

RISK AND RETURN IN ENVIRONMENTAL ECONOMICS*

by

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PRELIMINARY AND INCOMPLETE

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Abstract: This paper examines the risk/return tradeoff for environmental investments, and the implications of that tradeoff for policy choice. Consider a policy to reduce GHG emissions, and thereby limit global warming and the damages that might result. To what extent does the value of such a policy depend on the expected future damages from warming versus the uncertainty over those damages, i.e., on the expected benefits from the policy versus their riskiness? And in designing such a policy, to what extent should the objective be a reduction in the expected temperature increases versus a reduction in uncertainty over future temperature increases. Using a simple model of a stock externality (e.g., temperature) that evolves stochastically, I examine the “willingness to pay” (WTP) for alternative policies that would reduce the expected damages under “business as usual” (BAU) versus the variance of those damages. I also show how one can compute “iso-WTP” curves (social indifference curves) for combinations of risk and expected returns as policy objectives. Given cost estimates for reducing risk and increasing expected returns, one can then compute the optimal risk-return mix for policy. I illustrate these results by calibrating the model to data for global warming.

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1 Introduction.

The adoption of an environmental policy usually imposes costs on society, but is expected to yield a social return in the form of a stream of benefits, e.g., health benefits from less pollution. Those benefits, however, usually occur in the future and may be highly uncertain, so that (as with other private or public investments) the actual return is uncertain. This is especially true for environmental policies involving stock externalities, such as increases in greenhouse gas (GHG) concentrations, the acidification of lakes and oceans, and the accumulation of toxic waste. All of these problems involve long time horizons and uncertain impacts. This paper examines the risk/return tradeoff for environmental investments and the implications of that tradeoff for policy choice. I utilize a very simple and stylized model which focuses on climate change, because of the long time horizon and considerable uncertainty involved. However, the framework developed here could easily be applied to other environmental policy problems, such as the management of toxic (including nuclear) waste.¹

Consider a policy designed to limit the accumulation of atmospheric greenhouse gases. An example would be a simple abatement policy, implemented via the imposition of a carbon tax. That policy would have some cost to society, but by reducing GHG emissions it would reduce the extent of warming by some uncertain amount, and thus it would yield uncertain future benefits. The uncertainty would arise because (a) we have limited knowledge of the relationship between GHG concentrations and temperature; and (b) we know even less about how higher global temperatures would affect GDP growth or other indicia of welfare. Thus we could ask two questions. First, how large are the expected social benefits from a given policy relative to the risk involved? Second, to what extent is the social value of the policy (measured, e.g., by society's willingness to pay for it) driven by expected outcomes without the policy versus uncertainty over those outcomes.

Now consider the design of an environmental policy, such as one to limit global warming. Alternative policies might have different impacts on the *expected* change in temperature

¹For a general discussion of the nature of the uncertainties involved in environmental policies, whether or not stock externalities are involved, see Pindyck (2007).

versus the *variance* of that change. For example, rather than (or in addition to) GHG emissions abatement, money might be spent to improve our knowledge of how GHG emissions affect temperature, and how changes in temperature affect economic output. Therefore a purely research-focused policy might do little or nothing to reduce the expected change in temperature (and hence the expected reduction in GDP growth), but might reduce the variance of our estimates of future temperature and GDP growth. As a general matter, there is value in reducing the expected damages from GHG emissions and in reducing the variance of those damages. At issue is the tradeoff between the two. To what extent should policy be aimed at reducing the expected change in temperature (and thus expected damages) versus the variance of the change in temperature?

The framework I use to address these questions is “willingness to pay” (WTP). Consider a policy that would substantially reduce expected future temperature increases, and possibly reduce the uncertainty over those temperature increases. The willingness to pay for such a policy is the *maximum* percentage reduction in current and future consumption that society should be willing to give up in order to achieve those reductions in the expectation and/or variance of future temperature increases.² This does not mean it will be possible to achieve these reductions given the revenues from this WTP; doing so may be more costly, or less costly. WTP relates only to the *demand side* of policy, i.e., society’s reservation price for the achievement of a particular policy objective.

In Pindyck (2011a, 2012) I calculated WTP for GHG abatement policies that would limit future increases in temperature to some specified amount (e.g., 3°C), based on a probability distribution for the rate of warming absent the policy, i.e., under “business as usual” (BAU). Of course WTP depends on the probability distribution, and in particular, depends on both the *expected* increase in temperature under BAU and the *standard deviation* of the increase in temperature. I calculated the trade-off between the expected value of a temperature increase versus its standard deviation under BAU as determinants of WTP. That trade-off is the marginal rate of substitution along an “iso-WTP curve,” i.e., the locus of combinations

²I used a related measure of WTP to examine climate change policies under uncertainty; see Pindyck (2011a, 2012). WTP has also been used in the context of climate change by Heal and Kriström (2002).

of the expected value versus the standard deviation of the change in temperature such that the WTP (e.g., to prevent any temperature increase) is constant. The results showed that uncertainty over temperature change can be a stronger driver of WTP than expectations, and thus should be a major focus of climate change policy.

That earlier work focused on the *starting conditions* for policy, i.e., the relative contributions to WTP of the expected value versus the variance of the change in temperature under BAU. Here I turn to the *target* of policy, and examine the trade-off between reducing the expected rate of the increase in temperature versus reducing the variance of future increases in temperature. In particular, what combinations of drift reduction and variance reduction yield the same WTP? And then taking this one step further, suppose we can estimate the costs of drift reduction and variance reduction. If those costs are convex (as we would expect), we can determine the optimal risk versus expected return tradeoff.

In the next section I lay out the simple model that I will use to address these questions. Despite its simplicity, the model captures the key features of environmental policy risk and expected return that are of interest. In Section 3, I use the model to analyze risk and return for a policy that reduces the expected rate of temperature increase by some incremental amount. Section 4 addresses policies to change the expected rate of temperature increases and/or the variance of future temperature increases, and shows how we can calculate the willingness to pay for such policies. In Section 5 I calibrate this model to information on global warming and its impact, and then use it to address the risk-return tradeoff.

2 A Simple Model.

I use a simple model in which an environmental stock variable, in this case temperature change, follows an arithmetic Brownian motion (ABM) with positive drift and some volatility.³ The impact on consumption is through the real growth rate of GDP, which is linearly

³In the context of global warming, the actual stock variable is the atmospheric GHG concentration, which in turn drives temperature change, but with a lag. I ignore the lag, and thus can treat temperature change itself as the stock variable. In the context of toxic waste, the stock variable could be the quantity or concentration of waste material over some geographic area.

related to the change in temperature. Welfare is measured using CRRA utility. Various expectations and variances of utility can be found analytically, making the model very tractable.

2.1 Temperature and GDP Growth.

Let X_t denote the anthropomorphic increase in temperature from its current level. I assume that under “business as usual,” X_t follows an arithmetic Brownian motion (ABM) with drift and volatility α_X and σ_X respectively. I also assume that the impact of an increase in X_t is to reduce the real rate of growth of consumption, g_t . This differs from the damage function specifications used in most integrated assessment models of climate change, where X_t is assumed to reduce consumption directly.⁴ Theoretical arguments and empirical evidence support the view that higher temperatures (and environmental damage in general) should affect the growth rate of consumption rather than its level, as in the model I use here.⁵

The process for X_t is thus:

$$dX = \alpha_X dt + \sigma_X dz , \tag{1}$$

where dz is the increment of a Weiner process. The growth rate of consumption is given by the following linear relationship:

$$g_t = g_0 - \gamma X_t , \tag{2}$$

⁴Most economic studies of climate change relate the temperature increase T to GDP through a “loss function” $L(T)$, with $L(0) = 1$ and $L' < 0$, so GDP at a horizon H is $L(T_H)GDP_H$, where GDP_H is but-for GDP with no warming. These studies — see, e.g., Nordhaus (2008) — typically use an inverse-quadratic or exponential-quadratic function. The loss function $L(T)$ implies that if temperatures rise but later fall, GDP could return to its but-for path with no permanent loss.

⁵There are several reasons to expect higher temperatures to affect the growth rate of GDP as opposed to the level. First, some effects of warming will be permanent; e.g., destruction of ecosystems, extinction of species, and deaths from weather extremes. A growth rate effect allows warming to have a permanent impact. Second, there is theoretical support for a growth rate effect. The resources needed to counter the impact of warming will reduce those available for R&D and capital investment, reducing growth. Adaptation to rising temperatures is equivalent to the cost of increasingly strict emission standards, which, as Stokey (1998) has shown with an endogenous growth model, reduces the rate of return on capital and lowers the growth rate. (Also see the related study by Brock and Taylor (2010).) Finally, there is empirical support for a growth rate effect. Using historical data on temperatures and precipitation over 50 years for a panel of 136 countries, Dell, Jones, and Olken (2008, 2009) have shown that higher temperatures reduce GDP growth rates but not levels. Also, using economic and financial market data for a panel of 147 countries over the period 1950 to 2007, Bansal and Ochoa (2011a, b) show that increases in temperature have a strong negative impact on economic growth. For an analysis of the policy implications of direct versus growth rate impact, see Pindyck (2011a).

where g_0 is the growth rate absent any warming. Thus g_t also follows an ABM:

$$dg = -\gamma\alpha_X dt - \gamma\sigma_X dz \equiv -\alpha dt - \sigma dz . \quad (3)$$

Eqn. (2) implies that at any time s ,

$$g(s) = g_0 - \alpha s - \sigma \int_0^s dz = g_0 - \alpha s - \sigma z(s) ,$$

so consumption at a future time t can be written as:

$$C_t = C_0 e^{\int_0^t g(s) ds} = C_0 e^{g_0 t - \frac{1}{2} \alpha t^2 - \sigma \int_0^t z(s) ds} , \quad (4)$$

To measure welfare, I posit a constant relative risk aversion (CRRA) social utility function:

$$U(C_t) = C_t^{1-\eta} / (1-\eta) , \quad (5)$$

where η is the index of relative risk aversion (and $1/\eta$ is the elasticity of intertemporal substitution). At time 0, welfare (under business as usual) is then given by:

$$W_0 = \frac{1}{1-\eta} \mathcal{E}_0 \int_0^\infty C_t^{1-\eta} e^{-\delta(t-s)} dt \quad (6)$$

where δ is the rate at which utility is discounted.

2.2 Willingness to Pay.

Suppose that with some expenditure over time, society could reduce α_X and/or σ_X , i.e., reduce the expected rate of temperature change and/or the volatility of temperature change (and hence the variance of the change in temperature at any arbitrary date in the future). Let α_X and σ_X be the drift and volatility of X_t under BAU, and α'_X and σ'_X be the corresponding drift and volatility under a policy that has a permanent cost to society of w percent of consumption. Here $\alpha'_X \leq \alpha_X$ and $\sigma'_X \leq \sigma_X$. Willingness to pay is the maximum value of w that society would accept to reduce α_X to α'_X and reduce σ_X to σ'_X .

Note that WTP is society's reservation price for achieving the policy objective $(\alpha_X, \sigma_X) \rightarrow (\alpha'_X, \sigma'_X)$. However, it is a reservation price for a particular type of policy and a particular

form of pricing: At time $t = 0$ the policy is adopted (with no option to wait for more information), and the drift and volatility of X_t are immediately changed to their new values. The payment flow, however, starts at $t = 0$ and continues forever, i.e., takes the form of a permanent reduction of consumption of w percent (yielding a permanent flow of revenue to pay for the policy objective). One might naturally argue that this is unrealistic. First, we would expect that changing α_X and/or σ_X should take time, and perhaps considerable time. Second, there is little reason to expect that the flow cost of achieving this policy objective will be a fixed percentage of GNP or consumption. Thus WTP might instead be defined as the willingness to give up a particular time-varying percentage of consumption in return for a shift in α_X and/or σ_X that occurs over some period of time. However, the simplification that I use (α_X and/or σ_X change instantly and the flow cost is a constant percentage of consumption) adds clarity to the basic results.

Welfare under the policy is given by:

$$W_1(\alpha'_X, \sigma'_X) = \frac{(1-w)^{1-\eta}}{1-\eta} \mathcal{E}'_0 \int_0^\infty C_t^{1-\eta} e^{-\delta t} dt, \quad (7)$$

where \mathcal{E}'_0 denotes the expectation at $t = 0$ when the drift and volatility of X_t are α'_X and σ'_X . Under BAU (i.e., no policy), welfare is:

$$W_2 = \frac{1}{1-\eta} \mathcal{E}_0 \int_0^\infty C_t^{1-\eta} e^{-\delta t} dt, \quad (8)$$

where \mathcal{E}_0 is the expectation under the original drift and volatility, α_X and σ_X . Then WTP is the value w^* that equates W_1 and W_2 . Note that w^* is a percentage of consumption. (I will treat GDP and consumption as interchangeable, and assume that all losses from higher temperatures, including health and ecosystem damage, can be monetized and included in GDP.)

From eqns. (2) and (3), the drift and volatility of temperature correspond to a drift and volatility of the real growth rate g , i.e., $\alpha = \gamma\alpha_X$, $\sigma = \gamma\sigma_X$, $\alpha' = \gamma\alpha'_X$, and $\sigma' = \gamma\sigma'_X$. In what follows I will refer to changes in α and σ rather than α_X and σ_X .

Suppose that under BAU, the drift and volatility of g are α_0 and σ_0 respectively. If we specify a target drift and volatility, α' and σ' , we can calculate the WTP for a policy

that would achieve this target. Suppose that WTP is w_1^* . We are interested, however, in the tradeoff between drift reduction and variance reduction as targets of policy. That is, we want to know what combinations of drift reduction and variance reduction will yield the same WTP, w_1^* , assuming we start at α_0 and σ_0 . We can compute the locus of such combinations and thereby obtain an “iso-WTP curve,” i.e., combinations of α and σ for which, starting at α_0 and σ_0 , all have the same WTP.

An iso-WTP curve is illustrated in Figure 1. Point A is the starting drift and volatility, and points B and C are two targets, both of which have the same WTP of $w_1^* = .03$. Moving from A to *any* point on the curve will have the same WTP of .03. Note that α increases as we move *down* the vertical axis, so the curve represents a social indifference curve between expected return (a lower value of α) and risk (a higher value of σ). It is thus analogous to the return-risk indifference curve of an investor choosing a portfolio of equities and a risk-free asset. The figure also shows another iso-WTP curve, labelled $w_2^* = .05$; moving from α_0 and σ_0 to any point on this curve would have a WTP of .05.

If we knew the cost of reducing α and the cost of reducing σ , we could plot an iso-cost line. If those costs were linear in the change in α and the change in σ , the iso-cost line would be a straight line, as drawn in Figure 1. Its tangency with the iso-WTP curve $w_1^* = .03$ at point B is the cost-minimizing target combination of α and σ . There are other target combinations of α and σ that have a WTP of .03, but they would be more costly to achieve.

In Figure 1, moving from point A to point B has the same WTP = .03 as moving from point A to point C. We can also calculate the combination of *starting values* for α_0 and σ_0 that yield the same WTP = .03 when moving to point B. Doing so is useful because it allows us to determine combinations of temperature drift and volatility under BAU that have the same welfare implications. In other words, how important is the expected change in temperature (under BAU) relative to its variance as determinants of the demand for policy?

This is illustrated in Figure 2, which shows two iso-WTP curves. Curve 1 gives combinations of starting values α_0 and σ_0 such that the movement to point B has a WTP = .03. Thus $w^*(A \rightarrow B) = w^*(D \rightarrow B) = w^*(A \rightarrow C) = .03$. But $w^*(D \rightarrow A) = w^*(B \rightarrow C) = 0$, so $w^*(D \rightarrow C) = .03$. Thus moving from any point on Curve 1 to any point on Curve 2 will

have the same WTP.

The iso-WTP and iso-cost curves in Figures 1 and 2 are just illustrative examples. Later I will calibrate this simple model of global warming given by eqns. (1) to (5) to a set of estimates of temperature change and confidence intervals compiled by the IPCC (2007), and use the calibrated model to obtain iso-WTP curves along the lines of those shown in Figures 1 and 2.

3 Risk and Return for Policy Outcomes.

Consider an emissions abatement policy that reduces the expected growth rate of temperature and thereby increases the expected rate of growth of consumption. Putting aside the cost of the policy, we want to compare the expected benefit to the uncertainty over that benefit, i.e., compare the expected return to the riskiness of that return. To do this, I begin with a simple two-period example in which consumption today is known but consumption at a future time T is uncertain. I examine the benefit that results from a small increase in the drift of the growth rate (which would result from a small decrease in the expected rate of increase of temperature). I then turn to a continuous-time model, and again consider a policy that results in a small increase in the drift of the growth rate. Of interest is the expectation and variance of the cumulative return from the policy over the entire horizon.

3.1 Two Periods.

In this simple case, $C_0 = 1$ and C_T is uncertain, the outcome of the stochastic growth process, as given by eqn. (4). Given α and σ , welfare is stochastic and is given by:

$$\tilde{W}_T = \frac{1}{1 - \eta} \left[1 + (C_T^{1-\eta} e^{-\delta T}) \right] \quad (9)$$

Now consider a policy that results (at $t = 0$) in an incremental decrease in α . The (stochastic) return (in terms of increased welfare) from that policy is

$$\tilde{r}_{\alpha T} = -\frac{\partial \tilde{W}_T}{\partial \alpha} = -\frac{1}{1 - \eta} \frac{\partial}{\partial \alpha} C_T^{1-\eta} e^{-\delta T} . \quad (10)$$

Note that

$$C_T^{1-\eta} = e^{(1-\eta) \int_0^T g(s) ds} ,$$

and from eqn. (3), the growth rate $g(s)$ at time $s > 0$ is given by

$$g(s) = g_0 - \alpha s - \sigma \int_0^s dz = g_0 - \alpha s - \sigma z(s) .$$

Thus

$$C_T^{1-\eta} = e^{g_0 T - \frac{1}{2}(1-\eta)\alpha T^2 - \sigma(1-\eta) \int_0^T z(s) ds} , \quad (11)$$

and

$$-\frac{\partial}{\partial \alpha} C_T^{1-\eta} = \frac{1}{2}(1-\eta)T^2 C_T^{1-\eta} . \quad (12)$$

The (stochastic) return is thus

$$\tilde{r}_{\alpha T} = \frac{1}{2}T^2 C_T^{1-\eta} e^{-\delta T} . \quad (13)$$

We want the expectation and variance of this return. The Appendix gives formulas for the expectation of C_T and $C_T^{1-\eta}$. Using those formulas, the expected return is:

$$\begin{aligned} \bar{r}_{\alpha T} &= \frac{1}{1-\eta} \mathcal{E}_0 \left(\frac{\partial}{\partial \alpha} C_T^{1-\eta} e^{-\delta T} \right) = \frac{1}{2}T^2 \mathcal{E}_0(C_T^{1-\eta}) e^{-\delta T} \\ &= \frac{1}{2}T^2 e^{-\rho_0 T - \frac{1}{2}\alpha(1-\eta)T^2 + \frac{1}{6}\sigma^2(1-\eta)^2 T^3} \end{aligned} \quad (14)$$

where

$$\rho_0 \equiv \delta + (\eta - 1)g_0$$

. Note from eqn. (12) that

$$\mathcal{E}_0 \left(\frac{\partial}{\partial \alpha} C_T^{1-\eta} \right)^2 = \frac{1}{4}(1-\eta)^2 T^4 \mathcal{E}_0(C_T^{2-2\eta}) . \quad (15)$$

Using results in the Appendix to get $\mathcal{E}_0(C_T^{2-2\eta})$,

$$\mathcal{E}_0 \left(\frac{\partial}{\partial \alpha} C_T^{1-\eta} \right)^2 = \frac{1}{4}(1-\eta)^2 T^4 e^{-2\rho_0 T - \alpha(1-\eta)T^2 + \frac{2}{3}\sigma^2(1-\eta)^2 T^3} .$$

Thus the variance of the return is given by

$$\begin{aligned}
\mathcal{V}(r_{\alpha T}) &= \mathcal{E}_0(\tilde{r}_{\alpha T}^2) - \bar{r}_{\alpha T}^2 \\
&= \frac{1}{4}T^4 \mathcal{E}_0(C_T^{2-2\eta})e^{-2\delta T} - \bar{r}_{\alpha T}^2 \\
&= \frac{1}{4}T^4 e^{-2\rho_0 T - \alpha(1-\eta)T^2 + \frac{2}{3}\sigma^2(1-\eta)^2 T^3} - \frac{1}{4}T^4 e^{-2\rho_0 T - \alpha(1-\eta)T^2 + \frac{1}{3}\sigma^2(1-\eta)^2 T^3} \\
&= \frac{1}{4}T^4 e^{-2\rho_0 T - \alpha(1-\eta)T^2 + \frac{1}{3}\sigma^2(1-\eta)^2 T^3} [e^{\frac{1}{3}\sigma^2(1-\eta)^2 T^3} - 1] \\
&= \frac{1}{4}T^4 [\mathcal{E}_0(C_T^{1-\eta})e^{-\delta T}]^2 [e^{\frac{1}{3}\sigma^2(1-\eta)^2 T^3} - 1]
\end{aligned} \tag{16}$$

The standard deviation of the return is

$$\text{SD}(r_{\alpha T}) = \frac{1}{2}T^2 \mathcal{E}_0(C_T^{1-\eta})e^{-\delta T} [e^{\frac{1}{3}\sigma^2(1-\eta)^2 T^3} - 1]^{1/2} \tag{17}$$

One way to characterize the relative riskiness of this return is by its Sharpe ratio, i.e., the ratio of the expected return to its standard deviation. From eqns. (14) and (17), the Sharpe ratio is

$$S_{\alpha T} = \frac{\bar{r}_{\alpha T}}{\text{SD}(r_{\alpha T})} = [e^{\frac{1}{3}\sigma^2(1-\eta)^2 T^3} - 1]^{-1/2} \tag{18}$$

The Sharpe ratio is useful because it summarizes the effects of changes in various parameters, such as volatility, σ . As can be seen from eqns. (14) and (17), an increase in σ causes an increase in *both* the expected return and the standard deviation of the return. But the effect on the standard deviation is larger, so that an increase in σ causes a decrease in $S_{\alpha T}$.

Note the Sharpe ratio does not depend on the drift, α . The reason is that the exponential growth rates for both the expected return and the standard deviation include the factor $\frac{1}{2}\alpha(1-\eta)T^2$, so this factor cancels out of the ratio. Also note that if $\eta \neq 1$, $S_{\alpha T} \rightarrow 0$ as $T \rightarrow \infty$. The reason is that the standard deviation of the return grows faster with the time horizon than the expected return. Finally, as we would expect, an increase in the index of relative risk aversion, η , leads to a decrease in the Sharpe ratio.

3.2 Continuous-Time, Finite Horizon.

Now suppose we have a horizon T so that welfare is given by the integral of discounted utility over $t = 0$ to T . Taking future consumption as unknown, welfare is:

$$\tilde{W} = \frac{1}{1-\eta} \int_0^T C_t^{1-\eta} e^{-\delta t} dt \equiv \int_0^T \tilde{W}_t dt . \tag{19}$$

Thus \tilde{W}_t is (uncertain) discounted utility at time $t > 0$.

Once again we consider a policy that increases α incrementally. As before, it is easily seen that the (uncertain) return from this policy is

$$\tilde{r}_\alpha = \int_0^T \frac{1}{2} t^2 C_t^{1-\eta} e^{-\delta t} dt = \int_0^T \frac{1}{2} (1-\eta) t^2 \tilde{W}_t dt . \quad (20)$$

We can now examine returns over the full time horizon.

We want the expectation, standard deviation, and Sharpe ratio for the cumulative return over the horizon 0 to T , \tilde{r}_α . The expectation is straightforward; it is simply

$$\bar{r}_\alpha = \int_0^T \frac{1}{2} t^2 \mathcal{E}_0(C_t^{1-\eta}) e^{-\delta t} dt = \int_0^T \frac{1}{2} t^2 e^{-\rho_0 t - \frac{1}{2} \alpha (1-\eta) t^2 + \frac{1}{6} \sigma^2 (1-\eta)^2 t^3} dt . \quad (21)$$

The variance is $\mathcal{V}(\tilde{r}_\alpha) = \mathcal{E}_0(\tilde{r}_\alpha^2) - \bar{r}_\alpha^2$, so we need to find

$$\mathcal{E}_0(r_\alpha^2) = \mathcal{E}_0 \left(\int_0^T \frac{1}{2} t^2 C_t^{1-\eta} e^{-\delta t} dt \right)^2 \quad (22)$$

We can write the integral as:

$$\begin{aligned} r_\alpha^2 &\approx \left[\sum_{j=0}^N \frac{1}{2} j^2 C_j^{1-\eta} e^{-j\delta} \right]^2 \\ &\approx \sum_{i=0}^N \sum_{j=0}^N \frac{1}{4} i^2 j^2 C_i^{1-\eta} C_j^{1-\eta} e^{-\delta(i+j)} \end{aligned} \quad (23)$$

where $N = T/\Delta t$.

As shown in the Appendix,

$$\mathcal{E}_0(C_i^{1-\eta} C_j^{1-\eta}) = C_0^{2(1-\eta)} e^{(1-\eta)g_0(i+j)+b(i,j)} , \quad (24)$$

where

$$b(i,j) = \frac{1}{12} \sigma^2 (1-\eta)^2 (i+j)^3 - \frac{1}{2} \alpha (1-\eta) (i^2 + j^2) . \quad (25)$$

Setting $C_0 = 1$ and letting $\Delta t \rightarrow 0$,

$$\mathcal{E}_0(r_\alpha^2) = \int_0^T \int_0^T \frac{1}{4} t^2 s^2 e^{-\rho_0(t+s) - \frac{1}{2} \alpha (1-\eta) (t^2+s^2) + \frac{1}{12} \sigma^2 (1-\eta)^2 (t+s)^3} dt ds . \quad (26)$$

Using eqns. (21) and (26), we can find (numerically) the expectation, standard deviation, and Sharpe ratio for the cumulative return \tilde{r}_α . Figure 3 shows the Sharpe ratio as a function

of σ for three values of η , using a time horizon of $T = 150$ years. (The value used for α and the range for σ come from a calibration to climate change information that will be discussed later.)

4 WTP and Policy Targets.

In the previous section I considered marginal policies, i.e., policies that would result in a marginal reduction in α . This allowed for a simple characterization of such policies in terms of a Sharpe ratio. I turn now to policies that would result in a substantial change in the drift α and/or the volatility σ , and evaluate those policies in the context of willingness to pay. I begin with policies for which changes are made at one or more discrete points in time. This is simpler than the continuous-time case, which I turn to next.

4.1 WTP: Consumption Now and at Future T .

Again, $C_0 = 1$, and C_T is uncertain. Under BAU (no abatement policy), $\alpha = \alpha_0$ and $\sigma = \sigma_0$. We want to find the fraction of current and future consumption society would give up to decrease α and/or decrease σ . Define $a(\alpha_1, \sigma_1, T) = -(1/2)\alpha_1(1 - \eta)T^2 + (1/6)\sigma_1^2(1 - \eta)^2T^3$. Then to obtain the WTP to move from (α_0, σ_0) to (α_1, σ_1) , write the social welfare for this change as:

$$W_{1T} = \frac{[1 - w(\alpha_1, \sigma_1, T)]^{1-\eta}}{1 - \eta} [1 + e^{-\rho_0 T + a(\alpha_1, \sigma_1, T)}]. \quad (27)$$

Under BAU, welfare is

$$W_{2T} = \frac{1}{1 - \eta} [1 + e^{-\rho_0 T + a(\alpha_0, \sigma_0, T)}] \quad (28)$$

Equating W_{1T} and W_{2T} yields the WTP for this change:⁶

$$w^*(\alpha_1, \sigma_1, T) = 1 - \left[\frac{1 + e^{-\rho_0 T + a(\alpha_0, \sigma_0, T)}}{1 + e^{-\rho_0 T + a(\alpha_1, \sigma_1, T)}} \right]^{\frac{1}{1-\eta}}, \quad (29)$$

where $\rho_0 \equiv \delta + (\eta - 1)g_0$ as before.

⁶Of course it may or may not be possible to shift from (α_0, σ_0) to (α_1, σ_1) . This WTP is simply the most society would sacrifice for this change in α and σ , were it feasible to do so with the resulting revenue.

We would like to generate iso-WTP curves, i.e., different combinations of (α', σ') such that $w^*(\alpha', \sigma', T) = w^*(\alpha_1, \sigma_1, T)$. This would describe a “risk-return” trade-off, i.e., a trade-off between decreasing α and decreasing σ that yields the same WTP as for moving from (α_0, σ_0) to (α_1, σ_1) . For this simple two-period case, this is easily done analytically. Let $a(T)$ denote the function a with α and σ set to their starting (calibrated for BAU) values of α_0 and σ_0 . Then using eqn. (29), $w^*(\alpha', \sigma', T) = w^*(\alpha_1, \sigma_1, T)$ implies:

$$a(\alpha', \sigma', T) = a(\alpha_1, \sigma_1, T) , \quad (30)$$

which in turn implies that

$$\sigma' = \sqrt{\sigma_1^2 + \frac{3(\alpha' - \alpha_1)}{(1 - \eta)T}} , \quad (31)$$

or equivalently,

$$\alpha' = \alpha_1 + \frac{1}{3}(1 - \eta)T(\sigma'^2 - \sigma_1^2) . \quad (32)$$

Thus moving from (α_0, σ_0) to any combination (α', σ') that satisfies eqn. (32) will have the same WTP = $w^*(\alpha_1, \sigma_1, T)$.

The slope of the iso-WTP curve is the marginal rate of substitution (MRS) between the two policy targets, α' and σ' . From (32), the MRS is:

$$\text{MRS} = \frac{d\alpha'}{d\sigma'} = \frac{2}{3}\sigma'(1 - \eta)T . \quad (33)$$

Note that as T increases, the MRS becomes larger in magnitude, i.e., it takes larger and larger increases in α' to compensate for any given reduction in σ' . (This is because of the $\sigma^2 T^3$ term versus αT^2 in the function a .)

It is important to keep in mind that the (α', σ') combinations on the iso-WTP curve represent different *ending* positions that result from a policy. In other words, we are always starting from (α_0, σ_0) . All policies that move from this starting position to an ending position on the curve have the same WTP.

4.2 Multiple Time Periods; Action at $t = 0$.

Suppose there are n 50-year time intervals: $T_1 = 50$, $T_2 = 100$, ..., $T_n = 50n$, and suppose we can decrease α and/or reduce σ starting immediately by reducing consumption now (i.e.,

at $t = 0$) and at every future T_i by some fraction. In this case, to find the WTP to move from (α_0, σ_0) to (α_1, σ_1) , write the welfare measures W_1 and W_2 as:

$$W_1 = \frac{[1 - w(\alpha_1, \sigma_1)]^{1-\eta}}{1 - \eta} \left[1 + \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_1, \sigma_1, T_i)} \right] \quad (34)$$

$$W_2 = \frac{1}{1 - \eta} \left[1 + \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_0, \sigma_0, T_i)} \right] \quad (35)$$

Thus to obtain $w^*(\alpha_1, \sigma_1)$, just replace eqn. (29) with:

$$w^*(\alpha_1, \sigma_1) = 1 - \left[\frac{1 + \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_0, \sigma_0, T_i)}}{1 + \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_1, \sigma_1, T_i)}} \right]^{\frac{1}{1-\eta}}. \quad (36)$$

As before, we can generate “iso-WTP” curves, i.e., different combinations of (α', σ') such that $w^*(\alpha', \sigma') = w^*(\alpha_1, \sigma_1)$. These “iso-WTP” curves will depend on n because as we saw above, the $d\sigma^2/d\alpha$ that keeps W_T fixed varies with T . Also, we cannot generate these curves analytically. From eqn. (36), the (α', σ') that gives the same WTP as (α_1, σ_1) is the solution of:

$$\sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha', \sigma', T_i)} = \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_1, \sigma_1, T_i)} \quad (37)$$

Eqn. (37) can be solved numerically for combinations of α' and σ' .

Eqn. (37) applies to a specific WTP, namely the WTP that applies to moving from (α_0, σ_0) to (α_1, σ_1) . Starting from (α_0, σ_0) , however, we can find combinations of α' and σ' that yield the same *arbitrary* WTP. Denoting the arbitrary WTP by w^* , we find the combinations of α' and σ' by solving:

$$\sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha', \sigma', T_i)} = (1 - w^*)^{\eta-1} \left[1 + \sum_{i=1}^n e^{-\rho_0 T_i + a(\alpha_0, \sigma_0, T_i)} \right] - 1. \quad (38)$$

4.3 WTP: Continuous Time, with Immediate Action.

Welfare over some future time horizon, with no policy intervention, is given by

$$W_2 = \frac{1}{1 - \eta} \int_0^\infty e^{-\rho_0 t + a(\alpha_0, \sigma_0, t)} dt. \quad (39)$$

Now once again consider a policy that would move from (α_0, σ_0) to (α_1, σ_1) . Denote the WTP for this policy by w_1 , and note that with the intervention, welfare is:

$$W_1 = \frac{1}{1 - \eta} \int_0^\infty (1 - w_1)^{1-\eta} e^{-\rho_0 t + a(\alpha_1, \sigma_1, t)} dt, \quad (40)$$

where once again, $a(\alpha_1, \sigma_1, t) = -(1/2)\alpha_1(1 - \eta)t^2 + (1/6)\sigma_1^2(1 - \eta)^2t^3$. Equating W_1 and W_2 , we get:

$$w_1 = 1 - \left[\frac{G(\alpha_1, \sigma_1)}{G(\alpha_0, \sigma_0)} \right]^{\frac{1}{\eta-1}}, \quad (41)$$

where

$$G(\alpha_0, \sigma_0) = \int_0^\infty e^{-\rho_0 t + a(\alpha_0, \sigma_0, t)} dt, \quad (42)$$

and likewise for $G(\alpha_1, \sigma_1)$. Thus, given starting values of α and σ (based, say, on a calibration of a future T against the IPCC or other distributions), we can calculate the WTP to increase α and/or decrease σ .

As before, we would like to obtain iso-WTP curves, i.e., combinations of α' and σ' for which the WTP is again w_1 . This can be done numerically by finding combinations of α' and σ' that satisfy

$$G(\alpha', \sigma') = G(\alpha_1, \sigma_1). \quad (43)$$

We can also obtain combinations of α' and σ' for which the WTP is equal to some arbitrary number, w . From eqn. (41), we find those combinations that satisfy

$$G(\alpha', \sigma') = (1 - w)^{\eta-1} G(\alpha_0, \sigma_0). \quad (44)$$

Finally, note that the integration over time cannot be carried out over an arbitrarily large horizon. It is necessary to limit the time horizon to some number of years, e.g., 200 or 300.

5 Calibration and Results.

Although it is exceedingly simple, the model specified in Section 2 can be informative about risk and return in the context of climate change policy. In this section, I calibrate the model by determining values of the parameters α_X , σ_X , and γ that are consistent with recent studies of global warming and its impact. I then calculate iso-WTP curves and discuss their implications.

5.1 Calibration of α and σ under BAU.

The most recent IPCC report (2007) states that growing GHG emissions (i.e., under BAU) would likely lead to a doubling of the atmospheric CO_{2e} concentration relative to the pre-industrial level by mid-century, which would “most likely” cause an increase in global mean temperature between 2.0°C to 4.5°C by 2100, with an expected value of 2.5°C to 3.0°C. The IPCC report indicates that this range, derived from the results of 22 scientific studies the IPCC surveyed, represents a roughly 66- to 90-percent confidence interval, i.e., there is a 5 to 17-percent probability of a temperature increase above 4.5°C. The 22 studies surveyed by the IPCC also provide rough estimates of the outer tail of the temperature distribution. In summarizing them, the IPCC translated the implied outcome distributions into a standardized form that allows comparability across the studies, and created graphs showing multiple distributions implied by groups of studies. Those distributions suggest that there is a 5% probability that a doubling of the CO_{2e} concentration relative to the pre-industrial level would lead to a global mean temperature increase, T , of 7°C or more.

I will use, for a horizon of 100 years, $\mathcal{E}(X) = 3^\circ\text{C}$ and a 5% probability of a temperature increase $\geq 7^\circ\text{C}$. The 5% point is 1.65 standard deviations above the mean (because eqn. (1) implies that T_t is normally distributed), so one standard deviation is $4/1.65 = 2.42$. Thus $\alpha_X = 3/100 = .03$, and $\sigma_X = 2.42/\sqrt{100} = 0.242$. More recent studies have larger numbers for the expected temperature change and for the 5% point, but I will use these earlier (perhaps overly optimistic) estimates for now.⁷

We need the process for the growth rate g_t , and thus we need an estimate of γ . To obtain γ I use eqn. (51) for expected future consumption, combined with estimates of the expected loss of GDP at a specific temperature change. As in earlier work, I use the IPCC (2007) estimates that the loss of GDP resulting from a temperature change of 4°C is “most likely” in the range of 1% to 5%. I will use the upper end of that range, i.e., 5%. Thus, I take

⁷For more recent estimates of the temperature change distribution, see Sokolov et al (2009). For more recent — and much more pessimistic — estimates of the impact of higher temperatures on the growth rate, see Bansal and Ochoa (2011a, b).

$\alpha'_X = .04$, i.e., $\mathcal{E}_0(X_{100}|\alpha'_X = .04) = 4^\circ\text{C}$. Using eqn. (51), for $t = 100$,

$$\mathcal{E}_0(C_t) = e^{g_0t - (1/2)\alpha'_X\gamma t^2 + (1/6)\sigma_X^2\gamma^2 t^3} = .95e^{g_0t} ,$$

so that

$$-\frac{1}{2}\alpha'_X\gamma t^2 + \frac{1}{6}\sigma_X^2\gamma^2 t^3 = \ln(.95) . \quad (45)$$

Substituting $t = 100$, $\alpha'_X = .04$ and $\sigma_X = .242$ yields the following quadratic equation for γ :

$$\gamma^2 - .0205\gamma + .00000312 = 0$$

If $\sigma_X = 0$, $\gamma = .000152$, so we want the smaller root of this equation, which yields $\gamma = .00025$.

Thus we have $\alpha_X = .03$, $\sigma_X = .242$, and $\gamma = .00025$, so that $\alpha = \alpha_X\gamma = .0000075$ and $\sigma = \sigma_X\gamma = .000061$. These are the values for α and σ that correspond to the starting values (α_0, σ_0) when calculating WTP and iso-WTP curves.

Parameter Values. WTP also depends on the index of relative risk aversion η , the rate of time preference δ , and the base level growth rate g_0 . Thus we must select values for these parameters.

The finance and macroeconomics literatures have estimates of η ranging from 1.5 to 6, and estimates of δ ranging from .01 to .04. The per capita real growth rate g_0 , measured from historical data, is about .02. It has been argued that for intergenerational comparisons δ should be close to zero, on the grounds that society should not value the well-being of our great-grandchildren less than our own. Without taking sides in that (essentially ethical) argument, I set $\delta = 0$ for the calculations that follow. (For values of δ of .01 or higher, WTP is close to zero, and it is difficult to justify *any* abatement policy.)

Likewise, while values of η above 4 may be consistent with the (relatively short-horizon) behavior of investors, we might apply lower values to welfare comparisons involving future generations. In the (deterministic) Ramsey growth model, the optimal savings rate is $s^* = (R - \delta)/\eta R$, where R is the consumption discount rate and return on investment. If $R = .04$ and $\delta = .02$, $s^* = 1/2\eta$, suggesting η should be in the range of 2 to 4. Lower values of η result in a higher WTP, so I generally set $\eta = 2$.

5.2 Results.

I calculate iso-WTP curves using the parameter values discussed above. Figure 4 shows three iso-WTP curves, generated using starting values for α and σ based on the calibration discussed in the next section. Other parameters are $g_0 = .02$, $\delta = 0$, $\eta = 1.5$, and the horizon is $T = 300$ years. The starting drift and volatility are $\alpha_0 = .0000075$ and $\sigma_0 = .000061$, shown as the small circle at the bottom of the diagram. The WTP is calculated to reduce α to zero, but leave σ unchanged, and turns out to be .0450. (The WTP is large because I am using a relatively low value for the index of risk aversion, η .) The curve shows other combinations of α and σ that (relative to the starting values α_0 and σ_0) also have a WTP of .0450. The other two curves show combinations of α and σ that, starting from α_0 and σ_0 , have WTPs of .03 and .06. (Note that all values of α on the top curve are negative.)

Figure 5 shows an iso-WTP curve (labeled B) for the same starting value of the drift, $\alpha_0 = .0000075$, but a higher starting value for the volatility, $\sigma_0 = .00012$, and now $\eta = 2$. The WTP to reduce α to zero is now only .0144 (because of the larger value of η). The curve labeled A shows alternative combinations of starting values for α_0 and σ_0 that give the same WTP = .0144 for any target on curve B. Thus shifts from any points on curve A to any points on curve B are welfare-equivalent.

Finally, based on the original starting values $\alpha_0 = .0000075$ and $\sigma_0 = .000061$, with $\eta = 2$, Figure 6 shows an iso-WTP curve that plots combinations of target values of α and σ that have a WTP of .02. Using rough cost estimates based on the IPCC (2007), I have also plotted the iso-cost line that is tangent to this iso-WTP curve. The iso-cost line is:

$$C = .02 = 2350(\alpha_0 - \alpha') + 14(\sigma_0 - \sigma')$$

This iso-cost line is based on a relatively high cost of reducing σ (which might reflect the IPCC's focus on abatement policies. In any case, taking these cost numbers at face value, the cost-minimizing target would reduce σ only slightly and reduce α from .0000075 to about .00000715.

6 Concluding Remarks.

Using a simple model of a stock externality (e.g., temperature) that evolves stochastically, I examined the extent to which the value of a GHG abatement policy depends on the expected future benefits from the policy versus the riskiness of those benefits. I also presented a framework by which one can estimate the extent to which the objective of policy should be a reduction in the expected future damages versus a reduction in the uncertainty over those damages. The framework is based on the computation of “iso-WTP” curves (social indifference curves) for combinations of risk and expected returns as policy objectives. Given cost estimates for reducing risk and increasing expected returns, one can then compute the optimal risk-return mix for policy.

In the case of climate change, there is a broad consensus on the expected rate of temperature increase under BAU — about 3°C. There is considerable disagreement, however, over the nature and extent of uncertainty around that expected value. That disagreement relates to the choice of probability distribution that best applies to future temperatures (and in particular whether it should be a fat- or thin-tailed distribution), and to the parameterization of any particular distribution. I have used a simple (thin-tailed) normal distribution to describe temperature change, but the model can be extended to incorporate alternative distributions (resulting from alternative stochastic processes for X_t). The framework presented here can be used to evaluate the implications of alternative characterizations of uncertainty for determining the targets of climate change policy.

There are other ways in which the framework presented here can be expanded to treat more realistic policy options. First, one can make the change in α and σ occur gradually over some period of time, and one can make the flow cost of the policy time-varying (although both changes would be at the cost of greatly complicating the numerical solution of the model). Even maintaining the simple framework used here (in which the change in α and σ occur instantly and the flow cost is a constant percentage of GDP), the framework can be extended to making the timing of policy implementation endogenous.

Appendix: Some Expectations

There are some key expected values that are used throughout this paper. They are derived below, beginning with $\mathcal{E}_s(C_t^{1-\eta})$.

Expected Utility. Denote $F(C, g, s) = \mathcal{E}_s(C_t^{1-\eta})$, for $t > s$. Then F must satisfy the following partial differential equation (i.e., Kolmogorov Forward Equation):

$$dF = \frac{1}{2}\sigma^2 F_{gg} - \alpha F_g + gCF_C + F_s = 0 , \quad (46)$$

with boundary conditions $F(C, g, t) = C_t^{1-\eta}$ and $F(0, g, s) = 0$. Try, and then verify by substitution, a function of the form:

$$F(C, g, s) = C_s^{1-\eta} e^{(1-\eta)g_s(t-s)+a(s)} . \quad (47)$$

The partial derivatives are:

$$\begin{aligned} F_g &= (1-\eta)(t-s)F \\ F_{gg} &= (1-\eta)^2(t-s)^2F \\ F_C &= (1-\eta)F/C \\ F_s &= [-(1-\eta)g_s + a'(s)]F \end{aligned}$$

After substituting into eqn. (46) and dividing through by F , we have:

$$a'(s) = -\frac{1}{2}\sigma^2(1-\eta)^2(t-s)^2 + \alpha(1-\eta)(t-s) \quad (48)$$

so that the function $a(s)$ is:

$$a(s) = \frac{1}{6}\sigma^2(1-\eta)^2(t-s)^3 - \frac{1}{2}\alpha(1-\eta)(t-s)^2 . \quad (49)$$

Note that if $\eta > 1$ and $\alpha > 0$, $a(s) > 0$.

Thus starting at time $s = 0$ with $C_0 = 1$,

$$\mathcal{E}_0(C_t^{1-\eta})e^{-\delta t} = e^{-\delta t + (1-\eta)g_0 t - (1/2)\alpha(1-\eta)t^2 + (1/6)\sigma^2(1-\eta)^2 t^3} . \quad (50)$$

Note that if $\eta > 1$, as t increases, $\mathcal{E}_0(C_t^{1-\eta})$ first decreases and then eventually increases without bound. Thus the integration over time to compute welfare must be done for a finite horizon.

Expected Consumption. We will also need an expression for $\mathcal{E}_0(C_t)$. Following the same steps as above it is easy to show that if $C_0 = 1$,

$$\mathcal{E}_0(C_t) = e^{g_0 t - (1/2)\alpha t^2 + (1/6)\sigma^2 t^3} . \quad (51)$$

Note that if $\alpha^2 > 2g_0\sigma^2$, $\mathcal{E}_0(C_t)$ is first increasing in t up to a local maximum, then will decrease to a local minimum, and then increase without bound. But if $\alpha^2 < 2g_0\sigma^2$, then $\mathcal{E}_0(C_t)$ will always be increasing in t .

Expectations of Products. We will want to calculate the variance of the return from an abatement policy. To do this we need the expectation of $C_i^{1-\eta}C_j^{1-\eta}$ for two different future times i and j . Denote $G(C, g, s) = \mathcal{E}_s(C_i^{1-\eta}C_j^{1-\eta})$ for $i, j > s$. Then G satisfies the following Kolmogorov Forward Equation:

$$dG = \frac{1}{2}\sigma^2 G_{gg} + \alpha G_g + gCG_C + G_s = 0 , \quad (52)$$

Try and confirm that the solution has the form:

$$G(C, g, s) = C_s^{2(1-\eta)} e^{(1-\eta)g_s[(i-s)+(j-s)]+b(s)} . \quad (53)$$

Substituting the partial derivatives G_g, G_{gg} , etc. into eqn. (52) yields

$$b'(s) = -\frac{1}{2}\sigma^2(1-\eta)^2[(i-s)+(j-s)]^2 + \alpha(1-\eta)[(i-s)+(j-s)] \quad (54)$$

so that the function $b(s)$ is:

$$b(s) = \frac{1}{12}\sigma^2(1-\eta)^2[(i-s)+(j-s)]^3 - \frac{1}{2}\alpha(1-\eta)[(i-s)^2+(j-s)^2] . \quad (55)$$

I use this result when calculating the variance of incremental returns from an abatement policy.

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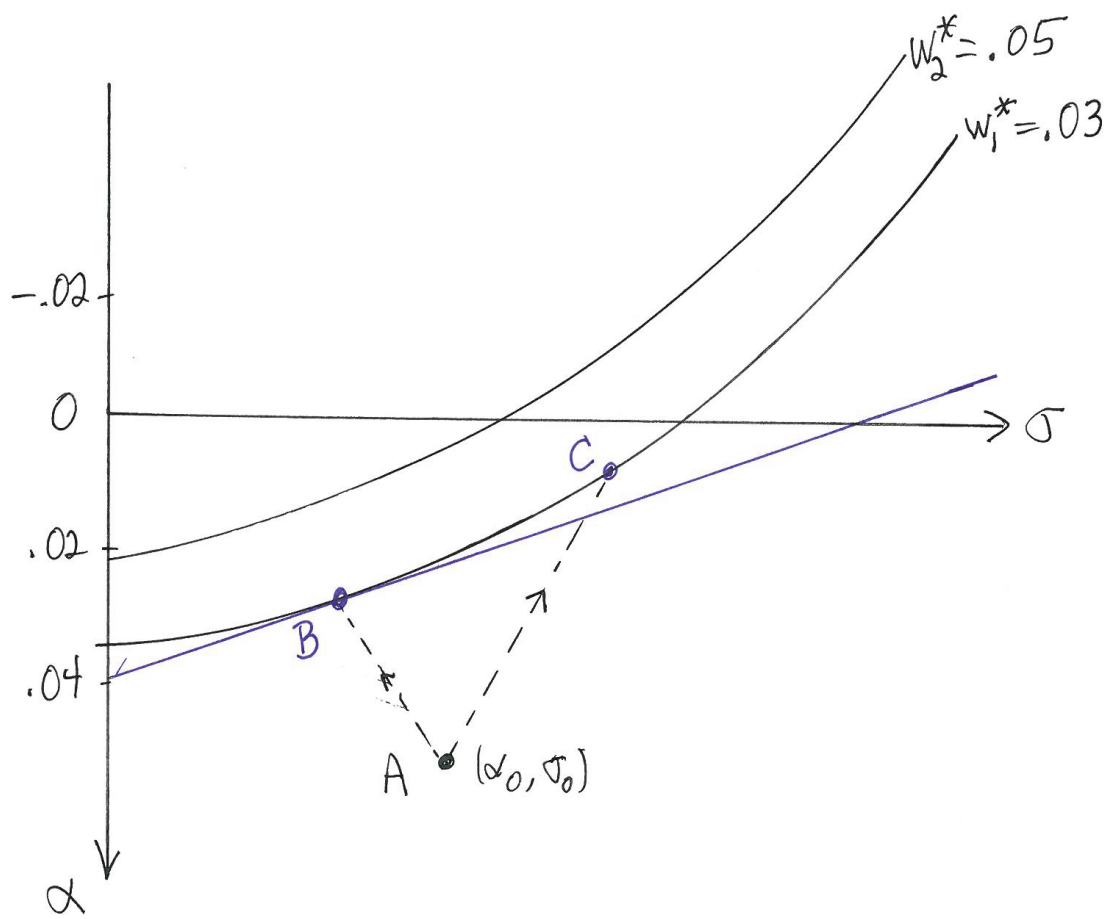


Figure 1: Iso-WTP Curves and Iso-Cost Line

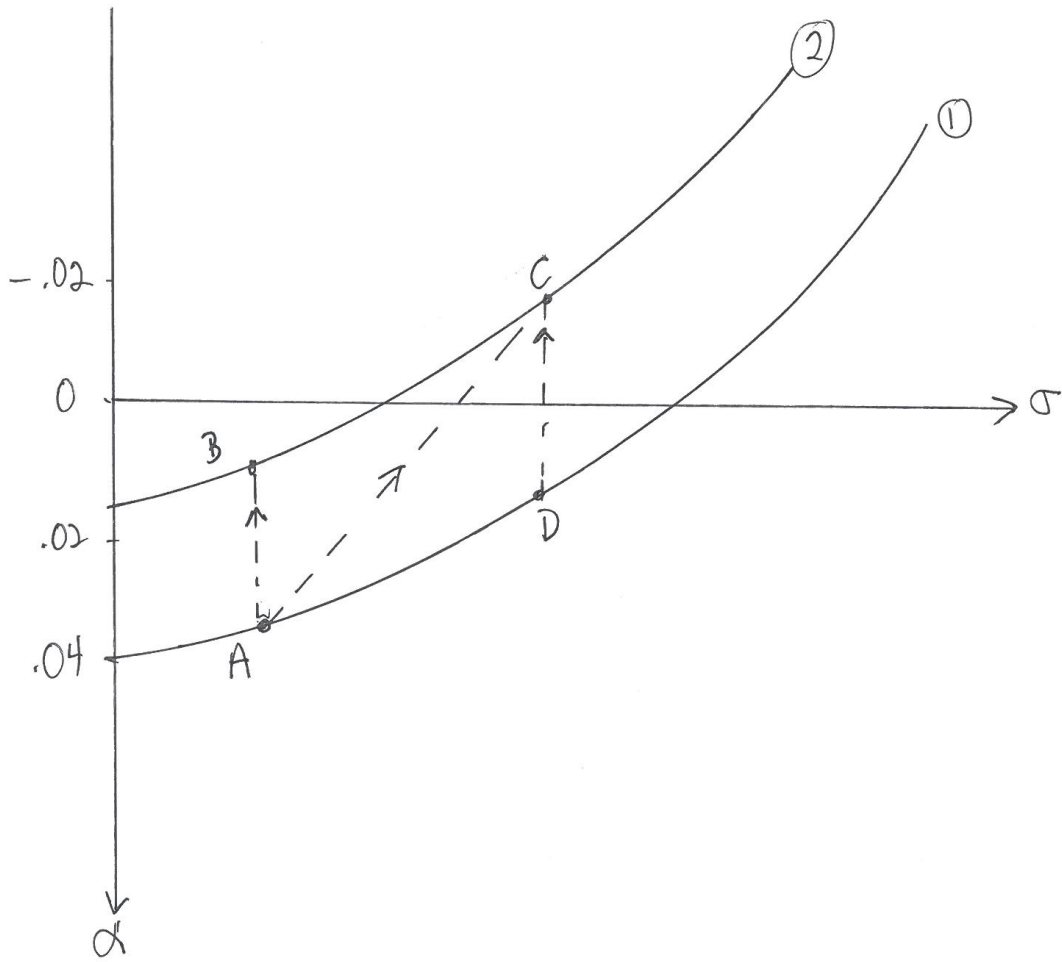


Figure 2: Iso-WTP Starting Curve and Target Curve

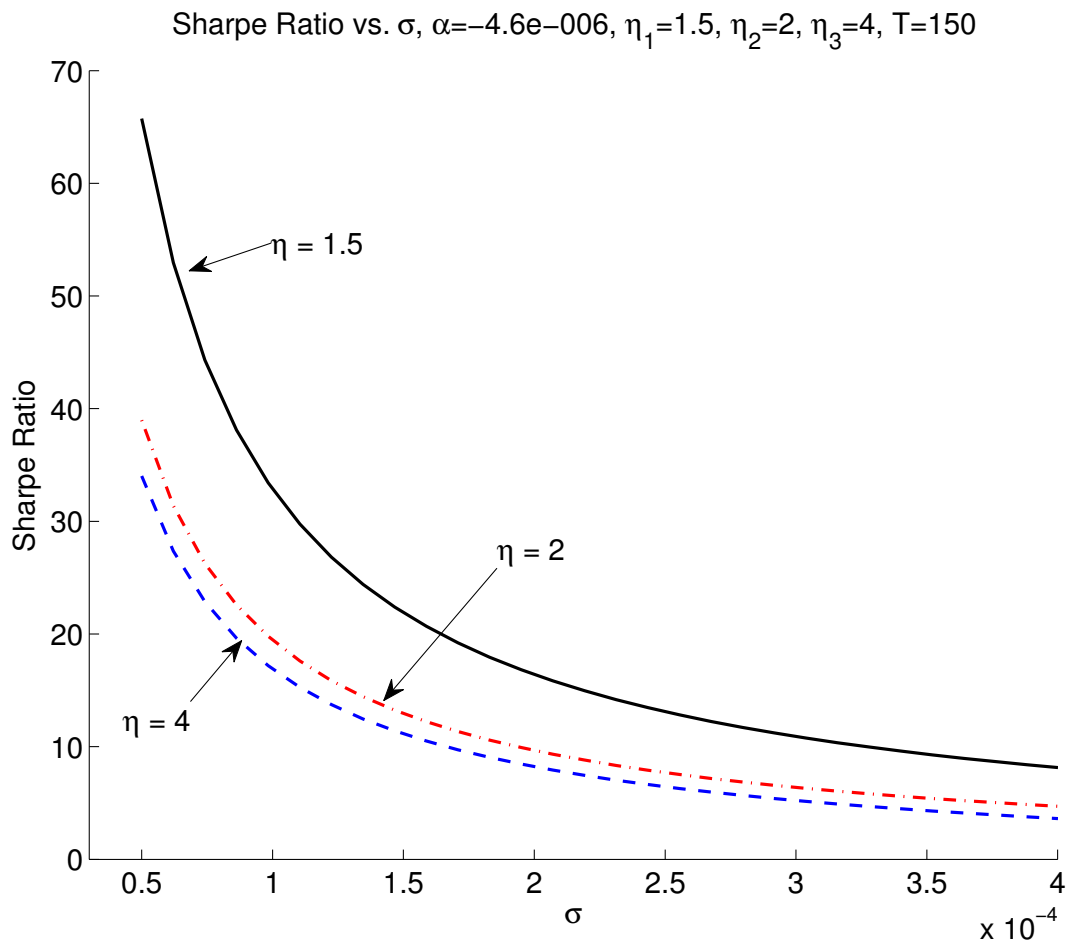


Figure 3: Sharpe Ratio vs. Sigma ($g_0 = .020$, $\delta = 0$, $\eta = 2, 3, 4$)

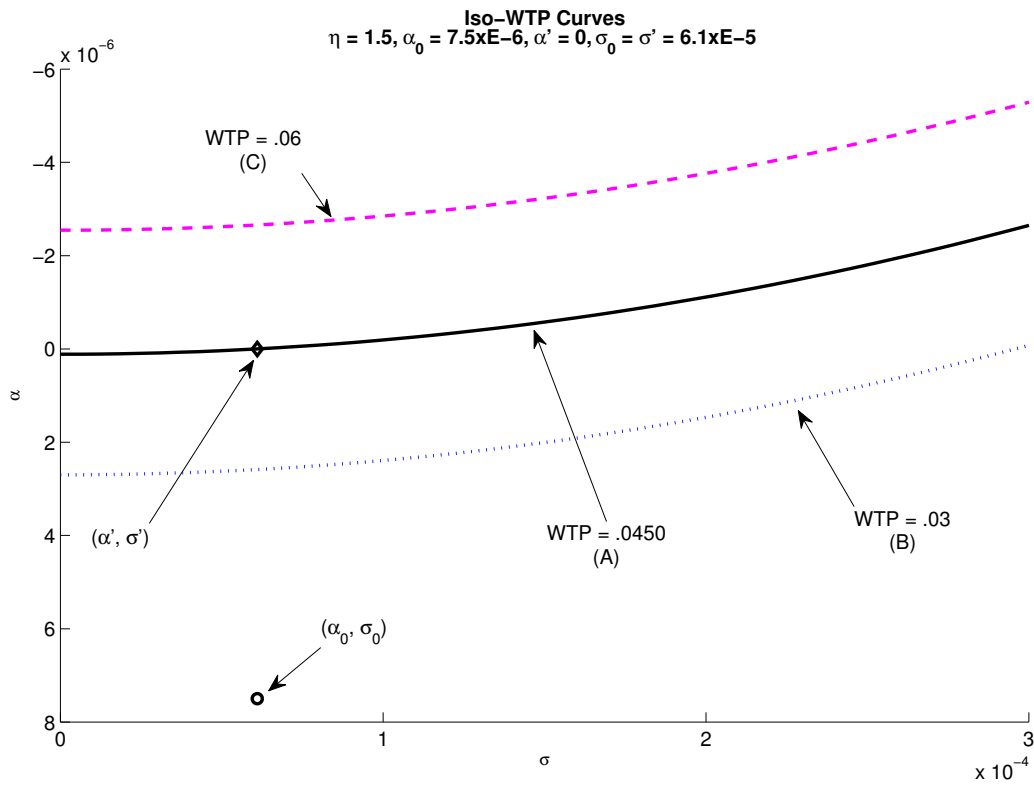


Figure 4: Iso-WTP Curves ($g_0 = .020, \delta = 0, \eta = 1.5$)

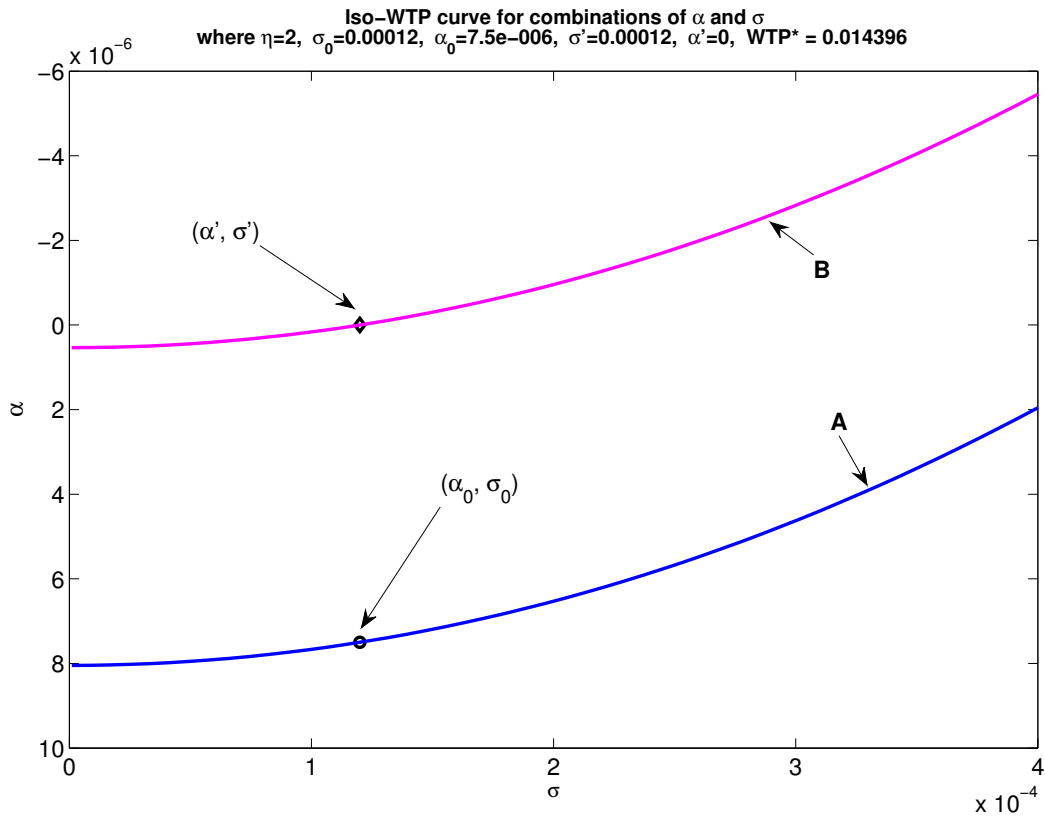


Figure 5: Iso-WTP Starting and Target Curves ($g_0 = .020$, $\delta = 0$, $\eta = 2$)

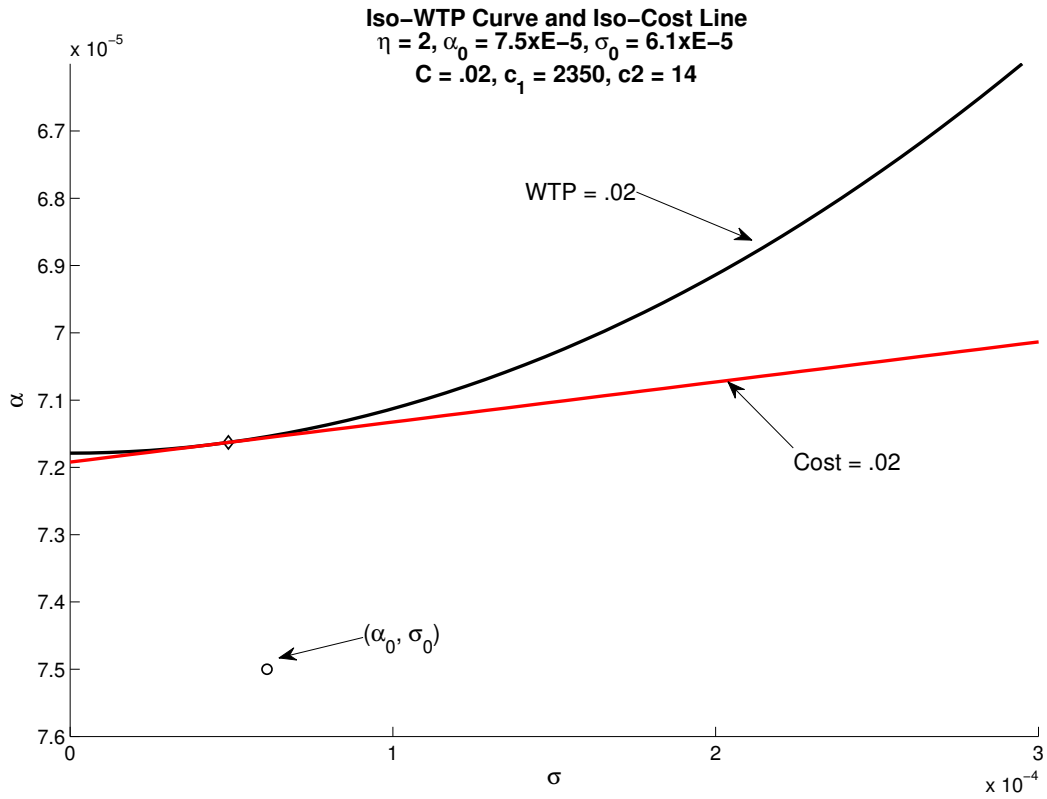


Figure 6: Iso-WTP and Iso-Cost Curves ($g_0 = .020, \delta = 0, \eta = 2$)