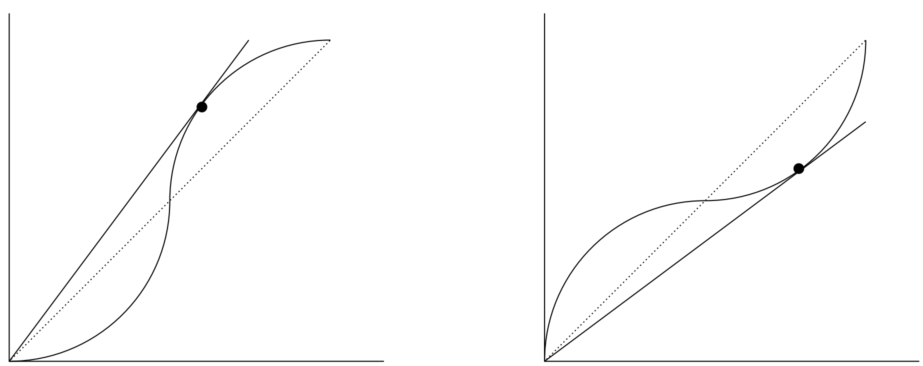
$$\alpha g''(1-\frac{1-\alpha}{\gamma}) + (1-\alpha)g''(\frac{1-\alpha}{\gamma}) < 0 \tag{9}$$

Consider first the case where $g(1-d) = 1 - g(d)$. We focus attention on two special cases of this form: (i) $g$ is convex on $[0, \frac{1}{2}]$ and concave on $[\frac{1}{2}, 1]$ and (ii) $g$ is concave on $[0, \frac{1}{2}]$ and convex on $[\frac{1}{2}, 1]$. Case (i) represents individuals who get most of their satisfaction when they become the majority in the mixed area and those who were the majority in the mixed area suffer the sharpest decline in utility when they become the minority there. Case (ii) represents the opposite attitude — individuals enjoy most the fact that their type "made it" into the segregated area, and suffer most from the first people of the other type to enter their domain.

If $g(1-d) = 1 - g(d)$, then $g'(1-d) = g'(d)$ and $g''(1-d) = -g''(d)$. Set $\gamma = 1$ and obtain from eqs. (6) and (7)

$$g'(\frac{\alpha}{\beta}) = \frac{g(\alpha/\beta)}{\alpha/\beta} \text{ and } g''(\frac{\alpha}{\beta}) < 0$$

(recall that $\alpha > \frac{1}{2}$ hence $2\alpha - 1 > 0$). An inner solution is thus possible in case (i), but not in case (ii) (see Fig. 1). Observe however that the solution is indeed an inner solution only if $\beta \in (\alpha, 1)$, that is, if $\frac{\alpha}{\beta} \in (\alpha, 1)$.



Case (i) Case (ii)

Figure 1: $g(1-d) = 1 - g(d)$

Alternatively, set $\beta = 1$ and obtain from eqs. (8) and (9)

$$g'(\frac{1-\alpha}{\gamma}) = \frac{g([1-\alpha]/\gamma)}{[1-\alpha]/\gamma} \text{ and } g''(\frac{1-\alpha}{\gamma}) > 0$$

Here an inner solution is possible in case (ii), but not in case (i). We obtain that if full segregation is not optimal, then in case (i) it is optimal to restrict the majority but not the minority (that is, set $\gamma = 1$ and $\beta < 1$), while in case (ii) it is optimal to restrict the minority but not the majority.

The symmetry assumption $g(1-d) = 1-g(d)$ seems strong, but it is more plausible in case (i) than in case (ii). If individuals care about their type being the minority or the majority in the mixed region, then we should expect $g$ to have the highest slope on both sides of $\frac{1}{2}$. In fact, We can get the optimum of $\gamma = 1 > \beta$ even if the crucial point is not $\frac{1}{2}$, for example, individuals may care most about whether their type consists of more or less than $\frac{2}{3}$ of the mixed region. Consider, for example, the function $g(d) = 2.5d^2 - 1.5d^3$ (panel (a) in Fig. 2). This function crosses the main diagonal at $d = \frac{2}{3}$. For $\alpha \geqslant \frac{2}{3}$, the optimal division is $\gamma = 1$ and $\beta = (12\alpha^2 - 6\alpha)/(9\alpha - 4)$ (see Appendix 2 for details). Observe that $\beta \geqslant \alpha$ iff $\alpha \geqslant \frac{2}{3}$.

Another type of optimum points is obtained for functions $g$ that have a bulge above the main diagonal for low values of $d$ and then coincide with the main diagonal. For such functions there are typically two policies that yield the maximum level of social welfare, one where $\beta = 1$ and $\gamma < 1$, and one where $\beta < 1$ and $\gamma = 1$. For example, for the function of panel (b) of Fig. 2,[11] we obtain by eq. (4) for $\alpha = 0.4$ that $W(0.6, 0.4) = W(1, 1) = 0.6^2 + 0.4^2 = 0.52$. It is easy to verify that $W(1, \gamma)$ is maximized at $\gamma = \frac{4}{9}$, $W(\beta, 1)$ is maximized at $\beta = \frac{2}{3}$, and that $W(\frac{2}{3}, 1) = W(1, \frac{4}{9}) = \frac{41}{75} \approx 0.547$. Observe that the parallel case, where $g$ coincides with the main diagonal except for a region where its graph is below the diagonal is covered by the analysis at the beginning of Section 4.

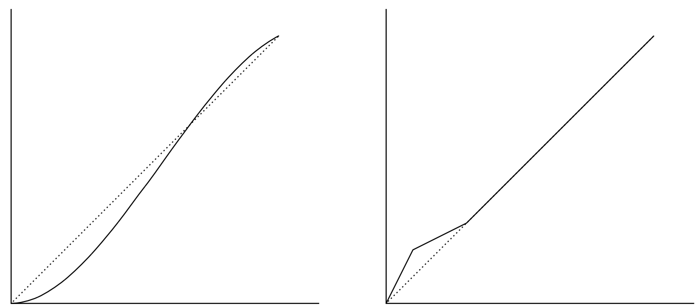
# 6 Concluding Remarks

This paper is not intended to explain why the current legal rules permit the establishment of minority-only institutions, neighborhoods, or associations and it certainly does not claim that utilitarian arithmetic explains such legislations. Nor is it our goal to compute optimal policies of land allocation.[12] Yet we maintain that the automatic resistance of utilitarians to the establishment of minority-only privileges is too hasty as our analysis dictates caution

[11] The function used in panel (b) is given by

$$g(d) = \begin{cases} 2d & 0 \leqslant d \leqslant 0.1 \\ 0.5d + 0.15 & 0.1 \leqslant d \leqslant 0.3 \\ d & 0.3 \leqslant d \leqslant 1 \end{cases}$$

[12] It may well be that maximal social welfare will obtain when the country is divided into two mixed areas with different densities.



(a) (b)

Figure 2: Two examples

on the part of those who use utilitarianism to attack progressive legislation establishing asymmetrical norms.

Our analysis establishes that asymmetric norms may maximize a utilitarian social welfare function in cases individuals benefit greatly from increasing their group's presence in a neighborhood above a certain threshold, e.g., 50%. Thus increasing their presence from 0 to 45% does not trigger a large rise in their well-being. Yet, the increase from 45% to 55% increases greatly their well-being. Of course the precise threshold and the impact on one's well-being may differ from one case to another. Yet we have established that under such circumstances utilitarian considerations may dictate the establishment of an affirmative action regime under which minorities are privileged over majorities. Is this assumption realistic?

One response is that our paper does not aim to establish that the circumstances which justify an asymmetrical norms hold in reality. We have no information concerning the utility functions of individuals and we cannot therefore affirm that such a system is indeed optimal. Yet, it is also evident that the more plausible the necessary circumstances are, the stronger is our case. Hence it is valuable to establish that the assumptions which give rise to the justifiability of the affirmative action scheme are indeed plausible even if not necessarily true.

We believe that the threshold conditions we posit are plausible for two reasons. The presence of a certain percentage, e.g., 50% of members of one's group guarantees certain impact on the political system. If the different groups have different political tastes, it follows that it will be crucial for a group to reach a threshold which entitles it to a share in the local government of the mixed area. At times the desirable threshold may be sufficient to block unfriendly amendments proposed by an adverse group. Alternatively the desirable threshold is sufficient to pass laws which promote the interests of the group (at the expense of the adverse group). Consequently a small increase in the presence of the relevant group around the threshold may trigger a great increase in the well-being of its members.

Secondly, the presence of a certain percentage of members of the relevant group may affect the supply of goods and services. The greater their presence, the more likely there will be businesses which provide goods and services catering to their taste. Such an advantage often depends on reaching a certain threshold of presence in the relevant neighborhood. One can for instance imagine that the presence of ten thousands Hispanics (comprising of 30% of the population) may make it profitable to open a book store specializing in books in Spanish. The presence of thirty thousand Hispanics makes it profitable to open a cinema specializing in Spanish-speaking movies etc. It seems therefore that a small increase in the percentage of Hispanics around the threshold may trigger a large increase in their well-being.

Therefore, although the paper shows that different utility functions may lead to different optimal allocations of rights, we believe that there are good reasons to assume that actual preferences are such that granting extra rights to minorities are optimal. A natural question this paper did not attempt to answer is how should society react to situations where a minority group is trying to exclude another minority group (for example, religious groups trying to exclude homosexuals, black colleges rejecting Hispanic candidates, etc.) Utilitarian calculus may answer such questions as well.

# Appendix 1: Axioms

In this appendix we offer axioms leading to the conclusion that the utility of each person from a policy leading to $(s, t, d)$ is

$$u(s, t, d) = s + tg(d)$$

where $g$ is strictly increasing, $g(0) = 0$ and $g(1) = 1$.

Consider the set $A = \{(s, t, d) \in [0, 1]^3 : s + t \leqslant 1\}$. From an individual point of view, the vector $(s, t, d)$ means that the group to which the individidual belongs can reside in $s + t$ part of the country, where in the $s$ part only this group resides while in the $t$ part both groups can reside, and the density of people of the individual's group in this $t$ area is $d$. Let $\succeq$ be a complete, transitive, and continuous preference relation over $A$. There is therefore a function $u$ representing these preferences. Assume further that $\succeq$ is increasingly monotonic in $s$, and for positive $t$ and $d$, it is increasing in these two variables as well.

When $d = 0$, there are no representatives of the individual's type in the mixed area, so this area does not really exist. This leads to the first restriction on $\succeq$:

A. For all $t \leqslant 1 - s$, $(s, t, 0) \sim (s, 0, 0)$.

Similarly, if $t = 0$ there is no mixed area, and $d$ doesn't matter. That is,

B. For all $d$, $(s, 0, d) \sim (s, 0, 0)$.

When $d = 1$, the mixed area is not mixed but is inhabited by members of the individual's group only. Hence

C. $(s, t, 1) \sim (s + t, 0, 0)$.

Next, we assume separability between the two areas. The evaluation of two possible mixed areas does not depend on the size of what is assigned to the exclusive use of the individual's group. Formally:

D. $(s, t, d) \succeq (s, t', d') \iff (s', t, d) \succeq (s', t', d')$.

Define $f(t, d)$ by $(0, t, d) \sim (0, f(t, d), 1)$ and obtain by the last two assumptions that

$$(s, t, d) \sim (s, f(t, d), 1) \sim (s + f(t, d), 0, 0) \tag{10}$$

Therefore $\succeq$ can be represented by

$$u(s, t, d) = s + f(t, d) \tag{11}$$

Observe that by assumption C above,

$$f(t, 1) = t \tag{12}$$

Finally, we assume size-homotheticity:

E. $(s,t,d) \succeq (s',t',d')$ iff for all appropriate[13] $\lambda$, $(\lambda s, \lambda t, d) \succeq (\lambda s', \lambda t', d')$. In particular, $(s,t,1) \sim (s,t',d)$ implies $(\lambda s, \lambda t, 1) \sim (\lambda s, \lambda t', d)$. By eqs. (10) and (12),

$$t = f(t,1) = f(t',d)$$
$$\lambda t = f(\lambda t,1) = f(\lambda t',d)$$

As $f$ is homogeneous of degree 1 in its first argument, it follows that $u(s,t,d) = s + tg(d)$ (see eq. (11) above). Since $u(s,t,0) = s$ it follows that $g(0) = 0$, and since $f(t,1) = t$ it follows that $g(1) = 1$.

# Appendix 2: The Case $g(d) = 2.5d^2 - 1.5d^3$

Consider the function $g(d) = 2.5d^2 - 1.5d^3$. Set $\gamma = 1$ and obtain from eq. (6)

$$\beta = \frac{12\alpha^2 - 6\alpha}{9\alpha - 4}$$

Observe that $\beta \geqslant \alpha$ iff $\alpha \geqslant \frac{2}{3}$. Substitute into the second order condition to obtain $-2\alpha + 1$ which is negative for all $\alpha > \frac{1}{2}$, hence $(\frac{12\alpha^2-6\alpha}{9\alpha-4}, 1)$ is a maximum point.

Next we check for the optimal value of $\gamma$ when $\beta = 1$. Similarly to the above analysis we get from eq. (8) that

$$\gamma = \frac{30\alpha - 54\alpha^2 + 42\alpha^3 - 12\alpha^4 - 6}{19\alpha - 23\alpha^2 + 9\alpha^3 - 5}$$

Substitute into the second order condition to obtain

$$\frac{(9\alpha - 5)^4}{432(2\alpha - 1)^3(1 - \alpha)}$$

which is positive for all $\alpha > \frac{1}{2}$ (and $\neq \frac{5}{9}$, where it is zero). In other words, when $\beta = 1$ the optimal value of $\gamma$ is either 1 (which is covered by the previous analysis), or $1 - \alpha$. We have therefore to check the value of $W(\beta, \gamma)$ at two points, $(\alpha, 1-\alpha)$ and $(\frac{12\alpha^2-6\alpha}{9\alpha-4}, 1)$. From eq. (5) we obtain

$$W(\frac{12\alpha^2 - 6\alpha}{9\alpha - 4}, 1) - W(\alpha, 1 - \alpha) =$$
$$\frac{(76\alpha - 171\alpha^2 + 135\alpha^3 - 12)\alpha}{72(2\alpha - 1)^2}$$

[13] That is, for all $0 < \lambda \leqslant \min\{\frac{1}{s+t}, \frac{1}{s'+t'}\}$.

The sign of this expression is the same as the sign of $76\alpha - 171\alpha^2 + 135\alpha^3 - 12$. This function is increasing on $[\frac{1}{2}, 1]$ and its value at $\alpha = \frac{2}{3}$ is positive. In other words, for all $\alpha > \frac{2}{3}$, the optimal policy is to set $\beta = \frac{12\alpha^2 - 6\alpha}{9\alpha - 4}$ and $\gamma = 1$.

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