Risk Sharing and the International Consumption Correlation Puzzle*

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Abstract

International real business cycle models often generate consumption correlations that exceed GDP correlations. In the data, however, the opposite is true. This paper uses a simple endowment model with trade costs to evaluate the role of risk sharing in generating the discrepancy between theory and data. I show that a two-country version of the model always generates consumption correlations that are greater than output correlations; however, this ranking can be inverted simply by adding countries to the model. Specifically, consumption correlations tend to be lower than output correlations when the latter are high. These results hold also after extending the model to include production; however, the comparable quantity to endowment in the production economy is GDP net of government spending and investment, not GDP. Notably, the pattern of correlations across the G7 countries is consistent with the prediction of the model. The quantitative results, however, are ambiguous. If preferences display sufficient non-separability between consumption and leisure then the model generates the correct ranking of correlations for most pairs of countries even in a frictionless environment. Separable preferences, on the other hand, require unrealistically high level of trade costs in order to produce similar results.

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1 Introduction

Standard international real business cycle models often generate cross-country consumption correlations that exceed GDP correlations. In the data, however, the opposite is true. This discrepancy between theory and data constitutes the international consumption correlation puzzle. The puzzle was introduced by Backus, Kehoe and Kydland (1992), and many subsequent studies have reinforced their findings[1].

Models with a complete set of contingent assets often generate high consumption correlations; as a result, attention has been directed towards financial markets imperfections and consumption co-movement for resolving the puzzle. Baxter (1995), Kollmann (1996), and Heathcote and Perri (2002), for example, conclude that models with incomplete markets perform better in generating international co-movements relative to their complete markets counterparts. It should be noted, however, that although models with incomplete markets do close some of the gap between consumption and GDP correlations, generally they are unsuccessful in producing the correct ranking.

This paper evaluates the role of risk sharing under complete financial markets in generating the puzzle. After documenting the co-movements of consumption and output across the G7 countries, I study the implications of risk sharing for international co-movements using a simple endowment model with trade costs and complete markets. Then, the model is extended to include production, and different simulations are analyzed for quantitative results. The qualitative analysis suggests that the cross-sectional pattern of international correlations in the data is consistent with the predictions of the model. However, the quantitative results are ambiguous as they are sensitive to the specification of preferences.

For the endowment economy, I find that a two-country version of the model produces consumption correlations that are always greater than output correlations. However, this ranking can be inverted, for some pairs of countries, simply by adding countries to the model. In a multi-country setting, the model is able to generate consumption correlations below output correlations when the latter are high.

In the model, trade is motivated entirely by risk sharing. For that reason, the worst countries can do is to avoid trade entirely. In that case, consumption correlations are equal to endowment correlations since residents of each country consume only their endowment. If we allow for trade, consumptions become smoother since trade is used to offset endowment shocks. As a result, consumptions become correlated with the aggregate endowment. In a two-country world, this necessarily increases the co-movement of consumption across countries, and specifically consumptions become more correlated than endowments. To see the effect of additional countries it is useful to consider an extreme case in which two countries, say A and B, have a perfectly correlated endowments process. If these are the only countries in the world, there are no gains from trade. Each

[1] See, for example, Stockman and Tesar (1995), Baxter and Crucini (1995), Kollmann (1996), Kehoe and Perri (2002), and Ravn and Mazzenga (2002).

country consumes its own endowment, and hence consumptions are perfectly correlated as well. Adding country C to the model can reduce the correlation of consumptions (between A and B) as long as C has a different effect on A than on B. This can happen if, for example, trade costs between A and C are different from those between B and C. Hence, this example demonstrates that consumption correlations can fall below output correlations when output correlations are high.

After extending the model to include production, similar results still hold; however it is shown that the comparable quantity to endowment in the production economy is not GDP but rather GDP net of investment[2]. A model with government would require netting out government spending as well. This result implies that for the evaluation of risk sharing in the data consumption correlations should be compared to the correlations of GDP net of government spending and investment (GDP-G-I hereafter)[3].

As mentioned, the paper also documents the cross-sectional correlations of consumption, GDP, and GDP-G-I across the G7 countries. I find that netting out government spending and investment inverts the ranking of correlations for many pairs of countries. I also find that the cross-sectional pattern of consumption and GDP-G-I correlations is consistent with the predictions of the model. Specifically, the data display a tendency of consumption correlations to be lower than the correlations of GDP-G-I when the latter are high.

Finally, the production economy version of the model is simulated. The model is able to match the magnitude of consumption correlations in spite of the complete markets assumption. It is unable, however, to generate the correct ranking of consumption and GDP correlations. In the model, the international correlations of GDP are too low[4]. Although these results may direct attention towards the co-movement of GDP, they shed little light on the source of the puzzle. In the model all quantities are determined simultaneously in a general equilibrium framework; it is therefore difficult to identify the forces behind the co-movement anomaly from simulations of the full model. In order to focus on the role of consumption co-movement and risk sharing, I substitute actual data in the equation that governs international allocations, and solve for consumption[5]. The results suggest that sufficient non-separability in the utility function between consumption and leisure can generate the observed ranking of correlations even in a frictionless environment; while separable preferences require unrealistically high trade frictions in order to produce similar results. In that sense non-separability is more important than trade costs for the resolution of the puzzle.

[2] This statement is accurate if utility is additively separable in consumption and leisure; otherwise the propositions derived from the endowment model are no longer true, although the simulations generate similar results.

[3] Obstfeld and Rogoff (2000) also argue that for the purpose of evaluating risk sharing government spending and investment should be subtracted from GDP.

[4] Low international correlations of GDP are common in the literature. See, for example, Backus, Kehoe and Kydland (1992), Baxter and Crucini (1995), Hethcote and Perri (2002), and Kehoe and Perri (2002).

[5] See Section 5.3.2. for details.

To summarize, the paper evaluates the role of risk sharing in generating the consumption correlation puzzle. I find that the cross-sectional pattern of correlations in the data is consistent with the predictions of a multi-country model. However, the quantitative results are ambiguous as they are sensitive to specification of preferences. Empirical work is therefore needed in order to reach a decisive conclusion.

The rest of the paper is organized as follows. Section 2 documents the correlations of consumption, GDP, and GDP-G-I across the G7 countries. Section 3 provides a simple presentation of the basic theory behind risk sharing and international consumption correlations. It also reviews mechanisms, suggested in the literature, to reduce these correlations. Section 4 develops an endowment economy model and derives the qualitative results discussed above. Section 5 adds production and investment and simulates the model. Section 6 concludes.

2 International Co-movements: Stylized Facts

This section documents the cross sectional correlations of consumption, GDP, and GDP-G-I across the G7 countries. These countries are Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States.

The consumption correlation puzzle is a statement about the international co-movement of consumptions relative to the co-movement GDPs. The theoretical analysis in this paper, however, suggests that risk sharing has direct implications on the co-movement of consumptions relative to the correlations of GDP-G-I, not GDP[6]. For that reason I add GDP-G-I to the analysis. Obstfeld and Rogoff (2000) also argue that government spending and investment should be subtracted from GDP. They claim that private consumers can share only the income remaining after investment and government consumption. A different way to make the argument is to think of net exports as the resources that are used for sharing risk internationally. In that sense, net exports are used as a shock absorber for smoothing private consumption. Consumption correlations should therefore be compared to the co-movements of consumption plus the resources that are used to smooth it in international markets, i.e. consumption plus net exports; which of course is the same as GDP-G-I.

Table 1 summarizes the international correlations of consumption (ρ_C), GDP (ρ_Y), and GDP-G-I (ρ_{Y-G-I}) across the G7 countries, and compares the results under different filtering procedures. The data are in annual frequency, and the sample period is 1970-2002[7]. The sample period was chosen based on data availability for all countries. Before filtering, all series were expressed in fixed prices, adjusted for purchasing power parity (PPP), and put into per-capita terms. The filters were applied on the natural logarithm of the series. The data appendix (Appendix 5) describes the data in details and provides complementary tables.

[6] See Section 5, and specifically equation 24.

[7] Data for Germany prior to 1991 are based on estimates for a unified Germany.

Table 1: International Co-Movements
G7 Countries, Summary Statistics

	HP Filter $\lambda = 100$	HP Filter $\lambda = 10$	First Difference	$BP_3\,(2,8)$
Weighted Average* ρ_C	0.38	0.39	0.36	0.37
Weighted Average* ρ_Y	0.50	0.52	0.48	0.51
Weighted Average* ρ_{Y-G-I}	0.38	0.40	0.40	0.43
$\rho_C > \rho_Y$	8	2	4	2
$\rho_C > \rho_{Y-G-I}$	14	11	9	5
$\rho_C > \rho_{Y-G-I}$				
s.t. $\rho_{Y-G-I} > Median$	5	2	3	1
s.t. $\rho_{Y-G-I} \leq Median$	9	9	6	4
$Corr\left(\rho_C - \rho_{Y-I-G}, \rho_{Y-I-G}\right)$	-0.65	-0.52	-0.42	-0.42

*Weighted by population. Letting i denote the population in country i and N the number of countries, the weight of country pair i,j is given by $n_i n_j / \sum_{l=1}^{N-1} \sum_{k=l+1}^{N} n_l n_k$

I consider three different filtering procedures: the Hodrick and Prescott (1997) filter (HP filter), first differencing, and the Baxter and King (1995) band-pass filter. Baxter and King (1995) argue that when using the HP filter in annual frequency, a smoothing parameter (λ, hereafter) of 10 is more appropriate than the commonly used value of 100. Table 1 presents the results under both values. Figure 1 provides a visual presentation of the correlations for the HP filter with $\lambda = 100$.

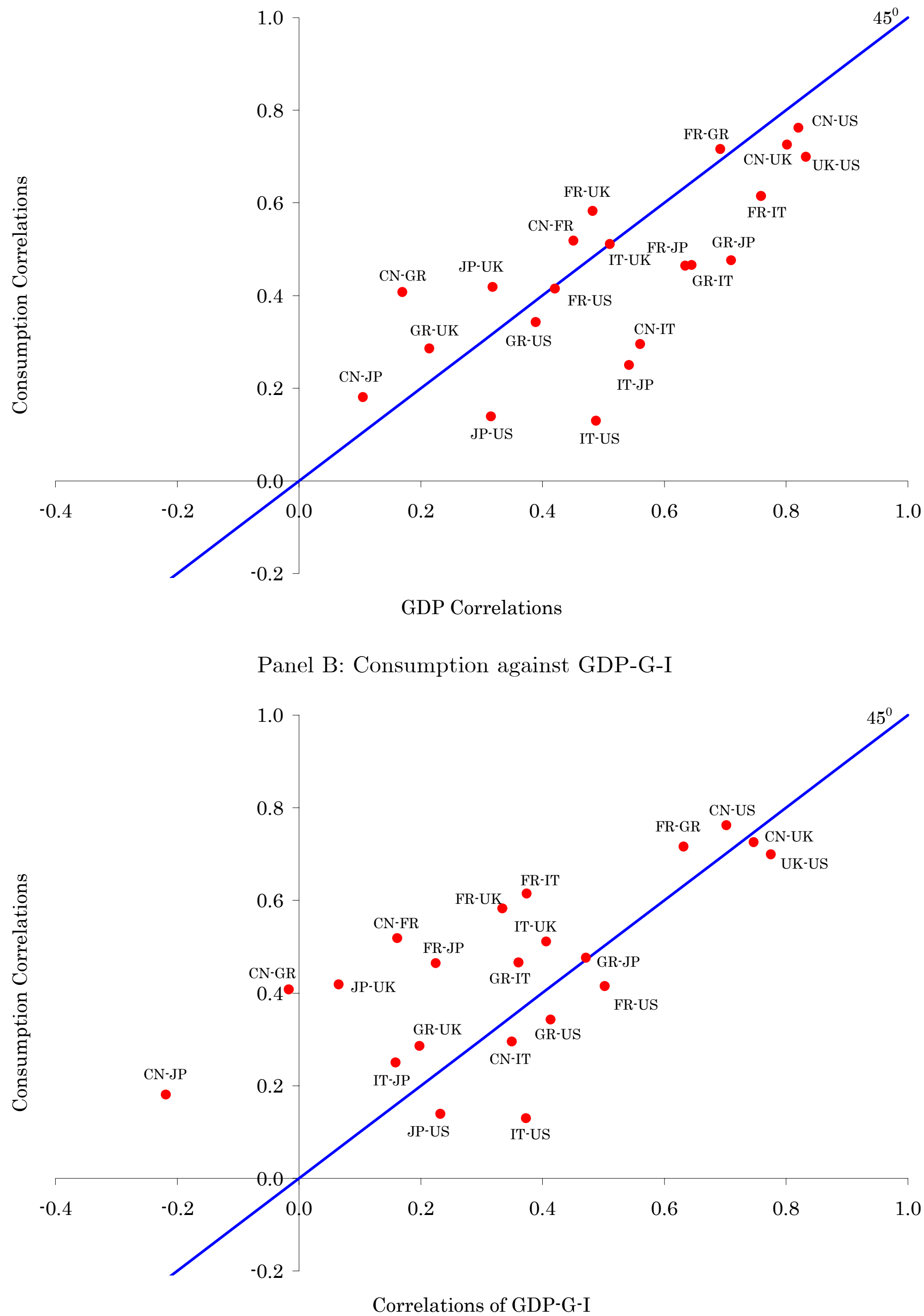
The data display a few robust features. All filtering procedures display, on average, the same ranking of correlations. The average GDP correlation is greater than the average consumption correlation. Note that although the correlations of GDP-G-I are also, on average, greater than consumption correlations, many country pairs display the opposite ranking; and there are always more pairs of countries with $\rho_C > \rho_{Y-G-I}$ than $\rho_C > \rho_Y$.

Next, consider the ranking of consumption and GDP-G-I correlations conditional on ρ_{Y-G-I}. In all cases there are more pairs of countries with $\rho_C > \rho_{Y-G-I}$ when ρ_{Y-G-I} is low (below its median) than when ρ_{Y-G-I} is high (above the median). This is also reflected in the correlation between the difference $\rho_C - \rho_{Y-G-I}$ and ρ_{Y-G-I}. This correlation is negative in all cases, suggesting that one is more likely to find ρ_C below ρ_{Y-G-I} when the latter is high.

These features can be seen in Figure 1. Panel A demonstrates the puzzle, since for most pairs of countries consumption correlations are lower than GDP correlations. Panel B shows that after subtracting investment and government spending, this is no longer the case. Furthermore, it also shows that one is more likely to find pairs of countries with ρ_C below ρ_{Y-G-I} when ρ_{Y-G-I} is high. Subsequent sections demonstrate that these features are consistent with risk sharing in a multi-country model.

Figure 1: International Co-Movements, G7 Countries, HP Filter $\lambda = 100$

Panel A: Consumption against GDP



3 The Puzzle: Basic Theory and The Literature

This section reviews the theoretical connection between cross-country consumption correlations and risk sharing; it then discusses different approaches that have been used in the literature to reduce consumption correlations in the models.

Standard international real business cycle models with complete markets often result in the following equation:

$$\frac{U_{C_1(s^t)}}{U_{C_2(s^t)}} = \kappa e_{1,2}\left(s^t\right) \qquad \forall s^t \tag{1}$$

Where U_{C_i} is the marginal utility of consumption in country i, $i = 1, 2$. $e_{1,2}$ denotes the price of consumption in country 1 in terms of country 2's consumption (i.e. the real exchange rate), s^t denotes the history of events from date zero to date t, and κ is a constant that depends on date zero marginal utilities. Equation (1) is derived by equating each country's marginal rate of substitution between date t and date zero consumptions to their relative price.

If we are willing to assume that the law of one price holds, i.e. $e_{1,2}(s^t) = 1$ for all s^t, we find that the marginal utilities of consumption are perfectly correlated. If we further assume that the utility function is additively separable, then consumptions are perfectly correlated across countries up to a first order approximation.

The driving forces behind this result are risk aversion and completeness of financial markets. By participating in the world's financial markets each country is able to hedge against output fluctuations. As a result, when one country faces an adverse shock relative to other countries, it receives transfers according to the contract implied by the relevant contingency. That is, by having access to financial markets, countries share risk and consequently consumptions across countries tend to move together, and specifically they become more correlated than output. Unfortunately, international consumption correlations in the data are not nearly as high as theory implies, and as was shown in the previous section, these correlations are typically lower than GDP correlations. As a result, the consumption correlation puzzle is often taken as evidence of a lack of risk sharing.

Generally speaking, there are two approaches for reducing theoretical consumption co-movements. The first is to deviate from the law of one price. Under this approach the real exchange rate is no longer constant, which by (1) reduces the link between consumptions across countries. The second approach is through the left hand side of (1). Non-separability in consumption and leisure, for example, reduces the link between consumptions even when the real exchange rate is constant.

As mentioned, one of the leading explanations for the puzzle is a lack of risk sharing. Consequently, the literature has investigated different environments of incomplete markets. Baxter and Crucini (1995) study the interaction of financial markets integration and the process of productivity shocks. They conclude that when either productivity shocks have low persistence or fast transmission across countries, the structure of financial markets does not matter and the puzzle remains. However, under high persistence or slow transmission, financial integration does matter. Specifically, under a non-contingent bond economy with random walks in the productivity process and no spillovers, they were able to generate GDP correlation that is greater than consumption correlation. Kollmann (1996) demonstrates that restricting assets to non-contingent bonds can substantially reduce consumption correlations relative to complete market economies. Heathcote and Perri (2002) conclude that financial autarkies generate results closer to the data relative to complete markets and non-contingent bond economies. Kehoe and Perri (2002) endogenize the incompleteness of financial markets. They conclude that this friction helps to reduce the gap between consumption and output correlations; however, they are unable to resolve the ranking of these correlations.

Another approach that uses fluctuations in the real exchange rate is to introduce trade frictions. Obstfeld and Rogoff (2000) suggest that trade frictions can account for many of the international economics puzzles, including the consumption correlation puzzle. Ravn and Mazzenga (2002) evaluate the performances of models with transportation costs. They find that the effect of transportation costs on international co-movement is limited. However, they were able to reverse the ranking of consumption and output correlations after introducing delivery lags in imports. That is, in their model, international trade is determined one quarter in advance. However, this explanation cannot account for the ranking of correlations in lower frequency data, as the lag becomes unrealistically long. Zimmermann (1997) experiments with a three-country model with transportation costs and finds no significant effect on consumption and output correlations.

The second approach to reducing consumption correlations is through the left hand side of (1). Stockman and Tesar (1995) experiment with non-separable utility functions in both leisure and non-tradable goods. They conclude that the effect of non-separability is too small to account for the puzzle[8]. They also conduct a more successful experiment by introducing taste shocks to their model. This modification, although successful in generating the correct ranking of correlations, does not shed much light on the reason for the low correlation. It simply indicates that consumptions do not move together because there is some exogenous shock that moves them this way.

To conclude, the consumption correlation puzzle is robust to many different environments and parameterizations. It has proved difficult to write down models that generate international consumption correlations smaller than output correlations.

In order to evaluate the role of complete financial markets in generating the puzzle, the next section studies the theoretical implications of risk sharing for international co-movement of consumption and output.

[8] This conclusion stems from the choice of preferences Stockman and Tesar (1995) made in their study. However, as Section 5 of the paper demonstrates, the results are sensitive to the specification of preferences.

4 A Multi-Country Endowment Model

There are two reasons for analyzing multi-country models. First, as Section 2 demonstrated, the data display some robust *cross-sectional* features. In order to produce similar results a multi-country model must be used. The second reason has to do with the ranking of correlations. As will be demonstrated below, a two-country version of the model always generates consumption correlation greater than output correlation. Adding countries can invert this ranking for some pairs of countries.

Since my objective is to study the implications of risk sharing, the theoretical model needs to incorporate complete financial markets. However, as explained in the previous section, generally some friction is needed in order to reduce international consumption correlations. I choose to use trade costs, although any other friction would probably work just as well, with the exception, of course, of incomplete markets. Appendix 1 provides some empirical evidence for the relevance of trade costs, or more precisely - distance, to the international co-movement of consumption.

4.1 The Model

4.1.1 The Environment

There are N countries, each populated with a large number of infinitely lived individuals. There is a single consumption good. In each period individuals are endowed with some level of the good. The endowment process is random and is assumed to follow a covariance stationary process. Endowments in the initial period (period zero) take their deterministic level with certainty. I further assume that countries start with no international assets or liabilities. Individuals in all countries have the same subjective discount factor and time separable utility; other individual characteristics may differ *across* countries. All countries are engaged in international trade. Transferring goods across borders is costly. Financial markets are complete.

The purpose of abstracting from multiple goods and production is to suppress any trade that is motivated by a love of variety and by relative advantage. In the model trade is motivated by risk sharing alone.

Since there are no distortions, the model can be solved using the social planner's problem. The advantage of doing so is that the solution of the model is characterized by a system of static equations. As a result we can easily obtain a closed form solution after taking first order approximation.

The planner's objective function is given by:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left\{ \sum_{i=1}^{N} \lambda_i n_i U \left[C_i \left(s^t \right) \right] \right\} \pi \left(s^t \right) \tag{2}$$

where β is the subjective discount factor, λ_i is the welfare weight of country i, n_i denotes the population mass of country i, C_i is *per-capita* consumption of individuals in country i, s^t is the history of events from period zero to date t, and $\pi(s^t)$ is its unconditional probability.

The resource constraint of country i after history s^t is given by:

$$n_i C_i\left(s^t\right) + n_i \sum_{j\neq i} X_{ij}\left(s^t\right) = n_i Y_i\left(s^t\right) + \sum_{j\neq i}\left\{n_j X_{ji}\left(s^t\right) - \tau_{ji}\Psi\left[n_j X_{ji}\left(s^t\right)\right]\right\} \tag{3}$$

where Y_i denotes *per-capita* endowment of country i. X_{ij} represents *per-capita* exports from country i to country j (per-capita in terms of the exporting country), and $\tau_{ij}\Psi(\cdot)$ is its related trade cost. I restrict X to take non-negative values. The τ_{ij}s are also non-negative parameters such that $\tau_{ij} = \tau_{ji}$. Finally, I assume that the function $\Psi(\cdot)$ satisfies:

$$\Psi : \mathbb{R}_+ \rightarrow \mathbb{R}_+,\ \Psi(0) = 0,\ \Psi'(\cdot) \geq 0,\ \Psi'(0) = 0,\ \Psi''(\cdot) > 0 \tag{4}$$

In the deterministic version of the model endowments are constant, and since the discount factors are the same across countries and initial assets are zero, the deterministic competitive equilibrium results in no trade. The assumption that $\Psi'(0) = 0$ is necessary in order to be able to approximate the solution around the deterministic competitive equilibrium.

From this point on, I will suppress the history notation.

4.1.2 Solution of The Model

First Order Conditions, and Direction of Trade: Combining the first order conditions with respect to consumption and exports, we get:

$$\lambda_j U_{cjt}\left[1 - \tau_{ij}\Psi'\left(n_i X_{ijt}\right)\right] \leq \lambda_i U_{cit} \qquad X_{ijt} \geq 0 \text{ w.c.s.} \tag{5}$$

Conditions (5) and (3) characterize the solution of the model.

Condition (5) has numerous implications for the direction of trade. First, due to the trade costs and the one-good nature of the model, trade always goes in one direction between any pair of countries for any given history.

Lemma 1 *If* $X_{ijt} > 0$ *then* $X_{jit} = 0$
Proof. *Suppose, by contradiction, that* $X_{ijt} > 0$ *and* $X_{jit} > 0$. *Using (5) we get that:*

$$1 - \tau_{ij}\Psi'\left(n_i X_{ijt}\right) = \frac{\lambda_i U_{cit}}{\lambda_j U_{cjt}} > 0, \text{ and} \tag{6a}$$

$$1 - \tau_{ji}\Psi'\left(n_j X_{jit}\right) = \frac{\lambda_j U_{cjt}}{\lambda_i U_{cit}} > 0 \tag{6b}$$

which implies:

$$\left[1 - \tau_{ij}\Psi'\left(n_i X_{ijt}\right)\right]\left[1 - \tau_{ji}\Psi'\left(n_j X_{jit}\right)\right] = 1 \tag{7}$$

However, since $\tau_{ij}\Psi'\left(n_i X_{ij}\right) > 0$ *for all strictly positive* X_{ij}*'s it follows that both* $\left[1 - \tau_{ij}\Psi'\left(n_i X_{ijt}\right)\right]$ *and* $\left[1 - \tau_{ji}\Psi'\left(n_j X_{jit}\right)\right]$ *must be negative in order for (7) to hold, but this contradicts (6). Therefore, if* $X_{ijt} > 0$ *then it must be the case that* $X_{jit} = 0$. ■

Second, (5) implies that in equilibrium there is no circular trade. That is, if country A exports to B and B exports to C, then C does not export to A. However, since some pairs of countries may not trade in equilibrium, it will be useful to define an indicator function that will arbitrarily identify one of them as an exporter. Define:

$$\Upsilon_{ijt} = \begin{cases} 1 & \text{if } X_{ijt} > 0 \\ 1 & \text{if } X_{ijt} = X_{jit} = 0 \text{ and } i > j \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Lemma 2 *If $\Upsilon_{ijt} = \Upsilon_{jkt} = ... = \Upsilon_{xyt} = \Upsilon_{yzt} = 1$ then $X_{zit} = 0$*
Proof. *Suppose, by contradiction, that $X_{zit} > 0$. By (5) it follows that: $\lambda_i U_{cit} \leq \lambda_j U_{cjt} \leq ... \leq \lambda_y U_{cyt} \leq \lambda_z U_{czt}$, which means $\lambda_i U_{cit} \leq \lambda_z U_{czt}$. But if $X_{zit} > 0$ then $\lambda_z U_{czt} \leq \lambda_i U_{cit}$. The two inequalities suggest $\lambda_z U_{czt} = \lambda_i U_{cit}$, which by using (5) again implies $\Psi' (n_i X_{zit}) = 0$ and by (4) we get $X_{zit} = 0$. This contradicts the initial assumption.* ■

Finally, (5) implies a specific pattern of trade. In any given state of nature there exists exactly one country that exports to $N-1$ countries; one country that exports to $N-2$ countries; one country that exports to $N-3$, and so on. Recall, however, that if a pair of countries do not trade, one of them is arbitrarily defined to be an exporter; this is spelled out in (8). The result is formalized in the following proposition.

Proposition 3 *Let Z_N denote the set of countries in the world economy, and let N be the number of elements in Z_N. Then, for each integer m, $m \in \{0, 1, ..., N-1\}$, $\exists$ a unique $\widetilde{z} \in Z_N$ such that:*

$$\sum_{z \in Z_N,\ z \neq \widetilde{z}} \Upsilon_{\widetilde{z}zt} = m$$

Proof. *See appendix 2* ■

Note that (5) defines $N(N-1)$ inequality conditions; Proposition 3 determines which combinations of these conditions can hold with equality in equilibrium.

Welfare Weights: As mentioned, the deterministic competitive equilibrium results in no trade. This allows us to pin down a ratio of welfare weights that is consistent with a competitive equilibrium. Using (5) and (4) it follows that:

$$\frac{\lambda_i}{\lambda_j} = \frac{U_{cj}\left(\overline{Y}_j\right)}{U_{ci}\left(\overline{Y}_i\right)} \tag{9}$$

where $\overline{Y}_i$ and $\overline{Y}_j$ are the deterministic levels of the per-capita endowments in country i and j respectively.

Log-Linearization: Let Ω_{it} denote the set of countries to which country i exports in period t. That is, $j \in \Omega_{it}$ if and only if $\Upsilon_{ijt} = 1$. Using this notation, (5) can now be written with equality for the relevant pairs of countries.

$$\lambda_j U_{cjt} \left[1 - \tau_{ij} \Psi' \left(n_i X_{ijt}\right)\right] = \lambda_i U_{cit} \qquad \forall j \in \Omega_{it} \text{ and } \forall i \tag{10}$$

Similarly, (3) can be written as:

$$n_i C_{it} + n_i \sum_{j \in \Omega_{it}} X_{ijt} = n_i Y_{it} + \sum_{j \notin \Omega_{it}} \left[n_j X_{jit} - \tau_{ji} \Psi \left(n_j X_{jit}\right)\right] \tag{11}$$

In the deterministic equilibrium, $X_{ij} = 0$ and therefore $\overline{C}_i = \overline{Y}_i$. After using (9), and linearizing (10) around the deterministic equilibrium, we get:

$$n_i X_{ijt} = \frac{\theta_i \left(\overline{Y}_i\right) \widetilde{C}_{it} - \theta_j \left(\overline{Y}_j\right) \widetilde{C}_{jt}}{\tau_{ij} \Psi'' (0)} \qquad \forall j \in \Omega_{it} \text{ and } \forall i \tag{12}$$

where tilde variables denote percentage deviations from deterministic equilibrium, and $\theta_i \left(\overline{Y}_i\right)$ is the relative risk aversion coefficient evaluated at $\overline{Y}_i$:

$$\theta_i \left(\overline{Y}_i\right) \equiv -\frac{U_{cci} \left(\overline{Y}_i\right)}{U_{ci} \left(\overline{Y}_i\right)} \overline{Y}_i \tag{13}$$

After linearizing (11) and substituting for exports using (12) we get:

$$n_i \overline{Y}_i \widetilde{C}_{it} + \sum_{j \in \Omega_{it}} \frac{\theta_i \left(\overline{Y}_i\right) \widetilde{C}_{it} - \theta_j \left(\overline{Y}_j\right) \widetilde{C}_{jt}}{\tau_{ij} \Psi'' (0)} = n_i \overline{Y}_i \widetilde{Y}_{it} + \sum_{j \notin \Omega_{it}} \frac{\theta_j \left(\overline{Y}_j\right) \widetilde{C}_{jt} - \theta_i \left(\overline{Y}_i\right) \widetilde{C}_{it}}{\tau_{ji} \Psi'' (0)} \qquad \forall i$$

Notice that the summations over exports and imports can be combined under one summation sign regardless of the direction of trade. That is, the specific combination of inequalities implied by (5) does not matter for the linearization; all combinations result in the same approximated solution. After rearranging the system boils down to:

$$\left(1 + \sum_{j \neq i} \frac{\theta_i \left(\overline{Y}_i\right)}{n_i \overline{Y}_i \tau_{ij} \Psi'' (0)}\right) \widetilde{C}_{it} - \sum_{j \neq i} \frac{\theta_j \left(\overline{Y}_j\right)}{n_i \overline{Y}_i \tau_{ij} \Psi'' (0)} \widetilde{C}_{jt} = \widetilde{Y}_{it} \qquad \forall i \tag{14}$$

This characterizes the solution for consumption given the realization of the endowment process in each period.

4.2 A Special Case: Two Countries

Consider now a special case of the model in which there exist only two countries, home and foreign. The variables of the foreign country will be denoted by an asterisk. From (14) the solution for consumptions is given by:

$$\widetilde{C}_t = \frac{\left[\theta^*\left(\overline{Y}^*\right)+\tau\Psi''(0)\,n^*\overline{Y}^*\right]\frac{n\overline{Y}}{n^*\overline{Y}^*}\widetilde{Y}_t+\theta^*\left(\overline{Y}^*\right)\widetilde{Y}_t^*}{\theta\left(\overline{Y}\right)+\theta^*\left(\overline{Y}^*\right)\frac{n\overline{Y}}{n^*\overline{Y}^*}+\tau\Psi''(0)\,n\overline{Y}} \tag{15a}$$

$$\widetilde{C}_t^* = \frac{\left[\theta\left(\overline{Y}\right)+\tau\Psi''(0)\,n\overline{Y}\right]\frac{n^*\overline{Y}^*}{n\overline{Y}}\widetilde{Y}_t^*+\theta\left(\overline{Y}\right)\widetilde{Y}_t}{\theta\left(\overline{Y}\right)\frac{n^*\overline{Y}^*}{n\overline{Y}}+\theta^*\left(\overline{Y}^*\right)+\tau\Psi''(0)\,n^*\overline{Y}^*} \tag{15b}$$

For $\tau = 0$ consumptions in both countries become:

$$\widetilde{C}_t = \theta^*\left(\overline{Y}^*\right)\cdot\frac{n\overline{Y}\widetilde{Y}_t+n^*\overline{Y}^*\widetilde{Y}_t^*}{\theta\left(\overline{Y}\right)n^*\overline{Y}^*+\theta^*\left(\overline{Y}^*\right)n\overline{Y}}$$

$$\widetilde{C}_t^* = \theta\left(\overline{Y}\right)\cdot\frac{n^*\overline{Y}^*\widetilde{Y}_t^*+n\overline{Y}\widetilde{Y}_t}{\theta\left(\overline{Y}\right)n^*\overline{Y}^*+\theta^*\left(\overline{Y}^*\right)n\overline{Y}}$$

That is:

$$\widetilde{C}_t = \widetilde{C}_t^*\cdot\frac{\theta^*\left(\overline{Y}^*\right)}{\theta\left(\overline{Y}\right)}$$

Therefore for $\tau = 0$ we get:

$$\rho_{\widetilde{C}_t,\widetilde{C}_t^*} = 1$$

Which is the standard result for frictionless environments, as was discussed in Section 3.

As $\tau \to \infty$, using L'Hospital's rule we get:

$$\lim_{\tau\to\infty}\widetilde{C}_t = \widetilde{Y}_t \qquad \lim_{\tau\to\infty}\widetilde{C}_t^* = \widetilde{Y}_t^*$$

That is, as τ increases the two countries eventually become autarkies. Therefore:

$$\lim_{\tau\to\infty}\rho_{\widetilde{C}_t,\widetilde{C}_t^*} = \rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$$

It comes, now, as no surprise that for a positive finite value of τ, consumptions are not perfectly correlated. However, consumption correlation is always greater than output correlation.

Proposition 4 *For* $\tau \in [0,\infty)$ $\rho_{\widetilde{C}_t,\widetilde{C}_t^*} \geq \rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$
Proof. *See appendix 3.* ■

Although the model used here is oversimplified, its ingredients are the building blocks of standard real business cycle models. The driving force behind the inequality of Proposition 4 exists in any model that allows for some risk sharing. It should be noted that this result is not unique to trade costs. For example, in a similar environment with no trade frictions but under exogenously imposed incomplete markets, it is straightforward to show that a two-period model generates the same inequality.

The next example demonstrates that simply by adding a country to the model the inequality of Proposition 4 can be inverted.

4.3 Correlations with Three Countries: An Example

Proposition 4 is an artifact of a two-country model. However, before adding countries it should be noted that in order to invert the ranking of correlations, some heterogeneity is necessary. This is formalized in the following proposition.

Proposition 5 *If* $\theta_i\left(\overline{Y}_i\right)=\theta_j\left(\overline{Y}_j\right)$, $n_i=n_j$, $\overline{Y}_i=\overline{Y}_j$, $\sigma_{\widetilde{Y}_{it}}=\sigma_{\widetilde{Y}_{jt}}$ *for all* i,j *and* $\tau_{ij}=\tau_{kl}$, $\rho_{\widetilde{Y}_{it},\widetilde{Y}_{jt}}=\rho_{\widetilde{Y}_{kt},\widetilde{Y}_{lt}}$ *for all* i,j,k,l *then* $\rho_{\widetilde{C}_{it},\widetilde{C}_{jt}}\geq\rho_{\widetilde{Y}_{it},\widetilde{Y}_{jt}}$ *for all* i,j.

Proof. See appendix 4. ■

Now, consider a world with three identical countries labeled A, B, and C, among which there exists a pair-wise heterogeneity in terms of trade costs and endowment correlations. I have conducted three experiments in order to determine the responsiveness of consumption correlations to changes in endowment correlations. Trade cost parameters are fixed in all experiments. Table 2 summarizes the parameter values of the different experiments. Figure 2 summarizes the results.

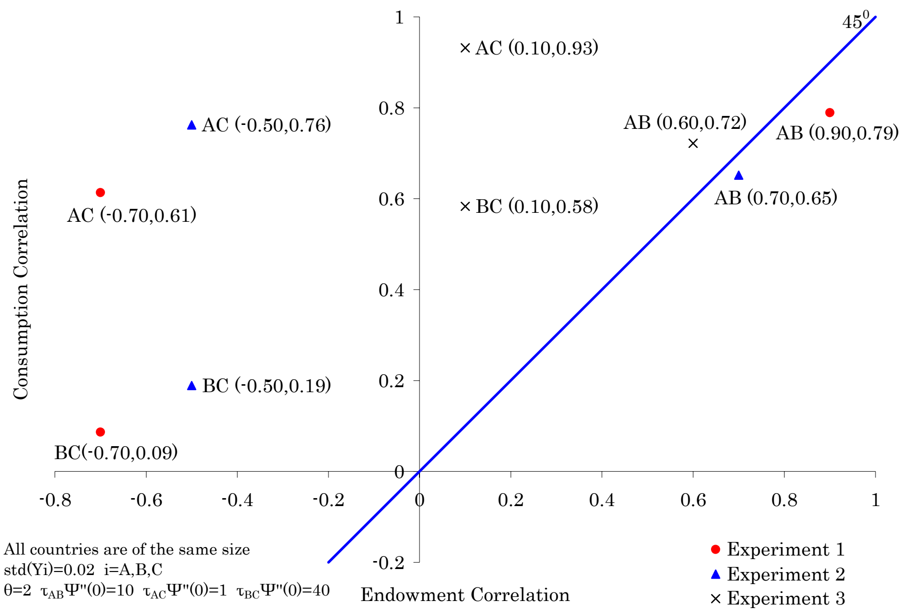
The most important result in Figure 2 is that with three countries, consumption correlation in the model can fall below output correlation (see experiments 1 and 2). Also note that this happens for the pairs of countries with high output correlation. This result is consistent with the pattern of correlations found in the data (see Table 1 and Figure 1).

Intuitively, the reason for this result is that countries A and B cannot share risk efficiently since their output is highly correlated. Introducing a third country, country C, with low endowment correlation disconnects the tight comovement of consumption in A and B as they both trade mainly with C. This is because C provides an opportunity for more efficient risk sharing. The assumption of different trade costs is important since it allows for a differentiated response from A and B to similar endowment shocks, which in turn reduces their consumption correlation.

Table 2: Three Country Model, An Example

Parameter Values			
Relative risk aversion	$\theta_A=2$	$\theta_B=2$	$\theta_C=2$
Standard deviation	$\sigma_A=0.02$	$\sigma_B=0.02$	$\sigma_C=0.02$
Trade costs	$\tau_{AB}\Psi''(0)=10$	$\tau_{AC}\Psi''(0)=1$	$\tau_{BC}\Psi''(0)=40$
Endowment Correlations			
Experiment 1	$\rho_{AB}=0.90$	$\rho_{AC}=-0.70$	$\rho_{BC}=-0.70$
Experiment 2	$\rho_{AB}=0.70$	$\rho_{AC}=-0.50$	$\rho_{BC}=-0.50$
Experiment 3	$\rho_{AB}=0.60$	$\rho_{AC}=0.10$	$\rho_{BC}=0.10$

Figure 2: Three Country Model, An Example



The last thing to point out is that as we move from experiment 1 to experiments 2 and 3, there is less pair-wise heterogeneity in terms of output correlations. Introducing the third country in experiment 3 is therefore not as helpful for sharing risk as in experiments 1 and 2. As a result, we are less likely to find consumption correlation below output correlation.

These results suggest that a multi-country model might be an important ingredient for the resolution of the consumption correlation puzzle.

5 Adding Production and Investment

Recall that the evidence presented in Section 2 distinguished between GDP and GDP net of investment (and government spending). In order to make a similar distinction in the model, production and investment must be added. This section, aside from simulating the model, demonstrates that GDP net of investment in a production economy is the comparable quantity to output in the endowment economy of Section 4.

5.1 The Environment

The environment is similar to the one of the endowment model with the following modifications: utility is a function of both consumption, C, and hours worked, H, and is denoted by $U(C, H)$. Production uses both capital, K, and hours worked, H, in a constant returns to scale production function, $F(K, H)$. Labor inputs are immobile across countries. The economy is subject to productivity shocks, A. Output in country i is given by:

$$Y_{it} = A_{it} F\left(K_{it}, H_{it}\right) \tag{16}$$

I assume that the natural logarithm of A, denoted by a, follows a covariance stationary $AR(1)$ process:

$$a_t = (I - R)\,\overline{a} + R a_{t-1} + \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} N(0, \Sigma) \qquad a_t, \varepsilon_t, \overline{a} \in \mathbb{R}^N \tag{17}$$

Notice that a_t in (17) denotes a vector of productivities in the different countries. Capital accumulation is subject to adjustment cost as in Hayashi (1982). Capital in country i accumulates over time according to:

$$K_{it+1} = (1 - \delta) K_{it} + \Phi\left(\frac{I_{it}}{K_{it}}\right) K_{it} \tag{18}$$

Where I denotes gross investment, δ is depreciation rate, and $\Phi(\cdot)$ is the capital adjustment cost function. I assume that $\Phi(\cdot)$ satisfies:

$$\Phi : \mathbb{R}_+ \to \mathbb{R}_+,\ \Psi'(\cdot) > 0,\ \Phi''(0) < 0,\ \Psi(\delta) = \delta,\ \Psi'(\delta) = 1 \tag{19}$$

Finally, the resource constraint is given by:

$$n_i C_{it} + n_i I_{it} + n_i \sum_{j \neq i} X_{ijt} = n_i Y_{it} + \sum_{j \neq i} \left[n_j X_{jit} - \tau_{ji} \Psi\left(n_j X_{jit}\right)\right] \tag{20}$$

5.1.1 Solution of The Model

First Order Conditions: From the first order conditions with respect to consumption and trade we get:

$$\lambda_j U_{cjt} \left[1 - \tau_{ij} \Psi'\left(n_i X_{ijt}\right)\right] \leq \lambda_i U_{cit} \qquad X_{ijt} \geq 0 \text{ w.c.s.} \tag{21}$$

This equation is identical to (5), and therefore all the results that followed from it regarding the pattern of trade (Lemma 1, Lemma 2, and Proposition 3) hold in this environment as well. Now, however, there are two additional optimality conditions with respect to labor and capital:

$$\begin{aligned} -\frac{U_{Hit}}{U_{Cit}} &= A_{it} F_{Hit} \\ U_{Cit} &= \beta E_t \left\{ U_{Cit+1} \frac{q_{it+1}}{q_{it}} \left[\frac{A_{it+1} F_{Kit+1}}{q_{it+1}} + 1 - \delta - \frac{1}{q_{it+1}} \cdot \frac{I_{it+1}}{K_{it+1}} + \Phi\left(\frac{I_{it+1}}{K_{it+1}}\right) \right] \right\} \end{aligned}$$

Where q is Tobin's q:

$$q_{it} \equiv \frac{1}{\Phi'\left(\frac{I_{it}}{K_{it}}\right)}$$

Note that under the assumptions specified in (19), in deterministic steady state $I = \delta K$ and Tobin's q takes a value of 1.

Welfare Weights: As before, there is no trade in the deterministic steady state, and the welfare weights are pinned down using (21):

$$\frac{\lambda_i}{\lambda_j} = \frac{U_{cj}\left(\overline{C}_j\right)}{U_{ci}\left(\overline{C}_i\right)} \tag{22}$$

Where $\overline{C}_i = \overline{Y}_i - \overline{I}_i$ is country i's consumption level in the deterministic steady state.

Log-Linearization: As before, let Ω_{it} denote the set of countries to which country i exports in period t. That is, $j \in \Omega_{it}$ if and only if $\Upsilon_{ijt} > 0$ (see equation (8)).

Linearization of (21) now yields:

$$\begin{aligned} n_i X_{ijt} \quad = \quad & \frac{\theta_i\left(\overline{C}_i, \overline{H}_i\right)\widetilde{C}_{it} - \theta_j\left(\overline{C}_j, \overline{H}_j\right)\widetilde{C}_{jt}}{\tau_{ij}\Psi''(0)} + \\ & \frac{1}{\tau_{ij}\Psi''(0)}\left\{\frac{U_{chj}\left(\overline{C}_j, \overline{H}_j\right)}{U_{cj}\left(\overline{C}_j, \overline{H}_j\right)}\overline{H}_j\widetilde{H}_{jt} - \frac{U_{chi}\left(\overline{C}_i, \overline{H}_i\right)}{U_{ci}\left(\overline{C}_i, \overline{H}_i\right)}\overline{H}_i\widetilde{H}_{it}\right\} \qquad \forall j \in \Omega_{it} \text{ and } \forall i \end{aligned} \tag{23}$$

where tilde variables denote percentage deviations from deterministic steady state, upper barred variables denote deterministic steady state quantities, and $\theta\left(\overline{C}, \overline{H}\right)$ is defined by:

$$\theta_i\left(\overline{C}_i, \overline{H}_i\right) \equiv -\frac{U_{cci}\left(\overline{C}_i, \overline{H}_i\right)}{U_{ci}\left(\overline{C}_i, \overline{H}_i\right)}\overline{C}_i$$

Linearizing the resource constraint and substituting for exports using (23) gives:

$$\begin{aligned} & \left(1 + \sum_{j\neq i}\frac{\theta_i\left(\overline{Y}_i - \overline{I}_i, \overline{H}_i\right)}{n_i\left(\overline{Y}_i - \overline{I}_i\right)\tau_{ij}\Psi''(0)}\right)\widetilde{C}_{it} - \sum_{j\neq i}\frac{\theta_j\left(\overline{Y}_j - \overline{I}_j, \overline{H}_j\right)}{n_i\left(\overline{Y}_i - \overline{I}_i\right)\tau_{ij}\Psi''(0)}\widetilde{C}_{jt} \\ & + \sum_{j\neq i}\frac{1}{n_i\left(\overline{Y}_i - \overline{I}_i\right)\tau_{ij}\Psi''(0)}\left[\frac{U_{chj}}{U_{cj}}\overline{H}_j\widetilde{H}_{jt} - \frac{U_{chi}}{U_{ci}}\overline{H}_i\widetilde{H}_{it}\right] \\ = \quad & \frac{\overline{Y}_i}{\overline{Y}_i - \overline{I}_i}\widetilde{Y}_{it} - \frac{\overline{I}_i}{\overline{Y}_i - \overline{I}_i}\widetilde{I}_{it} = \widetilde{Y_{it} - I_{it}} \qquad \forall i \end{aligned} \tag{24}$$

The importance of equation (24) is that it gives a theoretical support for subtracting investment from GDP in the empirical analysis. To see that, compare (24) with the equivalent equation in the endowment model, equation (14). Since the endowment economy does not have labor, ignore the term in the second line of (24), or alternatively assume that utility is additively separable so this term is zeroed out since $U_{ch} = 0$ in that case. Notice that wherever GDP appears in (14), equation (24) has GDP net of investment instead. Recall that equation (14) describes how countries share risk given the realization of endowments.

Equivalently, in the production economy, (24) describes how countries share risk given GDP net of investment.

Also note that if $U_{ch} = 0$, propositions 4 and 5 still hold in the production version of the model since these propositions rely entirely on (14). The only change in the propositions and the proofs would be to replace Y with $Y - I$. That is, under additively separable utility function a two-country version of the model always generates consumption correlation greater than the correlation of GDP net of investment. However, this ranking can be inverted simply by adding (heterogeneous) countries to the model.

Functional Forms: For the benchmark simulation I will assume a standard utility function:

$$U(C,H) = \frac{\left[C^{\eta}(1-H)^{1-\eta}\right]^{1-\theta} - 1}{1-\theta}$$

I will also experiment with GHH preferences:

$$U(C,H) = \frac{\left[C - \xi H^{\nu}\right]^{1-\theta} - 1}{1-\theta}$$

Under these preferences labor supply is only a function of current real wage; changes in wealth do not affect it[9]. This property will generate better inputs and GDP correlations across countries relative to the co-movement implied by standard preferences. Furthermore, given the parameter values used in the simulations, the GHH preferences exhibit a greater degree of non-separability; this in turn will prove to be important for the evaluation of the role of consumption co-movement in generating the consumption correlation puzzle.

Finally, technology is given by a standard Cobb-Douglas production function:

$$F(K,H) = K^{\alpha}H^{1-\alpha}$$

5.2 Calibration

The model is calibrated for the G7 countries using annual data from 1970 to 2002. The sample period was chosen based on availability of data for all countries. Some of the parameters are chosen based on commonly used values in the literature (α, β, δ, and θ), while others are based on the data and model's steady-state relationships. Parameter values are the same for all countries except for size, trade costs, and the productivity process. Table 3 presents their values. Estimated values of the productivity process are in the data appendix (Appendix 5).

[9] See Greenwood, Hercowitz, and Huffman (1988) and Correia, Neves, and Rebelo (1995) for a discussion about the properties of this function.

Table 3: Calibration of Parameter Values

Preferences							
Discount factor, β	0.96						
Risk aversion, θ	2						
Standard preferences, η	0.34						
GHH preferences, ν	1.68						
	CN	FR	GR	IT	JP	UK	US
GHH preferences, ξ	2.29	2.07	1.99	1.98	2.02	1.95	2.71
Technology							
Capital income share, α	0.36						
Depreciation rate, δ	0.10						
SS hours worked, $\overline{H}$	0.31						
Country size	CN	FR	GR	IT	JP	UK	US
Population, n	0.11	0.23	0.33	0.23	0.49	0.24	1.00
SS per capita GDP, $\overline{Y}$	0.85	0.76	0.73	0.73	0.75	0.72	1.00
Trade costs							
Proportionality factor, γ	4.9						
Distance (1000km)		CN	FR	GR	IT	JP	UK
	FR	5.64					
	GR	5.84	0.40				
	IT	6.73	1.12	1.08			
	JP	10.29	9.68	9.32	9.83		
	UK	5.35	0.34	0.51	1.45	9.53	
	US	0.74	6.16	6.39	7.22	10.88	5.89

Common parameters: The discount factor, β, is set to 0.96; depreciation rate, δ, is 0.1; capital income share, α, is 0.36; and the utility parameter, θ, is 2. These values are commonly used in the literature. See for example, Kydland and Prescott (1982)[10], Backus, Kehoe, and Kydland (1992), Heathcote and Perri (2002), and Kehoe and Perri (2002).

Steady state level of hours worked is set to 0.31 for all countries. This value is based on a weighted average of hours worked per employee in the sample period and it assumes a time endowment of 16 hours per day.

The consumption share parameter in the standard utility function, η, was calculated using the parameters above and the steady state version of the first order condition with respect to hours. η takes a value of 0.34.

For the GHH preferences, ν governs the intertemporal elasticity of substitution of labor supply. Its value was chosen to match the corresponding elasticity

[10] Kydland and Prescott (1982) use $\theta = 1.5$.

evaluated in steady state under standard preferences. ν takes the value 1.68[11]. Given ν, ξ is pinned down using the steady state version of the first order condition with respect to hours. ξ is country specific, as it depends on steady state level of per capita GDP.

Country size: Two parameters reflect size: population (n_i), and steady-state level of GDP per capita ($\overline{Y}_i$). Both are normalized to 1 for the US. For other countries these parameters are measured as the sample averages of each country relative to the US.

Trade cost: I assume that trade cost parameters are proportional to distance, that is:

$$\tau_{ij}\Psi''(0) = \gamma \cdot \text{distance}_{ij} \qquad \forall i, j$$

Distance is measured as the great-circle distance between capital cities[12]. The coefficient of proportionality is chosen to match the average standard deviation of net exports as a fraction of GDP when using the standard preferences specification. This value is set to 4.9.

Productivity process: Productivity shocks were measured using the Solow residuals:

$$\ln(A_{it}) = \ln(Y_{it}) - \alpha \ln(K_{it}) - (1-\alpha)\ln(H_{it}) \tag{25}$$

where Y is total GDP, K is capital stock, and H is total hours worked. The input series were constructed as follows. Total hours worked is the product of average hours worked per employee and total employment. Capital was constructed by assuming that in the initial period the world economy was in steady state, then in each period the change in capital was calculated using $\overline{I}_i \exp\left(\widetilde{I}_{it}\right) - \delta K_{it-1}$, where $\widetilde{I}_{it}$ is the log deviation of investment from its linear time trend, and $\overline{I}_i$ is its steady state level[13].

Finally, using $\ln(A_{it})$ I calculate the log deviation of productivity from a linear time trend, $\widetilde{A}_{it}$, and estimate the following unrestricted VAR:

$$\widetilde{A}_t = R\widetilde{A}_{t-1} + \varepsilon_t \quad \text{where } \varepsilon_t \overset{iid}{\sim} N(0, \Sigma) \tag{26}$$

Estimation results of (26) are presented in the data appendix (Appendix 5).

5.3 Results

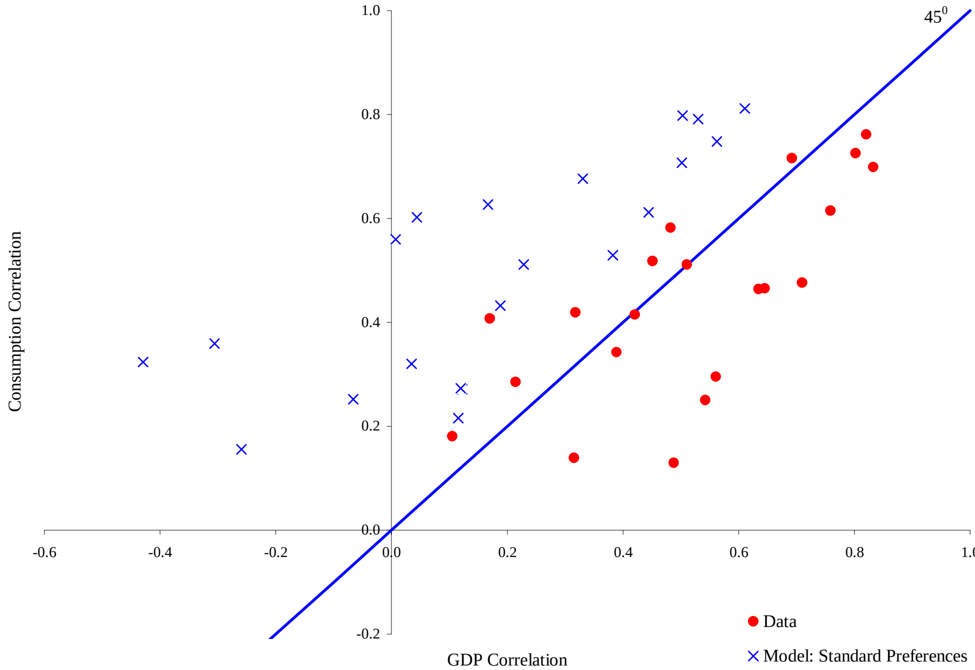
Table 4 summarizes simulation results with the two type of preferences. The table averages the statistics across countries (weighting by population). All statistics are calculated after removing HP trend using a smoothing parameter of 100. Figures in the table are averages over 10,000 simulations.

[11] This value is consistent with other studies. Greenwood Hercowitz and Huffman (1988), for example, set ν to take the value 1.7, Correia Neves and Rebelo (1995) use 1.5.

[12] For Germany I use Bonn instead of Berlin.

[13] Note that in (25), GDP and total hours are growing over time while the measure of capital is stationary by construction. This is not a problem because a time trend is removed from $\ln(A_{it})$ before estimating the time series process of productivity. The stationarity of capital affects only the coefficient of the time trend; it does not change the time series properties of the deviations of productivity from its trend.

Figure 3: Consumption vs GDP Correlations



5.3.1 The Benchmark Simulation: Standard Preferences

The standard preferences are used as a benchmark, since this utility function and similar parameter values are widely used in the literature. This subsection analyzes the results of the benchmark exercise.

Notably, the model produces realistic magnitudes of consumption correlations, 0.43 in the model versus 0.38 in the data. The correlation between the difference $\rho_C - \rho_{Y-I}$ and ρ_{Y-I} is somewhat too negative in the model, -0.78 versus -0.65 in the data. On the other hand, the model produces too low GDP correlations, 0.19 versus 0.50 in the data[14]. Therefore, although it seems that the data display features that are consistent with risk sharing, the puzzle remains, as the model produces the wrong ranking of consumption and GDP correlations. This is best seen in Figure 3.

Inspection of the results in Table 4 also reveals that the model fails to produce enough co-movement in hours and investment. This, in turn, may explain the low GDP correlations.

In the model, new capital tends to flow to the most productive location, resulting in low correlations of investment (0.18 versus 0.36 in the data).

[14] GDP correlations of this magnitude and lower are common in the literature. See for example, Backus Kehoe and Kydland (1992), Baxter and Crucini (1995), Hethcote and Perri (2002), and Kehoe and Perri (2002).

Table 4: Simulation Results, Summary Statistics*

	GDP	GDP-I**	Consumption	Investment	Hours	NX/GDP
International correlations						
Data	0.50	0.38	0.38	0.36	0.39	0.05
Standard preferences	0.19	−0.01	0.43	0.18	0.12	−0.21
GHH preferences	0.22	0.08	0.29	0.14	0.22	−0.21
$Corr\left(\rho_C - \rho_{Y-I}, \rho_{Y-I}\right)$						
Data		−0.65				
Standard preferences		−0.78				
GHH preferences		−0.61				
Standard deviation (in percents)						
Data	1.95	1.81	1.96	5.45	1.59	0.62
Standard preferences	1.89	1.28	0.58	5.45	0.96	0.63
GHH preferences	2.09	1.72	1.19	5.10	1.26	0.62
Correlation with GDP						
Data	1.00	0.87	0.87	0.88	0.77	−0.47
Standard preferences	1.00	0.81	0.90	0.87	0.99	0.53
GHH preferences	1.00	0.87	0.99	0.83	1.00	0.39
Autocorrelation						
Data	0.56	0.45	0.65	0.62	0.59	0.60
Standard preferences	0.41	0.53	0.47	0.33	0.40	0.60
GHH preferences	0.41	0.45	0.41	0.34	0.41	0.60

* Averages over 10,000 simulations, and across countries (weighted by population).

** For the actual data statistics are reported GDP-G-I.

Baxter (1995) and Kehoe and Perri (2002) suggest that the low international correlation in hours (0.12 versus 0.39 in the data) may be attributed to the complete market assumption of the model.

The reason is that a positive productivity shock in one country increases labor demand in that country and under complete markets it also induces trade from that country to others as implied by the relevant contingency. This in turn, increases consumption in other countries and reduces labor supply, resulting in negative international co-movement in hours worked. Note, however, that this effect can be reduced if labor supply does not react to consumption. One way of achieving this is by using GHH preferences, since labor supply under these preferences reacts only to fluctuations in current real wage and is not sensitive to movements in consumption[15].

[15] See Greenwood Hercowitz and Huffman (1988) and Correia Neves and Rebelo (1995).

Although these results may direct attention to the co-movement of production inputs and GDP across countries, they shed little light on the source of the puzzle. In particular, they do not help determining whether consumption co-movement in complete financial markets is the source of the puzzle, or maybe the low co-movement in production inputs and GDP generate the wrong ranking of correlations. This is because all quantities in the model are determined in a general equilibrium framework, once the model produces higher GDP correlations it is likely that consumption correlations will increase as well. The next section attempts to provide a better understanding for the role of consumption in a complete markets environment in generating the puzzle.

5.3.2 Isolating Consumption Co-movement

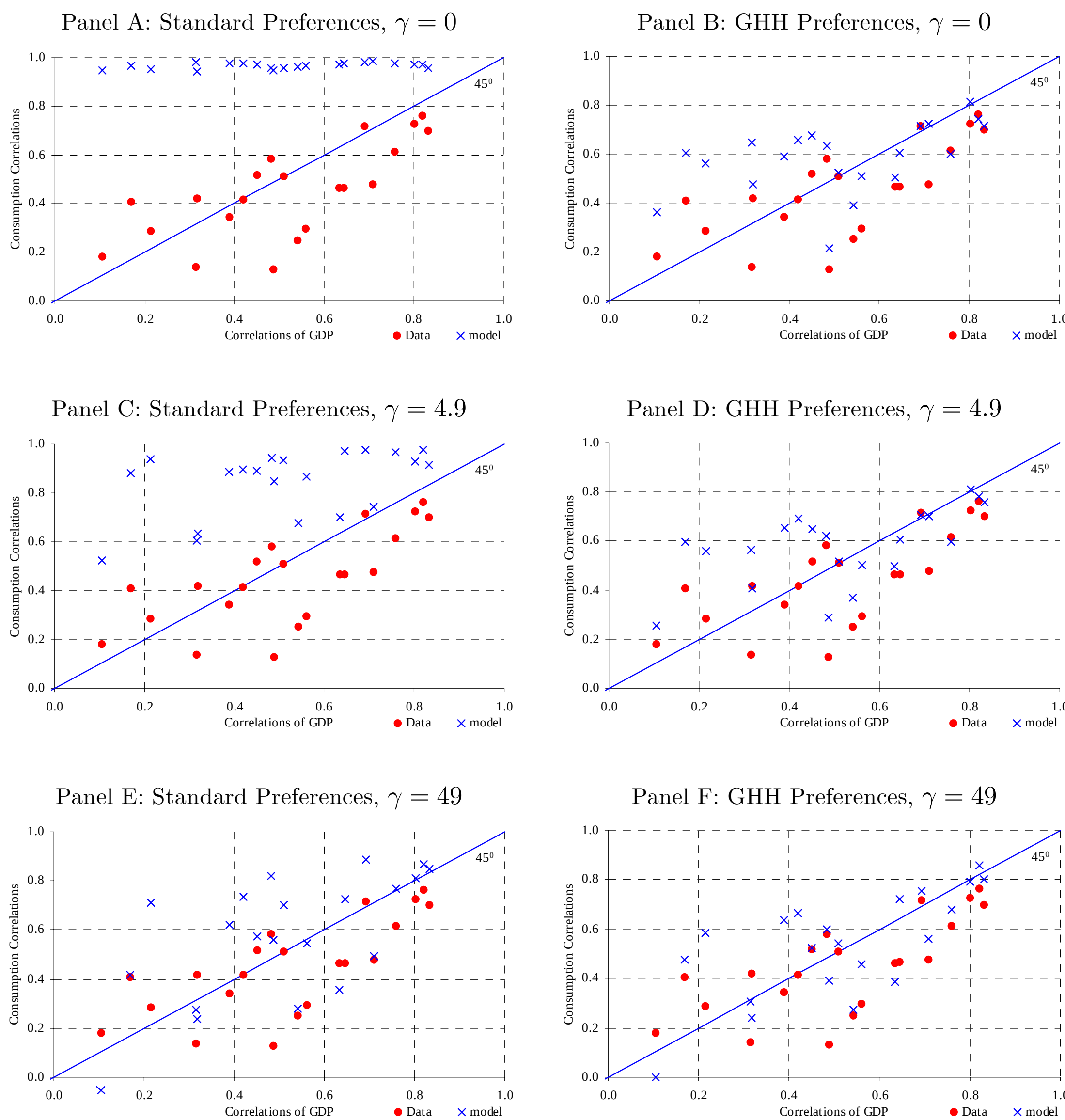
As discussed above the results of the benchmark simulation are ambiguous with regards to the forces that are responsible for the puzzle. This section attempts isolate the effects of consumption and risk sharing on international co-movements.

For that purpose, we can use equation (24). Recall that (24) is derived from the first order conditions with respect to consumption and exports together with the resource constraint, and it describes how consumption is allocated across countries taking GDP and production inputs as given. Also recall that in the model cross-country allocations are motivated solely by risk sharing. We can now use the actual data to substitute for production inputs and GDP in (24) and solve for consumption. This way consumption correlations cannot be reduced simply because the model under predicts the co-movement in output.

I conduct six experiments. The experiments use both types of preferences under three different levels of trade costs: free trade ($\gamma = 0$), "normal" trade costs where trade cost parameter is the same as in the benchmark simulation ($\gamma = 4.9$), and high trade costs ($\gamma = 49$). Table 5 reports model's statistics, and Figure 4 provides a graphical presentation of consumption and GDP correlations.

From Figure 4 it is clear that the results are sensitive to the specification of preferences. In particular what matters is the separability between consumption and hours. To see this recall that if utility is additively separable then (24) is identical to (14), and as we saw earlier, costless trade in the endowment economy results in perfect consumption co-movement across countries. Panel A in Figure 4 suggests that under the current parameterization, the standard preferences are very close to the separable case as consumptions are almost perfectly correlated across countries when trade is costless (average correlation is 0.97). Panel B, on the other hand, demonstrates that non-separability alone can significantly reduce consumption co-movement. Comparison of the coefficients of hours in (24) supports this conclusion. Under the standard preferences $\frac{U_{ch}}{U_c}\overline{H} = 0.29$ while under the GHH case the coefficient is more then 10 times bigger, $\frac{U_{ch}}{U_c}\overline{H} = 3.53$. If utility is additively separable then these coefficients are, of course, zero. Hence, GHH preferences display a greater degree of non-separability.

Figure 4: The Effects of Non-Separability and Trade Costs



It should be noted that this result is NOT in odds with the findings of Stockman and Tesar (1995) but rather complement them[16]. In their paper non-separability between consumption and hours is restricted to a Cobb-Douglas utility function, resulting in international co-movements as in Panel A.

[16]Stockman and Tesar (1995) conclude that non-separability in consumption and leisure has little effect on international consumption correlations.

Table 5: Isolating Consumption Co-Movement, Summary Statistics*

	International correlations		
	Consumption	NX/GDP	$Corr\left(\rho_C - \rho_{Y-I} < \rho_{Y-I}\right)$
Data	0.38	0.05	−0.65
Standard preferences, $\gamma = 0$	0.97	−0.26	−1.00
Standard preferences, $\gamma = 4.9$	0.81	−0.20	−0.84
Standard preferences, $\gamma = 49$	0.55	−0.04	−0.31
GHH preferences, $\gamma = 0$	0.58	−0.24	−0.82
GHH preferences, $\gamma = 4.9$	0.58	−0.21	−0.80
GHH preferences, $\gamma = 49$	0.52	−0.07	−0.51
	Standard deviations		
	Consumption	NX/GDP	$\rho_C < \rho_Y$
Data	1.96	0.62	13
Standard preferences, $\gamma = 0$	1.38	0.83	0
Standard preferences, $\gamma = 4.9$	1.43	0.55	0
Standard preferences, $\gamma = 49$	1.59	0.26	7
GHH preferences, $\gamma = 0$	1.60	0.83	8
GHH preferences, $\gamma = 4.9$	1.58	0.71	9
GHH preferences, $\gamma = 49$	1.61	0.44	11

* Averages across countries (weighted by population).

Next consider the effect of trade costs. Again, the effect turns out to be sensitive to the specification of the utility function. Under standard preferences trade costs can considerably reduce consumption co-movement across countries, this is demonstrated by comparing Panels A, C, and E in Figure 4. However, under GHH preferences consumption correlations hardly change (Panels B, D, and F). Intuitively, under separable preferences only consumption can be employed for smoothing marginal utilities, while under non-separable preferences movement in hours can be used for that purpose as well. As a result, the effect of trade costs on consumption co-movement is stronger when utility separable.

Overall, it seems that for the purpose of explaining the consumption correlation puzzle non-separability is more important than trade costs. Under GHH preferences equation (24) is able to generate the correct ranking of correlations for most pairs of countries, even in a frictionless environment. Standard preferences, on the other hand, require unrealistically high level of trade costs in order to produce similar results. Notice that under the high trade costs scenario ($\gamma = 49$), the model with standard preferences is able to invert the ranking of correlations for some pairs of countries; however, it is doing so by suppressing trade. The standard deviation of net exports, in this case, becomes less than a half of what is observed in the data (0.26 versus 0.62 in the data)[17].

[17] This result is similar to the findings of Kehoe and Perri (2002). They show that consumption correlations can be significantly reduced by endogenizing debt constraints in the

The statistics in Table 5 imply that equation (24) tends to generate excessive risk sharing across countries. This is reflected in the high level of consumption correlations, and in the correlations between the difference $\rho_C - \rho_{Y-I}$ and ρ_{Y-I} which tend to be too negative relative to the data. These results shed doubt on complete markets as being the source of the puzzle since the correct ranking of correlations can be produced even in the presence of excessive risk sharing.

5.3.3 Simulation with GHH Preferences

Since the GHH preferences produce better international co-movement in consumption, it is interesting to evaluate their performance in the full model. Table 4 presents the simulation results.

Not surprisingly, the simulation with GHH preferences produce lower consumption correlations. In fact the average correlation is now lower than the data, 0.29 versus 0.38. GDP correlations, although somewhat higher relative to the standard preferences case (0.22 versus 0.19), are still too low compared to the data, 0.22 versus 0.50. Taken together, the puzzle remains, the model still produces consumption correlations that exceed GDP correlations although the gap between them is now much smaller (0.07 versus 0.34 in the standard preferences simulation). It seems that the main problem is the low GDP correlations rather than consumption co-movements.

The GHH preferences also improve the predictions of the model with regard to hours co-movement and the volatility of hours and consumption. These results are related to the determination of labor supply. Letting w denote real wage, labor supply in the model is given by:

$$H = \left(\frac{w}{\nu\xi}\right)^{\frac{1}{\nu-1}} \qquad \text{GHH} \tag{27a}$$

$$H = 1 - \frac{1-\eta}{\eta} \cdot \frac{C}{w} \qquad \text{Standard Preferences} \tag{27b}$$

Labor supply under GHH preferences reacts only to current real wage, with standard preferences, on the other hand, labor supply moves also with consumption. As explained earlier, a positive productivity shock in one country increases labor demand in that country and consumption in others because of the complete markets assumption. This, in turn, reduces labor supply in those countries when we assume standard preferences. This effect reduces hours co-movement across countries. Under GHH preferences labor supply does not react to consumption, resulting in higher co-movement in hours.

Table 4 also demonstrates that GHH preferences induce higher volatility in hours and consumption relative to the standard preferences case. Notice, however, that the standard deviations of both variables are still too low relative to the data. Intuitively, within each country, a positive productivity shock increases labor demand and consumption. Under standard preferences, the

international credit markets. However, as a side product, their model also produces trade volatility that is significantly lower than the data.

increase in consumption moderates the increase in hours through the effect on labor supply. In the GHH case this secondary effect does not exist, resulting in higher volatility in hours. The volatility in hours is translated into higher volatility in consumption as consumers attempt to smooth their marginal utility of consumption ($U_{ch} > 0$).

6 Conclusions

This paper has attempted to evaluate the role of risk sharing in generating the consumption correlation puzzle. For that purpose I first study the implications of risk sharing on the international correlations of consumption and output in a simple endowment model with complete markets and trade costs. In the model trade is motivated solely by risk sharing. This analysis has generated a few qualitative results. First, a two-country version of the model always generates consumption correlations that exceed output correlations. However, this ranking can be inverted simply by adding countries to the model. Second, symmetry across countries also imposes consumption correlations that exceed output correlations. Lastly, a multi-country version of the model with heterogeneity tends to generate consumption correlations that are lower than output correlations when the latter are high.

Next, in order to compare the predictions of the model with the data, the model is extended to include production. I find that if utility is additively separable in consumption and leisure, then the results of the endowment economy still apply; however the comparable quantity to endowment in the production economy is GDP-G-I, not GDP. The paper then shows that, qualitatively, the cross-sectional pattern of correlations of the G7 countries is consistent with the prediction of the model. Specifically, in the data consumption correlations tend to fall below the correlations of GDP-G-I when the latter are high.

Simulations of the model have generated ambiguous quantitative results. The model is able to produce realistic co-movement in consumption in spite of the complete markets assumption; however, the puzzle remains as the model generates consumption correlations that are greater than GDP correlations due to low co-movement in GDP. The general equilibrium nature of the model makes it hard to identify the source of problem, since once the model is able to produce the correct GDP correlations it will probably generate higher consumption correlations as well.

Finally, the paper tries to isolate the effect of consumption and risk sharing on international co-movement. I use the actual data of GDP and production inputs in the equation that governs the allocation of consumption across countries, and solve for consumption. This way any forces that generate counterfactual co-movements in GDP are neutralized. The results are sensitive to the specification of preferences. GHH preferences are able to generate the correct ranking of correlations for most pairs of countries even in a frictionless environment; standard preferences (Cobb-Douglas in consumption and leisure), on the other hand, require unrealistically high level of trade costs in order to generate similar results.

The key feature that governs the difference in results is the degree of separability between consumption and leisure. Standard preferences are close to a separable case; therefore smoothing marginal utilities is achieved mainly by adjusting consumption, this in turn results in high consumption co-movement. Under GHH preferences, utility displays a higher degree of non-separability, suggesting that labor is used for smoothing marginal utilities in addition to consumption. As a result consumption movement across countries is less synchronized.

It should be stressed that GHH preferences are able to generate the correct ranking of correlations even though the experiments resulted in excessive risk sharing relative to the data. This is best seen in the high consumption correlations and in the correlations between $\rho_C - \rho_{Y-I}$ and ρ_{Y-I} that are too negative. This result casts doubt on excessive risk sharing as being the source of the puzzle, since it demonstrates that models can produce the correct international co-movement even in a frictionless environment.

To conclude, the question whether excessive risk sharing in the model is the main force behind the puzzle boils down to an empirical one. If preferences are proved to be close to the separable case then it is likely that the answer is positive. However, if preferences display sufficient non-separability then financial markets integration plays little role in generating the puzzle and attention should be directed towards the co-movement of GDP and production inputs.

7 Appendix 1: Consumption Correlations and Trade Costs

The objective of this appendix is to provide some empirical evidence for the relevance of trade costs to the international co-movements of consumption. By trade costs I mean any cost resulting from transferring goods across borders. These may include, among other things, transportation costs, insurance, tariffs, information costs, and costs associated with the use of different currencies. In a recent paper, Anderson and van Wincoop (2004) survey the measurement of trade costs. They conclude that these costs are large (up to 170 percent) even for integrated and developed economies.

Although trade costs may be large, it is not obvious that they are relevant to international consumption co-movement. It should be noted that the literature, and also this paper, has been generally unsuccessful in resolving the puzzle using this channel[18]. If trade costs are important, then one would expect that the greater the costs, the lower consumption correlations would be. In terms of equation (1), no cost implies that the law of one price holds and therefore consumption correlation should be high. As trade costs increase, risk sharing becomes more costly, and consumption movement across countries, in turn, becomes disconnected. This suggests a negative coefficient in a cross-sectional regression of consumption correlations on trade costs.

[18] See, for example, Backus, Kehoe and Kydland (1992), Zimmermann (1997), and Ravn and Mazzenga (2002).

Table A1.1: Regression Results
Dependant Variable $Corr(C_i, C_j)$

Estimation method		OLS	OLS	OLS	IV*	IV*
Specification		1	2	3	2	3
Constant	coef.	0.610	0.408	0.281	0.348	0.347
	std.	0.063	0.084	0.092	0.124	0.109
	prob.	0.000	0.000	0.007	0.011	0.005
Corr. GDP-G-I	coef.		0.405		0.525	
	std.		0.132		0.224	
	prob.		0.007		0.031	
Corr. GDP	coef.			0.535		0.428
	std.			0.129		0.161
	prob.			0.001		0.016
Distance	coef.	−0.030	−0.018	−0.020	−0.015	−0.022
	std.	0.010	0.009	0.007	0.010	0.008
	prob.	0.006	0.051	0.014	0.164	0.010
R^2		0.337	0.565	0.661	0.545	0.648
No. of obs.		21	21	21	21	21

* Instrumental variable: Cross-country correlations of Solow residuals (after removing HP trend)

As Anderson and van Wincoop (2004) indicate, empirically, distance is found to be an important factor in the determination of trade costs. Since data on bilateral trade costs are not readily available, I use a measure of distance between pairs of countries as a proxy[19].

Table A1.1 summarizes the estimation results of different regressions of the following form:

$$Corr\left(C_i, C_j\right) = \beta_0 + \beta_1 \text{distance}_{i,j} + \beta_2 Corr\left(Y_i, Y_j\right) + \varepsilon_{i,j} \qquad i, j \in G7 \quad (28)$$

where $Corr\left(C_i, C_j\right)$ is the correlation of consumption between countries i and j, $Corr\left(Y_i, Y_j\right)$ stands for output correlations where I experiment with both GDP and GDP-G-I[20]. The distance variable is the great circle distance between capital cities[21] expressed in thousands of kilometers. The regression is estimated for the G7 countries (21 pairs of countries).

Specification 1 regresses consumption correlations on distance and a constant ($\beta_2 = 0$). The distance coefficient is negative and significantly different from zero.

One might argue that consumption correlation is positively related to output correlation, and that the farther countries are from each other, the lower

[19] If one strongly believes that distance is unrelated to trade costs, then the empirical results of this appendix should be merely taken as evidence for the relevance of distance to consumption co-movement. Recall that in the calibration I used distance instead of trade costs. This, in turn, provides better comparability between the empirical evidence of this appendix and the model, and does not require the interpretation of distance as a proxy for trade costs.

[20] These are the same correlations as in Table 1 under the HP filtering with $\lambda = 100$.

[21] For Germany I use Bonn instead of Berlin.

their output correlation will be. Therefore, the negative distance coefficient, in specification 1 might merely reflect the fall in output correlation as distance increases. The next two specifications control for this problem. Specification 2 uses GDP net of investment and government spending while 3 uses GDP. Both distance coefficients are negative and significantly different from zero. Also, as expected, the coefficients of output correlations are positive and significant.

Using output correlations as a regressor might create endogeneity problems in an OLS estimation[22]. Equation (28) is also estimated using an IV estimation, where I use the cross-country correlations of Solow residuals as an instrument[23]. The last two columns of Table A1.1 report the results of the estimation. Again, both distance coefficients are negative, although this time the distance coefficient is insignificant under specification 2.

To conclude, the negative relationship between consumption correlation and distance seems to be a robust feature of the data. This suggests that trade costs may be a relevant factor in the determination of cross-country consumption comovement.

8 Appendix 2: Proof of Proposition 3

This appendix proves proposition 3. However, I will first review some of the notation and prove an additional lemma.

Recall that Z_N denotes the set of all countries in the world economy. We have already defined an indicator function for the direction of trade after an arbitrary history of events:

$$\Upsilon_{ijt} = \begin{cases} 1 & \text{if } X_{ijt} > 0 \\ 1 & \text{if } X_{ijt} = X_{jit} = 0 \text{ and } i > j \\ 0 & \text{otherwise} \end{cases}$$

For simplicity, I will drop the time index from this point onward.

Let $Z_\#$ denote an arbitrary subset of Z_N with at least two elements, where $\#$ denotes the number of elements in $Z_\#$, hence $\# \in \{2, 3, ..., N\}$.

Lemma 6 *If for an arbitrary* $\widetilde{N}$, $2 < \widetilde{N} < N$, *and for each integer* m, $m \in \left\{0, 1, 2, ..., \widetilde{N} - 1\right\}$, $\exists$ *a unique* $\widetilde{z} \in Z_{\widetilde{N}}$ *such that:*

$$\sum_{z \in Z_{\widetilde{N}},\ z \neq \widetilde{z}} \Upsilon_{\widetilde{z}z} = m$$

Then for any two countries A and B in $Z_{\widetilde{N}}$ *that satisfy:* $\sum_{z \in Z_{\widetilde{N}},\ z \neq A} \Upsilon_{Az} > \sum_{z \in Z_{\widetilde{N}},\ z \neq B} \Upsilon_{Bz}$, *it must be true that:*

$$\Upsilon_{AB} = 1$$

[22] If we assume that $Corr(Y_i, Y_j)$ is positively correlated with $\varepsilon_{i,j}$ and negatively correlated with distance, then the OLS estimator of β_1 is biased upward. That is, it is likely that the true coefficient is more negative than its OLS estimates in Table A1.1.

[23] See section 5.2 and the data appendix for the exact derivation of the Solow residuals.

Proof. *Since by assumption each country exports to a different number of countries in $Z_{\widetilde{N}}$; we can index countries using that number. In particular, assign the index i to the country that exports to $i-1$ countries:* $\sum\limits_{z \in Z_{\widetilde{N}},\ z \neq i} \Upsilon_{iz} = i-1$.

Note that for country 1: $\Upsilon_{1z} = 0\ \forall 1 < z \leq \widetilde{N},\ z \in Z_{\widetilde{N}}$, *by Lemma 1 this specifically implies* $\Upsilon_{21} = 1$.

Now assume that an arbitrary country, in $Z_{\widetilde{N}}$, *indexed by* k *satisfies:* $\Upsilon_{kz} = 1\ \forall 1 \leq z \leq k-1,\ z \in Z_{\widetilde{N}}$. *Consider, country* $k+1$. *We know that by assumption and by Lemma 1* $\Upsilon_{k+1k} = 1$. *Notice that by Lemma 2* $\nexists z < k,\ z \in Z_{\widetilde{N}}$ *such that* $\Upsilon_{zk+1} = 1$ *since that would create a circular trade. Therefore, by Lemma 1* $\Upsilon_{k+1z} = 1\ \forall 1 \leq z \leq k,\ z \in Z_{\widetilde{N}}$.

We can now conclude by induction that if $A, B \in Z_{\widetilde{N}}$ *and* $\sum\limits_{z \in Z_{\widetilde{N}},\ z \neq A} \Upsilon_{Az} > \sum\limits_{z \in Z_{\widetilde{N}},\ z \neq B} \Upsilon_{Bz}$ *then by the indexing convention* $A > B$, *and therefore* $\Upsilon_{AB} = 1$.
■

We are now ready for the proof of proposition 3. The proof is by induction on the number of countries, #.

Proof. For $\# = 2$ let $Z_2 = \{i, j\}$. By Lemma 1 we know that either:

1. $\Upsilon_{ij} = 1$ and $\Upsilon_{ji} = 0$, or
2. $\Upsilon_{ij} = 0$ and $\Upsilon_{ji} = 1$

Therefore, for $\# = 2$, for each integer m, $m \in \{0, 1\}$, $\exists$ a unique $z \in Z_2$ such that:

$$\sum_{z \in Z_2,\ z \neq \widetilde{z}} \Upsilon_{\widetilde{z}z} = m$$

Now, assume that for an arbitrary $\# = \widetilde{N}$, $2 < \widetilde{N} < N$, for each integer m, $m \in \left\{0, 1, 2, ..., \widetilde{N}-1\right\}$, $\exists$ a unique $\widetilde{z} \in Z_{\widetilde{N}}$ such that:

$$\sum_{z \in Z_{\widetilde{N}},\ z \neq \widetilde{z}} \Upsilon_{\widetilde{z}z} = m$$

I will refer to this assumption as the "induction assumption".

Consider $\# = \widetilde{N}+1$ such that $Z_{\widetilde{N}+1} = Z_{\widetilde{N}} \cup \widehat{z}$, $\widehat{z} \in Z_N$, $\widehat{z} \notin Z_{\widetilde{N}}$.

- Case 1: $\sum\limits_{z \in Z_{\widetilde{N}+1},\ z \neq \widehat{z}} \Upsilon_{\widehat{z}z} = 0$

  In this case, by Lemma 1, $\Upsilon_{z\widehat{z}} = 1$ for all $z \in Z_{\widetilde{N}+1}$, $z \neq \widehat{z}$, therefore using the assumption on $Z_{\widetilde{N}}$ we get that for each integer m, $m \in \left\{0, 1, 2, ..., \widetilde{N}\right\}$, $\exists$ a unique $\widetilde{z} \in Z_{\widetilde{N}+1}$ such that:

$$\sum_{z \in Z_{\widetilde{N}+1},\ z \neq \widetilde{z}} \Upsilon_{\widetilde{z}z} = m$$

- Case 2: $\sum_{z \in Z_{\widetilde{N}+1},\ z \neq \widehat{z}} \Upsilon_{\widehat{z}z} = \widetilde{N}$

  In this case, by Lemma 1, $\Upsilon_{\widehat{z}z} = 1$ and $\Upsilon_{z\widehat{z}} = 0$ for all $z \in Z_{\widetilde{N}+1}$, $z \neq \widehat{z}$; therefore using the assumption on $Z_{\widetilde{N}}$ we get that for each integer m, $m \in \left\{0, 1, 2, ..., \widetilde{N}\right\}$, $\exists$ a unique $\widetilde{z} \in Z_{\widetilde{N}+1}$ such that:

$$\sum_{z \in Z_{\widetilde{N}+1},\ z \neq \widetilde{z}} \Upsilon_{\widetilde{z}z} = m$$

- Case 3: $0 < \sum_{z \in Z_{\widetilde{N}+1},\ z \neq \widehat{z}} \Upsilon_{\widehat{z}z} < \widetilde{N}$

  Let $\widehat{m} \equiv \sum_{z \in Z_{\widetilde{N}+1},\ z \neq \widehat{z}} \Upsilon_{\widehat{z}z}$. Here I will use again the indexing convention of Lemma 6, under which country i satisfies $\sum_{z \in Z_{\widetilde{N}},\ z \neq i} \Upsilon_{iz} = i - 1$. Also, by the induction assumption we can employ Lemma 6 and conclude that $\Upsilon_{\widetilde{z}z^*} = 1$ for $\widetilde{z} > z^*$, $\widetilde{z}, z^* \in Z_{\widetilde{N}}$.

  Suppose by contradiction that $\Upsilon_{\widetilde{z}\widehat{z}} = 1$ for some $\widetilde{z} > \widehat{m}$, $\widetilde{z} \in Z_{\widetilde{N}}$. Since $\widehat{m} = \sum_{z \in Z_{\widetilde{N}+1},\ z \neq \widehat{z}} \Upsilon_{\widehat{z}z}$ it must be the case that $\exists z^* \leq \widehat{m}$ such that $\Upsilon_{\widehat{z}z^*} = 0$, which by Lemma 1 implies $\Upsilon_{z^*\widehat{z}} = 1$, but this contradicts Lemma 2 since by the indexing convention $\Upsilon_{\widetilde{z}z^*} = 1$.

  Therefore $\Upsilon_{\widetilde{z}\widehat{z}} = 1$ for all $\widetilde{z} > \widehat{m}$, $\widetilde{z} \in Z_{\widetilde{N}}$, but since $\sum_{z \in Z_{\widetilde{N}+1},\ z \neq \widehat{z}} \Upsilon_{\widehat{z}z} = \widehat{m}$, Lemma 1 implies that $\Upsilon_{z^*\widehat{z}} = 0$ for all $z^* \leq \widehat{m}$, $z^* \in Z_{\widetilde{N}}$.

  Therefore, by combining these condition with the induction assumption we conclude that for each integer m, $m \in \left\{0, 1, 2, ..., \widetilde{N}\right\}$, $\exists$ a unique $\widetilde{z} \in Z_{\widetilde{N}+1}$ such that:

$$\sum_{z \in Z_{\widetilde{N}+1},\ z \neq \widetilde{z}} \Upsilon_{\widetilde{z}z} = m$$

  We can now conclude by induction that each integer m, $m \in \{0, 1, 2, ..., N-1\}$, $\exists$ a unique $\widetilde{z} \in Z_N$ such that:

$$\sum_{z \in Z_N,\ z \neq \widetilde{z}} \Upsilon_{\widetilde{z}z} = m$$

■

9 Appendix 3: Proof of Proposition 4

This appendix proves proposition 4. However, before getting to the proof it is necessary to calculate the correlation between C and C^*.

Recall that in this case $0 < \tau < \infty$. For ease of notation define:

$$
\begin{aligned}
a &\equiv \left(\theta^* + \tau\Psi''(0)\, n^*\overline{Y}^*\right) \frac{n\overline{Y}}{n^*\overline{Y}^*} \\
a^* &\equiv \left(\theta + \tau\Psi''(0)\, n\overline{Y}\right) \frac{n^*\overline{Y}^*}{n\overline{Y}}
\end{aligned}
$$

Where θ, θ^*, and Ψ'' are evaluated at the deterministic equilibrium. Using these definitions together with (15), consumptions in the home and foreign countries can now be written as:

$$
\begin{aligned}
\widetilde{C}_t &= \frac{a\widetilde{Y}_t + \theta^*\widetilde{Y}_t^*}{\theta + a} \\
\widetilde{C}_t^* &= \frac{a^*\widetilde{Y}_t^* + \theta\widetilde{Y}_t}{\theta^* + a^*}
\end{aligned}
$$

Hence, the covariance of consumptions is given by:

$$
Cov\left(\widetilde{C}_t, \widetilde{C}_t^*\right) = \frac{a\theta\sigma^2_{\widetilde{Y}} + a^*\theta^*\sigma^2_{\widetilde{Y}^*} + (\theta^*\theta + aa^*)\,\sigma_{\widetilde{Y}_t,\widetilde{Y}_t^*}}{(\theta + a)(\theta^* + a^*)} \tag{29}
$$

And the variances are:

$$
\begin{aligned}
Var\left(\widetilde{C}_t\right) &= \frac{a^2\sigma^2_{\widetilde{Y}} + \theta^{*2}\sigma^2_{\widetilde{Y}^*} + 2a\theta^*\sigma_{\widetilde{Y}_t,\widetilde{Y}_t^*}}{(\theta + a)^2} \\
Var\left(\widetilde{C}_t^*\right) &= \frac{a^{*2}\sigma^2_{\widetilde{Y}^*} + \theta^2\sigma^2_{\widetilde{Y}} + 2a^*\theta\sigma_{\widetilde{Y}_t,\widetilde{Y}_t^*}}{(\theta^* + a^*)^2}
\end{aligned}
$$

where $\sigma^2_{\widetilde{Y}}$ and $\sigma^2_{\widetilde{Y}^*}$ are the variances of endowments in the home and foreign countries respectively, and $\sigma_{\widetilde{Y}_t,\widetilde{Y}_t^*}$ is the covariance of the endowments. Using these expressions, consumption correlation is given by:

$$
\rho_{\widetilde{C}_t,\widetilde{C}_t^*} = \frac{a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}} + (\theta^*\theta + aa^*)\,\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}}{\sqrt{\left(a^2\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + \theta^{*2}\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}} + 2a\theta^*\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right)\left(a^{*2}\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}} + \theta^2\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + 2a^*\theta\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right)}} \tag{30}
$$

Where $\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$ is the correlation coefficient between $\widetilde{Y}_t$ and $\widetilde{Y}_t^*$. Now we are ready to prove that $\rho_{\widetilde{C}_t,\widetilde{C}_t^*} \geq \rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$ for $\tau \in [0, \infty)$.

Proof. The proof is based on dividing the possible values of $\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$ into three regions. However, before considering the different cases, it will prove useful to calculate $\rho^2_{\widetilde{C}_t,\widetilde{C}_t^*}$. By squaring (30) and after some manipulations we get:

$$
\rho^2_{\widetilde{C}_t,\widetilde{C}_t^*} = \frac{\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}} + (\theta^*\theta + aa^*)\,\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right]^2}{\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}} + (aa^* + \theta\theta^*)\,\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right]^2 + (aa^* - \theta\theta^*)^2\left(1 - \rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right)} \tag{31}
$$

- Case 1: $\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*} \geq 0$

  Notice that a, a^*, θ, θ^* are all positive, therefore:

$$4aa^*\theta\theta^*\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*} = (aa^*+\theta\theta^*)^2\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*} - (aa^*-\theta\theta^*)^2\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*} \geq 0$$

  Adding two more non-negative terms gives us:

$$\begin{aligned} & \left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]^2 + 2\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right](aa^*+\theta\theta^*)\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*} \\ + \quad & (aa^*+\theta\theta^*)^2\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*} - (aa^*-\theta\theta^*)^2\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*} \geq 0 \end{aligned}$$

  Collecting terms and multiplying by $1-\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}$ gives:

$$\left(1-\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right)\left\{\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}} + (aa^*+\theta\theta^*)\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right]^2 - (aa^*-\theta\theta^*)^2\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right\} \geq 0$$

  Rearranging, we get:

$$\begin{aligned} \left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}} + (aa^*+\theta\theta^*)\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right]^2 \quad \geq \quad & \rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}} + (aa^*+\theta\theta^*)\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right]^2 \\ & + \left(1-\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right)(aa^*-\theta\theta^*)^2\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*} \end{aligned}$$

  Factoring out $\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}$ on the left hand side and rearranging yields:

$$\frac{\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}} + (aa^*+\theta\theta^*)\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right]^2}{\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}} + (aa^*+\theta\theta^*)\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right]^2 + \left(1-\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right)(aa^*-\theta\theta^*)^2} \geq \rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}$$

  which by (31) is just:

$$\rho^2_{\widetilde{C}_t,\widetilde{C}_t^*} \geq \rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*} \tag{32}$$

  Recall that in this case $\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*} \geq 0$, hence from (30) it follows that $\rho_{\widetilde{C}_t,\widetilde{C}_t^*} \geq 0$. Therefore, taking the square root of both sides does not invert the inequality, and we get:

$$\rho_{\widetilde{C}_t,\widetilde{C}_t^*} \geq \rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$$

- Case 2: $0 > \rho_{\widetilde{Y}_t,\widetilde{Y}_t^*} \geq -(aa^*+\theta\theta^*)^{-1}\left(a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}} + a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right)$

  In this case $\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$ is negative, however from (29) it follows that $Cov\left(\widetilde{C}_t,\widetilde{C}_t^*\right) \geq 0$ and hence $\rho_{\widetilde{C}_t,\widetilde{C}_t^*} \geq 0$. Therefore:

$$\rho_{\widetilde{C}_t,\widetilde{C}_t^*} \geq 0 \geq \rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$$

- Case 3: $\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*} < -\left(aa^*+\theta\theta^*\right)^{-1}\left(a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right)$

  Note that in this case $Cov\left(\widetilde{C}_t,\widetilde{C}_t^*\right)<0$ and hence $\rho_{\widetilde{C}_t,\widetilde{C}_t^*}<0$. Now we start from:

$$-\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}-a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]^2-4aa^*\theta\theta^*\left(1-\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right)\leq 0$$

  Therefore:

$$\begin{aligned}&-a^2\theta^2\frac{\sigma^2_{\widetilde{Y}}}{\sigma^2_{\widetilde{Y}^*}}-a^{2*}\theta^{2*}\frac{\sigma^2_{\widetilde{Y}^*}}{\sigma^2_{\widetilde{Y}}}+2aa^*\theta\theta^*-4aa^*\theta\theta^*+4aa^*\theta\theta^*\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\\ =\;&-\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]^2+4aa^*\theta\theta^*\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\leq 0\end{aligned}$$

  Adding and subtracting $\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]^2$ gives:

$$\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]^2+4aa^*\theta\theta^*\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}-2\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]^2\leq 0$$

  Multiplying and dividing the last term by $(aa^*+\theta\theta^*)$ gives:

$$\begin{aligned}&\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]^2+4aa^*\theta\theta^*\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\\ +\;&2\left(aa^*+\theta\theta^*\right)\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]\cdot\frac{-\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]}{(aa^*+\theta\theta^*)}\leq 0\end{aligned}$$

  Recall that in this case $\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}<-\frac{\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]}{(aa^*+\theta\theta^*)}<0$; therefore replacing $-\frac{\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]}{(aa^*+\theta\theta^*)}$ by $\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$ will make the left hand side even more negative and will not alter the inequality. That is:

$$\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]^2+4aa^*\theta\theta^*\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}+2\left(aa^*+\theta\theta^*\right)\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\leq 0$$

  Notice that $4aa^*\theta\theta^*\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}=\left(aa^*+\theta\theta^*\right)^2\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}-\left(aa^*-\theta\theta^*\right)^2\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}$, therefore:

$$\begin{aligned}&\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]^2+\left(aa^*+\theta\theta^*\right)^2\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\\ +\;&2\left(aa^*+\theta\theta^*\right)\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}\right]\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\leq\left(aa^*-\theta\theta^*\right)^2\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\end{aligned}$$

Which is the same as:

$$\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}+(aa^*+\theta\theta^*)\,\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right]^2\leq(aa^*-\theta\theta^*)^2\,\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}$$

Multiplying both sides by $1-\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}$ and rearranging gives:

$$\frac{\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}+(aa^*+\theta\theta^*)\,\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right]^2}{\left[a\theta\frac{\sigma_{\widetilde{Y}}}{\sigma_{\widetilde{Y}^*}}+a^*\theta^*\frac{\sigma_{\widetilde{Y}^*}}{\sigma_{\widetilde{Y}}}+(aa^*+\theta\theta^*)\,\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right]^2+\left(1-\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}\right)(aa^*-\theta\theta^*)^2}\leq\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*}$$

Using (31) this simplifies to:

$$\rho^2_{\widetilde{C}_t,\widetilde{C}_t^*}\leq\rho^2_{\widetilde{Y}_t,\widetilde{Y}_t^*} \tag{33}$$

Recall that in the case under consideration both $\rho_{\widetilde{C}_t,\widetilde{C}_t^*}$ and $\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$ are negative. Therefore, taking the negative square root from (33) inverts the inequality sign, and we get:

$$\rho_{\widetilde{C}_t,\widetilde{C}_t^*}\geq\rho_{\widetilde{Y}_t,\widetilde{Y}_t^*}$$

■

10 Appendix 4: Proof of Proposition 5

Before proving proposition 5, we need to derive some results for the symmetric case and to prove an additional lemma. Let:

$$\mu\equiv\frac{\theta\left(\overline{Y}\right)}{n\overline{Y}\tau\Psi''(0)}$$

Where the country indices were dropped since we consider a completely symmetric case. Equation (14) now becomes:

$$[1+(N-1)\,\mu]\,\widetilde{C}_{it}-\mu\sum_{j\neq i}\widetilde{C}_{jt}=\widetilde{Y}_{it}\qquad\forall i \tag{34}$$

Using (34) the variance and covariance of consumptions are given by:

$$\begin{aligned}\sigma^2_C &= \frac{\mu\,(N-1)\,(\mu N+2)\,\sigma_{YY^*}+[1+\mu\,(\mu N+2)]\,\sigma^2_Y}{1+\mu N\,(\mu N+2)}\\ \sigma_{CC^*} &= \frac{[1+\mu\,(N-1)\,(\mu N+2)]\,\sigma_{YY^*}+\mu\,(\mu N+2)\,\sigma^2_Y}{1+\mu N\,(\mu N+2)}\end{aligned}$$

Where $\sigma_Y^2 \equiv \sigma_{\widetilde{Y}_i}^2$ and $\sigma_{YY^*} \equiv \sigma_{\widetilde{Y}_i \widetilde{Y}_j}$ for all i, j, and N is the number of countries ($N \geq 2$). Using these results, the consumption correlation between any two arbitrary countries is:

$$\rho_{CC^*} = \frac{[1 + \mu (N-1)(\mu N + 2)] \rho_{YY^*} + \mu (\mu N + 2)}{\mu (N-1)(\mu N + 2) \rho_{YY^*} + 1 + \mu (\mu N + 2)}$$

And after some manipulations:

$$\rho_{CC^*} = 1 - \frac{1 - \rho_{YY^*}}{\mu (N-1)(\mu N + 2) \rho_{YY^*} + 1 + \mu (\mu N + 2)} \tag{35}$$

Lemma 7 *If Y_1, Y_2, ..., Y_N are N random variables such that $\sigma_{Y_i}^2 = \sigma_{Y_j}^2 \equiv \sigma_Y^2$ and $\rho_{Y_i Y_j} = \rho_{Y_k Y_l} \equiv \rho_{YY^*}$ for all i, j, k, l, then: $\rho_{YY^*} \geq -\frac{1}{N-1}$*

Proof. Notice that:

$$\begin{bmatrix} \sigma_Y^2 & & \sigma_{YY^*} \\ & \ddots & \\ \sigma_{YY^*} & & \sigma_Y^2 \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \left[\sigma_Y^2 + (N-1)\sigma_{YY^*}\right] \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

That is, $\sigma_Y^2 + (N-1)\sigma_{YY^*}$ is an eigenvalue of the variance-covariance matrix of the vector $(Y_1, Y_2, ..., Y_N)$. Since any variance-covariance matrix is positive semidefinite it must be the case that:

$$\sigma_Y^2 + (N-1)\sigma_{YY^*} \geq 0$$

Rearranging, we get:

$$\rho_{YY^*} = \frac{\sigma_{YY^*}}{\sigma_Y^2} \geq -\frac{1}{N-1}$$

■

Now we are ready for the proof of proposition 5.

Proof. By Lemma 7, we have that:

$$(N-1)\rho_{YY^*} \geq -1$$

Multiplying both sides by $\mu (\mu N + 2)$, adding 1, and rearranging, we get:

$$\frac{1}{\mu (N-1)(\mu N + 2) \rho_{YY^*} + 1 + \mu (\mu N + 2)} \leq 1$$

Multiplying both sides by $1 - \rho_{YY^*}$ and rearranging gives us:

$$1 - \frac{1 - \rho_{YY^*}}{\mu (N-1)(\mu N + 2) \rho_{YY^*} + 1 + \mu (\mu N + 2)} \geq \rho_{YY^*}$$

and by (35) we get that:

$$\rho_{CC^*} \geq \rho_{YY^*}$$

■

11 Appendix 5: Data Appendix

The source of the data is SourceOECD database. The data are in annual frequency and the sample period throughout the paper is 1970-2002. The sample period was chosen based on availability of the data for all G7 countries.

11.1 Data of Table 1

National accounts variables in fixed prices are taken from Annual National Accounts, volume I, Main aggregates, Vol 2004, release 03. Population data are from Annual National Accounts, volume I, Population and Employment, Vol 2004, release 03. GDP data in fixed prices and fixed PPP are taken from Annual National Accounts, volume I, Comparative Tables, Vol 2004, release 03.

Investment is measured as gross fixed capital formation. Consumption of non-profit institution was combined with government spending in all countries except for the US due to data availability.

Table A5.1: Cross-Country Correlations

Country	HP, $\lambda = 100$			HP, $\lambda = 100$			First Difference			$BP_3\,(2,8)$		
Pair	C	Y	Y-G-I	C	Y	Y-G-I	C	Y	Y-G-I	C	Y	Y-G-I
CN-FR	0.52	0.45	0.16	0.19	0.37	0.25	0.37	0.44	0.31	0.10	0.37	0.33
CN-GR	0.41	0.17	-0.02	0.28	0.30	0.18	0.30	0.29	0.25	0.20	0.29	0.27
CN-IT	0.30	0.56	0.35	0.26	0.55	0.50	0.24	0.47	0.40	0.24	0.52	0.56
CN-JP	0.18	0.11	-0.22	0.11	0.22	0.06	0.15	0.12	-0.01	0.08	0.18	0.06
CN-UK	0.73	0.80	0.75	0.64	0.70	0.73	0.53	0.58	0.61	0.57	0.58	0.65
CN-US	0.76	0.82	0.70	0.65	0.81	0.82	0.64	0.78	0.77	0.60	0.79	0.82
FR-GR	0.72	0.69	0.63	0.61	0.66	0.59	0.61	0.67	0.58	0.52	0.62	0.60
FR-IT	0.61	0.76	0.37	0.52	0.71	0.37	0.46	0.73	0.51	0.37	0.73	0.41
FR-JP	0.46	0.63	0.22	0.54	0.62	0.28	0.54	0.44	0.31	0.53	0.51	0.29
FR-UK	0.58	0.48	0.33	0.54	0.51	0.39	0.39	0.48	0.33	0.46	0.59	0.44
FR-US	0.41	0.42	0.50	0.39	0.45	0.43	0.40	0.43	0.40	0.41	0.47	0.43
GR-IT	0.47	0.64	0.36	0.44	0.63	0.50	0.45	0.69	0.63	0.40	0.68	0.54
GR-JP	0.48	0.71	0.47	0.30	0.62	0.50	0.49	0.61	0.48	0.32	0.59	0.59
GR-UK	0.29	0.21	0.20	0.26	0.30	0.23	0.15	0.32	0.31	0.16	0.31	0.28
GR-US	0.34	0.39	0.41	0.46	0.54	0.33	0.31	0.51	0.39	0.37	0.53	0.39
IT-JP	0.25	0.54	0.16	0.27	0.47	0.16	0.28	0.36	0.33	0.20	0.44	0.32
IT-UK	0.51	0.51	0.41	0.37	0.47	0.46	0.19	0.37	0.35	0.21	0.41	0.40
IT-US	0.13	0.49	0.37	-0.02	0.44	0.38	0.03	0.40	0.34	-0.05	0.44	0.42
JP-UK	0.42	0.32	0.07	0.62	0.41	0.22	0.42	0.36	0.20	0.61	0.44	0.29
JP-US	0.14	0.32	0.23	0.34	0.42	0.31	0.28	0.36	0.30	0.39	0.40	0.30
UK-US	0.70	0.83	0.78	0.57	0.76	0.72	0.65	0.69	0.62	0.58	0.72	0.66
Average*	0.38	0.50	0.38	0.39	0.52	0.40	0.36	0.48	0.40	0.37	0.51	0.43

*Weighted by population.

PPP values were approximated by measuring the share in GDP of each variable using the series expressed in domestic fixed prices, and multiplying the result by GDP in fixed prices and fixed PPP. To get per capita values, these quantities were divided by total population.

The series were filtered after taking the natural logarithm. The data for Germany before 1991 are based on estimates for unified Germany. In the text, Table 1 displayed average correlations; the complete set of correlations for every pair of countries is presented in Table A5.1.

11.2 Data of Table 4

The data for GDP consumption and investment is the same as in Table 1. The hours series are the product of average hours per employee and total employment. Both are taken from Economic Outlook: Annual and quarterly data, Vol 2004, release 01. Net export share is taken to be the external balance of goods and services divided by GDP; both are from the National Accounts tables.

Table A5.2: VAR Estimation of Productivity Process

Panel A: Estimates of coefficients (the matrix R)

Regressors:		Dependent Variable: Productivity shock of						
Lagged productivity of		CN	FR	GR	IT	JP	UK	US
Canada	coef.	0.700	0.109	-0.032	0.092	-0.265	-0.031	0.048
	std.	0.128	0.165	0.192	0.239	0.263	0.191	0.160
	prob.	0.000	0.514	0.867	0.704	0.324	0.872	0.764
France	coef.	-0.019	0.753	0.090	-0.190	0.544	-0.062	-0.140
	std.	0.175	0.226	0.263	0.328	0.361	0.262	0.219
	prob.	0.915	0.003	0.735	0.567	0.144	0.814	0.529
Germany	coef.	0.215	-0.132	0.565	0.183	-0.215	0.067	0.040
	std.	0.123	0.159	0.185	0.231	0.254	0.184	0.154
	prob.	0.094	0.414	0.005	0.435	0.406	0.718	0.795
Italy	coef.	-0.223	-0.097	-0.358	0.218	0.155	-0.063	-0.169
	std.	0.143	0.184	0.214	0.267	0.294	0.213	0.178
	prob.	0.129	0.603	0.108	0.422	0.602	0.770	0.352
Japan	coef.	-0.166	0.159	0.418	0.540	0.548	-0.052	0.013
	std.	0.137	0.177	0.206	0.256	0.282	0.204	0.171
	prob.	0.236	0.378	0.053	0.045	0.063	0.801	0.940
United Kingdom	coef.	0.467	0.217	-0.140	0.752	-0.267	0.845	0.170
	std.	0.162	0.210	0.244	0.304	0.334	0.242	0.203
	prob.	0.008	0.311	0.571	0.020	0.432	0.002	0.411
United States	coef.	-0.165	-0.081	0.246	-0.205	0.607	-0.354	0.344
	std.	0.205	0.265	0.308	0.384	0.422	0.306	0.256
	prob.	0.427	0.764	0.432	0.597	0.163	0.258	0.191
R^2		0.835	0.883	0.777	0.807	0.820	0.528	0.592

Table A5.2 Continued

Panel B: Residuals' standard deviations and correlations*

	CN	FR	GR	IT	JP	UK	US
Canada	0.77						
France	0.22	1.00					
Germany	0.39	0.40	1.16				
Italy	0.41	0.77	0.54	1.44			
Japan	0.27	0.59	0.66	0.64	1.59		
United Kingdom	0.28	0.48	0.02	0.56	0.23	1.15	
United States	0.66	0.14	0.51	0.34	0.40	0.31	0.96

*Standard deviations (in percents) along the diagonal, correlations off-diagonal.

11.3 The Solow Residuals

Productivity shocks were measured using the Solow residuals as described by equation (25):

$$\ln(A_{it}) = \ln(Y_{it}) - \alpha \ln(K_{it}) - (1-\alpha)\ln(H_{it})$$

After removing linear time trend from $\ln(A_{it})$, the time series process for the productivity shocks was estimated as an unrestricted $VAR(1)$, as described by equation (26). The estimation results are presented in Table A5.2. Panel A reports the estimates of the coefficients of the matrix R from equation (26), while Panel B reports the standard deviations and correlations of the residuals.

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