THE INFORMATION IN LONG-MATURITY FORWARD RATES:
IMPLICATIONS FOR EXCHANGE RATES AND THE FORWARD PREMIUM ANOMALY

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Abstract

In contrast to current thinking, exchange rates do reflect significant economic fundamentals. These fundamentals are also reflected in interest rates and expected inflation. Specifically, forward interest rate differentials between countries forecast both future spot interest rate and inflation rate differentials, as well as, and more important, future movements in exchange rates. For example, 20% of the variation of annual exchange rate changes can be explained, and in the *right direction*, by prior forward rate differentials. Previous violations of uncovered interest rate parity seem to be a short-horizon phenomenon.

I. Introduction

There are well over 100 papers documenting in some form or another the forward premium anomaly, namely that interest rate differentials across countries do not move one-for-one with future exchange rate changes (e.g., see Hodrick (1987) and Engel (1996) for survey evidence). In fact, they tend to move in opposite directions. Not surprisingly, over the last decade or so this anomaly has led to a plethora of papers that develop possible explanations in the early sections of their papers, only to shoot them down in later sections. It would not be unreasonable to conclude that this is one of the more robust puzzles in financial economics. The exchange rate puzzle is, in fact, even deeper. Parallel to the finance literature on the forward premium puzzle a large literature developed, starting with Meese and Rogoff (1983), on the macro side. That literature documents an even more startling puzzle—exchange rates do not seem to be related to fundamentals.[1] The random walk model has proven unbeatable using a variety of finance and/or macro variables.

This paper provides a novel way of looking at the forward premium anomaly by recasting the parity relation in terms of forward interest rate differentials across countries against future exchange rate movements. In contrast to existing evidence, we find much more support in favor of the theory. For example, the one-year forward interest rate differential one year hence has a coefficient of 1.03 with the annual change in the USD/DEM exchange rate in one year, the two years hence differential has a coefficient of 1.17, the three years differential hence of 2.55, and so forth. This compares with the well-documented negative coefficient of -0.55 between the one-year spot rate differential and the exchange rate change.

This paper complements the work of Fama and Bliss (1987) and Jorion and Mishkin (1991), who document that long-maturity forward rates forecast future interest rates in the U.S. and internationally, respectively. Interestingly, these papers show a dichotomous effect between short-maturity and long-maturity forwards, not unlike the forward premium result mentioned above. This is not by chance and relates directly to the commingling of the expectations hypothesis of the term structure and uncovered interest

[1] These findings are revisited and confirmed for recent data by Cheung, Chinn and Garcia Pascual (2003).

rate parity in exchange rates. We explore this link and argue that the breakdown in the term structure and exchange rate theories are most probably driven by the same source, i.e., short-term rates.

We are not the first paper to explore the links between the term structure and exchange rates (e.g., Campbell and Clarida (1987), Bekaert and Hodrick (2001), Bekaert, Wei and Xing (2002) and Chinn and Meredith (2003)). However, our paper's contribution is new in that we exploit the information in the forward curve about future interest rates and exchange rates. It is perhaps not a complete surprise then that our conclusions differ markedly from these papers.[2]

We document several important results. First, in contrast to current research on uncovered interest rate parity, prior forward rate differentials have strong forecasting power for exchange rates. Moreover, the direction of these forecasts coincides with fundamental economic intuition. Second, consistent with existing research, these same forward differentials forecast future expected spot rate and inflation differentials, thus, providing support for a fundamentally consistent economic framework. Third, we show that the breakdown of uncovered interest rate priority derives from a negative relation between future exchange rate changes and the component of interest rates changes outside the above fundamentals. This breakdown is related to the same driving force behind failures of the Fisher effect at short horizons. The existing exchange rate literature provides several possible explanations for these findings.

This paper is organized as follows. Section 2 describes the exchange rate parity relation in terms of forward rate differentials and presents surprising evidence *in favor of the exchange rate expectations hypothesis* in contrast to results using spot interest rate differentials. In Section 3 we derive the link between the expectations hypothesis of interest rates and the expectations hypothesis of exchange rates by decomposing interest rate differentials into their expected component and their innovation. We argue that the failure of the exchange rates expectations hypothesis is due to the unexpected component of interest rates. In Section 4, we provide an interpretation of the results as evidence of a link between exchange rates and fundamentals. Specifically, we show that interest forward differentials predict both future inflation differentials and exchange rate changes and is therefore consistent with long run purchasing power parity, while the role of the innovation, interpreted as the real rate differential, is consistent with the asset approach to exchange rate determination. Section 5 makes some concluding remarks.

II. Uncovered Interest Rate Parity and the Expectations Hypothesis

A. Existing Evidence

The expectations hypothesis for exchange rates is commonly written as

$$E_t s_{t+k} = f_t^k, \tag{1}$$

where s_{t+k} is the log of the spot price of foreign currency at time $t+k$ and f_t^k is the log of k-period forward exchange rate at time t. Assuming no arbitrage and covered interest rate parity (i.e., $f_t^k - s_t = i_{t,k} - i_{t,k}^*$, where $i_{t,k}$ is the domestic k-period interest rate at time t and the superscript * denotes the corresponding foreign interest rate), the expected change in the exchange rate equals the interest rate differential. Thus, one standard way of testing equation (1) for k=1 is to estimate the regression

$$s_{t+1} - s_t = \alpha + \beta(i_{t,1} - i_{t,1}^*) + \varepsilon_{t,1}. \tag{2}$$

Under uncovered interest rate parity, α and β should be 0 and 1, respectively. As mentioned in the introduction, equation (1) has been resoundingly rejected. The forward premium, $f_t^k - s_{t+k}$, has a systematic bias (i.e., α differs from zero), is predictable, and, most alarming, diverges, to the extent that β tends to be negative.[3]

A recent exception to these findings can be found in Chinn and Meredith (2003). They run regression equation (2) over multiple horizons, i.e., $s_{t+k} - s_t$ on $i_{t,k} - i_{t,k}^*$ where k varies from 5 to 10 years. Using the last 20 years of data, and across many countries, the point estimates of β tend to be positive and are statistically indistinguishable from one. The statistical properties of the regressions, however, are questionable. There are only between 2 to 4 nonoverlapping observations present in the data. The problems with such

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[2] Chinn and Meredith (2003) are an exception in that they find support for uncovered interest parity using long-horizon exchange rates.

[3] See, for example, Engle (1996) And Lewis (1995) for surveys of this literature.

regressions are by now well-known. For example, Richardson and Stock (1989) and Valkanov (2002) show that the estimators and test statistics are inconsistent, having non-normal limiting distributions. The intuition is that the frequent sampling of large partial sums of the data (i.e., in this case, 5 to 10 year exchange rate movements) takes on the properties of a nonstationary series and standard central limit theorems no longer apply.

One way to alleviate this issue is to use an alternative methodology, based on a specified vector auto regression (VAR), which exploits the short-run dynamics to infer long-range forecasts (see, for example, Hodrick (1992), and, more recently, in the context of exchange rates and the term structure, Bekaert, Wei and Xing (2002)). Bekaert, Wei and Xing (2002) argue this as one of the primary motivations in looking at long-horizon properties of uncovered interest rate parity. There are two problems with this approach in the context of equation (2). First, as pointed out in Boudoukh and Richardson (1994), the benefits underlying the VAR methodology quickly erode if the predictive variables are highly persistent. As has been documented elsewhere (and seen in Table 1A of this paper), interest rate differentials have a high degree of serial correlation. Second, and more important, the VAR specification imposes a particular model on the system at hand. If that model is incorrect, then it is unclear how to interpret the long-range forecast. For the exchange rate and term structure hypotheses, there is ample evidence that the short- and long-run behave quite differently (e.g., consider the findings in Fama (1984) and Fama and Bliss (1987)). In fact, as pointed out by Fama in the aforementioned work, and in our findings to follow on exchange rates, it is this divergence that is most interesting.

Nevertheless, as a first pass at the short-horizon version of equation (2), we collect exchange rate and interest rate data on the U.S., U.K. and Germany (or the Euro in the latter part of the sample) over the period 1976-2002. Table 1C reports estimates of equation (2) for annual rates on a monthly overlapping basis.[4] The β coefficients are -1.23 and -0.55 for the USD/GBP and USD/DEM, respectively, confirming the well-known negative relation between exchange rates and interest rate differentials. While the estimates are fairly noisy, joint tests of the coefficients being equal to one can be resoundingly rejected. The corresponding p-values for Wald and LaGrange multiplier tests

[4] The data used in this study is described in detail in Appendix A.

are 0.02 and 0.03, respectively.[5] Of some interest, and a key finding of our analysis to follow, are the low R^2s in the regressions.

The low R^2 is both disappointing and puzzling. The key fundamental underlying expected exchange rate movements are interest rate differentials between countries. These interest rate differentials, in theory, predict expected inflation rate differentials. Since inflation is fairly predictable (see, e.g., Fama and Gibbons (1984)), and inflation differentials are a fundamental driver of exchange rates via purchasing power parity, one would have expected a closer relationship.

B. Information about Exchange Rate Changes in Long-Maturity Forward Rates

Note that equation (1) can be used to define the difference of expected future spot exchange rates in terms of the difference in two forward exchange rates. That is,

$$E_t[s_{t+k} - s_{t+j}] = f_t^k - f_t^j, \tag{3}$$

where $k>j$. Thus, the expected depreciation in the $(k\text{-}j)$-period exchange rate as of today equals the difference in the corresponding forward rates under the expectations hypothesis.

Again under covered interest rate parity, we can replace the forward exchange rates in equation (3) by the interest rate differentials of the two countries, i.e.,

$$E_t[s_{t+k} - s_{t+j}] = (i_{t,k} - i^*_{t,k}) - (i_{t,j} - i^*_{t,j}). \tag{4}$$

Rearranging terms in equation (4) and using the definition of forward interest rates, we get

$$\begin{aligned} E_t[s_{t+k} - s_{t+j}] &= (i_{t,k} - i_{t,j}) - (i^*_{t,k} - i^*_{t,j}) \\ &= if_t^{j,k} - if_t^{*j,k}, \end{aligned} \tag{5}$$

where $if_t^{j,k}$ and $if_t^{*j,k}$ are the forward interest rates at time t from $t+j$ to $t+k$ for domestic and foreign currencies, respectively. Equation (5) forms the basis for much of the empirical analysis to follow. It says that the expected depreciation in future spot exchange

[5] Throughout this study, we employ both the Lagrange multiplier and Wald statistics for testing the joint hypotheses. As shown by Berndt and Savin (1977), there is a numerical ordering between these statistics which may lead to different inferences being drawn. For an especially relevant discussion, see Bekaert and Hodrick (2001) in the context of testing the expectations hypothesis of the term structure.

rates can be approximated by $if_t^{j,k} - if_t^{*j,k}$, what we call the *forward interest rate differential*.

Our alternative way to characterize regression (2) via equation (5) is to use the forward interest rate differentials, namely

$$s_{t+1} - s_t = \alpha + \beta(if_{t-j}^{j,j+1} - if_{t-j}^{*j,j+1}) + \varepsilon_{t-j,j+1}. \tag{6}$$

Under the expectations hypothesis of exchange rates, the exchange rate change from *t* to *t+1* should move one-for-one with the forward interest rate differential from *t* to *t+1* that was set at time *t-j*. That is, α and β should be 0 and 1, respectively. Equation (6) is a generalization of (2) ((6) yields (2) for *j=0*). In (6) we are exploiting the information in the entire forward curve. Note, however, that the error term now represents a *j*-step ahead forecast and therefore is serially correlated up to *j* observations.[6] Of course, standard techniques have been developed to adjust for this type of dependence (e.g., Hansen and Hodrick (1980) and Hansen (1982)).

One of the difficulties in studying multi-step ahead forecast regressions like those specified in equation (8) is the availability of data. While sophisticated econometrics have somewhat alleviated the problem (e.g., Hansen (1982), Newey and West (1987), and Andrews (1991)), the benefits are still constrained by the number of independent observations (e.g., Boudoukh and Richardson (1993) and Richardson and Stock (1989)). (See the discussion in Section II.A above.) Note that for the regression in (6), however, the degree of serial correlation in the errors depends upon the relative variance of exchange rates versus interest rate differentials and the correlation between unexpected shocks to these variables. There are strong reasons to suspect that these factors help mitigate the serial correlation. For example, Table 1A provides evidence that exchange rates are much more variable and relatively unpredictable. Thus, because the forecast update component of the residual in equation (6) as we move forward in time is likely to be small relative to the unpredictable component, the induced serial correlation in the errors will be correspondingly small and the effective number of independent observations will not be reduced substantially by the overlap.

[6] In our case *j* is measured in years and the data is samples monthly, so, in fact, the overlap will be *12*j*.

Equation (6) provides an alternative basis for testing the expectations hypothesis of exchange rates. Table 2 describes the estimation results over different horizons and across different currencies. In contrast to the results of Table 1C and the standard testing paradigm, Table 2 shows that equation (6) looks much more promising. Forward interest rate differentials are able to predict in the right direction changes in future exchange rates. For example, looking forward one to four years yields coefficients of 1.03, 1.17, 2.55 and 3.98 for the USD/DEM exchange rate. The results for the USD/GBP show a similar relation. The coefficients in this case are 0.89, 3.94, 1.73 and 3.09, respectively looking forward one to four years. Since the theory predicts a coefficient of 1, these results are quite different than those presented in Table 1C. One interesting aspect of these estimates is that they follow a similar pattern to those in the term structure regressions to be discussed below.

As is evident from these results, the coefficients, while in the right direction, are noisy. However, conducting joint tests that $\beta=1$ at each horizon across the currencies yields very few rejections using either the Wald or Lagrange multiplier tests. In particular, at the four horizons, none of the Lagrange multiplier tests generate rejections, with p-values ranging from 24% to 99%. While it is well known that Wald statistics are biased towards rejection (e.g., Bekaert and Hodrick (2001)), the rejections occur at only two of the horizons. These results are in stark contrast to the one-year horizon in Table 1B, which reports rejections of uncovered interest rate parity.

One interesting aspect of the results in Table 2 is the higher R^2s in the regressions. While the dependent variable, i.e., annual exchange rate changes, is the same, the forecasting variable differs. For the USD/DEM, the R^2s are higher in all of the forward interest rate differential regressions (eqn. (6)) than in the interest rate differential regression (eqn. (2)). For the USD/GBP, this is true for two of the four regressions. What is remarkable about this result is that the information is (i) old relative to current interest rates, and (ii) more subject to measurement error due to the formation of forward rates. We argue below that this finding is a direct result of the fundamental relation between exchange rates, inflation and interest rates, and the puzzling behavior of short-term interest rates.

One potential concern is that the R^2s are spuriously high due to small sample problems in the longer horizon regressions. We argue above that the overlap problem is not that serious due the relatively low level of predictability in the exchange rate change, but it is still worth verifying this conjecture. Consequently, we construct a bootstrap experiment where we estimate a VAR for the relevant forward rate differentials, interest rate differentials and changes in exchange rates, imposing the expectations hypotheses in both interest rates and exchange rates. We then simulate this model, drawing with replacement from the residuals of the constrained VAR, and replicate our regressions. The results indicate that there is a small upward bias in the R^2s at long horizons, but that this bias cannot explain the large differences in R^2s across horizons found in the data.

III. Insights from Linking the Expectations Hypothesis of Interest Rates with the Expectations Hypothesis of Exchange Rates

A. The Expectations Hypothesis of Interest Rates across Countries

A related literature investigating the expectations hypothesis of interest rates has evolved along a path similar to that of the exchange rate literature. In particular, according to the expectations hypothesis of interest rates, forward interest rates equal future expected spot rates:

$$E_t i_{t+k,1} = if_t^{k,k+1}. \tag{7}$$

Analogous to the exchange rate literature, the standard way of testing equation (7) is to regress the change in the interest rates on the forward interest rate premium (i.e., *if* - *i*). That is,

$$i_{t+k,1} - i_{t,1} = \alpha + \beta(if_t^{k,k+1} - i_{t,1}) + \varepsilon_{t,k}. \tag{8}$$

Under the expectations hypothesis, α and β should be 0 and 1, respectively.

A common finding from this literature is that equation (8) does poorly at short-horizons both in terms of explanatory power and in terms of the sign and magnitude of the β coefficient (e.g., Fama (1984a), Bekaert and Hodrick (2001)). However, the relation appears to hold much better at longer horizons. Table 3 replicates Fama and Bliss's (1987) and Jorion and Mishkin's (1991) regression analysis of forward interest rates on future spot interest rates across the U.S., U.K. and Germany for our sample period.

For example, over the period 1964 to 1984, Fama and Bliss document coefficients of 0.09, 0.69, 1.30 and 1.61 respectively for 1-year forwards on 1-year rates 1 to 4 years ahead. While our sample size covers a later period, i.e., 1976 to 2002, the results are very similar. For example, in U.S. data, the analogous coefficients for 1-year forwards 1 to 4 years hence are 0.02, 0.23, 0.65 and 1.13. These results are robust to countries as well. Both the U.K. and Germany show a similar pattern with 0.56, 1.03, 1.21 and 1.42, and 0.54, 0.89, 1.34 and 1.65, respectively for 1 to 4 years ahead, coinciding with the findings of Jorion and Mishkin (1991) whose sample ends in 1987.

One of the major findings of Fama and Bliss (1987) is that the R^2 also increases dramatically with the horizon, notably from 0% to 8% to 24% to 48% over 1 to 4 years ahead. Fama and Bliss suggest one possibility is that interest rates follow a slow mean-reverting process. Under that scenario, the variance of the expected interest change will become a more important component of the overall variability of interest rate changes as the horizon increases, consistent with their finding. An alternative explanation is that a wedge is drawn between short and long horizons for the expectations hypothesis to hold. In particular, if short term rates are governed not only by underlying fundamentals, but are also a function of policy variables (central banks' goals, e.g., affecting inflation or exchange rates via the short rate), then distortions are likely to occur.

As can be seen from Table 3A, the R^2s also increase with horizon for all three countries. For example, for the U.S., U.K and Germany, the increase is respectively from 0% to 19%, 12% to 46%, and 9% to 52%. The magnitude of the coefficients, and the higher R^2, suggest that there is considerable information in long-maturity forwards about expected future interest rates. Since these interest rate differentials in theory forecast future exchange rate movements under uncovered interest rate parity, there appears to be some hope that we can address the forward premium puzzle. The next section deals with this question.

B. The Relation Between the Expectations Hypothesis of the Interest Rates and Exchange Rates

The previous sections have presented several related stylized facts.

- At short-horizons (less than 1 year), both expected exchange rates and expected interest rates do not conform to their respective expectations hypotheses. In fact, for exchange rates the relation is the opposite of what one might expect.
- However, these same short-term exchange rates and interest rates are forecasted based on prior information contained in past forward interest rates. That is, past long-maturity forward rates contain information about both future interest rate and future exchange rate movements.
- Moreover, these long-maturity past forward rates have substantial power relative to near-term and more current forwards, i.e., much higher R^2s from the relevant regressions.

These results beg the question – how can old information about the term structure be so much "more informative" than current information?

To understand why this might be the case, note that equations (2) and (6) can be linked via the expectations hypothesis of the interest rates as described by equation (7). In particular, note that under this hypothesis the forward interest rate differential is related to expected future spot interest rate differentials via

$$if_t^{j,k} - if_t^{*j,k} = E_t[i_{t+j,k-j} - i^*_{t+j,k-j}]. \tag{9}$$

Equation (9) implies that the interest rate differential $i_{t+j,k-j} - i^*_{t+j,k-j}$ can be broken into its expected component, $if_t^{j,k} - if_t^{*j,k}$, and its unexpected component, denote $u_{t,j}^{k-j} = (i_{t+j,k-j} - i^*_{t+j,k-j}) - (if_t^{j,k} - if_t^{*j,k})$, where $u_{t,j}^{k-j}$ is the j-step ahead forecast error of a (*k-j*)-period interest rate. For *k=1*, and combining the above breakdown with equation (2), we obtain the regression:

$$s_{t+1} - s_t = \alpha + \beta(if_{t-j}^{j,j+1} - if_{t-j}^{*j,j+1}) + \gamma u^1_{t-j.j} + \varepsilon_{t-j.j+1}. \tag{10}$$

Under the expectations hypothesis of exchange rates, α should equal 0, and both β and γ should be 1. The expectations hypothesis of the term structure allows us to understand the source of forward premium anomaly in equation (2) by separating out the β and γ coefficients.

At first glance, it may seem that use of the expectations hypothesis of the term structure is misguided. After all, as mentioned above, this hypothesis has been soundly rejected in a variety of contexts. However, note that the precise formulation of the hypothesis in equation (9) and used in (10) relates forward rates to future spot rates of interest. Whether forward interest rates forecast future spot interest rates has been a much studied topic, see, for example, Fama (1984a), Fama and Bliss (1987) and Jorion and Mishkin (1991), and, for that matter, Table 3 above. Those papers, and Table 3, tend to conclude that the forecasts work poorly at short horizons but quite well at long horizons.

These results pose somewhat of a puzzle to researchers studying the expectations hypothesis of the term structure. On the one hand, the results using past forward interest rate differentials are in stark contrast to those using current interest rate differentials. On the other hand, there is considerable evidence that long-maturity forwards forecast future interest rates, which in turn should reflect the predictive component of exchange rate moves. It seems worthwhile, therefore, separating out these effects to further understand why we so soundly rejected the theory before, and now we accept it. Regression equation (10) provides us with the ability to do such an analysis by running the multiple regression of both (i) the difference between the interest rate and forward interest rate differential and (ii) the forward rate differential itself, on future exchange rate moves.

Table 4 provides results for the regression in equation (10). The results are reported per exchange rate and for each horizon. Because the forward interest rate differentials can be quite noisy at any given horizon, we also use the average past forward rate differential, i.e., $\frac{1}{4}\sum_{j=2}^{5}(if_{t-j}^{j,j+1} - if^{*j,j+1}_{t-j})$, which should also have a coefficient of one. Across all the horizons (including the average horizon) and across both currencies, the coefficient β is always positive and hovers (albeit nosily) around 1. In contrast, the γ

coefficients are all negative and of similar magnitude. The R^2s are on the whole quite impressive.

For example, consider the results for the USD/DEM exchange rate. At forward rate horizons of two to five years, the βs are 0.97, 0.78, 1.93 and 3.32, respectively, while the γs are -2.20, -1.01, -0.92 and -0.75. Similarly, for the average forward rate, β is 3.07 and γ is -1.71 with an R^2 of 20.05%. Table 4 documents that the same forecast power for future interest rates is also true for exchange rates. The negative γ explains, therefore, why the forward premium anomaly exists from a statistical viewpoint, i.e., why we get negative coefficients in Table 1 and low R^2s. Breaking up interest rates into the two components separates out the "fundamental" information contained in the forward curve and other, unspecified, information. The fundamental information by and large conforms to the expectations hypothesis of the term structure and the expectations hypothesis of exchange rates, while this other, unspecified, information does not. Of course, by including them together, the two information sources offset each other, leading to a low R^2. The next section tries to understand the sources of this information.

IV. Fundamental Determinants of Exchange Rates, Inflation and Interest Rates

Are fundamentals driving the effects we have documented above? We argue that dated (i.e., old) information in forward rates is important because these rates predict the fundamental component of future spot rates. An alternative story is that there is simply additional information in the term structure of interest rates about future exchange rates that has nothing to do with fundamentals. If this were the case, then presumably forward rates contemporaneous with the spot rates in the standard forward premium regression would provide better information since they contain newer information. Table 4B investigate this conjecture. The regressions are analogous to those presented above under the label "**avg**" (i.e., estimates from equation (10)), except that the regressor is the average of contemporaneous (time *t*) forward rates instead of lagged forward rates. The striking result is that the R^2s on these regressions, marked "**avcon**", are extremely low -

5.73% and 2.64% versus 24.15% and 20.05% for the regressions using lagged forward rate differentials. In fact, the predictive power is only a little higher than that using the spot interest rate differential alone (see Table 1C).

A second check of the fundamentals story is to look at the relation between forward rates and inflation rates, since inflation is one of the major fundamental factors in exchange rate determination. Table 5 presents results from three sets of regressions that examine this issue. Table 5A shows results from regressions of the future change in the inflation rate on the spread between the forward interest rate and the spot rate across countries and horizons, i.e., equation (8) with changes in inflation rates as the dependent variable

$$\pi_{t+k,1} - \pi_{t,1} = \alpha + \beta(if_t^{k,k+1} - i_{t,1}) + \varepsilon_{t,k}.$$

At longer horizons, changes in inflation move approximately one-for-one with forward rate differentials. Moreover, the R^2s also increase substantially as we move to longer horizons.[7] These results are consistent with the interest rate expectations hypothesis regressions in Table 3, and they suggest that those results are potentially driven by inflation predictability, or lack thereof, at different horizons.

Tables 5B and 5C examine similar regressions except that the dependent variables are cross-country differentials between inflation rates. In Table 5B the predictors are forward rate differentials, i.e.,

$$\pi_{t+k,1} - \pi^*_{t+k,1} = \alpha + \beta\left(if_t^{k,k+1} - if_t^{*k,k+1}\right) + \varepsilon_{t,k}$$

and in Table 5C we also add the difference between the spot interest rate differential and the forward rate differential (analogous to equations (6) and (10))

$$\pi_{t+k,1} - \pi^*_{t+k,1} = \alpha + \beta\left(if_t^{k,k+1} - if_t^{*k,k+1}\right) + \gamma u^1_{t-j\cdot j} + \varepsilon_{t,k},$$

The results for the USD/GBP are again consistent with the fundamentals story. The coefficient on the forward rate differential increases towards 1 at longer horizons and the R^2s also improve. Moreover, adding the unexpected component of the spot rate differential has little or no effect on predictive power, as this term appears to be uncorrelated with future differences of changes in inflation rates. This lack of predictive

[7] Jorion and Mishkin (1991) report similar results for an earlier sample period.

power also holds for the USD/DEM series, but the patterns in the coefficients on the forward rate differentials and the predictive power are not consistent with the earlier evidence.

Nevertheless, our empirical results suggest a relation between forward interest rate differentials and future changes short-term exchange rates, interest rate differentials and inflation rate differentials; in other words, the analogous theory to the expectations hypothesis for each of these quantities conforms well to the data over long horizons. These findings are in contrast to current thinking. Of course, the puzzle has not been completely resolved. As seen in Tables 3 and 5, short-term forward rates are much less related to next period's spot interest and inflation rates than longer term forward rates.

While we do not have an explanation for these findings, our results are informative within the context of the current literature. Note that there are two important features to our findings: (i) there is a breakdown between the short- and long-term, and (ii) the common element appears to be the "failure" of short-term interest rates to conform to the theory. Two popular (albeit unsuccessful) explanations for the forward premium puzzle are the existence of risk premia (e.g., Fama (1984), Bekaert (1996)) or expectational errors (statistical issues, such as peso problems, or behavioral phenomena, such as violations of rational expectations[8]). The fact that there is a breakdown between short and long horizons, however, would appear to impose an even greater burden on these theories to explain the data. An alternative view is provided in the macroeconomics literature, which explains violations of short-term Fisher effects in terms of liquidity and money demand (e.g., see Friedman (1968), Dornbusch (1976), Lucas (1982), Mussa (1986), and McCallum (1994), among others).

A recent paper by Alvarez, Atkeson and Kehoe (2002) argues that the asset market and goods market are segmented due to transactions costs in these markets, i.e., trading costs in assets and cash-in-advance constraints in goods. Under these circumstances, they show that open market operations by the government can move real interest rates and exchange rates in the same direction but inflation in the opposite direction. The intuition is that because of transactions costs only a segment of the market participates, so that these aggregate variables reflect only a portion of the aggregate households in the economy. Alternatively, one could impose frictions in the goods markets. For example, in Dornbusch's (1976) sticky-price monetary model, government market operations (such as a fall in the real money supply) lead to a rise in interest rates. This rate rise causes capital to flow into the country as investors try and take advantage of these high rates, resulting in an appreciation of the currency. Of course, as good prices begin to adjust, both rates and the currency begin to fall to their long-run equilibrium.

Froot and Thaler (1990) take a less formal approach and argue that central banks can move interest rates artificially. Investors take advantage of these opportunities, which lead to a corresponding, though only partial, change in exchange rates in the short run. Why don't exchange rates adjust immediately? Froot and Thaler argue that the responsiveness of investors might be slow or that there might be limited capital (due to risk aversion). Alternatively, one could argue that the central bank is a large trader who in theory has the power to counteract actions by investors (e.g., by devaluing the currency). This may impose too great a risk on most investors.

All these theories – segmentation in the asset market, frictions in the goods market, limited arbitrage in the foreign exchange market – are consistent with short-term deviations from *fundamentals*, with interest rates being the source. This is the precise finding of our empirical analysis.

Note that all the violations of uncovered interest rate parity, the Fisher effect and the expectations hypothesis of the term structure are driven by high relative *ex ante* real rates. Define the difference in *ex ante* real rates across countries as $E_t\left[ir_{t,t+1} - ir^*{}_{t,t+1}\right] = \left(i_{t,t+1} - i^*{}_{t,t+1}\right) - E_t\left[\pi_{t,t+1} - \pi^*{}_{t,t+1}\right]$, where we measure $E_t\left[\pi_{t,t+1} - \pi_{t,t+1}{}^*\right]$ from the fundamentals identified in this paper, namely the forward interest rate differential, i.e., $if_{t-j}^{j,j+1} - if^{*j,j+1}_{t-j}$. Our measure of ex ante real rates is, therefore, noisy in that it includes the true measure plus unexpected changes in expectations over the past j years.

[8] For the former, see Lewis (1995) for exchange rates and Bekaert, Hodrick and Marshall (2001) for interest rates. For the latter, see Froot and Ito (1989).

Consider equation (2), but now conditioning on the magnitude of $E_t\left[ir_{t,t+1} - ir^*_{t,t+1}\right]$. In particular, we rerun regression (2) for two subsamples of the data, separated by the absolute value of our measure of ex ante real rate differentials.

IV. Concluding Remarks

Forward interest rate differentials between countries forecast future spot interest rate differentials, inflation rate differentials, and movements in exchange rates. The forecasts are in the direction that is consistent with the international Fisher effect, expectations hypothesis of the term structure and uncovered interest rate parity, respectively.

The puzzle for all of these hypotheses remains at short horizons (in our study, one year). We provide some evidence (albeit preliminary) that the source of the short horizon violations are differentials in ex ante real rates. In particular, when real rate differentials tend to be high, uncovered interest rate parity performs poorly relative to other periods. This is consistent with a number of exchange rate determination theories laid out in the literature that relate to frictions in either the asset or goods markets, or irrationality on the part of investors.

Appendix A: Data

We collected monthly data from Datastream for the period January 76 to December 2002, for a total of 324 months. Data was available for the full period for exchange rates and CPI levels. Zero curve data was derived from LIBOR (6 months and 12 months) and swap (2,3,4,5 year s.a. swap rates) data[9]. Since swap data became available only in the late 1980's, we augment our zero curve data with data provided to us by Philip Jorion. Jorion and Mishkin (1991) collect and derive data for zero coupon bonds from one-month to five years for the U.S., U.K. and Germany starting in August 1973[10] to December 1988. Swap and LIBOR data is preferred to typical government data because quotes are more liquid, and are less prone to missing data, supply and demand effects, and tax-related biases. This is especially true in the context of our model, since most of our regressions (and certainly the important ones, e.g., (8) and (10)) use forward differentials. To the extent there is a swap spread in swap data, to a first approximation the effect of this spread is diminished by use of swap (or forward) differentials.

[9] Cubic spline functions were fitted each month for each country to create a zero curve for maturities 6,12,18,…,60 months. Technically, our spline function fits exactly points in which we have data, namely LIBOR rates for the 6m and 12m maturities, and semi-annual swap rates for maturities 24m, 36m, 48m, and 60m. The only maturities we need to spline are, therefore, 18m, 30m, 42m and 54m. We maximize the smoothness of the spline function over these unknowns, by minimizing sum of squared deviations (subject to available datapoints taken as given).

[10] We thank Philip Jorion for graciously providing us with the data.

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Table 1
Preliminaries

Table 1A: Summary Statistics – Exchange Rates

XR	mn(%)	std(%)	acr1	acr12	corr
USD/GBP	-1.98	11.29	0.92	-0.02	0.70
USD/DEM	-0.63	13.25	0.93	0.11	

Table 1B: Summary Statistics – Forward Rate Differentials

DF	mth	mn(%)	std(%)	acr1	acr12
USD-GBP	12	-1.79	2.20	0.95	0.54
	24	-1.09	1.34	0.88	0.42
	36	-1.34	1.38	0.90	0.46
	48	-1.31	1.38	0.85	0.43
	60	-1.45	1.50	0.88	0.49
USD-DEM	12	1.53	2.53	0.97	0.75
	24	1.77	1.88	0.96	0.73
	36	1.79	1.62	0.96	0.72
	48	1.75	1.60	0.96	0.71
	60	1.66	1.68	0.97	0.73

Table 1C: The Forward Premium Puzzle: 1yr Horizon

	a	(se)	b	(se)	r2
USD/GBP	-4.17	2.03	-1.23	0.91	5.69
USD/DEM	0.21	2.20	-0.55	0.67	1.10

b	se	df	LMtst	pval	Wtst	pval
=1		2	7.32	0.03	7.90	0.02
-0.74	0.67	1	0.62	0.43	0.61	0.44

For a description of the data please see the data appendix.
Table 1A: **XR** is the annual exchange rate change, sampled monthly.
Table 1B: **DF** is the interest forward differential t to t+12, t+12 to t+24,…, t+48 to t+60
Table 1C: The regression is

$$s_{t+1} - s_t = \alpha + \beta(i_{t,1} - i^*_{t,1}) + \varepsilon_{t,1}.$$

The first set of tests are LM and Wald tests for the joint hypothesis that beta=1 for both currencies. The second set of tests impose that the two betas are equal and provides an estimate of that beta.

Table 2
The Expectations Hypothesis of XRs

Table 2A: Regression Results

	j	a	(se)	b	(se)	r2
USD/GBP	0	-4.17	2.03	-1.23	0.91	5.69
	1	-1.02	2.84	0.89	1.37	1.10
	2	3.29	2.21	3.94	0.72	23.19
	3	0.28	1.86	1.73	1.09	4.45
	4	2.51	1.88	3.09	1.19	16.72
USD/DEM	0	0.21	2.20	-0.55	0.67	1.10
	1	-2.44	2.82	1.03	1.65	2.12
	2	-2.72	2.68	1.17	1.56	2.03
	3	-5.09	2.58	2.55	1.11	9.44
	4	-7.23	2.63	3.98	1.03	25.55

Table 2B: Horizon-specific Tests

j	b	se	df	LMtst	pval	Wtst	pval
0	=1		2	7.32	0.03	7.90	0.02
1	=1		2	0.01	0.99	0.01	0.99
2	=1		2	3.19	0.20	23.28	0.00
3	=1		2	1.06	0.59	1.96	0.38
4	=1		2	2.41	0.30	9.19	0.01
0	-0.74	0.67	1	0.62	0.43	0.61	0.44
1	0.93	1.29	1	0.01	0.92	0.01	0.92
2	2.99	1.08	1	2.32	0.13	4.53	0.03
3	2.06	0.89	1	0.43	0.51	0.45	0.50
4	3.66	1.00	1	0.90	0.34	1.49	0.22

Table 2C: Cross Horizon Tests

Currency	j	b	se	tst	df	pval
Both	01234	=1	-	4.86	10	0.90
Both	1234	=1	-	4.36	8	0.82
USD/GBP	1234	=1	-	2.91	4	0.57
USD/DEM	1234	=1	-	4.00	4	0.41
Both	01234	0.45	0.17	4.81	9	0.85
Both	1234	2.10	0.48	3.68	7	0.82
USD/GBP	1234	2.70	0.52	2.90	3	0.41
USD/DEM	1234	2.51	0.84	2.91	3	0.41

For a description of the data please see the data appendix.
Table 2A: Results of

$$s_{t+1} - s_t = \alpha + \beta(if_{t-j}^{j,j+1} - if^{*j,j+1}_{t-j}) + \varepsilon_{t-j.j+1}\ .$$

Table 2B: The first set of tests are Wald tests for the joint hypothesis that beta=1 for both currencies. The second set of tests imposes that the two betas are equal and provide an estimate of that betas.
Table 2C: cross horizon joint Wald tests.

Table 3

The Expectations Hypothesis of Interest Rates

Table 3A: The Expectations Hypothesis country-by-country

Contry	**K**	**a**	**se**	**b**	**se**	**R2**
USD	1	-0.15	0.37	0.02	0.32	0.01
	2	-0.40	0.90	0.23	0.54	0.80
	3	-0.96	1.13	0.65	0.40	6.21
	4	-1.87	1.14	1.13	0.23	18.71
GBP	1	-0.26	0.33	0.56	0.24	12.17
	2	-0.81	0.57	1.03	0.29	27.06
	3	-0.99	0.72	1.21	0.32	35.41
	4	-1.53	0.74	1.42	0.36	46.08
DEM	1	-0.36	0.33	0.54	0.30	8.68
	2	-0.90	0.57	0.89	0.35	17.45
	3	-1.58	0.57	1.34	0.31	36.09
	4	-2.33	0.51	1.65	0.30	52.01

Table 3B: The Expectations Hypothesis Across countries

Contry	**K**	**a**	**se**	**b**	**se**	**R2**
USD-GBP	1	-0.05	0.33	0.31	0.16	3.93
	2	-0.01	0.52	0.45	0.20	8.62
	3	-0.09	0.69	0.48	0.28	11.17
	4	-0.16	0.89	0.65	0.32	17.38
USD-DEM	1	-0.06	0.29	0.49	0.23	8.29
	2	-0.06	0.60	0.69	0.27	15.55
	3	-0.14	0.78	0.90	0.26	27.65
	4	-0.20	0.91	1.09	0.25	39.56

For a description of the data please see the data appendix.
The first three panels estimate the expectations hypothesis for interest rates country by country.

$$i_{t+k,1} - i_{t,1} = \alpha + \beta(if_t^{k,k+1} - i_{t,1}) + \varepsilon_{t,k}.$$

The latter two panels estimate the implication of (4) across countries.

Table 4
The Expectations Hypothesis of IRs and XRs

Table 4A: Imposing the null of the expectations hypotheses of IRs

	j	a	(se)	b	(se)	c	(se)	r2
USD/GBP	1	-2.93	2.71	0.12	1.47	-1.53	0.93	9.15
	2	1.11	2.60	2.72	1.20	-1.21	0.80	28.70
	3	-1.99	2.18	0.40	1.12	-1.10	1.01	8.99
	4	0.93	2.44	2.15	1.58	-0.65	0.96	18.17
	avg	2.62	2.67	3.96	1.63	-1.09	0.78	24.15
USD/DEM	1	-2.89	2.80	0.97	1.56	-2.30	0.94	11.36
	2	-2.31	2.21	0.78	1.49	-1.01	0.53	5.14
	3	-4.21	1.98	1.93	1.18	-0.92	0.64	12.39
	4	-6.23	2.25	3.32	1.07	-0.75	0.55	27.56
	avg	-6.35	3.43	3.07	1.89	-1.71	0.68	20.05

For a description of the data please see the data appendix.
Results of estimation of the regression

$$s_{t+1} - s_t = \alpha + \beta(if_{t-j}^{j,j+1} - if^{*j,j+1}_{t}) + \gamma u_{t-j.j}^{1} + \varepsilon_{t-j.j+1}.$$

avg denotes a regression on the average of the regressors in the first four regressions in each panel: $\frac{1}{4}\sum_{j=1}^{4}(if_{t-j}^{j,j+1} - if^{*j,j+1}_{t-j})$

Table 4B: the information in the average contemporaneous forward rate differential

	H	a	(se)	b	(se)	c	(se)	r2
USD/GBP	avcon	-4.22	2.11	-1.40	1.49	-1.11	1.35	5.73
USD/DEM	avcon	-1.35	2.50	0.27	1.55	-1.39	1.15	2.64

For a description of the data please see the data appendix.
avcon denotes the average contemporaneous interest forward spread: $\frac{1}{4}\sum_{j=1}^{4}(if_{t}^{j,j+1} - if^{*j,j+1}_{t})$

Table 5
Inflation

Table 5A:
Inflation predictability by forward rates country-by-country

Contry	k	a	se	b	se	R2 .
USD	1	-0.22	0.45	0.12	0.39	0.29
	2	-0.90	0.90	0.73	0.48	9.04
	3	-1.71	0.95	1.16	0.46	20.08
	4	-2.55	0.94	1.36	0.51	32.05
GBP	1	-0.55	0.49	0.29	0.37	1.61
	2	-1.41	0.78	0.81	0.46	9.38
	3	-1.69	1.00	0.86	0.31	11.81
	4	-2.29	1.18	0.78	0.27	11.18
DEM	1	-0.22	0.23	0.21	0.20	2.14
	2	-0.49	0.51	0.34	0.28	5.12
	3	-0.92	0.65	0.60	0.26	14.71
	4	-1.62	0.63	0.95	0.23	35.90

For a description of the data please see the data appendix.
Estimates for

$$\pi_{t+k,1} - \pi_{t,1} = \alpha + \beta(if_t^{k,k+1} - i_{t,1}) + \varepsilon_{t,k}.$$

Table 5B:
Inflation differential predictability by forward rate differentials

	k	a	(se)	b	(se)	r2
USD/GBP	1	-0.55	0.32	0.27	0.11	14.53
	2	-0.74	0.34	0.39	0.18	11.23
	3	-0.88	0.32	0.24	0.16	4.69
	4	-0.95	0.41	0.51	0.25	10.89
	5	-0.59	0.67	0.96	0.41	26.14
USD/DEM	1	0.79	0.28	0.44	0.09	32.19
	2	0.76	0.48	0.55	0.21	21.23
	3	0.96	0.76	0.50	0.30	12.22
	4	1.10	0.82	0.46	0.30	9.90
	5	1.35	0.77	0.36	0.25	6.79

For a description of the data please see the data appendix.
Estimates for

$$\pi_{t+k,1} - \pi^*_{t+k,1} = \alpha + \beta\left(if_t^{k,k+1} - if_t^{*k,k+1}\right) + \varepsilon_{t,k}$$

Table 5C:
Predicting inflation as in (10)

	k	**a**	**(se)**	**b**	**(se)**	**c**	**(se)**	**r2**
USD/GBP	1	-0.63	0.43	0.43	0.18	0.08	0.15	12.49
	2	-0.84	0.49	0.27	0.24	0.03	0.15	4.82
	3	-0.62	0.59	0.71	0.38	0.16	0.15	13.59
	4	-0.71	0.95	0.89	0.50	-0.05	0.15	26.28
USD/DEM	1	0.77	0.50	0.55	0.21	0.07	0.18	21.50
	2	0.93	0.73	0.53	0.31	0.06	0.20	12.57
	3	1.05	0.72	0.50	0.33	0.05	0.20	10.18
	4	1.27	0.67	0.41	0.36	0.06	0.22	7.15

For a description of the data please see the data appendix.
Estimates for

$$\pi_{t+k,1} - \pi^{*}_{t+k,1} = \alpha + \beta\left(if_t^{k,k+1} - if_t^{*k,k+1}\right) + \gamma u^{1}_{t-j.j} + \varepsilon_{t,k},$$