THE ULTIMATUM GAME: INTERDEPENDENT PREFERENCES IN EXPERIMENTAL SETTING

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Abstract. We approach the Ultimatum Game as a mechanism designed to elicit information about the preferences and beliefs of players. While remaining agnostic about the right way to interpret preferences, we maintain the assumption that preferences are interdependent - the utility of a player may be a function of other players' types. We explain the relation between the best known explanations of the experimental evidence to the proposed framework. We then illustrate how standard arguments can be used to extract the information conveyed by existing experimental results, and how the latter can restrict the set of plausible models of interdependence. We then conduct an experiment that tests the equilibrium predictions of the proposed model as well as models of social preferences and intention based reciprocity. We find that the experimental evidence is consistent with the equilibrium predictions of the model with interdependent preferences, while being inconsistent with the other models.

0.1. **Introduction and Discussion.** Many bargaining experiments have shown that in certain environments the equilibrium predictions of economic theory, which are based on sequential rationality and selfish preferences are inconsistent with the experimental results (see Roth [36], Camerer [10], Fehr and Schmidt [19] for extensive reviews). One of the earliest and most influential of those experiments is the Ultimatum Game: a "proposer" makes an offer of x (between 0 and $\bar{x}$). If the responder accepts the offer then the proposer receives $(\bar{x} - x)$ while the responder receives x. If the responder rejects - both receive zero. A well known backwards induction (or maybe just common sense) argument predicts that the responder should accept any positive offer and therefore the proposer should offer only a minimal offer. As is now well known (since Güth et al [25]), the experimental evidence refute this prediction.

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A variety of explanations have been offered to explain why responders reject and why proposers don't demand the whole surplus. The common approach nowadays is to assume that agents have "other-regarding preferences." This literature can be broadly divided into three models, that have substantial differences among them. Inequity aversion models (Fehr and Schmidt [18], Bolton-Ockenfels [9]) assume that the agent's utility may be a function of the resources allocated to other agents as well as to herself: she might be altruistic towards others if their resources fall below an equitable benchmark, and feel envy when their allocation exceeds this level. Intention-based reciprocity models (Rabin [34]) assume that the agent cares about her opponent's intentions and motives. She holds beliefs about the opponent's beliefs over her actions, and her utility of his action may depend if she believed he took it because he believed that she will be nice to him or because he believed she will hurt him. Interdependent preferences models (Levine [29], Gul and Pesendorfer [24]) models assume that the agent's preferences may depend on her opponent's type (e.g. altruistic, greedy). In these models the opponent's action may affect both the material allocation and the inference the agent makes about the opponent type. Another explanation (and maybe the oldest) is that experimental subjects simply don't react to the precise experimental set-up (Frank [20], Aumann [1]). Their strategies have been molded in repeated bargaining environments ("rule rationality") that they have experienced during their lifetime. In those environments, a behavior that seems to take into account fairness arguments is instrumental in achieving higher payoffs.[1] In some sense, the subjects don't believe that the experiment itself is all that that is taking place. This view is perfectly reasonable: there is at least one additional party in the experiment - the experimenter herself, who structured the experiment and her goals might be relevant the subjects' payoff. Another group (Binmore, Samuelson and others [22, 5, 4, 6]) has been arguing that excessive attention has been given to backwards induction arguments. They claim that since the set of Nash equilibria is much larger, one should try and describe the outcome of the game in terms of another process that leads to Nash equilibrium, but not necessarily to subgame perfect equilibrium - like an evolutionary model.

[1]Fehr and Fischbacher [17] find that explicitly allowing for reputation building increases the responder's acceptance threshhold. Although this is an important evidence against the "rule rationality" interpretation, it may be argued that some subjects in the treatment without explicit reputation still use a strategy that implicitly assumes reputation building.

The experimental data seem to require more than just a reason why offers might be rejected. A variety of experimental studies illustrate that the relation between the proposer's offer and his expected revenue is hump shaped. Furthermore, there is a substantial variation in offers that are made in experiments. Proposers do sometimes demand everything (make low offers) even though these offers are rejected. On the other hand, there seems to be a limit to proposers' generosity - they seem reluctant to offer much more than 50% - perhaps because these high offers are almost certainly accepted.

In this paper, we try to use standard game-theoretic tools to interpret this data in a way that will hopefully be applicable to some important ultimatum-like environments studied in Economics (e.g. mechanism design). It is not our intent to provide new explanations for why offers are rejected, or why proposers don't demand the whole surplus. Rather, we simply accept that experimental subjects behave in their best interest and ask whether there is a way to use experimental evidence to uncover the underlying distributions of preferences and beliefs. This problem is more subtle in a game theoretic experiment than it is in a simple decision theoretic experiment because of the common knowledge assumption involved in game theory, which leads us to try and model 'experiments' such as the ultimatum game.

The specific approach we take is to assume that preferences are 'interdependent'. More simply, experimental subjects care about other participants' types, not necessarily about their action or outcomes. We believe that this sort of interdependence is less likely to be affected by changes in the experimental design, and is more likely to reflect preferences outside the lab than preferences that depend on outcomes or actions alone. For example, the fact that a subject dislikes unequal shares in a lab experiment might not mean that he dislikes unequal shares in a market. However someone who dislikes a person who is greedy enough to want a high share is likely to dislike the same person in a market. For the purposes of this paper we can be agnostic about this, we simply want to analyze the consequences of this approach. Experimental evidence tends to support this view (see Camerer [10] pages 110-113; Blount [7], Falk, Fehr and Fischbacher [14, 15] and McCabe, Rigdon and Smith [32]), as well as intention-based reciprocity. One particular advantage of the standard approach pursued in this paper over the fairness and reciprocity approaches is that it does not rely on the the psychological games setup (Geanakoplos, Pearce and Stacchetti [23]) in which a player's payoff depends on her belief on other players' beliefs and are very game specific. These setups requires considerable theoretical novelty in defining and constructing appropriate equilibrium concepts in normal and extensive form games (e.g., Dufwenberg and Kirchsteiger[13], Battigalli and Dufwenberg [3], Falk and Fischbacher [16]).

A second advantage to our approach is that it allows us to interpret the experimental results in a way that can be readily adapted to other games and problems of mechanism design - perhaps the most common use of the ultimatum game. The methods that we use to interpret the information are quite standard. Furthermore, experimental results have and will continue to be used for purposes other than testing economic theory. The work of Henrich et al [26], for example, tries to find correlations between ultimatum game outcomes and measures of development in different cultures. Our interpretation of the variation in outcomes as being due to differences in beliefs lends itself quite readily to this kind of analysis.

Traditional explanations for behavior in the ultimatum game can be recast in terms of interdependence. For example, the type of the proposer could be imagined to be some measure of his greed. A responder will receive some utility from rejecting an offer from a greedy proposer rather than being concerned per se with the payoff difference. Similarly, Aumann and Frank's "rule rationality" basically presumes that traders think that their actions in the experiment have consequences that go beyond the experiment itself. For example, responders and proposers might interpret a one shot ultimatum game in terms of a more traditional offer-counteroffer game that they usually play. A rejection of an unfair offer is the response that would work best in everyday bargaining situations in which the rejection would be followed up with a counteroffer. How effective it would be to reject such an offer would depend on characteristics of the proposer that the responder can't know - his discount rate for example. The proposer's initial offer signals information about his unknown type. Alternatively the proposer's type might simply be information that he has that is interesting to both parties. For example, a proposer might believe that he can induce the experimenter to run another experiment with higher stakes if the responder rejects his proposal. He might signal this by making a high demand. Alternatively, an impatient proposer might convey this by making a low demand. Interdependence arises from the fact that the responder expects the type of the proposer to affect his payoff in the continuation game.

In either of these cases, the ultimatum game is a signaling game, which will have many equilibria. We are interested in characterizing these equilibria, and in particular, looking for the 'most informative' of these equilibria. One of our main results illustrates that the quality of the information conveyed by the ultimatum game depends on the nature of the interdependence. When high type proposers want responders to reject offers, and responders who know the proposer has a high type also want to reject offers, we say that preferences have *positive* interdependence. Altruism corresponds to this structure of interdependence: suppose that the lower the proposer's type, the more altruistic he is. For a responder who knows the proposer's type, the more altruistic the proposer is, the lower is the marginal utility of rejection. It follows that the more altruistic the proposer is (as he takes into account the responder's payoff) the utility loss as a result of a responder rejection would be higher. This case corresponds also with the situation mentioned above in which the proposer knows a second experiment with higher stakes will be run if there is a rejection in the first experiment. We show that for this case, the equilibrium path for the ultimatum game can contain no more than two distinct demands. This pooling of types means that the experimenter can at best partition the set of proposer types into two distinct sets. Furthermore, even if the experimenter assumes that all the outcomes across an array of experiments are being generated by the same equilibrium, he will not be able to identify the beliefs of the responder that give rise to those equilibrium outcomes as a result. It seems that existing experimental results are inconsistent with the model of positive interdependence.

Levine [29] was the first to apply interdependent preferences to account for experimental outcomes. He assumed a specific parametric form of preferences (capturing a form of altruism and spitefulness), and calibrated the extent of altruism/spitefulness using some experiments. The players' preferences in Levine's model are positively interdependent, but the equilibrium supports more than two demands. The crucial difference between the two approaches is that Levine [29] assumes heterogeneity in responders' types. Although we can purify our model with responder types, their type-distribution is determined in equilibrium. In Levine's model this distribution is determined exogenously, by using symmetry between the proposer and the responder type distribution. This allows him to support high demand that is accepted with very low probability, simply because their is a responder type who is altruistic enough to accept this demand, even after inferring that the proposer is extremely spiteful. This is an additional degree of freedom, that (as argued below) is unnecessary to rationalize the results in the ultimatum game. Furthermore, once this path is taken, it is not clear how much freedom one should allow in modeling responder heterogeneity, since responders in the ultimatum game cannot signal their type to proposers. There is no doubt, however, that in practice there is a considerable heterogeneity among responders, possibly beyond what can be accounted for using purification of the mixed strategies used in this paper.

When high type proposers want the responder to accept offers (but responders want to reject offers from high type proposers), we say that preferences have *negative* interdependence. This case is consistent with both other-regarding preferences and rule rationality. For example, if the proposer's type represents his greed, the higher his type the higher is his utility loss due to a responder rejection, while the responder's marginal utility of rejecting is increasing in the proposer's greed. Similarly, within the "rule rationality" framework one may think of the proposer's type as representing his discount rate. The more impatient the proposer is, the worse he will do in a continuation, and therefore the higher is his utility loss due to rejection. If the responder would know the proposer's impatience, his marginal utility of rejecting would increase in it. Furthermore, although our preferred interpretation within the other-regarding preferences is of greed, one may interpret negative interdependence as corresponding to spiteful behavior: the higher the proposer's type, the more spiteful he is. The responder's marginal utility of rejecting is increasing in the proposer's spite, but the more spiteful the proposer is (as he knows that the responder's payoff as a result of rejecting is increasing) - the more he would like the responder to accept.

When there is negative interdependence, perfect Bayesian equilibria from the ultimatum experiment involve pooling by the lowest type proposers (positive measure of proposer's type will demand the entire surplus), and virtually complete separation by the other proposer types. In this case, the dispersion of offers in the ultimatum game will reveal the distribution of proposer types.

The equilibrium path in the case of negative interdependence also captures many of the other features of the experimental data. Many offers are made along the equilibrium path and none but the lowest is accepted for sure. This means that acceptance probabilities decline smoothly as proposers increase their demands. The relationship between the proposer's offer and his expected revenue is also hump shaped. Finally, the equilibrium path imposes a limit on the experiment's surplus that the proposer will offer to the responder.

It should be emphasized that the current paper does not take a stand on the interpretation of negative interdependence, but allows a unified treatment of various explanations that have been suggested in the past. The outcome of this study lends support to the general family case of negative interdependence, but as argued above - researchers could disagree on a specific interpretation.

After characterizing the most informative Perfect Bayesian Equilibrium, we perform an experimental comparative static investigation: we study how the equilibrium (distribution of offers and conditional acceptance rates) changes if we set a lower limit to the offer made by a proposer. While intention-based reciprocity and inequity aversion models predict concentration of offers on the lower bound, and decrease or no change (respectively) in the acceptance rate, we show that the equilibrium with negative interdependence predicts higher demands (lower offers) and higher acceptance rate. The intuition behind this prediction is that if the subset of low types who make high demands will not increase, then responders (who have low marginal utility of rejecting offers made by low types) would accept these high demand in certainty. To maintain an equilibrium, the subset of proposers who make high demands must increase, and in order to give the new pivotal high type proposers an incentive to make the high demand - the acceptance probability must increase. We find that the experimental results are consistent with the negative interdependence model proposed here.

1. The Model

Let P be a finite collection of feasible demands (offers) for the proposer. These are normalized to lie between 0 and 1. Suppose these offers are indexed in such a way that $0 = p_1 < p_2 < \ldots < p_n = 1$. The lowest demand p_1 is assumed to give all the surplus in the experiment to the responder. The highest demand p_n is assumed to give all the surplus from the experiment to the proposer. The proposer is of type $s \in [\underline{s}, \overline{s}] \equiv S$, which affects the payoff of both players. The distribution of types is given by F, and is assumed to be continuous with full support. Let $\alpha \in \{0, 1\}$ denote the action of the responder, $\alpha = 1$ meaning that she accepts the proposal. The payoffs are treated asymmetrically. The payoff to the proposer is given by $u_p(p, \alpha, s)$ where s is his type.

Condition 1.1. Let $p' > p$. Then

(1) $u_p(p', 1, s) > u_p(p, 1, s)$ and $u_p(p', 0, s) \geq u_p(p, 0, s)$ for all $s \in S$;
(2) If $u_p(p, 1, s) > u_p(p, 0, s)$ for some $s \in S$ then $u_p(p, 1, s) > u_p(p', 0, s)$.

The first part of the condition states that independently of whether an offer is accepted or rejected, the proposer is always better off with a higher demand (lower offer). The second part states that if the proposer has type s and prefers the demand p to be accepted rather than being rejected, then he must also prefer that the demand p be accepted to a rejection of a higher demand. The combination of the two parts imply that if $u_p(p,1,s) > u_p(p,0,s)$ for some $s \in S$ then $u_p(p,1,s) > u_p(p',0,s) \geq u_p(p,0,s)$.

The following condition is also used repeatedly.

Condition 1.2. Let s' and s be such that

$$u_p(p,0,s) - u_p(p,1,s) < u_p(p,0,s') - u_p(p,1,s')$$

and suppose that for some $p_j > p_k$ and $q_j < q_k$,

$$q_j u_p(p_j,1,s) + (1-q_j)u_p(p_j,0,s) \geq q_k u_p(p_k,1,s) + (1-q_k)u_p(p_k,0,s)$$

Then the same inequality holds strictly for type s'.

This "single-crossing" condition states that if the proposer's utility loss as a result of being rejected at s' is lower than at s, and if the proposer's expected utility from a high demand (with a certain probability of acceptance) is at least as high as the expected utility of a lower demand (with higher probability of acceptance) at s, then he strictly prefers the higher demand at s'.

The payoff to the responder also depends on the proposer's type. This payoff function is given by $u_r(p,\alpha,s)$. We assume that:

Condition 1.3. The function $u_r(p,0,s) - u_r(p,1,s)$ is monotonically increasing in s and p, and is supermodular in p and s. For every s there is a $p > 0$ such that $u_r(p,0,s) - u_r(p,1,s) < 0$; $u_r(p,0,\overline{s}) - u_r(p,1,\overline{s}) > 0$ for some p, and $u_r(p,0,\underline{s}) - u_r(p,1,\underline{s}) < 0$ for all $p \in P$.

The assumption is made on the responder's marginal utility of rejection, if she knew the proposer's type s. It is assumed that the marginal utility of rejection is increasing in the type and the demand, and is supermodular in the proposer's demand and type. That is, if the responder would know the proposer's type, for higher proposer's type the change in the marginal utility of rejecting a higher demand is higher. No matter what are the responder's beliefs about the proposer's type, there is some demand the responder would accept. A responder who believes the proposer's type is $\overline{s}$ will reject some offer, while a responder who believes the proposer's type is $\underline{s}$ will accept any offer.

This game has many equilibrium outcomes. The nature of these outcomes depends on the function $u_p(p,0,s) - u_p(p,1,s)$. In Appendix A we analyze the case where this function is strictly increasing. Since this function is always increasing for the responder, we refer to this as a situation of *positive interdependence.* We prove that the equilibrium can contain no more than two distinct demands. This equilibrium seems inconsistent with experimental outcomes in which a variety of demands are made in equilibrium.

2. Equilibrium with Negative Interdependence

To provide an account of experimental results using a single equilibrium, we now consider the case where $u_p(p, 0, s) - u_p(p, 1, s)$ is monotonically decreasing in s. We refer to this as negative interdependence. Without committing to a specific interpretation, the higher the proposer's type, the greater is the utility loss due to a responder's rejection. The responder's marginal utility of rejection continues to increase in the proposer's type. As discussed in the Introduction, this formulation unifies the fairness and "rule-rationaliy" interpretations. Roughly speaking, the proposer now has an incentive to try to hide his information from the responder because their interests are not aligned.

The following assumption is required to construct an equilibrium:

Condition 2.1. For any offer p let s be such that $u_r(p', 1, s) < u_r(p', 0, s)$ for each $p' > p$. Then

$$u_p(p, 1, s) > u_p(p, 0, s)$$

That is, if for every $p' > p$, a *responder* who knows the proposer's type prefers to reject p', then the *proposer* prefers p to be accepted. In other words: let $\hat{s}$ be the solution to $u_r(p, 0, s) = u_r(p, 1, s)$. This is the proposer's type such that if the responder believed the proposer had that type for sure, she would be just indifferent between accepting and rejecting the demand p. Roughly the Condition above states that provided a type s isn't too much lower than $\hat{s}$, the proposer of type s would strictly prefer to have the demand p accepted.

The following condition is intuitive and strengthens the first part of Condition 1.1.

Condition 2.2. The payoff to the proposer from a rejected offer is independent of p. That is, $u_p(p, 0, s) = u_p(p', 0, s)$ all $p, p' \in P$ and $s \in S$.

Under these assumptions, equilibrium satisfies the following condition:

Theorem 2.3. *Let $p' > p$ be two offers made on the equilibrium path. The probability with which the offer p is accepted is at least as large as the probability with which p' is accepted.*

Proof. Let q and q' be the acceptance probabilities associated with p and p' respectively, and suppose to the contrary that $q' > q$. In particular, this means that $q < 1$. Let $S(p)$ be the set of proposer types who make the demand p with positive probability on the equilibrium path. Since $q < 1$ there must be some type $s \in S(p)$ such that $u_r(p, 0, s) > u_r(p, 1, s)$. Since the responder's marginal utility of rejection is increasing in p, this same inequality must be true for every $p'' > p$. Then by Condition 2.1, $u_p(p, 1, s) > u_p(p, 0, s)$. This is a contradiction since a proposer of type s could then strictly increase his payoff by demanding p' which is accepted with higher probability. □

We now identify all the equilibrium outcomes for the game with negative interdependence. We first prove that a sequence of demands can be supported in a Perfect Bayesian Nash Equilibrium only if the highest demand is the whole pie, and the demands partition the proposer's types in a way that if the responder knows that a certain offer is made by an interval of proposer's types - she is indifferent between accepting and rejecting the offer (except possibly the lowest demand).

Theorem 2.4. *Suppose that $u_p(p, 0, s) - u_p(p, 1, s)$ is strictly decreasing, that $u_p(p_n, 0, \underline{s}) - u_p(p_n, 1, \underline{s}) > 0$, and that Conditions 1.1, 1.2, 1.3, 2.1 and 2.2 hold. Then an ascending sequence of demands $(\pi_1, \ldots, \pi_K)$ can be supported as a Perfect Bayesian Nash Equilibrium demands if*

(1) $\pi_K = 1$*; and*
(2) *there exists a strictly descending sequence of* $K+1$ *types* $(s_1, \ldots, s_K, s_{K+1})$ *with* $s_1 = \overline{s}$ *and* $s_{K+1} = \underline{s}$ *satisfying*

$$\int_{s_{k+1}}^{s_k} \{u_r(\pi_k, 0, s) - u_r(\pi_k, 1, s)\}\, dF(s) \leq 0$$

with equality holding for all k *except possibly for* $k = 1$.

Proof. The proof involves constructing a Perfect Bayesian Nash Equilibrium. Begin with the lowest demand π_1. Since

$$\int_{s_2}^{s_1} \{u_r(\pi_1, 0, s) - u_r(\pi_1, 1, s)\}\, dF(s) \leq 0$$

this demand is acceptable to a responder who believes that the proposer who makes it has a type in the interval $[s_2, s_1]$. Set $q_1 = 1$ so that the lowest demand is surely accepted. Proposers whose types are in the interval $[s_2, s_1]$ will make demand π_1 and responders will accept this offer with probability 1.

Now for each $k > 1$, select q_k such that

(2.1)
$$q_k u_p(\pi_k, 1, s_k) + (1 - q_k) u_p(\pi_k, 0, s_k) = q_{k-1} u_p(\pi_{k-1}, 1, s_k) + (1 - q_{k-1}) u_p(\pi_{k-1}, 0, s_k)$$

That is, q_k is chosen such that a proposer of type s_k is indifferent between demanding π_k and π_{k-1}.

We need to show that (2.1) has a positive solution. Observe that the inequalities

$$\int_{s_k}^{s_{k-1}} \{u_r(\pi_{k-1}, 0, s) - u_r(\pi_{k-1}, 1, s)\} \, dF(s) \leq 0$$

and

$$\int_{s_{k+1}}^{s_k} \{u_r(\pi_k, 0, s) - u_r(\pi_k, 1, s)\} \, dF(s) \leq 0$$

imply that a responder who believes that the offer comes from a proposer of type s_k must want to accept π_{k-1} and reject π_k and every higher demand. Then by Condition 2.1,

$$u_p(\pi_k, 1, s_k) > u_p(\pi_k, 0, s_k) = u_p(\pi_{k-1}, 0, s_k)$$

So from Conditions 2.2 and 1.1, (2.1) has a positive solution. Let proposers whose type is in the interval $[s_{k+1}, s_k]$ make the demand π_k, and suppose this is accepted with probability q_k.

From this construction, a proposer whose type is s_k is just indifferent between demanding π_k and π_{k-1}. By the single crossing Condition 1.2, proposers whose types are below s_k strictly prefer the demand π_k to the demand π_{k-1}. On the other hand, if a proposer whose type exceeds s_k strictly prefers to make the demand π_k instead of π_{k-1}, then a proposer whose type is s_k must also by Condition 1.2. Applying this argument at each value of k, it follows that the best equilibrium path offer for a proposer whose type is in the interval $(s_{k+1}, s_k]$ is the demand π_k.

To deal with off equilibrium offers, observe that the lowest offer π_1 that is made on the equilibrium path leads responders to believe that the proposer has a type in some interval $[s_2, \overline{s}]$ such that

$$\int_{s_2}^{\overline{s}} \{u_r(\pi_1, 0, s) - u_r(\pi_1, 1, s)\} \, dF(s) \leq 0$$

If this inequality is strict, then the offer is accepted with probability 1. In that case, suppose that lower offers are treated the same way - i.e., they lead to the same inference about the proposer's type, and are accepted with probability 1. Since proposer's payoff is strictly increasing as the size of an accepted demand increases, it will not pay any proposer to make a demand below π_1.

On the other hand, let p' be an off equilibrium demand that exceeds π_1. Suppose that π_k is the highest equilibrium path offer that is less than p'. By assumption, a responder who thinks that the proposer's type is s_{k+1} is willing to accept the offer π_k but wants to reject every higher equilibrium path offer. Let $[s', s'']$ be any interval such that

$$\int_{s'}^{s''} \{u_r(p', 0, s) - u_r(s', 1, s)\} \, dF(s) = 0$$

Now choose q' as above such that

$$q_k u_p(\pi_k, 1, s_{k+1}) + (1-q_k) u_p(\pi_k, 0, s_{k+1}) = q' u_p(p', 1, s_{k+1}) + (1-q') u_p(p', 0, s_{k+1})$$

Then as along the equilibrium path, if responders believe the proposer's type is in the interval $[s', s'']$ when p' is offered, and accept the demand with probability q', then proposers will all find higher payoffs with equilibrium path offers. □

We verify below that the theorem isn't vacuous in the sense that interesting equilibria of this kind always exist. Trivial equilibrium of this kind might exist. For example, if the highest demand is acceptable to the responder given her prior beliefs, then there is a simple equilibrium in which the proposer always demands the entire surplus. We next show that all equilibria must look like this.

Theorem 2.5. *Under the Assumptions of Theorem 2.4, the ascending sequence of demands $\{\pi_1, \ldots, \pi_K\}$ can be supported as equilibrium offers in some Perfect Bayesian Nash Equilibrium in which every demand is accepted with positive probability only if Conditions 1 and 2 of Theorem 2.4 hold.*

Proof. Condition 1: Let π_K be the highest demand and suppose it is accepted with probability q_K. If $\pi_K < 1$, then the off equilibrium demand 1 must be accepted with probability at least q_K to prevent the proposer with type $\underline{s}$ (who prefers every demand to be rejected) from deviating. This requires that for every proposer type s in the set of proposer types $S(\pi_K)$ who make the offer π_K in equilibrium, $u_p(1, 0, s) > u_p(1, 1, s)$, else one of these proposer types would deviate.

Now from Condition 2.1, $u_r(\pi_K, 1, s) > u_r(\pi_K, 0, s)$ for every $s \in S(\pi_K)$ (if equality holds for some s then a responder who believed the proposer's type were s would reject any higher demand requiring a proposer of that type to want the demand 1 to be accepted). As a consequence, the proposal π_K, and also $p_n = 1$, must be accepted for sure. Since the payoff to acceptance is increasing as the demand rises, every type in $S(\pi_K)$ will want to deviate which is inconsistent with equilibrium.

Condition 2: Any array of K distinct offers made on the equilibrium path partitions the interval $[\underline{s}, \overline{s}]$ into K subsets through the inference that the responder makes from price. No two distinct demands which are accepted with positive probability can be accepted with the same probability in equilibrium, because proposers prefer higher demands. The single crossing condition can then be used as in the proof of Theorem 2.4 to show that all the subsets in the partition are intervals. The requirement that all demands are accepted with positive probability then gives Condition 2. □

The equilibrium with negative interdependence is considerably different from that with positive interdependence. As illustrated, single price and two price equilibrium may exist. If they do, they always involve the highest demand. From (1), some proposer types must demand 1 and this must be weakly acceptable given responders beliefs when they see this demand. So there must be an interval of types $[\underline{s}, s_m]$ who demand 1. Let $\overline{p}$ satisfy

$$u_r(\overline{p}, 1, s_m) = u_r(\overline{p}, 0, s_m)$$

No demand between $\overline{p}$ and 1 will be made with positive probability. At the other extreme, there may be a demand $\underline{p}$ such that $u_r(\underline{p}, 1, \overline{s}) = u_r(\underline{p}, 0, \overline{s})$. This demand is acceptable to the responder no matter what his or her beliefs. No demand below this can be sustained in any equilibrium in which higher demands are accepted with lower probability.

2.1. **Maximally Dispersed Equilibrium.** In spite of the partial pooling present in every equilibrium, in what follows we focus on one particular equilibrium which is the most informative. The *Maximally Dispersed Equilibrium* is constructed by generating a particular sequence of demands, and the intervals associated with them. Begin by setting $\pi_m = p_n = 1$. Select an interval $[\underline{s}, s_m)$ with $s_m < \overline{s}$ such that

$$\int_{\underline{s}}^{s_m} \{u_r(\pi_m, 0, s) - u_r(\pi_m, 1, s)\} \, dF(s) = 0$$

if such an s_m exists. If the expression above is non-positive for all s_m, then the equilibrium is complete and all proposer types demand $p_n = 1$ (the whole pie) in the Maximally Dispersed Equilibrium.

Otherwise, assume a sequence $\{(\pi_m, s_m), (\pi_{m-1}, s_{m-1}), \ldots, (\pi_{k+1}, s_{k+1})\}$ has been constructed for $m, m-1, \ldots, k+1$, with $\pi_{k+1} > 0$ and $s_{k+1} < \overline{s}$. Let π_k be defined to be

$$\pi_k := \max\{P \ni p < \pi_{k+1} : u_r(p, 0, s_{k+1}) - u_r(p, 1, s_{k+1}) < 0\} \tag{2.2}$$

This price exists because by Condition 1.3, there is some offer that is acceptable to the responder no matter what her beliefs. Now select s_k

such that

$$\int_{s_{k+1}}^{s_k} \{u_r(\pi_{k+1}, 0, s) - u_r(\pi_{k+1}, 1, s)\} \, dF(s) = 0$$

if such an s_k exists. Otherwise set $s_k = \overline{s}$ and stop the construction.

Repeat this procedure until $s_k = \overline{s}$. Then re-index the demands and cutoffs such that m is the number of demands in the sequence.

The demands and cutoffs satisfy the Conditions of Theorem 2.4 by construction. The construction itself illustrates that such a sequence always exists. If responders want to reject the highest demand given their prior beliefs, then this sequence has at least two demands. At each step in the construction, the next highest demand is always chosen to be the highest demand that is consistent with conditions (1) and (2).

Figure 2.1 illustrates how these demands are constructed.

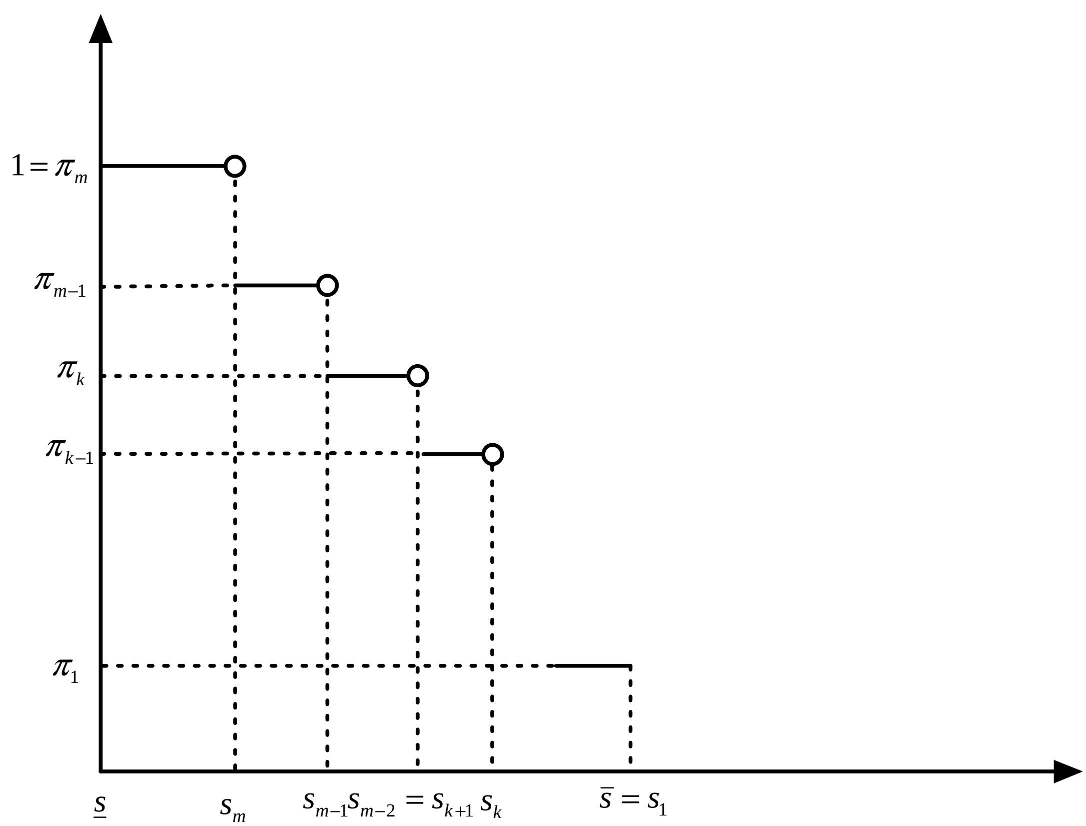

FIGURE 2.1. Construction of Demands in Equilibrium

To make any more progress characterizing the equilibrium, we need to put a little more structure on the feasible offers. Specifically

Condition 2.6. For any feasible offer p_k, let $\theta(p_k)$ be the type for the proposer such that

$$u_r(p_k, 1, \theta(p_k)) = u_r(p_k, 0, \theta(p_k))$$

Then

$$\int_{\theta(p_{k+2})}^{\theta(p_k)} \{u_r(p_{k+1}, 1, s) - u_r(p_{k+1}, 0, s)\} \, dF(s) \leq 0$$

If the function $\theta(p)$ is monotonically decreasing, then it will always be possible to construct a grid that satisfies this assumption. Let $[\underline{s}, s_m]$ be the interval described in the construction of the Maximally Dispersed equilibrium such that

$$\int_{\underline{s}}^{s_m} \{u_r(1, 0, s) - u_r(1, 1, s)\} \, dF(s) = 0$$

Now choose any price p_{m-1} such that

$$u_r(p_{m-1}, 0, s_m) - u_r(p_{m-1}, 1, s_m) < 0$$

This price is the first important element in the grid, since offers between p_{m-1} and 1 will be strictly dominated in equilibrium.

Now

$$\int_{s_m}^{\theta(p_{m-1})} \{u_r(p_{m-1}, 0, s) - u_r(p_{m-1}, 1, s)\} \, dF(s) > 0$$

So pick $s' > \theta(p_{m-1})$ such that

$$\int_{s_m}^{s'} \{u_r(p_{m-1}, 0, s) - u_r(p_{m-1}, 1, s)\} \, dF(s) = 0$$

and select any price $p_{m-1} < p : \theta(p) = s'$ as the next point in the grid of feasible demands. Repeating this procedure for lower demand generates a set of feasible demands satisfying Condition 2.6. So this Condition imposes a restriction on the set of feasible demands, not on the preferences or beliefs of the players.

Theorem 2.7. *If the grid of feasible demands satisfies Condition 2.6, and the Assumptions of Theorem 2.4 hold, then the Maximally Dispersed Equilibrium supports the demand 1 and every feasible demand in the interval $(\underline{p}, \overline{p})$ being made with positive probability on the equilibrium path.*

Proof. The proof simply involves showing that the sequential construction of demands in the definition of the Maximally Dispersed Equilibrium must cover every price in the interval. First note that if the demand 1 is acceptable to the proposer given his prior beliefs, then $\overline{p}$

and $\underline{p}$ coincide. Then the theorem follows trivially since there aren't any feasible demands in the interval $\left(\underline{p}, \overline{p}\right)$.

Now consider the second highest demand π_{m-1}. This is the highest feasible demand that is strictly acceptable to a responder who believes the proposer's type is s_m. Since $\overline{p}$ is defined such that a responder who believes the proposer's type is s_m is just indifferent between accepting and rejecting, a responder with the same belief will strictly accept the highest demand in the grid that is less than $\overline{p}$.

So let π_k be a feasible demand, and suppose that for each of the feasible demands above π_k there is an interval of types satisfying (2). In particular, there is some type s_{k+1} such that proposer types above s_{k+1} are assigned to demands above π_k, and if π_{k+1} is the next highest feasible demand, then

$$u_r\left(\pi_{k+1}, 1, s_{k+1}\right) < u_r\left(\pi_{k+1}, 0, s_{k+1}\right)$$

Then by Condition 2.6

$$\int_{s_{k+1}}^{\theta(\pi_{k-1})} \left\{u_r\left(\pi_k, 1, s\right) - u_r\left(\pi_k, 0, s_k\right)\right\} dF\left(s\right) < 0$$

Hence $s_k > \theta\left(\pi_{k-1}\right)$, so that

$$u_r\left(\pi_{k-1}, 1, s_k\right) > u_r\left(\pi_{k-1}, 0, s_k\right)$$

This means that π_{k-1} is the next demand used in construction of the Maximally Dispersed equilibrium.

Since this construction continues until s_k hits the boundary $\underline{s}$, every feasible demand above $\underline{p}$ will appear in this construction. □

2.2. **Humped Shaped Payoffs.** One property of particular interest is the expected payoff to proposers associated with different offers. In a collection of experimental results, for example, one might check empirically how often an offer p_k is accepted, then compute the product of p_k and the observed acceptance probability in order to compute an expected payoff.

Theorem 2.8. *Suppose that $u_p\left(p, 1, s\right) = p\phi\left(s\right)$ for some strictly positive function ϕ and that there is some proposer type $s < \overline{s}$ such that $u\left(0, 0, s\right) < 0$. Then the function $q_k p_k$ is decreasing when $u_p\left(p, 0, s\right)$ is positive and increasing otherwise.*

Proof. From (2.1) in the proof of Theorem 2.4

$$q_{k+1}p_{k+1}\phi\left(s_{k+1}\right) + (1 - q_{k+1})u_p\left(p_{k+1}, 0, s_{k+1}\right) =$$

$$= q_k p_k \phi\left(s_{k+1}\right) + (1 - q_k)u_p\left(p_k, 0, s_{k+1}\right)$$

Re-arranging and using Condition 2.2 gives

$$\{q_{k+1}p_{k+1} - q_k p_k\}\, \phi\,(s_{k+1}) = (q_{k+1} - q_k) u_p\,(p_k, 0, s_{k+1})$$

By Theorem 2.4, $q_{k+1} \leq q_k$. The sign of $q_{k+1}p_{k+1} - q_k p_k$ is then determined by the sign of $u\,(p_k, 0, s_{k+1})$. □

3. Comparative Static Experiment

We now turn to an experimental investigation of the proposed equilibrium with negative interdependence. As demonstrated in the previous section, a Perfect Bayesian Nash Equilibrium of the ultimatum game with negative interdependence can account for the known experimental regularities of the game. We were able to characterize the equilibrium based on basic assumptions on the underlying preferences, without assuming specific utility function. In this section we ask whether the model with negative interdependent preferences have testable implications that can differentiate it from other models of other-regarding preferences, and in particular intention-based reciprocity.

3.1. **Theoretical Predictions.** Consider the following slight variation of the ultimatum game: instead of allowing the proposer to offer the responder anything between 0 and $\bar{x}$, allow only offers between $k\bar{x}$ and $\bar{x}$. That is, the lower bound on an offer is a proportion k of the surplus. For small enough k (e.g. 10%), it is well known from the existing experimental literature, that only very few offers are made in the range between 0 and $k\bar{x}$.

The effect of truncating the range of offers within the model of social preference is straightforward. Proposers who would otherwise propose less than $k\bar{x}$ would offer $k\bar{x}$, and the acceptance probability should not change. Any model of intention-based reciprocity that is based on psychological game [23], would predict that the conditional acceptance probability would (weakly) fall, and equilibrium offers would (weakly) increase. The intuition is simple: any offer (especially close to $k\bar{x}$) reflects lower kindness of the proposer, since the set of alternative low offers is smaller. Therefore the responder will reciprocate to a given offer with a lower probability of acceptance.

The effect of setting an upper bound on the proposer's demand (a lower bound on his offer) in the Maximally Dispersed Equilibrium (MDE) with negative interdependent preferences is more subtle. We demonstrate the arguments in Figure 3.1 using $m = 3$. The top part corresponds to the standard MDE: start with $\pi_3 = 1$ and choose s_3 such that a responder who receives a demand of 1 will believe that it came from a proposer whose type is in the interval $[s_4, s_3)$ and will be indifferent between accepting and rejecting the offer. The responder will choose q_3 (the probability of accepting a demand of 1) such that a proposer of type s_3 will be indifferent between demanding 1 and $\pi_2 < 1$. This latter demand is made by proposers whose type is in $[s\_3, s_2)$, so the responder is indifferent between accepting and rejecting π_2. Now q_2- the probability of accepting π_2 is determined by making a proposer of type s_2 indifferent between demanding π_2 and $\pi_1 < \pi_2$. Finally, π_1 is made by proposer whose type is in $[s_2, s_1]$, and a responder who observes this demand would weakly prefer to accept.

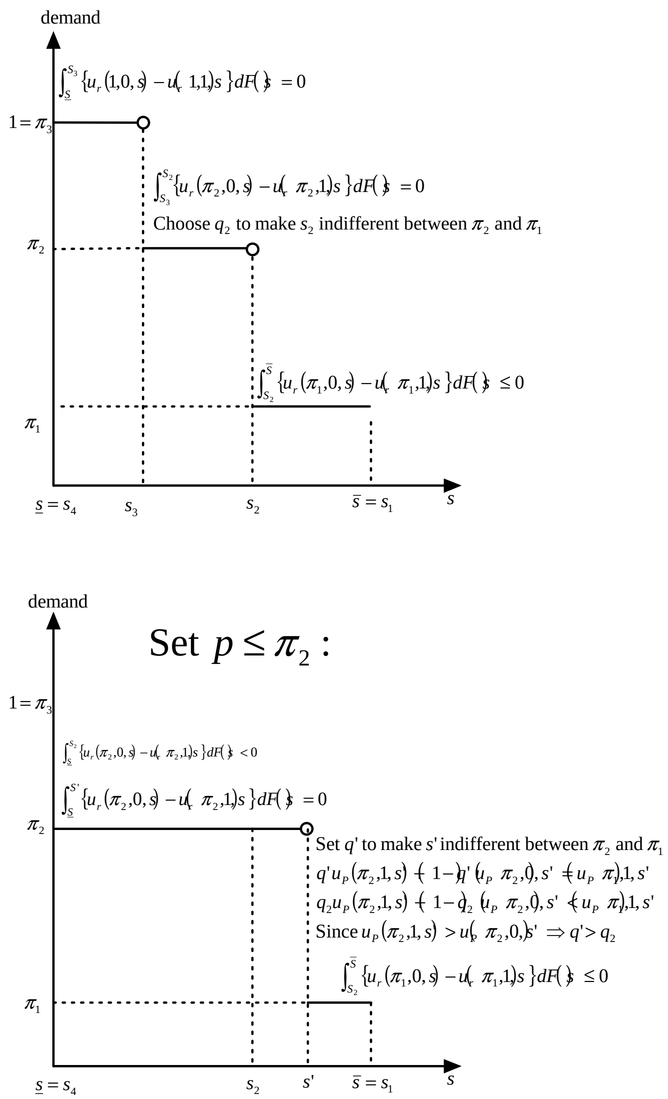

Figure 3.1. Maximally Dispersed Equilibrium with a Maximal Demand

When an upper bound on the proposer's demand is set by $p \leq \pi_2$, proposer whose type is lower than s_3 can demand at most π_2. Remembering that the responder's marginal utility of rejecting is increasing in the proposer's type, a responder who receives such a demand and had believed that it came only from a proposers whose type is in $[s_4, s_2)$ would accept π_2. Therefore, to make the responder indifferent between accepting and rejecting π_2, the set of proposer's type who demand π_2 must be $[s_4, s')$ where $s' > s_2$. That is, the subset of proposers $[s_2, s')$ who demanded π_1 before setting the bound would demand now $\pi_2 > \pi_1$. The probability of accepting π_2 is now determined by the pivotal type s'. From the construction of the original MDE, s' strictly preferred to demand π_1 to demand π_2, when the probability of acceptance is q_2. In order to make q' indifferent between the two demands, the probability of accepting π_2 must increase to $q' > q_2$. The same argument can be made when the grid of possible demands is finer, and the effect should continue beyond the upper bound itself.

To summarize, the predictions of the MDE with negative interdependent preferences is that when a minimal offer is set then: lower offers will be made, and the probability of acceptance of these offers will increase relative to the base. These predictions are in opposite directions of the predictions derived from models of social preference and intention-based reciprocity, and can serve as a simple experimental method to differentiate between these theories.

3.2. **Experimental Design and Implementation.** Subjects were recruited from the undergraduate body at the University of British Columbia by sending an e-mail message to a random group of students. After signing a consent form, the subjects received a detailed explanation about the experiment. After the subjects read the instructions, they were asked to answer some questions to verify all understood how the payment will be implemented. Those subjects who didn't fully understand the implementation, received a detailed explanation from research assistants. Only after confirming all subjects understood the procedures, the experiment started. In order to allocate the subjects to a "proposer" and "responder" role, they all participated in an "I Spy" contest. Those who found more items were designated a "proposer" and received \$5. The rest were asked to move to a nearby room and were designated a "responder." The surplus to be allocated was \$55. In order to allow both sides to quickly "learn" without creating reputational effect and to maintain the sequential structure of the experiment, each group (proposers and responders) was divided to two. Each proposer made offers to half of the responders. Each responder received offers from half of the proposers and chose whether to accept or reject each offer. After this first round each proposer made offers to the second half of the responders, and each responder received offers from the proposers he had not interacted with before. This method allows a proposer to experiment in the first round, and very fast learning. The payment was determined by choosing at random one offer for each proposer and each responder. In the control group the offers were allowed to vary between \$0 and \$55, and in the limit treatment the offers were between \$5 and \$55. From an experimental design point of view there are several points to point out: in both treatments there was a short contest to determine the role in the allocation; the design maintained anonymity and was double-blinded; the multiple offer made learning quick and efficient; there was no show-up fee for the experiment: the payment was completely determined by the performance in the experiment.

3.3. **Results.** Table 1 reports summary statistics of the base treatment and the limit treatment. A total of 52 subjects took part in the two sessions: 24 in the base (B) treatment and 28 in the limit (L) treatment. Each proposer in the base treatment made 12 offers: 6 in each round, when the offers in round 2 (R2) were made after observing the acceptance/rejection of offers in round 1 (R1). Similarly, each proposer in the limit treatment made 14 offers - half in the second round.

	B-R1	B-R2	L-R1	L-R2
Number of proposers	12	12	14	14
Average offer	19.07	21.21	15.31	15.00
Average acceptance rate	0.63	0.88	0.87	0.90
Within SD of offers	2.26	1.47	3.18	2.09
Total SD of offers	8.15	6.64	6.45	5.78

TABLE 1. Summary Statistics

Table 1 indicates the main finding of the investigation: setting a lower limit to the offer caused the mean offer to fall (by about 30%) from \$21.21 to \$15. In spite of the lower offers the average acceptance rate was marginally higher (90% in the limit treatment and 88% in the base treatment), implying that the conditional acceptance rate increased substantially. The learning and experimentation from the first to the second round could be seen by the decrease of about 35% of the within proposer standard deviation.

Table 2 reports the distribution of offers and acceptance rate. Although Table 2 reports the results for intervals, it is important to note that about 90% of offers were made in multiples of \$5. The table reveals the effect of setting a lower limit to the offers: the conditional acceptance rate increases and the frequency of low offers increases.

offer	%	Base-R1	Base-R2	Limit-R1	Limit-R2
\$0 to \$4	offers	0	0	0	0
	acceptance				
\$5 to \$9	offers	8	8	12	5
	acceptance	33	17	50	60
\$10 to \$14	offers	25	0	31	34
	acceptance	17		80	76
\$15 to \$19	offers	8	13	17	33
	acceptance	50	78	94	100
\$20 to \$24	offers	18	39	28	20
	acceptance	77	93	100	100
\$25+	offers	39	40	12	8
	acceptance	96	100	100	100

TABLE 2. Distribution of Offers and Acceptance Rate by Treatment and Round

3.3.1. *Offers.* In order to test whether the limit effect has a significant effect on offers we conduct a random-effect GLS regression. The negative effect of the limit treatment on second-round offers is significant at 1% (including both rounds the effect is significant at 5%).

# of observation=	170		Obs per group			
# of Groups=	26		min=	6		
			max=	7		
Random effect	$u_i \sim Gaussian$		Wald χ^2 (1) =	6.56		
Corr$(u_i, X) = 0$	(assumed)		$Prob > \chi^2 =$	0.01		
offer	*coef*	*SE*	*z*	$P > \|z\|$	*[95%*	*CI]*
Limit treatment	-6.208	2.423	-2.56	0.010	-10.958	-1.457
Constant	21.208	1.779	11.92	0.000	17.720	24.695
$\sigma_u = 6.1171$						
$\sigma_e = 1.8599$						
$\rho = 0.915$	(fraction of variance due to u_i)					

TABLE 3. Second-Round Offers: Random-Effect GLS

3.3.2. *Acceptance Rate.* As noted above, 90% of offers are made at multiples of \$5. This implies that using parametric assumptions, would extends those observations to intervals were offers have rarely been made.

Instead, we compare (non-parametrically, using Fisher exact test) the acceptance rate at offers of \$5, \$10, \$15, \$20 between the base treatment and the limit treatment. Note that we use both rounds since there is no significant difference between the conditional acceptance rates at different rounds, within the same treatment (for both the base and the limit treatments). Since we simultaneously test four hypotheses, care should be taken not to reject the joint null hypothesis of "no limit treatment effect" when it is true. That is, the p-values need to be adjusted such that the probability that at least one of the tests in the family would exceed the critical value under the joint null hypothesis of no effect is less than 5%. We use the most conservative approach - the Bonferroni adjustment (Savin [37, 38]), in which each p-value is multiplied by the number of tests (four in our case). It should be noted that we take a very conservative approach of using the Fisher exact test and the Bonferroni adjustment, that treats the acceptance rate at different offers as independent.

offer	B accept	B reject	L accept	L reject	Single p-value
5	2	6	6	5	0.208263
10	3	15	49	14	0.000003
15	6	2	46	1	0.052297
20	35	5	39	0	0.029196

TABLE 4. Fisher Exact p-value (one-sided) for the effect of Limit Treatment on conditional acceptance probability

As Table 4 clearly reveals, the null hypothesis that limiting the offer did not have an effect on the acceptance probability is rejected at 1%. The strongest and most dramatic effect occurred at \$10: in the first round, 25% and 31% of the offers in the base and the limit treatments, respectively, were made at that level. However, the acceptance rate in the base treatment was only 17% while in the limit treatment the acceptance rate of those offers was 80%. The experimental design allowed the proposers to learn this behavior, and in the second round there were no offers of \$10 in the base treatment, while 34% of the offers in the limit treatment were made at \$10. It is of interest to note that the proposer's expected revenue in the base treatment is maximized at an offer of \$20 (\$30.625) - which is the mode of the offer distribution, while in the limit treatment the expected revenue are maximized at an offer of \$15 (\$39.15), although the mode of the offer distribution is at \$10.

3.4. **Conclusion from the experiment.** We conclude that the outcome of the experiment is consistent with the MDE of the model with negative interdependence, while being inconsistent with models of social preference and intention-based preference.

Although we are not aware that this type of argument has been used before in the bargaining literature in Economics, it seems that economic agents are well aware of this phenomenon. For example, the Limit Treatment is strategically equivalent to allowing the offers to vary between \$0 and \$50, and giving the responder a "signing" bonus of \$5 if she accepts an offer. This contract structure is quite common in labor agreements and other contracts, and allows the proposer (employer, retailer, marketing agent) to achieve higher expected revenue.

4. Concluding comments

The arguments above illustrate that it is easy to interpret the results of the ultimatum game experiments using standard reasoning. Hopefully this interpretation is one that has some application beyond the experiments themselves. For example, it would seem possible to incorporate negative interdependence into a standard principal agent incentive problem. Another possible application can be in an auction design. In this case it is reasonable to expect that the seller has some private information that is of interest to the buyers. Conditional on this privae information which is of common interest, the buyers may have independent private valuations. The seller sets a reservation price, that acts similarly to the demand in the ultimatum game. If a buyer accepts this reservation price, she can bid in the auction. The structure of negative interdependence lends itself naturally to this problem. The insights suggested by the analysis of the ultimatum game, and in particular the equilibrium played, can be applied to this problem.

One way to view the approach in this paper is to think of the experiment as a kind of mechanism designed to extract preference and type information from participants. This is all that the model above does. There are a couple of dimensions in which the ultimatum game may not be the perfect experiment. One obvious deficiency is that it provides no information about the types of responders simply because they are only given a binary signal. This is the reason that we ignore a possible responder type in our analysis above. A second problem is that the ultimatum game creates a kind of informed principal problem for the proposer which supports multiple equilibrium outcomes. We ignore this above and simply assume that players are playing what we referred to as the Maximally Dispersed Equilibrium. Since this equilibrium supports the largest variety of proposer behavior of all the equilibria, it must be the most informative.

The ultimate experiment would presumably have proposers and responders declare their types then structure payoffs so that truthful reporting of type is a dominant strategy. But even this sidesteps a basic problem - an experiment is actually a Bayesian game between three players - the proposer, the responder, and the experimenter. The experimenter is the one for whom the stakes in the game are actually highest. The same sort of type dependencies ought to exist between the experimenter and subjects. Of course, a single experiment contains no variation in experimenter behavior that would make it possible to uncover this information, so the subjects' interpretation of the experimental design and its influence on them presents a much more complicated problem.

With this in mind, one may ask how the proposers modeled in the current study would play the Dictator game in which the proposer selects a demand then gets it for sure, and the 'responder' simply receives whatever the proposer offers. Since our proposers are better off with higher demands conditional on them being accepted, they should presumably demand all the surplus from the experiment for themselves. Of course, the reason that this doesn't happen is that exactly the same type dependence exists between the proposer and the experimenter - both the fact that the experimenter suggests a Dictator game instead of an Ultimatum game, and the other characteristics of the experiment alter the proposers perception of the payoffs in the experiment. For example, Hoffman, McCabe and Smith [27] and Cherry, Frykblom and Shogren [11] showed that implementing a subject-experimenter anonymity and generating the the pie through earned income, caused almost all dictators to keep the surplus to themselves. Furthermore, Bradsley [2] and List [31] showed that changing the dictator's strategy set to include negative giving (taking) caused almost all dictators to behave selfishly. Dana, Weber and Kuang [12] showed that many dictators were willing to leave the experimenter part of the surplus, instead of facing the choice how much to allocate to a passive responder - possibly showing preference to share with the experimenter than with the other subject (see also Lazear, Malmendier and Weber [28]). It may be impossible to control all aspects, but using the theoretical methods described in this study, it would presumably be possible to interpret the impact that the experimental design has on outcomes.

Appendix A. Equilibrium with Positive Interdependence

We assume:

Condition A.1.

$$\int_{\underline{s}}^{\overline{s}} \{u_r(p,0,s) - u_r(p,1,s)\}\, dF(s) < 0 \text{ for every } p < 1$$

This assumption implies that a responder using her prior beliefs would accept every offer except possibly the offer $p = 1$ (where the proposer gets the entire surplus from the experiment).[2] The assumption supported by experimental results of Blount [7] who showed that when demands are made by a random number generator, great majority of responders (80%) tend to accept even very high demands (95% of the pie and higher).

Theorem A.2. *Suppose that $u_p(p,0,s) - u_p(p,1,s)$ is strictly increasing in s and that $p < 1$ is a demand such that $u_p(p,1,\overline{s}) > u_p(p,0,\overline{s})$ and $u_r(p,1,\overline{s}) < u_r(p,0,\overline{s})$. Then there is a Perfect Bayesian Equilibrium in which all proposers make the offer p and this is accepted.*

Proof. By Condition A.1, the responder wants to accept every offer except possibly 1 given her prior beliefs. Let $p' > p$, and suppose that the responder believes that such a proposal is made by a proposer whose type lies in the interval $(s^*, \overline{s}]$ where s^* is chosen such that

$$\int_{s^*}^{\overline{s}} \{u_r(p',0,s) - u_r(p',1,s)\}\, dF(s) = 0$$

Since $u_r(p,1,\overline{s}) < u_r(p,0,\overline{s})$, the same inequality must hold for p'. Then from Condition A.1 and the continuity of F, such an s^* always exists. Then a responder who receives the offer p' will be just indifferent about whether or not to accept it. Choose q' such that

$$q' u_p(p',1,\overline{s}) + (1-q')u_p(p',0,\overline{s}) = u_p(p,1,\overline{s})$$

q' is chosen such that a proposer who received a type $\bar{s}$ is indifferent between demanding p and p'. By Condition 1.1 and the assumption that $u_p(p,1,\overline{s}) > u_p(p,0,\overline{s})$, $u_p(p',1,\bar{s}) > u_p(p,1,\bar{s}) > u_p(p',0,\bar{s})$, so a unique q' satisfying this condition exists. Then if any proposer whose type is less than $\overline{s}$ strictly prefers to offer p', then by Condition 1.2, the highest type proposer must also. This contradiction proves that all proposer types at least weakly prefer the offer p to p'. A similar argument is used to make downward deviations unprofitable. □

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[2] Note that Condition A.1 together with monotonicity imply that $u_r(p,0,\underline{s}) - u_r(p,1,\underline{s}) < 0$ for all $p < 1$.

It is possible to support equilibrium with multiple offers in this model, but in a restricted way.

Theorem A.3. *Suppose that $u_p(p,0,s) - u_p(p,1,s)$ is strictly increasing in s. Let $p_j > p_k$ be a pair of demands and s^* the solution to*

$$\int_{s^*}^{\overline{s}} \{u_r(p_j,0,s) - u_r(p_j,1,s)\}\, dF(s) = 0.$$

If $u_p(p_j,1,\overline{s}) > u_p(p_j,0,\overline{s})$, $u_p(p_k,1,s^*) > u_p(p_k,0,s^*)$, and $u_r(p_k,1,\overline{s}) < u_r(p_k,0,\overline{s})$, then there is a perfect Bayesian Nash equilibrium in which proposers whose type is in the interval $[s^*,\overline{s}]$ demand p_j, while proposers whose type is below s^* demand p_k. Responders accept p_k for sure and reject p_j with positive probability.

Proof. By Conditions 1.3 and A.1, and the assumption that $u_r(p_k,1,\overline{s}) < u_r(p_k,0,\overline{s})$, there is a unique type s^* satisfying the property

$$\int_{s^*}^{\overline{s}} \{u_r(p_j,0,s) - u_r(p_j,1,s)\}\, dF(s) = 0.$$

If the responder believes that the demand p_j comes from a proposer whose type is in the interval $[s^*,\overline{s}]$, then she is indifferent between accepting and rejecting the demand. By Condition A.1, the responder will strictly prefer to accept the demand p_k if she believes that it is made by proposers whose type is in the interval $[\underline{s}, s^*)$.

Since $u_p(p_k,1,s^*) > u_p(p_k,0,s^*)$, by Condition 1.1, $u_p(p_j,1,s^*) > u_p(p_k,1,s^*) > u_p(p_j,0,s^*)$. Thus there is a unique q_j satisfying

$$u_p(p_k,1,s^*) = q_j u_p(p_j,1,s^*) + (1-q_j)u_p(p_j,0,s^*) \tag{A.1}$$

Then if the responder accepts the demand p_j with probability q_j, Condition 1.2 implies that since a proposer of type s^* weakly prefers to make the higher demand (with lower probability of acceptance) p_j, every proposer of a higher type strictly prefers to demand p_j.

In a similar fashion, for each off equilibrium offer p', find x' such that the responder is indifferent between accepting and rejecting p' if she thinks that the demand was made by a proposer whose type is in the interval $[x',\overline{s}]$. The acceptance probability now depends on the offer. If $p' > p_j$, then repeating the arguments above and using the assumption that the highest proposer type prefers to have p_j accepted, it is possible to choose the acceptance probability such that a proposer of the highest type is indifferent between the demands p_j and p'. Similarly, if $p_j > p' > p_k$, then the acceptance probability is set so that a proposer of type s^* is indifferent between the demand p_j (and p_k) and the demand p'. If $p' < p_k$, then the acceptance probability is chosen such that a proposer of type $\underline{s}$ (who prefers all offers to be accepted) at least weakly prefers p_k to p'. This makes it possible to check deviations to off equilibrium prices. There are three cases, $p' > p_j$, $p_j > p' > p_k$, and $p' < p_k$. In the first case, the highest type proposer $\overline{s}$ weakly prefers the offer p_j to the offer p' by the choice of q'. If there is a proposer type $s < \overline{s}$ who strictly prefers p' to p_j, then the same must be true for a proposer of type $\overline{s}$ by Condition 1.2. Thus the deviation must be unprofitable for all proposer types. If $p' < p_k$, then since the proposer with the lowest type is indifferent, all proposer types must at least weakly prefer p_k to p' by Condition 1.2. If $p_j > p' > p_k$, then all proposer types above s^* prefer p_j by the single crossing condition. By Condition 1.2 if there is a proposer of type lower than s^* who strictly prefers the proposal p' to p_k, then type s^* must also have this strict preference if holds. This contradiction implies that types below s^* prefer p_k to p'. □

If the responder prefers to reject all offers when she believes that the proposer has the highest type $\overline{s}$, and the highest type proposer wants all offers to be accepted, then any non-zero offer or pair of offers can be supported as equilibrium offers. Furthermore, if $u_p(p_j, 0, s^*) > 0$ then (A.1) implies that the measured expected revenue of high demands - $q_j p_j$, is lower than the revenue of low demands - p_k. This finding is consistent with experimental finding. However, this is not the most interesting case: it seems more reasonable to study the possibility that proposers who have very high types would strictly prefer that some very low demands that they might make would be rejected. Also responders who believe the proposer has the highest type might well be willing to accept some very low demands by the proposer.

One standard property of experimental results is that higher demands are less likely to be accepted. The positive interdependence model is ill suited to explain the experimental results in a way that is consistent with a single equilibrium being played. The following theorem illustrates.

Theorem A.4. *Suppose that $u_p(p, 0, s) - u_p(p, 1, s)$ is strictly increasing. Then all perfect Bayesian equilibrium outcomes in which higher demands are accepted with lower probability involve at most two demands that are accepted with strictly positive probability.*

Proof. Suppose more than two demands are accepted with positive probability in some perfect Bayesian equilibrium. Let p_k and p_j be the lowest and next lowest demands made with strictly positive probability in this equilibrium, and let p' be any other demand that is accepted with strictly positive probability. Let q_k, q_j and q' be the probabilities with which these offers are accepted along the equilibrium path. By hypothesis, $q_k > q_j > q'$.

Let $\tilde{s}$ be the supremum of the the set of types for the proposer that offer p_k with strictly positive probability. If any lower type proposer weakly prefers to demand p_j or p', then by Condition 1.2, a proposer of type $\tilde{s}$ must strictly prefer the higher demand. Thus $\tilde{s} < \overline{s}$, and for each type below $\tilde{s}$,

$$q_k u_p(p_k, 1, s) + (1 - q_k) u_p(p_k, 0, s) > q_j u_p(p_j, 1, s) + (1 - q_j) u_p(p_j, 0, s)$$

and similarly for p'. The strict inequality is reversed for types $s > \tilde{s}$. Repeat the argument for the supremum of the set of types, say $\hat{s}$, for the proposer who demand p_j. Exactly as above, $\hat{s} < \overline{s}$. Then if p_j is demanded on the equilibrium path, the responder must believe that the proposer's type is in the interval $(\tilde{s}, \hat{s}]$. If the proposer makes the demand p', the responder must believe the proposer's type is in the interval $(\hat{s}, \overline{s}]$.

Now since $0 < q_j < q_k$, responders who see the offer p_j must be just indifferent about whether or not they accept it. As a consequence, a responder who thought that the proposer's type was $\hat{s}$, would strictly prefer to reject. Since $u_r(p, 0, s) - u_r(p, 1, s)$ is supermodular by Condition 1.3, the higher demand coupled with beliefs that the higher demand is made by a proposer of a higher type implies that the responder must strictly prefer to reject the demand p', a contradiction. □

The theorems above don't provide enough richness to explain existing experimental results as a single equilibrium since experimental results involve many demands and declining acceptance probabilities. In the single price equilibrium, all demands are accepted. In the two price equilibrium, the higher demand is rejected with a probability that depends on the other demand. If one intends to explain the variety of different offers by allowing many different equilibrium to be played, then there is no guarantee that higher demands will consistently be rejected with higher probability.

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