Technical Change as Factors' Shifting Impacts:

Harrod Neutrality, with a (Non-Neutral) "Twist"

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Abstract:

This paper reconciles seemingly contradictory dominant features of growth by reinterpreting the CES "distribution-parameters" as time-varying weights signifying factors' "impacts" on production, such that technical-change corresponds to a decrease in labor's impact, along with an increase in capital's. This is viewed as a natural outcome of the fact that ideas are embedded within capital. The proposed non-neutral mechanism is shown to be "quasi-equivalent" to standard Harrod-neutrality, due to the fact that as capital's impact asymptotes to one, changes in it become increasingly negligible and balanced-growth is attained. During industrialization non-neutrality is still significant, interpreted as a "capital-using" mechanization process where capital "replaces" labor, and is thus "*dis*-augmented"; the result being constant factor-shares. This resolves a recent controversy regarding the measurement of TFP growth, specifically in East Asian NICs. In addition, non-neutrality is linked with time-ranked "appropriate-technologies", whereby poverty-traps may emerge as an interaction between technology and institutional arrangements such as credit-constraints. Thus the model is also broadly consistent with under-development.

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1. **Introduction**

Following the major headway due to the "growth renaissance" of the 1990's, recent work has begun delving deeper into the micro-foundations of the aggregate production function and the process of its transformation through time, shedding new light on this most fundamental macro-economic entity. The current paper follows this trend, based on what boil-down to the following three seemingly contradictory features of growth:

i. Balanced growth with constant factor shares in an industrialized economy.
ii. Constant factor shares along an industrialization path (rise in the capital-output ratio).
iii. Systematic "medium-run" factor share trends in the industrialized economies.

The first feature necessitates strict labor augmenting technical change (LATC) or, equivalently, Harrod neutrality.[1] Such technical changes are compatible with any constant returns to scale function of labor and capital; in particular Cobb-Douglas, with its technologically pinned-down factor shares. Now whereas the second feature indeed strengthens the case for Cobb-Douglas, the third, in addition to consistently estimated low elasticities, weakens it.[2] However, applying a low substitution form in an effort to reconcile the later two features is bound to induce capital "*dis*-augmentation", which of-course counters the neutrality required by the first.[3]

The model presented here provides a simple and intuitive mechanism, yielding non-neutral technical changes which *tend* towards neutrality. The basic idea is to refine the deeply-rooted

[1] See Harrod (1937), Robinson (1938), Uzawa (1961) and Barro & Sala-i-Martin (2004, ch 2). For recent compelling rationalizations of LATC based on micro-founded aggregative functions, see Acemoglu (2003) and Jones (2005).
[2] See estimations by David & van de Klundert (1965), Lucas (1969) and most recently Antràs (2004). Jones (2005) derives an inter-technique Cobb-Douglas form from indeed low-substitution individual processes, in a model where factor shares display significant random variation through time due to stochastic properties. In contrast, Bentolila & Saint-Paul (2003) and Acemoglu (2003) show what appear to be deterministic "hump-shaped" patterns of the labor-share in 20th century post-war data. Indeed, so persuasive was the consistent rise in the U.S. labor share during the 1950's that scholars back then started doubting Paul Douglas's famed stylized fact (see e.g. Solow, 1958).
[3] Relaxing the Cobb-Douglas restriction and applying a CES form, Caselli (2005) is among the first to willingly accept a negative correlation between the "efficiency" or "productivity" indices of labor and capital, indeed referring to such non-neutral differences in technology as: "stunning".

dichotomous notion of "factor efficiency augmentation", by reinterpreting the CES "distribution parameter" of labor as the "weight", or "impact" of labor on the production process. Technical change is assumed to correspond to a decrease in the impact of labor, which also means a simultaneous increase in the impact of capital (one minus labor's impact).

For high levels of technology the proposed mechanism is shown to be quasi-equivalent to Harrod neutrality, due to the fact that as capital's impact asymptotes to one, changes in it become increasingly negligible. We thus have asymptotic LATC where balanced growth is attained in the limit, due to the ever-significant diminishing impact of labor, which tends to be offset by capital deepening. In this context, the current model with its featured "twisted CES" function does not intend to serve as a "practical" alternative to a standard CES-LATC formula, but rather to justify the later as a valid approximation at high technological levels. In addition, following the "induced innovation" theoretic literature due to Samuelson (1965) and others, which has recently been revived by Acemoglu (2003), Caselli (2005) and Jones (2005), the model provides a possible explanation for the "labor-bias" of technical change.

At low levels of technology, when capital's impact is sufficiently far from one, non-neutrality is substantial and interpreted as a process of mechanization where capital "replaces" labor.[4] This enables the consistency of low factor substitution with the experience of industrializing countries where factor shares are more-or-less invariant even as the capital-output ratio rises.[5] The idea is that the rapid capital deepening during initial industrialization is at least partially offset by the actual process of mechanization ("capital dis-augmentation").[6] The paper thus helps resolve a

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[4] The verb *replace*, referring to a dynamic process, is distinguished here from *substitute*, a static property of a given technique. To grasp the inherent non-neutral nature of "replacement", suppose a hydraulic excavator and its driver have the same output as 200 workers with shovels. If the excavator's value is greater than 200 shovels (which is clearly an understatement) then capital "*dis*-augmentation" is induced. Such a (one shot) replacement of "workers" by "machines" is highlighted by Zeira (1998). See historical accounts cited by Habakkuk (1962).

[5] See Young (1995) and Gollin (2002) in longitudinal and cross-sectional contexts, respectively; but see footnote 6.

[6] According to Abramovitz & David (1973), the aggregate capital share in the U.S. during the 19th century actually *rose* (from 0.23 to 0.37), which in the terms above implies that capital deepening was far from "catching up" with

recent controversy, referring to East Asian newly industrialized countries, whereby Cobb-Douglas based growth accounting is claimed to understate the contribution of technical change, but low substitution seemingly induces groundless non-neutrality.

Besides the above features, the model is compatible with the experience of least developed economies as well. In particular, the inherent non-neutrality generates the result by which no technique dominates another, but rather the optimal technique is a function of the capital-labor ratio; thus an "appropriate technology". The recent work of Banerjee & Duflo (2004) highlights the interaction of institutional arrangements, such as credit constraints, with appropriate technologies, induced by rising fixed-costs, in generating capital misallocation and thus under-development. Though the current paper formulates fully convex technologies, the intuition here is similar. In particular, capital's replacement of labor can be seen as a dynamic equivalent of a fixed cost, thus an "*inter-temporal* non-convexity".

At a deeper level, one may ponder regarding the assumed process of technical change, as well as the fundamental postulate regarding the "direction" of impact shift (i.e. from labor to capital). A more elaborate, yet still simple, idea-based model is thus presented with an underlying assumption whereby production labor is the "basic" factor of production, empowered merely by the capital it operates.[7] The essence of the motor or cognitive services provided by labor is thus assumed to remain relatively unchanged through time, whereas capital undergoes persistent qualitative changes due to its enhancement by new innovations; thereby increasing its impact on production while eroding labor's.[8]

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technical change. The authors interpret this as evidence for what they somewhat vaguely refer to as: "capital deepening technical change". The current paper does not view such technical change as a distinct phenomenon. Rather, "labor augmentation" and "capital *dis*-augmentation" are shown to be two sides of the same coin.

[7] As in other idea-based models, human capital, though obviously a necessary ingredient, is assumed to have but a secondary role in the process of growth; at least concerning the production, rather than R&D, sector.

[8] Existing models which formalize qualitative changes in capital are based on a multiplicative form. Relaxing their basic Cobb-Douglas assumption results in a reversion to their featuring of qualitative changes in *labor* (e.g. compare the original Romer, 1990, Cobb-Douglas model with its "nested-CES"-form extension by Acemoglu, 2003).

This extended mechanism features some interesting "by-products". For example, new ideas not only reduce the impact of labor, but of previous ideas as well. Thus a form of obsolescence is combined with a conceptually measurable "stock of ideas". In addition, the model is consistent with the stylized-fact concerning a constant flow of patents (new ideas) amidst exponential productivity growth. Indeed the model, though fully consistent with existing endogenous growth models, highlights the fact that the measure of productivity is *not* the conceptual entity measuring the stock of ideas.

The paper is organized as follows. The next section introduces the basic shifting impacts model, which is a standard CES function, but with a "twist" whereby constants are made time dependant, and vice-versa. This is followed by a parameter transformation, showing the quasi-equivalence to a Harrod neutral model with a "quasi-steady-state". The third section emphasizes the non-neutrality aspects of the model, theoretically and empirically relevant for industrializing as well as under-developed countries. The fourth section introduces the elaborate idea-based growth model. The fifth section concludes.

2. Basic Shifting Impacts Model

2.1 Dynamic CES, with a "Twist"

Consider the following CES production function of labor (L) and (physical) capital (K):

$$Y_t = \left[m_t(\lambda L_t)^\rho + (1-m_t)(\mu K_t)^\rho\right]^{\frac{1}{\rho}}. \tag{1}$$

While this formulation is similar to the standard one, introduced by Arrow, Chenery, Minhas & Solow (1961), three points should be stressed. First, the parameter ρ is restricted to being strictly negative and finite, that is a low elasticity of substitution: $\sigma \equiv \frac{1}{1-\rho} \in (0,1)$.[9] Second, the strictly positive parameters λ and μ are *not* to be seen as time-varying "factor efficiency" indices, but rather as constant "unit adjustments".[10]

The most fundamental point concerns m ($m \in (0,1]$), or $1 - m$, which Arrow et al. (1961) originally termed the "distribution parameter(s)"; these being the actual factor shares, regardless of the capital-labor ratio, only in the limiting Cobb-Douglas case ($\rho \to 0$). Here m and $1 - m$ are seen as indicating the *weights*, or *impacts* of labor and capital on production, respectively. Furthermore, technical change is viewed as a reduction in m; thus an increase in $1 - m$. Specifically, we shall assume an exponential decline of m, given an initial m_0 (possibly 1):

$$m_t = m_0 e^{g_m t}\ ;\ g_m < 0\,, \dot{g}_m = 0\ \ \forall t, \qquad (2)$$

where g_a denotes the exponential growth rate ($\frac{\dot{a}}{a}$) for any variable ("a") used hereafter.

Somewhat equivalent to the question which motivated the "induced innovation" literature of the 1960's (e.g. Samuelson, 1965), namely why is labor "augmented" rather than capital, one may ponder regarding the assumptions explicit in (2), namely, why is labor's impact decreasing, rather than increasing? Section four provides an attempt to rationalize this form of technical change, based on more elaborate micro-foundations. It shall be argued that new knowledge is

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[9] Low elasticity reflects the limited ability to substitute factors within any given technique; as opposed to factors' "replacement" induced by technical change. The "neo-classical" requirement for $\sigma > 0$ is justified by "allowing" a certain degree of flexibility in the manner of applying the given technique; e.g. shift work, plant configuration, etc.

[10] Arrow et al. (1961) originally did not incorporate such "efficiency" indices, but rather a Hicks neutral one. But as we shall see below, λ is indeed of little importance, while "back of the envelope" calculations give: $\mu \cong 1$. Thus in the more elaborate model of section four these parameters shall be omitted for clarity.

always embedded within capital, while the basic services provided by production labor remain unchanged; thus technical change increases capital's impact at the expense of labor's.

With a less-than-unit elasticity of substitution, not only do the marginal products tend to zero, but also, unlike the Cobb-Douglas case, the level of output is bounded when either of the two factors is being held constant; even as the input of the other factor increases. Though the point above, as well as the analyses below will be valid for any strictly negative value of ρ, it is helpful to keep in mind the simple example of $\rho = -1$ ($\sigma = 0.5$), in which case production takes the weighted harmonic mean form:[11]

$$Y_t = \left(\frac{m_t}{\lambda L_t} + \frac{1 - m_t}{\mu K_t} \right)^{-1} . \tag{1'}$$

With this simple specification we clearly see how the constancy of either factor, in particular the non-accumulable labor factor, serves as a "drag" on the benefits from the accumulation of the other. It is thus natural to consider technical change as working through an alleviation of this limit, namely decreasing the impact, or weight of the hindering factor.

Based on this interpretation we can now give somewhat alternative intuitions for the experience of industrialized economies, characterized by balanced growth and correspondingly constant factor shares. Given a rate of technical change that decreases labor's impact and relaxes its drag on output, capital's accumulation soon follows; the result indeed being balanced growth: a complementary process of technical change and capital deepening. Since a decrease in labor's impact, induced by technical change, makes labor more redundant, while capital deepening,

[11] Careful econometric estimates of the elasticity of substitution typically narrow it to the 0.3 – 0.7 range. See David & van de Klundert (1965), Lucas (1969) and Antràs (2004). See also review in Acemoglu (2003, footnote 3).

given low substitution, makes labor scarcer, the constancy of factors shares is, again, a balance between these two forces.

2.2 "Quasi Equivalence" to Harrod Neutrality

We now turn to applying a simple parameter transformation, which will show how the basic factor's shifting impact model is "quasi-equivalent" to the standard Harrod neutral or LATC model. We shall define the following "synthetic" entities, which have no conceptual basis and are merely convenient transformations of the impact parameters:

$$B_t \equiv m_t^{\frac{1}{\rho}}; \tag{3}$$

$$Q_t \equiv (1-m_t)^{\frac{1}{\rho}} = (1-B_t^{\rho})^{\frac{1}{\rho}}. \tag{4}$$

The revised production function and the capital share are thus, respectively:

$$Y_t = \left[(\lambda B_t L_t)^{\rho} + (\mu Q_t K_t)^{\rho}\right]^{\frac{1}{\rho}}; \tag{5}$$

$$s_t^K = \left[1 + \left(\frac{\lambda B_t L_t}{\mu Q_t K_t}\right)^{\rho}\right]^{-1} = \frac{(\mu Q_t K_t)^{\rho}}{(\lambda B_t L_t)^{\rho} + (\mu Q_t K_t)^{\rho}} = \left(\frac{\mu Q_t K_t}{Y_t}\right)^{\rho}. \tag{6}$$

The expression for the labor share simply swaps the terms $\lambda B_t L_t$ and $\mu Q_t K_t$.

The parameter B corresponds to the standard "labor efficiency" term, the source for balanced growth, whereas Q ("capital efficiency") is the source for non-neutrality and the reason this model is said to be *quasi*-equivalent to LATC.[12]

From the definition of B and Q in (3) and (4) we can easily express their growth rates as:

$$g_B = \tfrac{1}{\rho} g_m ; \tag{7}$$

$$g_Q = -\tfrac{1}{\rho} g_m \left(\tfrac{m_t}{1-m_t}\right) = -g_B \left(B_t^{-\rho} - 1\right)^{-1} . \tag{8}$$

Now, recall our prior, as expressed in (2), being a constant exponential decrease in m. Keeping in mind that ρ is negative, the growth rate of the "labor augmenting" entity B, evident in (7), is thus indeed exponentially increasing, that is $g_B > 0$ with $\dot{g}_B = 0$, $\forall t$.

What is no less important to notice here, though, is that in parallel to the rise in B, not only does Q decrease, approaching one in the limit, but as clear from (8) so does its growth rate (in absolute value), which tends to zero. Thus we have a rapid decline of Q, reaching the vicinity of one "very quickly", while B perpetually increases at a constant rate.[13] When Q is "sufficiently" close to one, the economy approaches a balanced growth path, with a constant growth rate: g_B. In addition, as can be seen in (6), factor shares are constant as well.

It should be noted, however, that in the transitional stage too, when Q is still "significantly" larger than one (but decreasing), the factor shares may be constant. What is needed for this, as

[12] We can obtain the standard CES form: $Y_t = [\alpha(D_tK_t)^\rho + (1-\alpha)(A_tL_t)^\rho]^{1/\rho}$, by defining: $A_t \equiv \frac{\lambda B_t}{(1-\alpha)^{1/\rho}}$ and $D_t \equiv \frac{\mu Q_t}{\alpha^{1/\rho}}$ for some arbitrary constant $\alpha \in (0,1)$. But a major point of the paper, elaborated in sub-section 3.2, is that A and D (or rather B and Q) are inherently related to one another, and thus constitute but a *single* entity, or degree of freedom.

[13] The intuition can be traced back to changes in m vs. $1 - m$. Suppose m declines from 0.002 to 0.001. While negligible in percentage points, this is a significant 50% drop. Conversely, the corresponding change in $1 - m$: 0.998 to 0.999, is negligible in any sense. The same reasoning holds *a fortiori* for even smaller levels of m.

apparent from (6), is that the growth rate of per worker capital exceeds that of B; though converging to it as Q approaches one. Specifically, it must be that: $g_B \cong g_{K/L} + g_Q$; which, given (8), means: $g_B \cong (1-m)g_{K/L}$. This implies that output growth is less than capital growth, but converging to it, or in other words: the output-capital ratio is converging "from above" to a constant value. Such a phenomenon is indeed a stylized fact of growth, associated with the "transitional dynamics" of the neo-classic growth model. We will elaborate on this in sub-section 3.2, below.

2.3 "Quasi Steady State" in a Solow Growth Model

One can proceed to analyze the balanced growth path/steady-state of a standard neo-classic growth model. Define "per efficiency unit" variables: $\tilde{y} \equiv \frac{Y}{BL}$ and $\tilde{k} \equiv \frac{K}{BL}$, so that (5) becomes:

$$\tilde{y}_t = \left[\lambda^\rho + \left(Q_t \mu \tilde{k}_t\right)^\rho\right]^{\frac{1}{\rho}}. \tag{9}$$

With a constant saving rate s (as also in a Ramsey model on its balanced growth path) and the (slightly manipulated) law of motion: $\frac{1}{s}\dot{\tilde{k}}_t = \tilde{y}_t - E\tilde{k}_t$, where $E \equiv \frac{g_B + g_L + \delta}{s}$ (δ is the depreciation rate), the steady-state value of $\tilde{k}$ is easily obtained.[14] But here this will not be a true steady state since Q is time-dependant and thus the dynamical system can not be made fully autonomous. Therefore we shall refer to a "quasi steady-state" (QSS) and continue maintaining time

[14] Assuming $\rho < 0$ in (1), or (5), gives: $\lim_{K\to 0}\partial Y/\partial K = \mu Q < \infty$ (unless $m = 1$), violating the Inada condition for this limit, thus potentially leading to the "trivial" (zero) steady state (see Barro & Sala-i-Martin, 2005). A *sufficient* restriction in order to avoid this is thus: $\mu > (\delta + g_L)/s$.

subscripts for the QSS values, denoted by "hats". Solving the law of motion equation for the (instantaneous) steady-state "per efficiency unit" level of capital we get:

$$\hat{k}_t = \lambda\left[E^{\rho} - (Q_t\mu)^{\rho}\right]^{-\frac{1}{\rho}}; \tag{10}$$

and following the law of motion, we have: $\hat{y}_t = E\hat{k}_t$. Equation (10) clearly shows that as Q tends to one, the model approaches an autonomous state with a "true" balanced growth path, where "tilde", and thus "hat" variables are constant and time indices are redundant.

Substituting the steady-state output-capital ratio (i.e. E) in (6) we get a simple expression for the QSS capital share:

$$\hat{s}_t^K = \left(\frac{Q_t\mu}{E}\right)^{\rho} = \left(\frac{g_B + g_L + \delta}{sQ_t\mu}\right)^{-\rho}. \tag{11}$$

It should be noted that (11) is *not* the capital share during "transitional dynamics", since $\dot{\tilde{k}} > 0$ at that stage. Thus (11) is actually valid only for the "true" (i.e. limiting) steady-state, where we get: $\hat{s}^K \equiv \lim_{t\to\infty} \hat{s}_t^K = \left(\frac{g_Y+\delta}{s\mu}\right)^{-\rho}$, from which we can "back out" μ according to:

$$\mu = \left(\frac{\hat{g}_Y + \delta}{\hat{s}}\right)\left(\hat{s}^K\right)^{\frac{1}{\rho}}, \tag{12}$$

where $\hat{g}_y$ and $\hat{s}$ denote steady-state (balanced growth) levels of per worker output and the savings rate, respectively. Since the terms in both brackets are typically around one third, we have:

$\mu = \left(\frac{1}{3}\right)^{\frac{1+\rho}{\rho}}$ or $\mu^{\rho} = \left(\frac{1}{3}\right)^{1+\rho}$. Thus with σ = 0.5, or ρ = –1, we back out μ = 1. However, the estimation in (12) is somewhat sensitive to the chosen elasticity. For example: $\sigma = \frac{1}{3}$ (ρ = –2) sets $\mu = \frac{1}{\sqrt{3}} \cong 0.58$, while $\sigma = \frac{2}{3}$ (ρ = –0.5) sets μ = 3.

3. Non-Neutrality

3.1 The Rotating Production Function

The previous section has shown the tendency of the simple factors' shifting impacts model for an asymptotic balanced growth path with constant factor shares. Nevertheless, we should bear in mind that that was a good approximation for an economy in which the level of technology (inversely related to labor's impact) and the capital-labor ratio are high. It is thus the initial phases of development, or the lack of it, which ought to be relatively interesting in the context of the current paper; these are the cases where the non-neutrality aspect of the model is most apparent.

The following proposition establishes the non-neutral, or "rotation" property of (1), distinguishing it from the standard case of neutral technical change.

PROPOSITION 3.1: *For any combination (L,K) satisfying $\frac{K}{L} = \frac{\lambda}{\mu}$, the level of output is the same for any level of technology (m). For any combination (L,K) satisfying $\frac{K}{L} > \frac{\lambda}{\mu}$, output is higher when m is lower (higher technologic level). For any combination (L,K) satisfying $\frac{K}{L} < \frac{\lambda}{\mu}$, output is lower when m is lower (higher technologic level)*

Proof: in the appendix.

Proposition 3.1 is depicted in figures 1 and 2. Figure 1 shows the production function (1) undergoing a discrete technical change in the isoquant plane. Unlike neutral changes, where isoquants are proportionally *compressed* along the dimension of the "augmented" factor, here they are *rotated*, such that same-level isoquants before and after the change intersect. The counterclockwise rotation in the (L,K) plane means that for capital-labor ratios above λ/μ the new technology indeed raises output for a given level of factors, or can maintain the level of output for a strict reduction in both factors. Furthermore, we know from section two that for high technology levels and capital-labor ratios the change will tend to resemble a neutral one. A corresponding rotation is shown in figure 2, where labor is normalized (say, to one) and per-worker output (y) is the "intensive form" function of per-worker capital (k).

However, as apparent in both figures, for capital-labor ratios which are below λ/μ, the new technology *reduces* output. This "adverse technical change" indeed seems rather strange at first glance and is certainly counter to the standard view of overall benefits from "better" technology, as implied by neutral technical change. But given the mechanization-induced replacement of labor by capital, featured in this low-substitution model, then the hypothetical adoption of a new technology without an appropriate increase in the level of capital may very well reduce output since labor will not be efficiently utilized. The notion of "appropriate technologies" will be further discussed in sub-section 3.3 below.

3.2 Non-Neutrality and Industrialization Accounting

Quantitative analysis of technical change is highly dependent on the assumptions regarding the "shape" of the production function, specifically whether or not it is Cobb-Douglas, with its unit elasticity of factor substitution. The debate over the elasticity of substitution's significance has surfaced recently in the context of the East Asian newly industrialized countries (NICs). Nelson

& Pack (1999), Rodrik (1998) and Hsieh (2000) all make the claim by which standard growth accounting, assuming Cobb-Douglas, overstates the contribution of factor deepening to growth, while understating the *direct* contribution of technical change.[15] Their alternative framework is a low substitution (high curvature) function undergoing LATC.

More specifically, Nelson & Pack (1999), Rodrik (1998) and Hsieh (2000) use the formula:

$$g_{S^K} = \left(1 - s^K\right)\rho\left[\left(g_K - g_L\right) - \left(g_A - g_D\right)\right], \tag{13}$$

where *A* and *D* are pure "labor efficiency" and "capital efficiency" terms, respectively, similar to *B* and *Q* in (5) above (see footnote 12), and $\rho = \frac{\sigma - 1}{\sigma} < 0$. These authors calculate TFP growth as the difference between *actual* output growth between two periods and the growth which *would have* occurred in the absence of technical change ($g_A = g_D = 0$); in which case the capital share *would have* fallen, due to the effect of capital deepening when factor substitution is low.

Young (1998) implies at least two problems with such an analysis. The first concerns the somewhat casual treatment of the "capital efficiency" term. Indeed, Nelson & Pack, Rodrik and Hsieh are right in claiming that (13) shows how LATC offsets capital deepening in preserving constant factor shares. But as clearly seen in (6), this constancy means that *strict* LATC will hold only during balanced growth (i.e. $g_Y = g_K$). Otherwise (i.e. $g_Y < g_K$) there has to be negative "capital efficiency" growth. As Young clearly explains: "In order for the Nelson-Pack-Rodrik-Hsieh framework to explain the facts of East Asian growth, it is not only necessary that factor augmenting technical change offset the growth of the capital-labor ratio, it is also necessary

[15] Of-course one can claim technical change has also the *indirect* effect of leading to the mere capital deepening.

(given the relatively slow growth of output relative to factor accumulation) that the production function *rotate*" (1998, pp. 4-5, italics in the original text).

The second, related, point concerns the "path dependency" of TFP growth estimates; as rigorously shown by Hsieh (2000). The implication is that if one rejects Cobb-Douglas, or more generally Hicks-neutrality, then the measuring of TFP growth is highly problematic. As Young (1998) points out, there is indeed no reason to prefer one "path", say "capital deepening first and then technical change" over another, say "technical change first". Young (1998) refers to TFP measures along these two paths as resembling "Paasche" and "Laspeyres" indices, respectively. Moreover, as implied by Young, the standard (Cobb-Douglas) method in some sense provides an "average" of the TFP growth estimates obtained from both these "extreme-case" paths; as can be seen in table 1 here, which is Young's (1998) table 2.

TFP measure	Hong Kong	Singapore[16]	South Korea	Taiwan
"Paasche" ($\sigma = 0.3$)	3.4	1.8	3.3	3.5
Standard ($\sigma = 1$)	2.4	0.1	1.6	2.1
"Laspeyres" ($\sigma = 0.3$)	0.9	–1.6	–1.4	–1.1

Table 1: TFG growth rates (%) in NICs, under different methods (1966-90). Source: Young (1998), table 2.

In three out the four NICs we see that the "Laspeyres" estimates shows up as negative, implying highly dominant "capital dis-augmentation". These results are explained graphically by Young (1998) and superimposed here in figure 2. Suppose the two points of actual estimation are: I and J. The so-called "Paasche" index of TFP growth corresponds to the vertical distance between J and the hypothetical point $\tilde{J}$ obtained by applying formula (13) under the assumption of no technical change. The "Laspeyres" index corresponds to the vertical distance from the

[16] Subsequent work has revealed potential problems with Singaporian data.

point I to the hypothetical point $\widetilde{I}$ obtained too by applying (13) without technical change; but "in reverse". Figure 2 shows the *hitherto* puzzling case of a negative "Laspeyres" index. But this need not always be the case, and it is indeed not surprising that Hong Kong, being initially the most developed of the four NICs, has all indices positive.

Nelson & Pack (1999), Rodrik (1998) and Hsieh (2000) rely on a long interval (extreme case) path; specifically the "capital accumulation first" ("Paasche") path. But one may ponder whether the claim by which TFP growth depends on the elasticity of substitution applies for the more realistic infinitesimal, or yearly, conjoined path. Recall that changes in TFP correspond to "shifts in the production function", such that: $g_Y = g_{TFP} + s^K g_K + (1 - s^K) g_L$, or: $g_y = g_{TFP} + s^K g_k$, under the standard assumptions. It is straightforward to show that given "factor augmenting" technical change, where output is a constant returns to scale function of "efficiency units" of capital ($D_t K_t$) and labor ($A_t L_t$), we obtain:

$$g_{TFP} = s^K g_D + (1 - s^K) g_A . \tag{14}$$

Now, if one does not consider "capital (dis-)augmentation" then of-course (14) simplifies to:

$$\widetilde{g}_{TFP} = (1 - s^K) g_A ; \tag{14'}$$

where the elasticity of substitution, or curvature, does not seem to affect the parsing of growth into technical change and capital accumulation. But there are two possible reasons for this. The first, which does not seem to be the case, is that factor shares are not constant. Thus overlooking their change in (14') conceals the actual dependence on the elasticity of substitution.

But the second reason for the elasticity of substitution's lack of appearance in (14') is that it is a biased formula, as it does not consider "capital (dis-)augmentation"; which in-fact it *should*, given the relative constancy of factor and given (6). The model here easily corrects this bias, and **without adding a degree of freedom** in the form of a separate "capital efficiency/productivity" entity. Rewriting (14) in terms of the synthetic parameters B and Q, defined in (3) and (4), we get, given (7) and (8):

$$g_{TFP} = \left(1 - \tfrac{s^K}{1-m}\right) g_B . \tag{15}$$

With constant factor shares we can estimate: $g_B = g_y$. Since $g_B > 0$ ($g_m < 0$), it is clear that (15) is consistent with either a positive or a negative growth rate of TFP, depending on whether the technological level is high (m closer to 0) or low (m closer to 1), respectively. Applying (6) and (4), then (15) can be rewritten as:

$$g_{TFP} = \left[1 - \left(\tfrac{\mu K}{Y}\right)^{\rho}\right] g_B .\text{[17]} \tag{15'}$$

The Nelson-Pack-Rodrik-Hsieh claim by which the elasticity of factor substitution ought to negatively affect TFP growth (i.e. low substitution means high TFP growth) is supported by (15'); but only if $\mu K > Y$, which holds only for relatively technologically advanced countries. If $\mu K < Y$ then the lower is the elasticity the smaller is TFP growth; which may even be negative, due to the dominance of "capital dis-augmentation". As stated previously in the paper this is interpreted as an indication of intense mechanization (i.e. "capital *using*").

[17] For example, if $\rho = -1$ and thus $\mu \cong 1$, we have $g_{TFP} = (1 - \frac{Y}{K}) g_B$.

3.3 Non-Neutrality and Low Development

In a long-anticipated attempt to bridge growth theory and development economics, Banerjee & Duflo (2004) have skillfully synthesized the idea of "appropriate technologies" with micro-level studies, inferring the occurrence of "traps"; though on an intra- rather than inter-country level. Relying on fixed costs as the cause for appropriate technologies, along with decreasing returns to scale, these authors show how imperfections, mainly credit constraints, generate a misallocation of capital and lack of sufficient technology adoption. More specifically, if there is heterogeneity in the credit given to various agents and if a "better" technology requires a greater fixed cost then there will be heterogeneity in the level of technology employed in the economy. Conversely, if credit rationing is relatively low, as in developed economies, then fixed costs hardly matter and the latest technologies will most often be adopted.

Rather than assuming fixed-costs at a static level (i.e. within a given technique), the non-neutrality of the shifting impacts model, which means requiring more capital in order to produce the same level of output, can be thought of as a dynamic equivalent. This feature can be considered as an "inter-temporal non-convexity". Thus, in essence, shifting impacts model too generates the result by which no technique dominates another; despite its static convexity. Rather, there is an "ideal" technique which is a function of the capital-labor ratio: an "appropriate technology".[18] Indeed, the sign of the "technology index derivative" will vary here across the production function's domain, which differs from the standard assumption that an "increase" in technology will always increase output.[19] A "classic" reference to appropriate technologies is Atkinson & Stiglitz (1969), who criticize the idea of neutral technical change,

[18] The basic model introduced in section 2 and discussed so far is of an "all or none" type appropriateness, but the more elaborate model introduced in the next section has an infinite number of appropriate technologies.

[19] Still, as shown in the next section, a marginally higher technological level will indeed (potentially) increase output *on the balanced growth path*.

due to its dubious implication by which a new innovation boosts production levels for *all* capital-labor ratios.[20]

A glance at figure 2 reveals that in similar to the approach undertaken by Banerjee & Duflo (2004), and in stark contrast to standard (neutral) formulations, for low levels of investment a newer technology is inferior in terms of its output per worker, as well as in its marginal product of capital per worker. Formally, finding the expression for the cross-derivative of the intensive form of (1): $\frac{\partial^2 y}{\partial k \partial m}$, which is non-monotone in *k*, gives the unique value of *k*: $\left(1-\frac{\rho}{1-m}\right)^{1/\rho}\frac{\lambda}{\mu}$, which is smaller than $\frac{\lambda}{\mu}$, below which a rise (fall) in *m* raises (lowers) the marginal product of capital per worker, and above which a rise (fall) in *m* lowers (raises) it.

As stressed by Banerjee & Duflo (2004), a Cobb-Douglas production function (or, as should be added, any function in which technical change is neutral) can not easily reconcile differences between rich and poor countries in their output-capital ratios, as well as their ratios of rates of return. The reason is that in standard models the required level of the technology gap induces a lower than observed ratio of returns when fitting the observed output ratio. Alternatively, one will obtain lower than observed output ratio when fitting the observed rate of return ratio. The underlying reason is that both the rate of return and output are increasing in these models with the level of technology.

The implication of time-ranked non-neutrality, as featured here, is that economic agents facing high interest rates will in-fact prefer investing in old technologies, even if newer ones are fully at their disposal.[21] The picture depicted here (e.g. figure 2) and formalized above, is of older

[20] Basu & Weil (1998) are renowned for reviving the idea of appropriate technologies, though the interpretation in the current paper is more in line with Zeira (1998), whose model indeed implies a non-neutral (albeit discrete) technical change. Caselli (2005) and Jones (2005) too link non-neutrality and appropriate technologies, though in their view, these are alternatives along a concurrent, specifically the frontier, "technology menu", rather than a time ranking of techniques.

[21] According to Banergee & Duflo (2004), for example, the median textile "firm" in India is a tailor using primitive technology, despite the existence of more modern (capital intensive) firms; perhaps due to the median entrepreneur's

technologies being characterized by initially soaring marginal productivity of capital, which levels-of rather fast.

A more fundamental implication is that neither technological differences, nor "institutional arrangements" alone can provide an explanation to underdevelopment. Rather, there has to be some sort of interaction between these two elements.[22]

4. An Elaborate Shifting Impacts Model

4.1 Production Technology

Somewhat inspired by the Classical approach to production, where labor is the "basic" factor of production, the formulation below assumes that production is the result of a low-substitution process by which capital "empowers", or "intensifies" workers' vital motor or cognitive services. Abstracting from important issues concerning human capital acquisition, the model implicitly assumes that workers are always knowledgeable as to the means by which to operate the contemporary technology, embedded within capital.

More specifically, it is assumed that capital empowers labor through a sequential process of n succeeding tasks, each with its own amount of "capital input". A higher number of tasks corresponds to a more complex type of capital, or equipment, embodying a higher technological level. These tasks seemingly bear resemblance to the intermediate capital-good variants of the expanding-variety growth model, originally due to Romer (1990). There are several main

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limited access to capital. Though these authors stress internal reasons for high interest rates, a contributing factor may be the interest rate relevant for foreign loans, which may be high due to various risk related factors. High interest rates as a possible cause for deficient capital flows are also highlighted by Lucas (1990).

[22] The dependence on the interaction between technology *and* credit constraints is dominant it much of the poverty traps literature. See Galor & Zeira (1993)

differences, though, which shall be highlighted below; one of which is that the model here assumes a low, rather than high elasticity of substitution among the different tasks.

Suppose each i-th task ($2 \leq i \leq n$) can augment the value of a given amount of the previous task's output (x_{i-1}), by using it as an input, to be combined with its own amount of capital (k_i), to produce intermediate output (x_i). The first task, x_1, augments the "basic input" of the production process, which is the labor input. Thus, in contrast to the product variety model, here labor is inherently embedded within the complexity of the production process. Omitting time subscripts for clarity, suppose the series of intermediate output $\{x_i\}_{i=1}^{n}$ in the process described above is recursively defined by the following CRS-CES function per each task, with $x_0 \equiv L$:

$$x_i = \left[\gamma x_{i-1}^{\rho} + (1-\gamma) k_i^{\rho}\right]^{\frac{1}{\rho}}, \tag{16}$$

where in similar to section two: $\rho \in (-\infty, 0)$, or in terms of the elasticity of substitution: $\sigma \equiv \frac{1}{1-\rho} \in (0,1)$; but the so-called "distribution parameter" $\gamma \in (0,1)$ is now constant.

Assume further that k_i is a "composite capital", comprised of κ capital types, defined by yet another CES function (per each production task):

$$k_i = \left[\sum_{j=1}^{\kappa} \beta^j \left(k_i^j\right)^{\varepsilon}\right]^{\frac{1}{\varepsilon}}, \tag{17}$$

where the superscripts denote capital type, the β-s are respective weights ($\Sigma\beta^j = 1$, $\beta^j \geq 0\ \forall j$), and $\varepsilon \leq 1$ determines the elasticity of substitution ($\frac{1}{1-\varepsilon}$) between them. A strong simplifying assumption is, of course, that ε and the β-s are identical across tasks.

Recursive substitution in (16), and defining the final task's output (the final output) as Y, yields the following $n + 1$ inputs CES production function:

$$Y = \left[\gamma^n L^\rho + (1-\gamma)\sum_{i=1}^{n} \gamma^{n-i} k_i^{\ \rho} \right]^{\frac{1}{\rho}}. \tag{18}$$

One can already observe how the assumption regarding labor as basic input will bring about its decreasing impact when the task number increases as technology advances.

Besides the inherent inclusion of labor, as well as low substitution among tasks, two additional differences between (18) and the standard expanding-variety growth model are apparent. First, notice the weight per each specific task, which is dependant on n. This shall manifest itself in the existence of an "appropriate" number of tasks per a given capital-labor ratio (an "appropriate technology"), as opposed to an unambiguous global optimality of a higher technology index in a variety setting; or in any less micro-founded model of neutral technical change. The additional difference is a non-symmetry property, evident in the appearance of i in the weighs; in contrast to the symmetric, or commutative, feature of the variety model. Thus capital in different tasks, though (in principle) homogeneous, is non-symmetric in its impact on final output.

Let us assume the usual simplifying assumption whereby the various k-s are homogeneous in that we can specify: $\Sigma k^j = K^j$, where K^j is "raw (j-type) capital", that is "forgone consumption" measured in units of output. Now regardless of whether it is done by the final goods' producers themselves or by intermediate goods' producers, there is a question of how should a given stock of "raw capital", be allocated among the various *n* tasks.

PROPOSITION 4.1: *If maximizing the output of (18), given a constraint on total amounts of capital utilized, then for all j capital types and any n,* $\{k_i^j\}_{i=1}^n$ *progresses geometrically, with a quotient:*

$$q \equiv \left(\frac{1}{\gamma}\right)^{\sigma} > 1 . \tag{19}$$

Proof: in the appendix.

From (19) we can write the exact definition of the optimal allocation values as:

$$k_i^j = q^{i-1}\left(\frac{q-1}{q^n - 1}\right)K^j , \tag{20}$$

where $K^j \equiv \sum_{i=1}^n k_i^j$ is the total amount of the *j*-th type capital. The increasing result should come as no surprise given the assumptions of this section, namely that the earlier tasks (as well as the basic labor input) undergo further enhancement by later ones, which thus diminish the formers' impact; whereas the impact of later tasks on final output is more salient. Qualitatively, this increasing pattern is similar to the results obtained in other serial-production models, such as those by Locay (1990) and Kremer (1993).

Substituting (20) back into (18) yields the aggregative specification:

$$Y = \left[\left(\gamma^n\right) L^\rho + \left(1-\gamma^n\right)\left(\frac{G}{G_n} K\right)^\rho \right]^{\frac{1}{\rho}}, \tag{21}$$

where: $G \equiv \left[\frac{1-\gamma^\sigma}{(1-\gamma)^\sigma}\right]^{\frac{1}{1-\sigma}}$, $G_n \equiv \left[\frac{1-\gamma^{n\sigma}}{\left(1-\gamma^n\right)^\sigma}\right]^{\frac{1}{1-\sigma}}$ and based on (17): $K \equiv \left[\sum_{j=1}^{\kappa} \beta^j \left(K^j\right)^\varepsilon\right]^{1/\varepsilon}$ is the "aggregate composite capital". If we define labor's impact: $m_t \equiv \gamma^{n_t}$, then (21) bears resemblance to (1) and thus, following (3), we can define the (synthetic) "labor efficiency/productivity" term:

$$B_t \equiv \left(\gamma^{\frac{1}{\rho}}\right)^{n_t} = e^{\frac{\ln\gamma}{\rho} n_t}; \tag{22}$$

where $\frac{\ln\gamma}{\rho} > 0$ is important for converting "ideas" into "productivity", as shown below.

Notice that G is constant, while G_n tends to one as n tends to infinity. Thus, following the analysis of section 2, the additional term G_n reinforces the "capital efficiency" term, here given (4): $Q_t \equiv \left(1-\gamma^{n_t}\right)^{1/\rho}$, as both tend to one and changes in them become increasingly negligible. In terms of an accounting exercise, with respect to the growth rate of "labor productivity", B, we get an expression which is slightly different than that in (14); though still larger than the estimate of the standard exercise.[23]

As opposed to the formulations of section 2, the inclusion of $\frac{G}{G_n}$ in (21) allows for an infinite number of appropriate technologies, as seen in figure 3, depicting (21) with various levels of n.

[23] Given (21) and (22), the equivalent of (15) is: $g_{TFP} = (1 - \frac{s^K}{1-m^\sigma})g_B$.

Formally, optimizing (21) with respect to n gives the (continuous) "ideal" or "appropriate" technology as an increasing function of the capital-labor ratio:[24]

$$n^* = \log_q\left(G\frac{K}{L}+1\right). \tag{23}$$

This expression can be substituted in (21), generating: $Y^*(L,K) \equiv \max_n \{Y(L,K;n)\} = GK + L$, the upper-envelope function seen in figure 3. The "appropriate technology" specified in (23) will, at least along the (asymptotic) balanced growth path, *not* be the "actual" one employed, or at all in existence, as we shall see below.

4.2 Technical Change along the (Asymptotic) Balanced Growth Path

Rather than complicating the current framework with the product-variety settings central to the, by-now classic, models of Romer (1990), Grossman & Helpman (1991) and Aghion & Howitt (1992), we shall adopt a more reduced-form analysis.[25] Following Jones (1995), define the following "idea flow" function of existing knowledge and researcher input:

$$\dot{n} = \xi \phi^n L_R^\eta, \tag{24}$$

[24] Not surprisingly, the function is homogeneous of degree zero in L and K, meaning that the appropriate technology is not dependant on scale; consistent with the CRS property of the production function (16).

[25] Such a complication would require, for example, assuming final output is produced by highly substitutable parallel processes as described by (16) – (18). As discussed below, the model can be intuitively viewed as combining features of both the expanding-variety (Romer) and the quality-ladder (static variety) models.

with all parameters (ξ, ϕ and η) strictly positive. This differs from Jones (1995) in that existing knowledge enters through an exponential, rather than a power function. But as shall be discussed below, the formulas are in-fact fully equivalent; whereas the difference is conceptual.

Following (22), then on a balanced growth path it must be that n is a linear (or rather, an affine) function of time; namely: $\ddot{n} = 0$. Therefore, similar to Jones' (1995) methodology, we ought to differentiate (24) and equate to zero, giving the solution:

$$\dot{n} = \frac{\eta}{\ln\left(\phi^{-1}\right)} g_{L_R} . \tag{25}$$

Thus, from (22), given (25), we get the economy's (asymptotic) balanced growth rate:

$$g_B = \frac{\ln \gamma}{\rho} \frac{\eta}{\ln\left(\phi^{-1}\right)} g_{L_R} . \tag{26}$$

In a fully specified Ramsey model, equation (26) can be substituted in the Euler equation, solving for the interest rate and thus also the allocation of labor to production and R&D. As we are following Jones' (1995) critique, whose implication is that the number of researchers only affects the *level* of the balanced growth path but not the growth rate itself, we do not need to simultaneously solve for the growth rate *and* interest rate, as in models exhibiting a "scale effect" (in growth rates). Therefore (26) is determined regardless of preference-related parameters.[26]

[26] The fertility rate could be endogenized in a Beckerian type model. But it seems a significant gap still exists in the literature, as the growth rate of R&D-engaged labor in G7 countries, shown by Jones (1995), Kortum (1997) and Segerstrom (1998), has been greater than the growth rate of the labor force, which itself has been greater than the fertility rate (e.g. due to women's increasing labor force participation). Thus the growth rate of researchers can not be purely due to population growth *per se*.

Recall that the current model implies the existence of appropriate technologies, following (23), which means that given prevailing levels of labor and capital an increase in the technological level, as defined by *n*, or *B*, may either increase or decrease output. But the following proposition shows that along the (asymptotic) balanced growth path the "actual" technological level is always smaller than the "appropriate" level. In other words, despite the potential for an "overly optimal" technology, the latest technologies are always dominated by at least a sub-set of those not yet invented. Thus: $\frac{\partial Y}{\partial n} > 0$, or $\frac{\partial Y}{\partial B} > 0$, like in the standard (neutral) model of technical change.

PROPOSITION 4.2: *Along the (asymptotic) balanced growth path:* $n_t < n^*_t$

Proof: in the appendix

4.3 Implications Concerning Knowledge Accumulation

As apparent from (25) or (26), it must be that: $0 < \phi < 1$, in order to avoid "explosive" growth. Thus, as is clear from (24), the model implies negative inter-temporal knowledge spillovers ("fishing out" of ideas). While this restriction diminishes the generality of the current analysis, it should be noted that the elaborately micro-founded models of technical change due to Kortum (1997) and Segerstrom (1998) generate an equivalent implication. A main empirical justification of Kortum (1997) and Segerstrom (1998) for the negative inter-temporal knowledge spillover is that in parallel to the exponential increase of R&D-engaged workers in industrialized countries, not only have productivity growth rates remained relatively constant, but so has the flow of new patents. The current model too is consistent with these stylized facts, though there is a conceptual difference here between the stock of ideas (*n*), and "productivity" (*B*), a reciprocal of labor's impact. These facts necessitate, as in Kortum (1997) and Segerstrom (1998), that new patents, or ideas, become increasingly more valuable with time.

The fact that ideas in the current model are accumulated linearly along a balanced growth path is derived from the formalization *assumed* in (24). As was stated above, (24) differs from Jones' (1995) formulation by relating the flow of new ideas to the existing stock of ideas through an exponential, rather than a power function. But it should be stressed that there is no contradiction between the models. In-fact, given (22) we can transform (24), expressed in terms of ideas, into a "productivity-level terms" specification, as follows:

$$\dot{B} = \Xi B^{\Phi} L_R^{\eta}, \tag{27}$$

where $\Xi = \xi \frac{\ln\gamma}{\rho} > 0$ and $\Phi = \frac{\rho \ln\phi}{\ln\gamma} + 1$. This expression is equivalent to the one proposed by Jones (1995), where the result required $\Phi < 1$; indeed so if (in the current model) $0 < \phi < 1$. Notice that for the range $0 < \Phi < 1$, rather than $\Phi < 0$, (27) can appear as if exhibiting slight *positive* past-research spillovers. What is actually happening in this range, as clear from (24), is that if ϕ is relatively close to one then an increase of n only slightly decreases $\dot{n}$ (*ceteris paribus*, i.e. with no change in L_R), whereas B is an exponential function of n. Thus since $\dot{n}$ is still positive, $\dot{B}$ (but not $\frac{\dot{B}}{B}$) may still (slightly) increase.

Once acknowledging the difference between "productivity levels" and "ideas", we can also say that there is no contradiction with the models due to Kortum (1997) and Segerstrom (1998). The former, for example, shows balanced growth as a possible outcome of productivity levels being drawn from a "thick-tailed" Pareto stationary search distribution; such as: $F(B) = 1 - \left(\frac{B}{B_0}\right)^{-\Omega}$, with $\Omega > 0$ (or $\Omega > 1$ if one wishes a finite mean). Given the relationship (22), it can easily be shown that this is equivalent here to assuming that the ideas themselves (i.e. n) are being drawn from an *exponential* stationary search distribution, with the CDF: $F(n) = 1 - e^{-\omega(n-n_0)}$, where $\omega = \frac{\ln\gamma}{\rho}\Omega > 0$ and n_0 relates to B_0 according to (22).[27]

On a more conceptual level, the model presented here combines basic intuitions from both the expanding-variety as well as the quality-ladder models. Equation (18), incorporating an additive separable term with a varying number of components, clearly resembles the former, while the inherent non-symmetry of the model and the fact that new ideas diminish the significance of previous ones resembles a Schumpeterian "creative destruction"-type process. In a sense, the current model can thus be seen as merging the orthogonal, but by no means contradictory, views of knowledge implicit in both.

The expanding-variety model highlights the fact that the technology index, corresponding to the "stock of knowledge", measures the cumulation of ideas or designs (which are modeled as symmetric for reasons of tractability). As such and as explicitly stated by Romer (1990, p. 79): "each new unit of knowledge corresponds to a design for a new good, so there is no conceptual problem measuring [the technology index]. It is a count of the number of designs". The quality ladder approach highlights the dominance of new ideas; though, as in the original neo-classic growth model, refraining from an attempt to map the quality or technology index onto some well-defined "real-life" accumulable stock (such as: "the number of ideas").

A combination of the two approaches requires a slight refinement of the notion of an "idea", freeing it of the commutativity, or symmetry feature inherent in the expanding-variety model. An idea is both an individual "sub-design" and an integrated "meta-design", which includes previous ideas as well. Knowledge is thus conceived here as a *hierarchical* cumulation of the set of ideas

[27] In Kortum's (1997) model, where researchers directly sample the *productivity levels*, an exponential (rather than a Pareto) stationary search distribution delivers counterfactual linear (rather than exponential) growth.

existing at a point in time.[28] This conceptualization fits well with the fundamental "standing on the shoulders of giants" property of knowledge.

5. Concluding Remarks

This paper has offered a refinement of the deeply-rooted "factor augmentation" concept. More specifically, the CES "distribution parameters" are viewed here as time-varying weights reflecting factor's "impacts" on production; where technical change corresponds to the shift of impact from labor to capital. While it is by no means the intention of the paper to downplay the fundamental role of "labor augmentation" in economics, it does suggest several insights which are masked if one interprets this concept *literally*.

Technical changes correspond here to a dynamic *replacement* process, whereby producing the same level of output requires less labor ("labor saving") but *more* capital ("capital using"); though as the technical level rises and capital's impact tends to unity, strict "labor augmentation" is eventually approached. The upshot is that the seemingly orthogonal "capital-" and "labor augmentation" facets of technology are shown here to be two sides of the same coin, which means that there is but a single technological entity and a single "direction" for technical change; not just in practice, but in potential as well. This implies that inventions perhaps need not be

[28] This can be explained with the help of the following simplistic example. A hybrid car incorporates many different ideas, most of which were added on along a time line. These would include: wheel, axel, chassis, shaft, piston, *internal* combustion, power charge, planetary gear set and power split device. On one hand, the knowledge incorporated in such cars is a cumulation of *all* these ideas, and in that sense resembles the expanding variety view (albeit with *low* substitution among the ideas). But despite their vitality, these ideas are not symmetric in their impact, in that the introduction of a new one creates a *superior* "meta-design" driving-out the previous one.

classified as either “capital-” or “labor augmenting”, where the former type are either ignored or altogether precluded in equilibrium.[29]

The elaborate version of the model also highlights the fact that capital (equipment), rather than labor, is the factor which embeds new ideas or designs. The upshot here is that technical change refers to qualitative changes in *capital*, rather than in labor. This is indeed what basic common sense would lead us to think; though the intuition is not conveyed by standard modeling, unless restricted to multiplicative forms.

Beyond the conceptual level, the model provides a simple and intuitive mechanism which rationalizes, as well as *reconciles*, three main growth related features. First are the stylized facts of industrialized economies, displaying balanced growth with constant factor shares, which are known to be consistent solely with strict “labor augmentation”. But such sort of technical change is asymptotically equivalent to shifting impacts, as capital’s impact tends to one.

Secondly, the relative constancy of factor shares during industrialization is explained here as the result of significant non-neutrality induced by factor replacement. In addition, the initially significant non-neutrality is also linked with *time-ranked* “appropriate technologies”, which through their potential interaction with certain “institutional arrangement” such as credit constraints, may lead to under-development.

Finally, relaxing the Cobb-Douglas assumption enables the deterministic and technology-based analysis of what seem to be pronounced “medium run” trends in factor shares, discussed by

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[29] Samuelson (1965, p. 355) puts it most bluntly (all punctuation and styling as in the original text):

> For the most part, labor-saving innovation has a spurious attractiveness to economists because of a fortuitous verbal muddle. When writers list inventions, they find it easy to list labor-saving ones and exceedingly difficult to list capital-saving ones. (Cannan is much quoted for his brilliance in being able to think up wire*less* as a capital-saving invention, the syllable “less” apparently being a guarantee that it does save capital!). That this is all fallacious becomes apparent when one examines a mathematical production function and tries to decide in advance whether a particular described invention changes the partial-derivatives of marginal productivity imputations one way or another.

Blanchard (1997) and Bentolila & Saint Paul (2003), among others; though such analyses typically require some sort of “friction”. These trends in factor shares perhaps indicate a complex innovation-accumulation interrelationship, inducing “technological cycles”, by which a few years may elapse before capital deepening and technical change are actually aligned. More specifically, from the rather hump-shaped pattern of labor’s share during the 2nd half of the 20th century one might speculate for instance that the rate of capital deepening initially overtook the rate of technical change; after which a reversal of trends occurred.

This dynamic innovation-accumulation relationship may perhaps be further extended in order to analyze issues concerning skilled versus unskilled labor. For example, so-called “skilled-biased” technical changes may refer to episodes when capital equipment providing cognitive type services is accumulated, relative to being innovated, at a faster rate than capital equipment providing physical or motor type services.

A further, perhaps related, extension concerns the issue of durable investment goods’ declining price, as notably emphasized by Greenwood, Hercowitz & Krusell (1997), which some may interpret as capital “saving” or “augmenting” technical change. Alternatively, following Whelan (2003), the decline in the price of durables can be analyzed by a two sector model, distinguishing between equipment, on one hand, and non-durables and structures, on the other. Technical change, interpreted as the decrease in labor’s impact, may (for reasons which are yet to be fully understood) be more rapid in the former sector.

Appendix: Proofs of Propositions

PROPOSITION 3.1:

We seek combinations of factors (L,K) which produce the same output under two different technologies: m_1 and m_2. Thus: $\left[m_1(\lambda L)^\rho + (1-m_1)(\mu K)^\rho\right]^{\frac{1}{\rho}} = \left[m_2(\lambda L)^\rho + (1-m_2)(\mu K)^\rho\right]^{\frac{1}{\rho}}$, which with a bit of algebra becomes: $(m_1 - m_2)(\lambda L)^\rho = (m_1 - m_2)(\mu K)^\rho$. Thus we have: $\frac{K}{L} = \frac{\lambda}{\mu}$, which is independent of the technologies, and is therefore valid for any m_1 and m_2.

Without loss of generality, assume: $m_2 < m_1$ (i.e. $B_2 > B_1$), thus "2" is the higher technological level. We now seek combinations of (L,K) where $Y_2 = Y(L,K;m_2)$ is greater (smaller) than $Y_1 = Y(L,K;m_1)$. Applying the algebra as in the previous paragraph but with an inequality < (>), gives: $\frac{K}{L} > \frac{\lambda}{\mu}$ ($\frac{K}{L} < \frac{\lambda}{\mu}$).

PROPOSITION 4.1:

Differentiating (18) by k_i^j and k_{i-1}^j (for any factor type j) given a constrained total amount of it, and dividing the constrained problem's first order conditions gives (with a little algebra):

$$\left(\frac{1}{\gamma}\right)^{\frac{1}{\varepsilon-1}} \left[\frac{\sum_{f=1}^{\kappa} \beta^j \left(k_i^f\right)^\varepsilon}{\sum_{f=1}^{\kappa} \beta^j \left(k_{i-1}^f\right)^\varepsilon}\right]^{\frac{\rho-\varepsilon}{\varepsilon(\varepsilon-1)}} \frac{k_i^j}{k_{i-1}^j} = 1 \qquad \text{(A1)}$$

Differentiating (18) with respect to any other (constrained total amount) *k'*-th type factor gives the same expression, except for the third LHS term, which is a ratio of $k^{j'}$-s. Therefore:

$\frac{k_i^j}{k_{i-1}^j} = \frac{k_i^{jw}}{k_{i-1}^{j'}} \equiv q_i$ for any two factor types: *j, j'*. Substituting k_i^j for $q_i k_{i-1}^j$ in (A1) collapses the second LHS term to $q_i^{(\rho\text{-}\varepsilon)/(\varepsilon\text{-}1)}$. The same applies when substituting $k_i^{j'}$ for $q_i k_{i-1}^{j'}$. Additional algebra shows that the subscript *i* is no longer needed, eliminates *ε*, and yields the proposition.

PROPOSITION 4.2:

Along a balanced growth path the growth rate of "labor efficiency/productivity" equals the growth rate of the capital-labor ratio *k*, thus given (22): $g_k = g_B = \frac{\ln \gamma}{\rho} \dot{n}$. Differentiating (23) by time yields: $\dot{n}^* = \frac{1}{\ln q} \frac{G\dot{k}}{Gk+1}$. Given (19) and multiplying by $\frac{k}{k}$ we get: $\dot{n}^* = \frac{1}{-\sigma \ln \gamma} \left(\frac{Gk}{Gk+1} \right) g_k$. Noting that: $\lim_{t \to \infty} \frac{Gk}{Gk+1} = 1$, and given the above expression for g_k, we have: $\lim_{t \to \infty} \dot{n}^* = \frac{1}{1-\sigma} \dot{n}$. This implies that the growth of *n* is always smaller than the growth of *n**. Thus even if $n^*_0 < n_0$, then *n** must surpass *n* at some finite time (specifically at $t = -\rho \frac{n_0 - n^*_0}{\dot{n}}$, where $\dot{n}$ is given by equation 25); as formally stated in the proposition.

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Figure 1: discrete technical change (“high *m*” to “low *m*”) in the isoquant plane

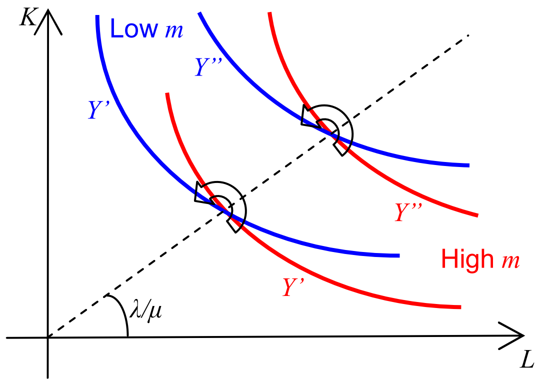

Figure 2: discrete technical change (“high *m*” to “low *m*”) in per-worker terms (“intensive form”)

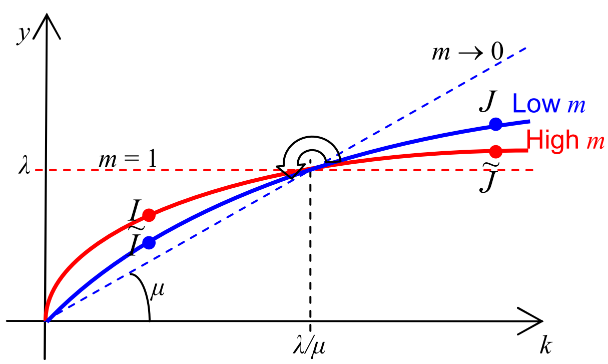

Figure 3: output as a function of capital, as in (21), given various technologies (levels of *n*)