STICKY PRICES IN A CASH-IN-ADVANCE MODEL: DOES MONEY MATTER?

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Preliminary and incomplete.

1. INTRODUCTION

Friedman's argument for a stable rate of growth in the money supply (a k% rule) is based on the claim that money plays an important role in causing the business cycle. Since his pioneering work with Anna Schwartz (Friedman and Schwartz [1963]) the importance of money for causing the business cycle has been questioned first by Tobin (1970) and later by the real business cycle literature. For example, King and Plosser (1984) argue that "Given the controversies surrounding the main contending hypotheses concerning money and business cycles...it seems worthwhile to consider alternative hypotheses". Lately there has been a renewed interest in monetary policy partly due to empirical work that find a significant effect of money on real variables. See for example, Clarida, Gali and Gertler (1999) and the references in this article.

Does the policy recommendation for a k% rule depend on the strength of the correlation between money surprises and real variables? Here I attempt to answer this question using a sticky

[1] I would like to than Boyan Jovanovic and Bob Lucas for lengthy discussions which led to a major revision of this paper.

price framework in which sellers choose both prices and quantities optimally. Thus I attempt to integrate the disequilibrium literature pioneered by Patinkin (1965), Clower (1965) and Barro and Grossman (1971) with the new Keynesian economics literature.

The new Keynesian economics literature that began with Svensson (1986) and Blanchard and Kiyotaki (1987), assumes that prices are chosen optimally whenever these choices are made and the firm must satisfy demand at its pre-announced price. Allowing firms to choose both prices and quantities is achieved at the cost of added complexity. There are two main benefits for doing that. First we can use such models for policy analysis because models that are based on optimization are immune to the Lucas (1976) critique. Second, the models shed new light on the does money matter question. It is shown that money may matter from a welfare point of view even in the absence of a correlation between money surprise and employment.

The plan of the paper is as follows. I start with a simple new Keynesian model. I then add the quantity to the list of choice variables. This is done in two ways. The first uses the standard Dixit-Stiglitz approach for modelling monopolistic competition. The second uses McFadden's random utility maximization approach. The equilibrium correlation between money surprises and employment is different across the three cases we consider but in all cases a k% rule is optimal.

2. A SIMPLE NEW KEYNESIAN TYPE MODEL

This model is based on the work of Blanchard and Kiyotaki (1987) and Woodford (2001). They assume that money is in the utility function. Here I use a cash-in-advance model.

There is a continuum of measure one of infinitely lived households, where a household is a worker/shopper pair. The shopper takes the available cash and spends all of it. The worker produces and sells his output for cash. At the end of the period both members of the houshold reunite and consume whatever the shopper has bought.

Each household produces a different good and consumes all goods. His single period utility function is of the Dixit and Stiglitz (1977) type:

$$(1) \qquad [\Sigma_{j=0}^{1} (y_j)^{\gamma}]^{1/\gamma} - v(L),$$

where $0 < \gamma < 1$ and $v(L) = (1/2)L^2$.

The typical household i starts the period with m_i normalized dollars and gets a transfer of x normalized dollars, where a normalized dollar is the beginning of the period money supply.[1] The amount of transfer x is an i.i.d random variable with a density function $\phi(x)$. It is assumed that $\beta - 1 \leq x < \infty$ so the money growth rate cannot be below the Friedman rule.

The buyer takes the normalized prices $(p_0,...,p_1)$ as given and spends the entire available amount of $m_i + x$ normalized dollars on all goods. Buyer i solves:

$$(2) \qquad \max_{y_j} [\Sigma_{j=0}^{1} (y_j)^{\gamma}]^{1/\gamma} \text{ s.t. } \Sigma_{j=0}^{1} p_j y_j = m_i + x.$$

The first order condition for the buyer's problem (2) are:

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[1] Thus, we divide all nominal magnitudes by the pre-transfer money supply.

(3) $$(y_1/y_j)^{\gamma-1} = p_1/p_j.$$

I use $\theta = 1/(\gamma - 1) < 0$ and (3) to solve for y_j as a function of (y_1, p_j, p_1). This yields:

(4) $$y_j = y_1(p_j/p_1)^{\theta}.$$

We now substitute (4) in the budget constraint ($\Sigma_j p_j y_j = m_i + x$) to get:

(5) $$y_1 = (m_i + x)(p_1)^{\theta}/\Sigma_j (p_j)^{1+\theta}.$$

Using $z = 1/\Sigma_j (p_j)^{1+\theta}$ and symmetry, household's i demand for product j is $y_j = z(p_j)^{\theta}(m_i + x)$ and his utility from consumption is:

(6) $$F(m_i + x, p^{-i}, p_i) = \{\Sigma_j[(m_i + x)z(p_j)^{\theta}]^{\gamma}\}^{1/\gamma},$$

where $p^{-i} = (p_0,...,p_{i-1},p_{i+1},...,p_1)$ is the prices posted by other sellers. Since there is a continuum of agents the effect of any single price on F is small. We therefore write $F(m + x, p^{-i})$ instead of $F(m + x, p^{-i}, p_i)$.

Our normalization implies $\Sigma_j m_j = 1$. Nominal spending per household is therefore 1 + x and the aggregate demand for product i is:

(7) $$y_i = z(p_i)^{\theta}\Sigma_j (m_j + x) = z(p_i)^{\theta}(1 + x).$$

Using the aggregate demand (7), we compute next period's balances (in terms of next period's normalized dollars):

(8) $$m' = p_i y_i/(1 + x) = z(p_i)^{1+\theta}.$$

Note that next period's money does not depend on x. It depends only on the relative price: $(p_i)^{1+\theta}/\Sigma_j\ (p_j)^{1+\theta}$.

The individual seller takes p^{-i} and z as given and assumes that the normalized prices charged by others will not change over time. He chooses his price p_i by solving the following Bellman's equation:

(9) $$V(m;\ p^{-i}) = E_X\{F(m + x,\ p^{-i})\}$$

$$+ \max_{p_i}\ E_X\{-\ v[(1 + x)z(p_i)^{\theta}]\} + \beta V(z(p_i)^{1+\theta};\ p^{-i}),$$

where E_X denotes expectations with respect to x. The first order condition for this problem is:

(10) $$E_X\{v'[(1 + x)z(p_i)^{\theta}](1 + x)(p_i)^{-1}\} =$$

$$= \beta V'[z(p_i)^{\theta}](1+\theta)/\theta = \beta V'(z(p_i)^{\theta})\gamma.$$

To provide an intuitive explanation of (10) we consider the effect of a small increase in the price p_i for a given realization of x. By increasing the price p_i the seller reduces demand and production. The amount of output reduced is given by the absolute value of the derivative of (7) which is: $\theta z(p_i)^{\theta-1}(1 + x)$. The cost reduction benefit is therefore: $-\ (v')\theta z(p_i)^{\theta-1}(1 + x)$. The increase in the price p_i will also lower next period's balances by the absolute value of the derivative of (8), which is: $-\ (1 + \theta)z(p_i)^{\theta}$.

The loss of utility associated with that is: $-\beta V'(1+\theta)z(p_i)^{\theta}$. Since at the optimum, the cost reduction benefits must equal the loss of utility due to the loss of revenues we get (10).

Equilibrium:

In equilibrium $p_j = p$ for all j, $z = 1/\Sigma_j (p_j)^{1+\theta} = p^{-(1+\theta)}$ and $zp^{\theta} = p^{-1}$. Substituting this in (6) we get the utility from consumption:

(11) $[\Sigma_j (z(m+x)(p_j)^{\theta})^{\gamma}]^{1/\gamma} = [(z(m+x)p^{\theta})^{\gamma}]^{1/\gamma} = (m+x)/p.$

It follows that $V' = 1/p$. Substituting $zp^{\theta} = p^{-1}$ and $V' = 1/p$ in (10), we get the equilibrium condition:

(12) $$E_x\{v'[(1+x)/p](1+x)\} = \beta\gamma.$$

Since $v'' > 0$, the left hand side of (12), $a(p) = E_x\{v'[(1+x)/p](1+x)\}$, is decreasing in p and we get a unique solution as in Figure 1.

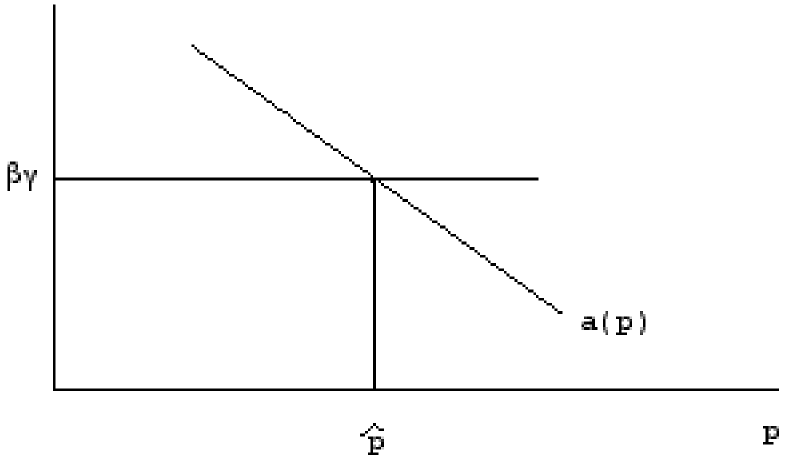

Figure 1

Note that the demand elasticity in absolute value $|\theta = 1/(\gamma - 1)|$ is an increasing function of γ. Thus high γ means low monopoly power. We can use Figure 1 to do some comparative statics: Increasing $\beta\gamma$ leads to a reduction in the equilibrium level of p and to an increase in average production. Thus, not surprisingly, low monopoly power is associated with high output. Low time preference is also associated with high output because of the delay in the payment for the labor effort assumed in the cash-in-advance model.

Substituting $v(L) = (1/2)L^2$ and $v'(L) = L$ in (12) leads to: $p = E_x(1 + x)^2/\beta\gamma$. The equilibrium labor supply is therefore:

$$(13) \qquad L(x) = (1 + x)/p = \beta\gamma(1 + x)/E_x(1 + x)^2.$$

<u>Optimal monetary policy:</u>

A social planner in this environment will solve:

$$(14) \qquad \max\ [\Sigma_j\ (y_j)^\gamma]^{1/\gamma} - \Sigma_j\ [v(y_j) = (1/2)(y_j)^2]$$

The solution to the planner's problem is: $y_j = 1$ for all j.

The equilibrium outcome (13) coincides with the planner's optimum (14) only if we choose a non-random rate of change $1 + \mu = \beta\gamma$. This policy works in our model because we assumed that buyers spend whatever they have. It will not work if we relax this assumption because the rate of return on money when $1 + \mu = \beta\gamma$ is $1/\beta\gamma > 1/\beta$. Therefore the buyer will not spend in this case. This means that equilibrium does not exist when $1 + \mu = \beta\gamma$. We therefore assumed that $\beta - 1 \leq x < \infty$ so the money growth rate cannot be below the Friedman rule.

The optimal policy under the constraint that $1 + \mu \geq \beta$ is the Friedman rule: $1 + \mu = \beta$. To see this claim note that at the Friedman rule (16) implies $L(x) = \gamma < 1$. This means that the marginal cost is less than the social benefit and therefore increasing μ is not desirable.

Can we benefit by increasing the variance of x and keeping $L(x) = \gamma$ on average? The answer to this question is also in the negative because the cost function is convex and the equilibrium utility from consumption is linear.

<u>Are the implied quantities optimal?</u>

Ex-post after observing the realization of x and after the commitment to price was already made, the seller will want to satisfy demand if:

(15) $$v'[(1 + x)/p] \leq \beta/(1 + x).$$

To see this claim, note that supplying an additional unit will bring p current normalized dollars in additional revenues or $p/(1 + x)$ next period's normalized dollars. The marginal benefit from doing it is therefore $\beta V' p/(1 + x) = \beta/(1 + x)$. Condition (15) says that this is worth doing if the marginal cost, $v'[(1 + x)/p]$, is less than the marginal benefit.

Substituting $p = E_x(1 + x)^2/\beta\gamma$ and $v'(L) = L$ in (15) leads to the following condition:

(16) $(1 + x)^2/E_X(1 + x)^2 \leq 1/\gamma$ for all possible realizations of x.

When γ is close to unity and monopoly power is low, then condition (16) will be violated for relatively large realizations of x.

To illustrate, I consider now the special case in which x may take two possible realizations: x = 0 and x = g with equal probability of occurrence. I calculate the level of γ (γ^c) for which (16) holds with equality for the following four cases: g = 0.1, 0.2, 0.3, 0.5. As can be seen from Table 1 the critical value of γ is in the range: 0.91 to 0.72. The last row in the Table is a calculation of the expected markup (EM) for $\beta = 0.96$ and for the appropriate γ^c.

Table 1*: The critical value of γ and the expected markup (EM)

g =	0.1	0.2	0.3	0.5
Ex = 0.5g	0.05	0.1	0.15	0.25
γ^c =	0.91	0.85	0.8	0.72
EM =	1.5	2.6	2.7	4.6

* The first row is the choice of the higher realization of x (g). Since the lower realization of x is zero the expected value of x is 0.5g. This is given by the second row. The third row computes the level of γ for which (16) holds with equality. The expected markup is calculated in the last row according to:
$EM = Ep/v'[L(x)] = Ep/L(x) = Ep/(1 + x)/p = p^2E[1/(1 + x)]$.
Substituting $p = E_X(1 + x)^2/\beta\gamma$ yields:
$EM = E[1/(1 + x)][E_X(1 + x)^2]^2/(\beta\gamma)^2$. We use $\beta = 0.96$ and $\gamma = \gamma^c$ to get: $EM = E[1/(1 + x)][E_X(1 + x)^2]^2/(0.96\gamma^c)^2$.

Christiano, Eichenbaum and Evans (1997) calibrate a sticky price model and say that there is little independent evidence on the value of γ ($1/\mu$ in their notation). In their benchmark parameterization they follow Hornstein (1993) and use $\gamma = 0.83$. The implied average markups seems to be high even when we choose $\gamma = 0.91$. Therefore, a choice of $\gamma > 0.91$ seems appropriate. This choice suggests that condition (16) is likely to be violated even if the expected rate of change in the money supply is less than 5%.

3. STICKY PRICES WITH OPTIMAL CHOICE OF QUANTITIES: THE PRODUCTION TO ORDER CASE

In the above new Keynesian model, there is a positive relationship between money surprises and real output and Friedman's k% rule is optimal. I now examine these conclusions under the assumption that the quantity supplied is optimal from the seller's point of view.

I start by focusing on the choice of quantities and assume that the normalized price, p, is exogenously given and constant over time (this means that the regular dollar price $P_t = pM_t$ responds with a one period lag to changes in the money supply). Since we do not allow for a choice of price we do not need the differentiated commodities structure. I therefore assume at this stage a single consumption good and a risk neutral utility function: $c - v(L)$, where c is the quantity consumed.

Figure 2 describes the sequence of events. The money supply transfer is realized after the price ($P_t = pM_t$) is exogenously set. Then sellers receive orders and choose whether to satisfy some or all of the orders they receive.

Figure 2

As before the typical buyer receives a transfer payment of x normalized dollars, where $\beta - 1 \le x < \infty$ is an i.i.d random variable with a density function $\phi(x)$.

In equilibrium, the typical seller receives an order of 1 + x normalized dollars. The revenue of the representative seller is therefore min(pL, 1 + x) normalized dollars if he chooses to produce L units.

Buyers arrive sequentially in an order that is determined randomly by an i.i.d lottery. Buyers who arrive late may not be able to buy. The probability that the buyer will make a buy depends on the realization x and is denoted by $\Pi(x)$. The household takes p and the probability $\Pi(x)$ as given and solves the following Bellman's equation:

(17) $$V(m; p) = \int_{\beta-1}^{\infty} \Pi(x)[(m + x)/p]\phi(x)dx$$

$$+ \int_{\beta-1}^{\infty} \{\max_L \; - v(L)$$

$$+ \Pi(x)\beta V[\min(pL, 1 + x)/(1 + x); p]$$

$$+ [1 - \Pi(x)]\beta V[(m + x + \min(pL, 1 + x))/(1 + x); p]\}\phi(x)dx.$$

The first row is the expected consumption for a household that starts with m normalized dollars. Then we have the expected value of the labor choices that are made after observing the realization of x.

The constant marginal utility of money, V', is:

(18) $V' = \int_{\beta-1}^{\infty} \{\Pi(x)/p + [1 - \Pi(x)]\beta V'/(1 + x)\}\phi(x)dx = \pi/p(1 - \sigma\beta)$,

where $\pi = \int_{\beta-1}^{\infty} \Pi(x)\phi(x)dx$ and $\sigma = \int_{\beta-1}^{\infty} \{[1 - \Pi(x)]/(1 + x)\}\phi(x)dx$.

To derive (18) note that an additional unit of money will yield 1/p utils if it buys in the current period. If it does not buy it will be carried to the next period yielding $\beta V'/(1 + x)$ utils.

Since the utility function is linear in consumption, there is no wealth effect (V' is a constant) and we can write the labor supply choice problem (for a given x) as:

$\max_L - v(L) + \beta V[\min(pL, 1 + x)/(1 + x)]$. The first order conditions for this problem are:

(19) $$pL \le 1 + x \;;$$

(20) $v'(L) \le p\beta V'/(1 + x) = A/(1 + x)$ with equality when $pL < 1 + x$,

where $A = \beta\pi/(1 - \sigma\beta)$ and (18) is used to substitute for V'.

Condition (19) says that it is not optimal to produce more than the quantity demanded. Condition (20) says that the marginal cost must be less than the marginal benefit from selling the good and must equal to it when there is excess demand.

Equilibrium for a given normalized price p is a pair of functions $[L(x), \Pi(x)]$ such that:

(a) Given $\Pi(x)$, the first order conditions (19) and (20) are satisfied;

(b) $\Pi(x) = \min\{1, pL(x)/(1 + x)\}$.

The requirement (b) says that in case of excess demand, the probability of making a buy is equal to the ratio of nominal supply to nominal demand.

Solving for equilibrium:

We guess a cut-off point ζ such that $\Pi(x) = 1$ if $x \leq \zeta$ and $pL(x)/(1 + x)$ otherwise. (When $x \leq \zeta$ there is excess demand and when $x > \zeta$ there is excess supply). To solve for ζ we start by treating A as a constant and define the notional supply $S(x; A)$ by the solution to: $v'(L) = A/(1 + x)$.

The notional supply $S(x; A)$ is a decreasing function of x as in Figure 3. The intuition is that when x increases, the relevant real price $P_t/P_{t+1} = pM_t/pM_t(1 + x) = 1/(1 + x)$, decreases. The demand is the upward sloping line $(1 + x)/p$ in Figure 3. Supply equals demand when $x = \zeta$.

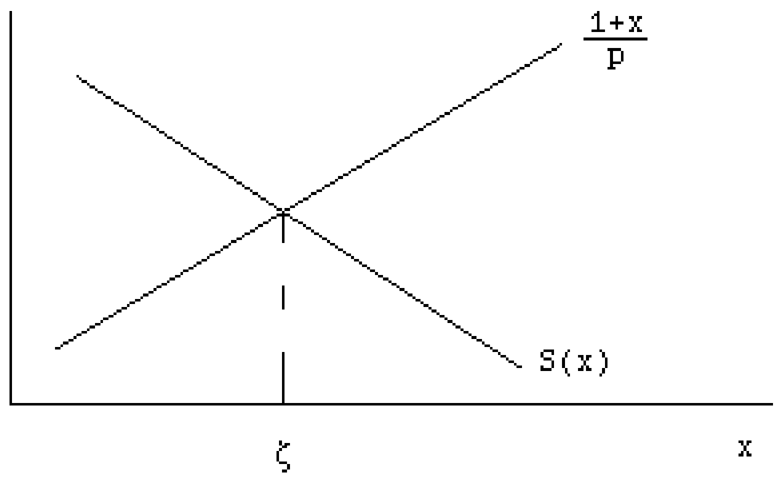

Figure 3

The actual amount traded is the minimum between supply and demand:

$$L(x; A) = \min\{S(x; A), (1 + x)/p\}. \quad (21)$$

Note that $L(x; A) = (1 + x)/p$ when $x \leq \zeta$ and $L(x; A) = S(x; A)$ when $x > \zeta$. It therefore has a "tent like shape" as in Figure 4.

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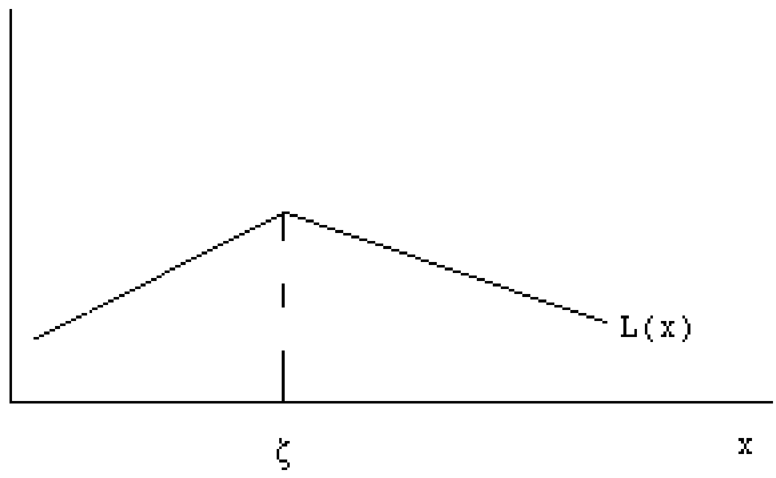

Figure 4

The cutoff point ζ is defined by the solution to:

$1 + \zeta = pL(\zeta)$. Using $v'(L) = L$ and (21) this leads to:

$\zeta = (pA)^{0.5} - 1$. The probability of making a buy is given by:

$$(22) \qquad \pi(A, p) = \text{Prob}(x \le \zeta) + \int_{\zeta}^{\infty} [pL(x)/(1 + x)]\phi(x)dx$$

$$= \text{Prob}[x \le (pA)^{0.5} - 1] + \int_{(pA)^{0.5}-1}^{\infty} [pA/(1 + x)^2]\phi(x)dx.$$

We now compute:

$$(23) \qquad \sigma(A, p) = \int_{\beta-1}^{\infty} \{[1 - \Pi(x)]/(1 + x)\}\phi(x)dx$$

$$= \int_{\zeta}^{\infty} [(1+x)^{-1} - pL(x)(1 + x)^{-2}]\phi(x)dx$$

$$= \int_{(pA)^{0.5}-1}^{\infty} [(1+x)^{-1} - pA(1 + x)^{-3}]\phi(x)dx.$$

We now look for a solution to the following equation:

$$(24) \qquad A = \beta\pi(A, p)/[1 - \sigma(A, p)\beta] \ge \beta^2/p.$$

The inequality is required to insure that $\zeta = (pA)^{0.5} - 1 \ge \beta - 1$. In the Appendix it is shown that a solution to (24) exists if p is not too small.

Adding the choice of price to the analysis turns out to be rather complicated. This is done in Appendix B under the assumption that buyers maximize a Dixit-Stiglitz CES utility function. Unlike the case in section 2 a single price equilibrium may not exist. But the qualitative result about the non-monotonic labor supply function still holds.

4. THE PRODUCTION TO MARKET CASE

I now consider the case in which producers do not receive orders and the amount produced is simply put on the market. In this case, typically only some of the output produced is sold and money surprises affect capacity utilization.[1] It is shown that the change in the assumption about selling leads to a qualitatively different equilibrium labor supply function. But the policy implications are the same as in the previous two models.

4.1 THE CHOICE OF QUANTITIES WHEN THE PRICE IS EXOGENOUSLY GIVEN

As in the previous section I start by assuming that the normalized price, p, is exogenously given. After observing the realization of the money supply (x) the seller chooses how much to produce and put his output on the market for sale. Buyers arrive and buy part or all of the available supply. Figure 5 describes the sequence of events.

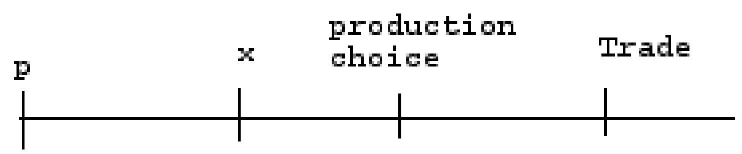

Figure 5

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[1] The model has some features in common with the literature on sequential trade (see, for example, Prescott [1975], Rotemberg and Summers [1990] and Eden [1990]). But unlike this literature, here the assumption of price rigidity is critical and there is a single price equilibrium rather than an equilibrium price distribution.

As in the previous section buyers who arrive late may not make a buy and $\Pi(x)$ denotes the probability of making a buy. All sellers sell the same fraction, $0 \leq \Omega(x) \leq 1$, of their output. The average revenue received by the seller for a unit produced is therefore: $W(x) = p\Omega(x)$. The amount of money that the household will have at the beginning of next period is: $LW(x)/(1 + x)$ if the buyer made a buy and $[LW(x) + m + x]/(1 + x)$ if the buyer did not make a buy.

Given the functions $\Pi(x)$, $W(x)$ the Bellman equation which describes the household's choice problem is:

$$(25) \qquad V(m; p) = \int_{\beta-1}^{\infty} [(m + x)\Pi(x)/p]\phi(x)dx$$

$$+ \int_{\beta-1}^{\infty} \{\max_L \; - v(L) + [\Pi(x)]\beta V[LW(x)/(1 + x); p]$$

$$+ [1 - \Pi(x)]\beta V[(m + x + LW(x))/(1 + x); p]\}\phi(x)dx.$$

Since the utility function is linear in consumption and V' is a constant, we can write the maximization problem in (25) as: $\max_L \; - v(L) + \beta V[LW(x)/(1 + x)]$. The first order condition for this problem is:

$$(26) \qquad v'(L) = \beta[W(x)/(1 + x)]V' = A\Omega(x)/(1 + x),$$

where the second equality uses $V' = \beta\pi/p(1 - \sigma\beta) = A/p$ and $W(x) = p\Omega(x)$.

Equilibrium for a given normalized price p is a vector of functions $[L(x), \Pi(x), \Omega(x)]$ and a triplet
$[\pi = \int_{\beta-1}^{\infty}\Pi(x)\phi(x)dx,\ \sigma = \int_{\beta-1}^{\infty}\{[1 - \Pi(x)]/(1 + x)\}\phi(x)dx,$
$A = \beta\pi/(1-\sigma\beta)]$ such that:

(a) $v'(L) = A\Omega(x)/(1 + x)$

(b) $\Pi(x) = \min\{1, pL(x)/(1 + x)\}$ = probability of making a buy;

(c) $\Omega(x) = \min\{1, (1 + x)/pL(x)\}$ = fraction of output sold.

Solving for equilibrium:

As in the previous production to order case, the labor supply in the excess demand region $x \geq \zeta$ is given by the solution, $L(x)$, to $v'(L) = L = A/(1 + x)$. The cutoff point ζ is given by the solution to: $1 + \zeta = pL(\zeta)$. This leads to: $\zeta = (pA)^{0.5} - 1$. In the excess demand range $\Omega(x) = 1$. When $x \leq \zeta$ there is excess supply, $\Omega(x) = (1 + x)/pL(x) \leq 1$ and we can write (26) as:

$$v'(L) = A/pL \text{ for } x \leq \zeta. \qquad (27)$$

The solution to (27) does not depend on x and is given by: $\hat{L} = (A/p)^{0.5}$. The intuition is as follows. An increase in x has two effects on the wage in terms of next period's normalized dollars: The increase in the fraction of output sold, $\Omega(x) = (1 + x)/p\hat{L}$, and the reduction in the value of a current normalized dollar. These two effects exactly offset each other and therefore the effective wage in terms of next period's normalized dollars, $W(x)/(1 + x) = 1/\hat{L}$ does not depend on x. Figure 6 illustrates the equilibrium labor supply function.

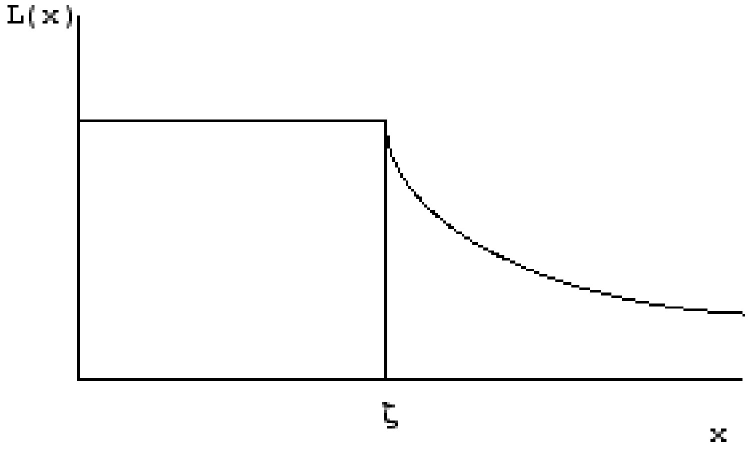

Figure 6

We now turn to a monopolistic competition environment in which p is a choice variable.

4.2 MONOPOLISTIC COMPETITION WITH RANDOM UTILITY MAXIMIZING CONSUMERS

I use McFadden's random utility maximization framework.[1] Buyers come to the market observe all remaining price offers and choose a single offer. They may choose a relatively expensive offer by "mistake". This mistakes may reflect unobserved attributes of the goods such as location. As a result of these mistakes a unit may be sold even if its price is higher than the price of other units.

When s units are available, the probability of choosing unit i is: Prob(i) = $w_i/\Sigma_{j=1}^{s} w_j$, where,

(28) $$w_i = \exp[\alpha(P/p_i)],$$

[1] For surveys of this approach see McFadden (2000) and Manski (2000).

$\alpha > 0$ is a parameter, p_i is the price of unit i and P is the average price of all offers (units).

Units are sold sequentially. The first unit is sold in the first round or market. Then a second unit is sold in the second market and so on. We now compute the probability of selling unit i at the price p_i when there are s-1 other units offered at the price p.

The average price when there are s - 1 units offered at the price p and one unit offered at the price p_i is:

(29) $$P(p_i, p, s) = [(s - 1)p + p_i]/s.$$

To simplify notation, I write P(s) instead of $P(p_i, p, s)$ to denote this average. The probability of selling unit i in the first market is:

(30) $$\omega_1(p_i, p, s) = w_i/\Sigma_{j=1}^{s} w_j$$

$$= \exp[\alpha P(s)/p_i]/\{(s-1)\exp[\alpha P(s)/p] + \exp[\alpha P(s)/p_i]\}.$$

If the unit was not sold in the first market it may be sold in the second market. Since s - 1 units are offered in the second market, the probability that the unit will be sold in the second market is:

(31) $$\omega_2 = (1 - \omega_1)w_i/\Sigma_{j=1}^{s-1} w_j =$$

$$(1 - \omega_1)\exp[\alpha P(s-1)/p_i] / \{(s-2)\exp[\alpha P(s-1)/p] + \exp[\alpha P(s-1)/p_i]\}.$$

Similarly, the probability that the unit will be sold in the third market is:

$\omega_3 = (1-\omega_1)(1-\omega_2)\exp[\alpha P(s-2)/p_i]/$
$\{(s-3)\exp[\alpha P(s-2)/p] + \exp[\alpha P(s-2)/p_i]\}$.

Letting $\omega_0 = 0$, we can write the probability that the unit will be sold in the jth market as:

(32) $$\omega_j = [\Pi_{k=0}^{j-1}(1 - \omega_k)]\exp[\alpha P(s-j+1)/p_i]/$$

$$\{(s-j)\exp[\alpha P(s-j+1)/p] + \exp[\alpha P(s-j+1)/p_i]\}.$$

In equilibrium, the number of units on the market is $s(x) = L(x)$. The number of units demanded is: $(1 + x)/p$. The number of rounds is the minimum between supply and demand: $S = \mathrm{Min}\{(1 + x)/p, s(x)\}$ rounds. The probability of selling a unit in excess supply situations, when $S = (1 + x)/p < s(x)$, is:

(33) $$\omega(p_i, p, s) = \Sigma_{j=1}^{S} \omega_j$$

$$= \Sigma_{j=1}^{S} [\Pi_{k=0}^{j-1}(1 - \omega_k)]\exp[\alpha P(s-j+1)/p_i]/$$

$$\{(s-j)\exp[\alpha P(s-j+1)/p] + \exp[\alpha P(s-j+1)/p_i]\}.$$

In excess demand situations when $S \geq s(x)$, the probability of selling the unit is 1 because if the unit was not sold in the first s-1 rounds it will be sold in round s when it is the only unit on the market. The probability of making a sale is therefore:

(34) $$\Omega[p_i, p, s(x), x] = \omega[p_i, p, s(x)]$$

when $s(x) > (1 + x)/p$ and 1 otherwise.

It is assumed that the household may put a different price tag on each unit and commit to sell the first unit at the price p_1, the second unit at the price p_2 and so on. The household takes the price posted by others (p), the supply schedule (s[x]), the probability of making a sale, $\Omega[p_i, p, s(x), x]$, and the probability of making a buy, $\Pi(x)$, as given and solve the following Bellman's equation:

(35) $$V[m; p, s(\bullet)] = \int_{\beta-1}^{\infty} [\Pi(x)(m + x)/p]\phi(x)dx +$$

$$+ \max_{p_1\ p_2\ \ldots} \int_{\beta-1}^{\infty} \{\max_L - v(L)$$

$$+ \Pi(x)\beta V\{\Sigma_{i=1}^{L}\Omega[p_i, p, s(x), x]p_i/(1 + x); p, s(\bullet)\}$$

$$+ [1-\Pi(x)]\beta V\{[m+x + \Sigma_{i=1}^{L}\Omega[p_i, p, s(x), x]p_i]/(1+x); p, s(\bullet)\}\}\phi(x)dx$$

We now define equilibrium as follows.

Equilibrium is a scalar (p) and a vector of functions $[\Pi(\bullet), \Omega(\bullet, \bullet, \bullet, \bullet), L(\bullet)]$ such that:

(a) $\Pi(x) = \min\{1, pL(x)/(1 + x)\}$ = probability of making a buy;

(b) The probability of making a sale, $\Omega(\bullet, \bullet, \bullet, \bullet)$, is computed according to (29) - (34);

(c) Given p, $s(x) = L(x)$ and $\Omega(\bullet, p, L(x), x]$, $p_i = p$ and $L(x)$ is a solution to (35).

We already solved for the equilibrium magnitudes as a function of p. We now turn to solve for the equilibrium level of p.

We consider the choice of the price of unit i when all other units are priced at p. The expected next period consumption that the seller can get if he sells unit i at the price p_i is:
$\int_{\beta-1}^{\infty} [\Omega(p_i, p, x)p_i/(1 + x)]\phi(x)dx$, where
$\Omega(p_i, p, x) = \Omega[p_i, p, s(x), x]$ is used for convenience. The seller will therefore choose the price of unit i by solving:

(36) $$\max_{p_i} \int_{\beta-1}^{\infty} [\Omega(p_i, p, x)p_i/(1 + x)]\phi(x)dx.$$

The first order condition for this problem is:

(37) $$\int_{\beta-1}^{\infty} [\Omega_1(p_i, p, x)p_i + \Omega(p_i, p, x)]/(1 + x)]\phi(x)dx = 0.$$

In equilibrium this first order condition can be written as:

(38) $$G(p) = \int_{\beta-1}^{\infty} -\Omega_1(p, p, x)pdx =$$

$$g(p) = \int_{\beta-1}^{\infty} \{\min[1, (1 + x)/pL(x)]/(1 + x)\}\phi(x)dx$$

Claim: There exists a solution to (38).

Proof is under construction.

Conclusions:

We relaxed the demand satisfying assumption made in the new Keynesian literature and found that the money/output relationship is not robust to changes in this assumption. When x is small it is optimal to satisfy demand but when x is large it is not. In the range when it is not optimal to satisfy demand more money means a lower real wage and therefore leads to less real output.

When x is small, more money may have no effect on labor supply. This occurs in the production to market case. In this case more money means that a larger fraction of output produced will be sold but the real price at which it is sold is reduced because of the expected inflation in the next period. The two effects on the relevant real wage exactly cancel each other and therefore there is no effect of money on employment in this range.

But even in the production to market case money is not neutral. It affects the fraction of output sold (capacity utilization). If we measure output only by the amount sold (as is the case in the service sector) we will find a positive effect of money on output in the range $x \leq \zeta$ and a positive effect of money on productivity in this range. But the correlation between money and output may be low because of the negative relationship between production and money in the $x > \zeta$ range.

The labor supply function is different across the three models studied. It is an increasing function of x when sellers are committed to satisfy demand (in the new Keynesian model). It has a "tent like shape" in the production to order case and it is weakly declining in the production to market case. However variations in x are harmful in all three models.

APPENDIX A

<u>Claim:</u> if $p > (1+\rho)/\{\text{Prob}[x \le 0] + \int_0^\infty [1/(1+x)^2]\phi(x)dx\}$, then there exists a solution to (24).

To show this claim we write (24) as

(A1) $$F(A) = \beta\pi(A, p)/A[1 - \sigma(A, p)\beta] = 1.$$

We will show that $F(A) > 1$ for $A = 1/p$ and $F(A) < 1$ for large A. Therefore continuity implies the existence of a solution $A(p)$ to (A1).

We start by showing that $F(1/p) > 1$. When $A = 1/p$, $\zeta = (pA)^{0.5} - 1 = 0$. Using (22) and (23) this implies:

(A2) $$\pi(1/p, p) = \text{Prob}[x \le 0] + \int_0^\infty [1/(1+x)^2]\phi(x)dx;$$

(A3) $$\sigma(1/p, p) = \int_0^\infty [(1+x)^{-1} - (1+x)^{-3}]\phi(x)dx.$$

Substituting (A2) - (A3) in (A1) leads to:

(A4) $$F(1/p) = \beta\{\text{Prob}[x \le 0] + \int_0^\infty [1/(1+x)^2]\phi(x)dx\}/$$
$$\{1 - \beta\int_0^\infty (1+x)^{-1}\phi(x)dx + \beta\int_0^\infty (1+x)^{-3}]\phi(x)dx\}(1/p).$$

We now observe that $F(1/p) > 1$ if:

(A5) $$p > \{1 - \beta\int_0^\infty (1+x)^{-1}\phi(x)dx + \beta\int_0^\infty (1+x)^{-3}]\phi(x)dx\}/$$
$$\beta\{\text{Prob}[x \le 0] + \int_0^\infty [1/(1+x)^2]\phi(x)dx\}.$$

The assumption

$p > (1 + \rho)/\{\text{Prob}[x \le 0] + \int_0^\infty [1/(1 + x)^2]\phi(x)dx\}$ guarantees the inequality (A5).

We have thus shown that $F(1/p) > 1$. We now show that $F(A) < 1$ for large A. For this purpose we note that (23) implies:

(A6) $$\sigma(A) = \int_{(pA)^{0.5}-1}^{\infty} [(1 + x)^{-1} - pA(1 + x)^{-3}]\phi(x)dx$$

$$< \int_{\beta-1}^{\infty} [(1 + x)^{-1}]\phi(x)dx \le 1/\beta.$$

Therefore, $1 - \sigma(A, p)\beta > 0$ and is bounded away from zero. Since $\beta\pi(A, p) < 1$, it follows that $F(A) = \beta\pi(A, p)/A[1 - \sigma(A, p)\beta]$ is small when A is large.

We have thus shown that when p is not too small, $F(1/p) > 1$ and $F(A) < 1$ for large A. Therefore by continuity there exists a solution to $F(A) = 1$. □

APPENDIX B: MONOPOLISTIC COMPETITION WITH DIXIT-STIGLITZ CONSUMERS

We adopt the Dixit-Stiglitz framework in section 2 and assume a large number of N infinitely lived households each producing a differentiated good and consuming all goods. The household's utility function is: $[\Sigma_j (y_j)^{\gamma}]^{1/\gamma} - v_i(L)$, where $0 < \gamma < 1$. To simplify the analysis I assume that some sellers can costlessly produce and therefore always satisfy demand. Households are indexed $i = 1-\lambda, \ldots, N$. It is assumed that $v_i(L) = 0$ for $i < 1$ and $v_i(L) = (1/2)L^2$ for $i \geq 1$. Thus, λ households can costlessly produce and $N - \lambda$ households produce at a cost.

Seller i chooses a price, p_i, and a labor supply function, $L_i(x)$. For $i \geq 1$ this choice implies a cut off point ζ_i, where demand is fully satisfied at the range $x \leq \zeta_i$ and rationing occurs when $x > \zeta_i$. I assume that low index sellers set a low price and a relatively low cut-off point: $p_i \leq p_j$ and $\zeta_i \leq \zeta_j$ for $i < j$. It will be shown that in equilibrium sellers who set a relatively low price will also have a relatively low cut-off point.

Buyers' (orders) arrive sequentially. Buyers find all goods when $x \leq \zeta_1$. When $\zeta_1 < x \leq \zeta_2$ buyers who arrive early find all goods but buyers who arrive late find only N - 1 goods (they do not find good 1). When $\zeta_2 < x \leq \zeta_3$ buyers who arrive early find all N goods. Those who arrive second find N - 1 goods and those who arrive last find only N - 2 goods.

From the buyer's point of view there are $N - \lambda + 1$ possible characterizations of the availability of goods, indexed $i = 1, \ldots, N+1$. We refer to these characterizations as market conditions. In market condition 1 all goods are available. In market condition 2 only goods indexed $j \neq 1$ are available. And in general, in market condition $1 \le s \le N$ only goods indexed $j \le 0$ and $j \ge s$ are available. In market condition $N + 1$ only goods indexed $j \le 0$ are available.

A buyer who chooses to spend d normalized dollars in market condition s solves:

$$\text{(B1)} \qquad F(d, s) = \max_{y_j} \left[\Sigma_{j=1-\lambda}^{0} (y_j)^{\gamma} + \Sigma_{j=s}^{N} (y_j)^{\gamma}\right]^{1/\gamma}$$

$$\text{s.t. } \Sigma_{j=1-\lambda}^{0} p_j y_j + \Sigma_{j=s}^{N} p_j y_j = d.$$

Here $F(d, s)$ is the expected current utility when the buyer arrives in market condition s. Let $z_s = 1/[\Sigma_{j=1-\lambda}^{0} (p_j)^{1+\theta} + \Sigma_{j=s}^{N}(p_j)^{1+\theta}]$ where as before $\theta = 1/(\gamma - 1) < 0$ denote the appropriate deflator for this case. The buyer's demand in market condition s can be derived in a way which is similar to (7) in the text and is given by:

$$\text{(B2)} \qquad z_s(p_i)^{\theta} d(x, s),$$

where $d(x, s)$ is the amount that the buyer will spend in market condition s.

We now turn to compute the fraction of buyers who will be in market condition s, $\pi(x, s)$. When $x \le \zeta_1$ all buyers find all goods and $\pi(x, 1) = 1$ ($\pi(x, s) = 0$ for $s > 1$). When $x > \zeta_1$ only a fraction $\pi(x, 1) = p_1 L_1(x)/Nd(x, 1)z_1(p_1)^{\theta}$ of the buyers find all goods.

Similarly, the fraction of buyers in market condition 2 (when $N - 1$ goods are available) is:

(B3) $$\pi(x, 2) = 1 - \pi(x, 1) \text{ if } x \le \zeta_2;$$
$$\pi(x, 2) = p_2L_2(x)/D_2(x) - \pi(x, 1) \text{ if } x > \zeta_2,$$

where $D_2(x) = N\{\pi(x, 1)d(x, 1)z_1(p_2)^\theta + [1 - \pi(x, 1)]d(x, 2)z_2(p_2)^\theta\}$ is the total demand for good 2. (This is the amount that will be sold of good 2 if there was no rationing of goods indexed $j \ge 2$). This says that when $x \le \zeta_1$ all buyers find all N goods and therefore the probability of finding only N - 1 goods is zero. When $\zeta_1 < x \le \zeta_2$ a fraction $\pi(x, 1)$ of the buyers finds all goods and the rest will find N - 1 goods, because at this range of x only good 1 is rationed. When $x > \zeta_2$ only a fraction $p_2L_2(x)/D_2(x)$ of the demand for good 2 will be satisfied. Thus a fraction $p_2L_2(x)/D_2(x)$ of the population will find either all N goods or N-1 goods. To arrive at the fraction that finds N - 1 goods we need to subtract the fraction of the population that finds all N goods ($\pi(x, 1)$).

In general, for $s \le N$ we have:

(B4) $$\pi(x, s) = 1 - \Sigma_{j=1}^{s-1} \pi(x, j) \quad \text{if } x \le \zeta_s;$$
$$\pi(x, s) = p_sL_s(x)/D_s(x) - \Sigma_{j=1}^{s-1} \pi(x, j) \quad \text{if } x > \zeta_s,$$

where,

$$D_s(x) = N\Sigma_{j=1}^{s-1} \pi(x, j)d(x, j)z_j(p_s)^\theta + N[1 - \Sigma_{j=1}^{s-1} \pi(x, j)]d(x, s)z_s(p_s)^\theta$$

is the demand for product s when goods indexed $1 \le j < s$ are rationed and goods indexed $j \ge s$ are not rationed. The fraction of buyers who are in market condition $s = N + 1$ is defined by:

$$\pi(x, N+1) = 1 - \Sigma_{j=1}^{N} \pi(x, j).$$

We now turn to the seller's choice. The demand for seller's i product is given by:

$$(p_i)^{\theta} z(x), \tag{B5}$$

where $z(x) = N\Sigma_{s=1}^{N+1} \pi(x, s)d(x, s)z_s$.

It is assumed that α is large and therefore we may neglect the individual seller's effect on z(x). Note however that the fraction α/N may be small.

At the time labor choices are made, seller i takes p_i and z(x) as given and choose labor $L_i(x)$ under the constraint that he does not produce more than the quantity demanded:

$$L_i(x) \leq (p_i)^{\theta} z(x). \tag{B6}$$

The household takes z(x) as given and solves the following Bellman's equation:

$$V(m) = \max_{p_i} \int_{\beta-1}^{\infty} \{\max_{d\ L} \Sigma_{s=1}^{N+1} \pi(x, s)\{F(d, s) \tag{B7}$$

$$- v(L) + \beta V[(m + x - d + p_i L)/(1 + x)]$$

$$\text{s.t. } d \leq m + x \text{ and } (30)\}\}\phi(x)dx$$

I now define equilibrium as follows.

Equilibrium is a vector of functions $[d(x, s), \pi(x, s), L_i(x), z(x)]$ and a vector of scalars (p_i, ζ_i) such that

(a) $p_i \leq p_{i+1}$ and $\zeta_i \leq \zeta_{i+1}$;

(b) $\pi(x, s)$ is calculated by (A4) and $z(x) = N\Sigma_{s=1}^{N} \pi(x, s)d(x, s)z_s$;

(c) Given $z(x)$, $[p_i, \zeta_i, L_i(x), d(x, s)]$ is a solution to (A7) for $m = 1$.

Claim 1: When $x > \zeta_i$ and the constraint (B6) is not binding $L_i(x)$ is a decreasing function.

This follows from the first order condition:

(B8) $$v'(L) = L = \beta V' p_i/(1 + x).$$

We can now solve for the cut-off point.

Claim 2: If p_i is a solution to (B7) then:

(B9) $$p_i = [z(\zeta_i)(1+\zeta_i)/\beta V']^{1/(1-\theta)}.$$

To show this claim note that at the cut-off point ζ_i demand is satisfied ($L = (p_i)^{\theta} z(\zeta_i)$) and the first order condition (B8) is satisfied ($L = \beta V' p_i/(1 + \zeta_i)$). (Thus, at this point constraint (B6) is not binding and the seller satisfies demand willingly). This leads to: $z(\zeta_i)(1+\zeta_i) = \beta V'(p_i)^{1-\theta}$ which leads to (B9).

Note that equilibrium condition (a) requires that a choice of a relatively high price is associated with a relatively high cut-off point. Since V' is a constant condition (a) will be satisfied if $z(x)$

is a non-decreasing function. I now show that this is the case when $d(x, s) = 1 + x$ and the liquidity constraint is always binding.

Claim 3: $z(x)$ is a non-decreasing function.

Proof: We start by noting that $z_s = 1/[\Sigma^{\theta}_{j=1-\lambda}(p_j)^{1+\theta} + \Sigma^{N}_{j=s}(p_j)^{1+\theta}]$ is an increasing function of s. We now define:

$$\text{(B10)} \qquad z^s(x) = \{\Sigma^{s}_{j=1} \pi(x, j)z_j + [1 - \Sigma^{s}_{j=1} \pi(x, j)]z_{s+1}\}$$

$$= z^{s-1}(x) + [1 - \Sigma^{s}_{j=1} \pi(x, j)](z_{s+1} - z_s).$$

Note that $z(x) = N(1 + x)\Sigma^{N+1}_{s=1} \pi(x, s)z_s = N(1 + x)z^N(x)$. It is therefore enough to show that $z^s(x)$ is an increasing function for all x. We show this by induction. We start from:

$$\text{(B11)} \qquad z^1(x) = \pi(x, 1)z_1 + [1 - \pi(x, 1)]z_2.$$

An increase in x from $x \le \zeta_1$ to $x > \zeta_1$ lowers $\pi(x, 1)$ from unity to $\pi(x, 1) = p_1L_1(x)/N(1 + x)z_1(p_1)^{\theta}$. In the range $x > \zeta_1$, $L_1(x)$ is a decreasing function and therefore $\pi(x, 1)$ is a decreasing function. Since $z_1 < z_2$ it follows that (B11) is an increasing function. I now turn to show that

$$\text{(B12)} \qquad z^2(x) = \pi(x, 1)z_1 + \pi(x, 2)z_2 + [1 - \pi(x, 1) - \pi(x, 2)]z_3$$

$$= z^1(x) + [1 - \pi(x, 1) - \pi(x, 2)](z_3 - z_2)$$

is increasing in x. I start by showing that the sum $\pi(x, 1) + \pi(x, 2)$ is weakly decreasing in x. Using (B3) we see that when $x \leq \zeta_1$ this sum is unity. When $x > \zeta_2$, $\pi(x, 2) + \pi(x, 1) = p_2L_2(x)/D_2(x)$, where $D_2(x) = N(1 + x)\{\pi(x, 1)z_1(p_2)^\theta + [1 - \pi(x, 1)]z_2(p_2)^\theta\}$. Since $\pi(x, 1)$ is decreasing and $z_1 < z_2$, $D_2(x)$ is an increasing function. Since $L_2(x)$ is a decreasing function it follows that $\pi(x, 2) + \pi(x, 1) = p_2L_2(x)/D_2(x)$ is a decreasing function.

Since $z^1(x)$ and $1 - \pi(x, 1) - \pi(x, 2)$ are increasing functions and $z_2 < z_3$, (B12) is an increasing function.

In general we assume that $z^{s-1}(x)$ is an increasing function, and show that (B10) is an increasing function. For this purpose, we use (B4) to show that $\Sigma_{j=1}^{s} \pi(x, j)$ is a weakly increasing function. When $x \leq \zeta_s$, $\Sigma_{j=1}^{s} \pi(x, j) = 1$. When $x > \zeta_s$, $\Sigma_{j=1}^{s} \pi(x, j) = p_sL_s(x)/D_s(x)$. Claim 1 implies that $L_s(x)$ is a decreasing function. The maintained assumption that $z^{s-1}(x)$ is an increasing function implies that $D_s(x) = N(1 + x)(p_s)^\theta\{\Sigma_{j=1}^{s-1} \pi(x, j)z_j + [1 - \Sigma_{j=1}^{s-1} \pi(x, j)]z_s\}$ $= N(1 + x)(p_s)^\theta z^{s-1}(x)$ is an increasing function. It follows that $\Sigma_{j=1}^{s} \pi(x, j) = p_sL_s(x)/D_s(x)$ is a decreasing function and since $z_{s+1} - z_s > 0$, (B12) is an increasing function.

It follows that: $z(x) = N(1 + x)\Sigma_{s=1}^{N+1} \pi(x, s)z_s = N(1 + x)z^N(x)$ is an increasing function. □

The individual seller's ($i \geq 1$) labor supply is $L_i(x) = (p_i)^\theta z(x)$ in the range $x \leq \zeta_i$. Claim 3 implies that $L_i(x)$ is increasing in this range. When $x > \zeta_i$ the individual labor supply is given by the decreasing function (B8). The individual labor supply may therefore look like the function L(x) in Figure B1.

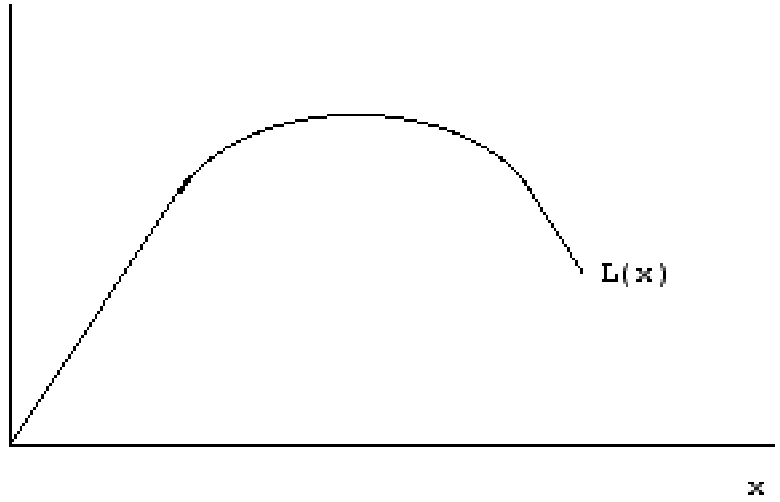

Figure B1

An equilibrium in which all sellers choose the same price ($p_i = p$ for all $i \geq 1$) may not exist. It is therefore possible that to get the aggregate labor supply we will have to add sellers with different prices and cut-off points. The qualitative shape of the aggregate labor supply function will remain as in Figure B1.

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