

The Price of Talent

How are wages related to abilities?

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How are wages related to abilities?

Schumpeter (1916) wrote:

"The great idea of investigating the relationship between wage differences and differences in ability opens a vast perspective. The new trail is steep and stony, but it must be followed."

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Being a wage setter in the late 90's Hi-tech sector

- ▶ Competitive labor market
 - ▶ Typical wage of university graduate software eng. in 1995 : 8,000 NIS
 - ▶ Typical wage of university graduate software eng. in 2000 :16,000 NIS
 - ▶ Comparative wage tables were available on the internet and updated monthly
 - ▶ About 250 interviews for about 50 positions.
- ▶ How was the wage setting procedure?
 - ▶ The bargaining set was determined by market prices
 - ▶ Upper limit: How much can the worker produce for our firm?
 - ▶ Lower limit: How much can the worker earn elsewhere?
- ▶ **But how was the market price set?**

In short

- ▶ Search and matching frictional labor market.
- ▶ Workers are **exogenously heterogeneous** in their abilities.
- ▶ Ex-ante identical firms endogenously create vacancies with different skill requirements.
- ▶ Axiomatic Nash bargaining is used for wage setting.
- ▶ Characterization of the stationary equilibrium.
- ▶ **Only some production functions can generate equilibrium (e.g. they must exhibit a Pareto tail)**
- ▶ Study of equilibrium relationships between worker abilities and job requirements, wages, vacancies and unemployment.

Shape of wage distribution

Mortensen (1999) states:

Any interesting theory of wage dispersion must be able to explain the shape as well as the extent of the dispersion of wages paid

- ▶ Pareto's (1895) discovered that distribution of income is strongly skewed
- ▶ Other social science disciplines consider ability to be normal.
- ▶ The question of measuring one's ability is largely an open question in economics.
- ▶ There are many attempts to produce explanations to the relationship between ability and wage.

- ▶ Teulings (1995) uses the concept of comparative advantage production function of Sattinger (1975)
 - ▶ Describes wage distribution of single dimensional heterogeneous agents in **frictionless** labor market.
- ▶ Shimer and Smith (2000) characterize the first general search and matching equilibrium for exogenously heterogeneous agents
 - ▶ Assume that the production function is **symmetric** in job and worker characteristics, quadratic matching technology.
- ▶ Albrecht and Vromman (2002) describe an economy in which workers are of **two types**
 - ▶ Production function is associated with job complexity alone, but more talented workers are capable to fulfill any job a less talented person can.

The economy - search and matching

- ▶ Discrete time, infinite horizon economy, with full information
- ▶ Two types of economic agents: firms and workers.
- ▶ Agents are risk neutral, infinitely lived and discount the future
- ▶ Workers are **exogenously heterogeneous** in their abilities
- ▶ Firms are ex-ante identical. but **ex-post can create vacancies with different skill requirements.**
- ▶ There is free entry of firms.
- ▶ **Productivity is attached only to the job characteristics.** (All workers holding the same job will produce the same.)
- ▶ Axiomatic Nash bargaining is used for wage setting.

The equilibrium

- ▶ Measure $0 < u < 1$ of unemployed workers distributed F_U in workers' ability
- ▶ Measure $(1 - u)$ of employed workers distributed F_W in workers' ability.
- ▶ Measure $(1 - u)$ of filled jobs distributed G_J in jobs' ability requirements.
- ▶ Measure $v > 0$ of open vacancies distributed G_V in jobs' ability requirements.
- ▶ Workers' matching strategy $a \rightarrow x_L(a)$ the lowest job the workers is still willing to take.

The workers

- ▶ Let us normalize the measure of workers to 1 indexed by $i \in [0, 1]$.
- ▶ Each worker is endowed with ability a_i
- ▶ The ability is distributed with some cumulative distribution function (**Uniform**) $F \equiv U [0, 1]$.
- ▶ Each worker may be employed or unemployed.
- ▶ Only unemployed workers search
- ▶ An unemployed worker gets flow utility $Z(a_i)$ from leisure and unemployment income, (Will be set to 0).
- ▶ Worker i employed by firm j receives wage w_{ij} determined by Nash bargaining (using their outside options).
- ▶ let us denote the measure of unemployed workers as u .
- ▶ The distribution of ability in unemployed workers is determined endogenously $a \sim F_U$
- ▶ The distribution of ability in employed workers is determined endogenously $a \sim F_W = \frac{F-uF_U}{1-u}$

The firms

- ▶ Each firm can have at most one job opening, so I use the terms "job" and "firm" interchangeably.
- ▶ A job is first opened as vacancy and sustaining it open is associated with a cost per period
- ▶ After it is matched by a suitable worker, it produces until destroyed by nature.
- ▶ Each firm has access to production technologies using only labor.
- ▶ **Each such production technology is characterized by a triplet $(x, C(x), P(x))$.**
 - ▶ The ability requirement $0 \leq x \leq 1$ determines the minimum ability required from a worker in order to fill the job
 - ▶ The per period flow cost of opening a vacancy is $C(x)$
 - ▶ When the job is filled it produces each period a unit whose price is $P(x)$

Job creation and destruction

- ▶ There is free entry of firms.
- ▶ Let us denote the measure of vacancies to be v
- ▶ Every period, at probability $1 > \delta > 0$ a filled job are destroyed
- ▶ Job destruction leave the worker unemployed and the firm with nothing.
- ▶ The endogenous threshold distribution of vacant jobs $x \sim G_V$
- ▶ The endogenous threshold distribution of $(1 - u)$ filled jobs is $x \sim G_J$

Search technology

- ▶ Only unemployed workers search for jobs.
- ▶ Every period at probability λ an unemployed worker is matched with a vacancy sampled from vacancy distribution.
- ▶ Search technology is called **quadratic search** technology.
- ▶ The probability of a job being matched by a some worker is $\lambda^J = \lambda u$
- ▶ The probability of a worker being matched with a some vacancy is $\lambda^W = \lambda v$.

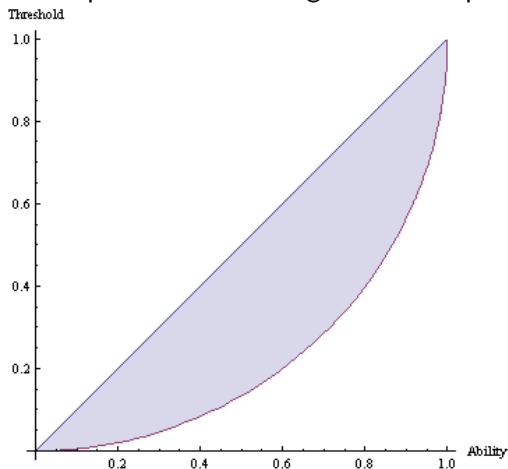
Wage setting

- ▶ The wage is determined by Nash bargaining with parameter b , dividing the surplus from the contact.
- ▶ The outside option of the worker is the value of being unemployed plus the value of continuing searching.
- ▶ The outside option of firm is the value of continuing searching (which is 0 due to free entry condition).
- ▶ Some workers matched can't fill the vacancy because their ability is below the required threshold.
- ▶ The maximum possible wage paid in a job is ceiled by the productivity associated with that job.

The matching set

The matching set are the types of jobs and workers that form a match.

The equilibrium matching set can be pictured as follows:



Firm's Bellman equations

- ▶ The **value of vacancy** with ability requirement x

$$V(x) = -C(x) \cdot L_P^C(x) + \beta \left(\lambda^J \int_{a'=x}^{a_H(x)} J(x, a') dF_U(a') \right) + \beta V(x) \left(1 - \lambda^J + \lambda^J \left(\int_{a'=0}^x dF_U(a') + \int_{a'=a_H(x)}^1 dF_U(a') \right) \right)$$

Where:

$$a_H(x) = a | P(x) = w(x, a)$$

$L_P^C(x)$ is a linear operator to be defined later.

- ▶ **Value of filled job** with ability requirement x held by a worker with ability a :

$$J(x, a) = P(x) - w(x, a) + \beta(1 - \delta) J(x, a)$$

- ▶ Free entrance guarantees that

$$V(x) = 0$$

Workers's Bellman equations

- ▶ The value of being **unemployed** for a worker with ability a

$$U(a) = Z(a) \cdot L_P^Z(a) + \beta \lambda^W \int_{x'=x_l(a)}^a W(x', a) dG_V(x') \\ + \beta U(a) \left(1 - \lambda^W + \lambda^W \left(\int_{x'=0}^{x_L(a)} dG_V(x') + \int_{x'=a}^1 dG_V(x') \right) \right)$$

- ▶ where

$L_P^Z(a)$ is a linear operator of P to be defined later.

$$x_L(a) = x \mid W(x, a) = U(a)$$

- ▶ The value of a **job with ability requirements** x for a worker with ability a :

$$W(x, a) = w(x, a) + \beta ((1 - \delta) W(x, a) + \delta U(a))$$

Bargaining equation

- ▶ I use axiomatic Nash bargaining for the division of surplus
- ▶ Bargaining wage under Nash bargaining with parameter b yields the following
$$W(x, a) - U(a) = b(W(x, a) - U(a) + J(x, a))$$
- ▶ and alternately
$$J(x, a) = (1 - b)(W(x, a) - U(a) + J(x, a))$$

Total surplus

$$\blacktriangleright T(x, a) \equiv W(x, a) - U(a) + J(x, a) = \frac{P(x) - P(x_L(a))}{(1 - \beta(1 - \delta))}$$

The surplus is a **separable** function of job's characteristics and worker's characteristics

- ▶ The wage solution:

$$w(x, a) = bP(x) + (1 - b)P(x_L(a)) = P(x_L(a)) + b(P(x) - P(x_L(a)))$$

The wage is therefore a weighted average between the productivity of current job and the lowest agreed job.

The wage is a **separable** function of job's characteristics and worker's characteristics

Firms' and workers' conditions

Firm's equal (zero) profit condition

$$\blacktriangleright C(x) \cdot L_P^C(x) =$$

$$\frac{\beta \lambda^W (1-b)}{(1-\beta(1-\delta))} \int_{a'=x}^{a_H(x)} (P(x) - P(x_L(a'))) dF_U(a')$$

The equal profit condition is also zero profit condition.

The cost of opening a vacancy equals to the expected value of the firm's share of the possible surplus

\blacktriangleright Where:

$C(x)$ - The cost of holding a vacancy open

λ^W - The probability the firm is matched

$\frac{\beta}{(1-\beta(1-\delta))}$ - The effective discount factor

$(1-b)$ - The firm's share in the Nash bargaining

$dF_U(a')$ - The distribution of abilities the unemployed

$P(x) - P(x_L(a')) = P(x) - w(x, a)$: The instantaneous surplus generated from the match

$a_H(x)$ - The ability of the most skilled worker the firm can hire

Firms' and workers' conditions

Worker's threshold condition

$$\blacktriangleright P(x_L(a)) = Z(a) \cdot L_P^Z(a) + \frac{b\lambda^J}{(1-\beta(1-\delta))} \int_{x'=x_L(a)}^a (P(x') - P(x_L(a))) dG_V(x')$$

The productivity of the job still taken by a worker with ability a , must be equal to the unemployed benefits plus the workers share out of possible surplus gained.

► Where:

$Z(a)$ -The value of being unemployed (will be taken to be 0)

λ^J -The probability the unemployed worker is matched

$\frac{\beta}{(1-\beta(1-\delta))}$ - The effective discount factor

b -The worker's share in the Nash bargaining

$dG_V(x')$ -The distribution of vacancies

$P(x') - P(x_L(a)) = P(x) - w(x, a)$:The instantaneous surplus generated from the match

$x_L(a)$ - The skill requirement of the lowest paying job the worker will accept

Worker and job flow equations

In stationary equilibrium, the stationarity of unemployed workers' ability distribution and vacant jobs' ability requirement distribution yield

- ▶ For all ability requirements the number of lost jobs is equal to the number of filled vacancies i.e.

$$\begin{aligned}\delta(1-u)G_J(x) &= \\ v\lambda^J \int_{x'=0}^x \left(\int_{a'=x'}^{a_H(x)} dF_U(a') \right) dG_V(x') \\ &= v\lambda^J \int_{x'=0}^x (F_U(a_H(x')) - F_U(x')) dG_V(x')\end{aligned}$$

- ▶ For all abilities the number of people losing their jobs is equal to the number of people finding new jobs

$$\begin{aligned}\delta(1-u)F_W(a) &= \\ u\lambda^W \int_{a'=0}^a \left(\int_{x'=x_L(a')} dG_V(x') \right) dF_U(a') \\ &= u\lambda^W \int_{a'=0}^a (G_V(a') - G_V(x_L(a')))) dF_U(a')\end{aligned}$$

End conditions and tail behaviors

► **Conditions on production function:**

Production function is monotonic, starts with 0 and must end at infinity, some tail behavior are mandatory.

$$P(0) = 0$$

$$P(1) = +\infty$$

$$P'(x) > 0,$$

$$P'(x)_{x \approx 0} \approx \frac{1}{a_H(x)}$$

$$P(x)_{x \approx 1} \approx \frac{1}{1-x}$$

► The simplest production function that fulfills these criteria is $\sqrt{x} + \frac{1}{1-x}$

► Although $P(x)$ is unbounded there must exist **bounded average productivity**

► That means that every person have finite productivity

The wage distribution in the model

- ▶ The right tail of the wage distribution exhibit a pareto tail between -2 and $-\infty$

$$h(w) \simeq w^{-\frac{1}{\alpha}-1}$$

- ▶ The average wage of a worker with ability a is

$$\bar{w}_a(a) = P(x_L(a)) \left(1 + \frac{1}{k(vGv(a) - vGv(x_L(a)))} \right)$$

- ▶ Average wage paid in job with threshold x is

$$\bar{w}_x(x) = P(x) - \frac{(1-b)x}{uFu(a_H(x)) - uFu(x)}$$

This defines the average markup

Generality of solution

- ▶ Population ability distribution was assumed to be **uniform**
- ▶ This is not restrictive since for **any** ability distribution $a' \sim F(a')$ can be transformed to $a = F(a')$
- ▶ Any production function $P(x)$ that create an equilibrium in the latter can be transformed to the former. by $x' = F^{-1}(x)$
- ▶ All **observables** remain the same (**including the wage distribution**)
- ▶ Matching set is transformed accordingly.

Existence & Uniqueness

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Definitions

Under some regularity conditions on $P(x)$

Theorem

The following conditions

$\mathbf{P}(0) = \mathbf{0}, \mathbf{P}(1) = +\infty, \mathbf{P}'(x) > \mathbf{0}, \lim_{x \rightarrow 1} (1-x)P(x) <$

$\infty, \lim_{x \rightarrow 1} \left(\frac{1}{P(x)} \right)' < \mathbf{0}$ are **necessary & sufficient** for the
existence of an equilibrium

Proof.

By Schauder fixed point theorem

Every continuous function from a convex compact subset K of a Banach space to K itself has a fixed point. □

Uniqueness is yet to be shown

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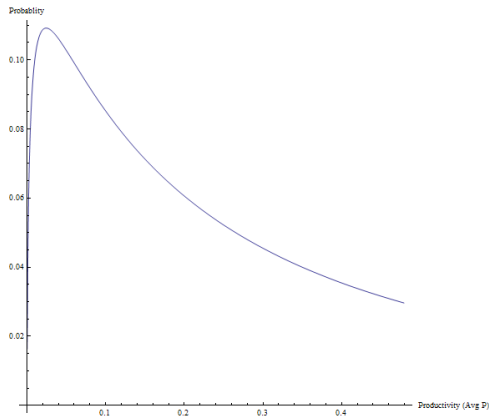
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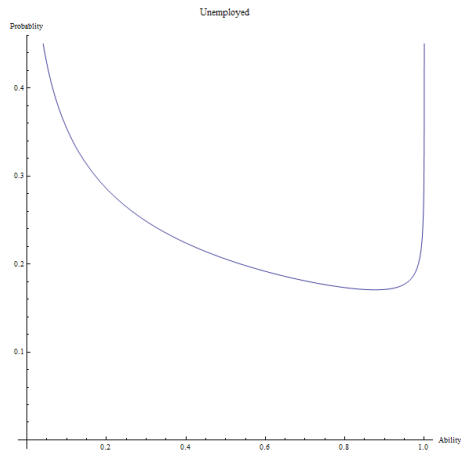
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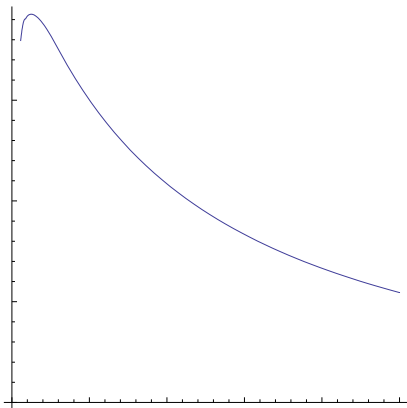
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Model features

- ▶ The model generates endogenous unemployment and vacancy measures
- ▶ The model creates wage distribution for identical workers
- ▶ The model defines the relation between ability and wage
- ▶ The model anticipates the shape of wage distribution and enables the study of inequality.
- ▶ Model parameters can suggest:
 - How bargaining power influence the wage distribution?
 - How the matching frequency can influence the labor market outcomes?

Next steps

- ▶ Verifying proof for existence of equilibrium
- ▶ Uniqueness of equilibrium
- ▶ Comparative statics of model parameters
- ▶ Introductions of measures of inequality

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