Private Investments in Higher Education: Comparing Alternative Funding Schemes

by

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Abstract: We consider an OLG economy with endogenous human capital formation. Young individuals make decision about their investment in higher education only after they obtain some signal, correlated to their ability. We examine three different funding regimes, each requires governmental intervention but not funding, available to students if they choose to invest in higher education. The economic implications of such funding schemes in equilibrium are studies, in particular the effects on accumulation of human capital and income inequality.

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1. Introduction

The important role played by higher education in the development process of countries and its significant impact on the distribution of income has long been recognized by economists. Researchers demonstrated cross-country evidence that show positive association between investment in education and economic growth [see, e.g., Barro (1998) and Bassinini and Scarpenta (2001)]; particularly, those OECD countries who exapanded higher education more rapidly from the 1960's experienced faster growth. The level of education has been shown to have positive effect on physical capital investments. Higher education has been expanded considerably in the OECD countries during the second half of the twentieth century; this is in terms of aggregate numbers of students as well as the total funding coming from public and private sources. The most striking example is the UK's higher education system: In 1960 there were 400,000 full time students compared to 2 million in the year 2000 [see, Greenaway and Haynes (2003)]. As a result of this expansion, all major industralized countries in Europe and elsewhere have been grappling with financing the rising costs of of higher education/ training systems. Due to fiscal pressures, we now observe a process of shifting part of this financial burden from public funding towards the students.[1] We also see a shift from income support transfers to programs based on students loans, which has resulted in a significant decline in the public funding per student. Clearly, the cost of higher education extends well beyond payments of tuitions, students in universities do not live with their parents anymore. Access to attarctive loans, to be invested in education, which are used for tuition, housing, equipment etc. also allow students to devote more time to improving their scholar achievements.

[1]Even in Russia 47% of the students in the higher education system must finance their tuition fees (which are significant), as well as other related costs from their own resources [see Kaganovich (2005)].

Although, there are good reasons for governmental intervention in the provision of higher education, there are some justifications to proceed with shifting part of the financial load to personal funding. In certain countries, due to the socio-economic background of the students, subsidizing heavily the higher education results in adverse effect on the distribution of income. On the other hand, it is desirable to remove the financial barriers to enhance participation of the younger generation in the higher education system. Due to the well-documented market failure in financing higher education, some other alternatives should be examined. Milton Friedman (1955, 1962) was the first to raise this issue and to suggest some solutions. Friedman was the first to offer income-contingent financing of students' investments in higher education. After pointing out the empirical evidence that indicates the underinvestment phenomena in higher education, he proposes to create financial instruments that allow investors to "buy" part of a student's future income: "....for education would be to "buy" a share of an individual's earning prospects" [Friedman (1962), p. 103]. These methods of financing schemes are designed to reduce the risk that students face, since such an arrangement will provide a hedge against low or no income in the future. In other words, we need to consider not a 'mortgage-type' contracts, or graduate taxes (even though it is income-dependent), but rather income-contingent paytment scheme. Such ideas were described in Barr and Crawford (1998), in Greenaway and Haynes (2003) and, in a more extensive way, by Lleras (2004) who focuses on the implementation of various ways of funding higher education via the private sector. Lleras considers few possibilities to carry out the income contingent funding schemes, including the case where we take into account ability and future labor possibilities among students. The design of such student loan program, repayment terms and debt default, as well as the international experience, has been discussed by Woodhall (1988). The problems involving cost of such programs and evidence related to loans collectibilty has been discussed by Albrecht and Ziderman (1993). Do such ideas really work? let us describe specific programs for private funding of higher education implemented in some countries. The role of student financial aid in the US has been studied by Dynarski (2001), where it was shown that in the last two decades of the twentieth century "offering $1000 of grant aid increased the probability of attending college by 3.6% and years of completed schooling by a tenth of a year"[2].

In 1989 Australia was the first to initiate income contingent students loans program [for details, see Chapman (1997)], which turned out to be a successful experiment. Following Australia came Ghana, Sweden, Chile, New Zealand. and the UK [see Lleras (2004)]. The uniqueness of the Australian model was not only in converting the paybacks of the education loans to be income-contingent, but also in using the existing tax authorities as a collection agency, which was unprecedented. The Australian system has been successful, in part, due to the well organized collection system that operates at low marginal cost. More precisely, the collection cost of loan repayments is 0.5% in this case, which is , clearly, a low cost. Moreover, taking legal actions agaist individuals is, relatively cheap under this collection mechanism. A reliable low cost loan repayment system is an essential component in such funding program.

These ideas were adopted by the other countries, including the UK. In the UK, as pointed out by Greenaway and Haynes [(2003), p.F162], "after 1998 income contingency applies and students become liable for repayment of maintenance loans once their gross income exceeds £10,000 per annum. Beyond this, graduates pay 9% of their marginal income, collected by the Inland Revenue and passes on to the Student Loans Company". Clearly, such a mechanism also minimizes debt default. In the

[2]Evidence in the US show a drastic increase in borrowing for higher education during the last decade of the twentieth century. Data can be found in the web site of the National Postsecondary Student Aid Study (http://nces.ed.gov/pubsearch/getpubcats.asp?sid=013)

US there has been recently some changes in the sudent loans repayment program. The ecactment of the Higher Education Reconciliation Act (HERA) of 2005 modifies the Direct Loan program which offers contingent repayment plans as well as 'income-sensitive' repayments. In some special cases interest rates are subsidized, and under some circustances HERA strengthens the teacher loan forgiveness plan.

Our aim in this paper is to use a general equilibrium framework in which we can analyze the economic consequences of various schemes to funding private expenditures during the higher education period. The framework used in this work is an overlapping generations model with endogenous human capital formation. This type of models were used, for example, by Azariadis and Drazen (1990), Orazem and Tesfatsion (1997), Viaene and Zilcha (2002). Heterogeneity in our case is introduced via random innate abilities in each generation. Throughout our analysis we take the investment made by the government in higher education to remain fixed. However, individual's own investment in higher education takes place after observing a signal correlated to his/her ability (or the random future income).

We shall consider three different funding schemes and examine their equilibrium effects on aggregate investments in human capital, on accumulation of human capital (and hence the impact on growth) and the compare their implication to income inequality. The three funding schemes we consider here are:

(a) Access to Credit Markets: guaranteeing that young individuals, who wish to invest in higher education, have unrestricted access to the existing credit markets.

(b) Full Risk Sharing: payments of education loans are income contingent where risk about future income (due to random abilities) is shared by all students of the same cohort,

(c) Partial Risk Sharing: future payments are contingent on the random future income, but only within groups of students who are of the 'same type' when they enter the higher education system.

Case (a) does not contain any element of repayment being income-dependent; in fact, each student needs to pay back his/her loans under the credit market terms. Thus, implementation in this case requires overcoming hurdles that prevent free access to credit during the studying period. Case (b) is a mechanism of risk sharing of the uncertainty about future income within the students population of the same cohort, where loan paybacks are fully linked to the (random) future income. Thus no external funds are needed to subsidize this program, it only requires implementation via certain mechanism that guarantees revelation of information about income and debt collection (see the Australian example). Case (c) contains partial risk sharing : each group (or 'type') will have its own risk sharing: payments of debt is income contingent within that type of students. There is no risk sharing among students with "different ability charecretistics"; this restricts the 'cross subsidization' between groups characterized by different 'signals' (which are correlated to incomes).

The paper is organized as follows. We present our model and the above mentioned three financing regimes in Section 2. Section 3 examines the implications of these financing schemes to human capital accumulation. Section 4 compares the welfare implications these funding schemes. The implications to income inequality are studied in section 5. Section 6 concludes the paper. Some proofs are relegated to the Appendix.

2. The Model

We consider an overlapping-generations economy with a single commodity, say, physical capital, which can be consumed or invested in production. Individuals live for three periods: the 'youth period', where each individual is supported by parents. In this period, the agent takes out a loan and makes a capital investment in education in order to acquire skills; the 'middle period', where individuals work, earn labor income, consume and save. Labor income depends on each agent's skills, or human capital, which is assumed to be observable. Part of the labor income is earmarked for the repayment of the loan. Finally, the 'retirement period' in which individuals consume their total savings. There is no population growth and each generation G_t (i.e., all individuals born at date $t-1$), $t = 0, 1, 2...$, consists of a continuum of agents with (Lebesgue-) measure 1, say the interval $[0, 1]$.

Our framework is characterized by heterogenous individuals in each generation, where heterogeneity is generated by random innate ability. While nature assigns abilities to individuals at birth, no individual knows exactly his own ability when, at young age, he invests privately in education. Therefore, the investment decision, x, is made under uncertainty. In the next period, the agent learns his ability A. We denote by $\nu(A)$ the time-invariant density of agents with ability A, where $A \in \mathcal{A} = [\underline{A}, \bar{A}] \subset \mathbb{R}_{++}$. From the perspective of a young individual, ability is random as it is the realization of a random variable with distribution $\nu(\cdot)$. Yet, there is no aggregate uncertainty in the economy, i.e., the ex post distribution of abilities across the members of a generation is exactly ν. Our modeling approach follows the technique suggested in Feldman and Gilles [1985, Proposition 2], where uncertainty exists at the individual level but in the aggregate there is no uncertainty.

The production function of human capital is, in general, a complex function which depends on individual, family, and other parameters. We shall restrict the structure of the human capital formation process, in order to make our equilibrium comparative dynamics analytically manageable. We assume that the level of human capital, or skills, of an individual $i \in G_t$, denoted by h_t^i, depends on the (random) innate ability A^i, the private investment in education $x^i \in \mathbb{R}_+$, and the *average* human capital of the older generation, denoted by H_{t-1} (which may represent the human capital of 'teachers'). Namely,

$$h^i = \varphi(A^i) g(x^i, H_{t-1}). \tag{1}$$

Public investment in individual education, which is assumed to be the same for all agents, is included in the accumulation function, g, through some implicit additive component. We make the following assumption about this process:

Assumption 1. *$\varphi(A)$ is an increasing and differentiable function. $g(x, H)$ is twice differentiable, strictly increasing and concave in the first argument, and satisfies $\lim_{x\to 0} g'_1(x, H) = \infty$ for $H > 0$. Also $g'_1(x, H)$ is non-decreasing in H. Furthermore, $g(x, H)$ exhibits decreasing concavity with respect to x, meaning that*

$$K(x, H) := -\frac{g''_{11}(x, H)}{g'_1(x, H)} \tag{2}$$

is decreasing in x, i.e., $K'_1(x, H) \leq 0 \; \forall x, H$.

Each agent chooses private investment in education after he has received a publicly observable signal $y \in Y \subset \mathbb{R}$. Within the group of agents with ability A, the signals are distributed according to the density $\nu_A(y)$. By assumption, the distribution of signals and abilities are correlated. Hence, the signal assigned to an agent can be used as a screening device for his unknown ability. Based on the screening information conveyed by the signal, the agent forms expectations about his ability in a Bayesian way. The distribution of signals received by agents in the same generation has the density $\mu(y) = \int_{\mathcal{A}} \nu_A(y)\nu(A)\,\mathrm{d}A$. Average ability of all agents who have received the signal y is $\bar{\varphi}(y) := E[\varphi(\tilde{A})|y] = \int_{\mathcal{A}} \varphi(A)\nu_y(A)\,\mathrm{d}A$, where $\nu_y(A)$ denotes the density of the conditional distribution of A given the signal y.

In our model both the signals and the investments made by individuals in their education are publicly observable. We assume throughout the paper that the Monotone Likelihood Ratio Property (MLRP) holds, i.e., the signals are ordered in such a way that $y' \geqslant y$ implies that the posterior distribution conditional on y' dominates the posterior distribution conditional on y in the first-degree stochastic dominance. In this sense, higher signals are ‘good news’ [see, Milgrom (1981)].

Each young individual needs a loan in order to finance his investment in education. The terms of repayment are subject to government intervention. We shall consider three different forms of government intervention in the market for education loans:

1. Regime I (Unrestricted Access to Credit Markets): Under this regime the government guarantees each student unrestricted access to credit markets for funds needed to finance higher education. The government also guarantees enforcement of debt collection.

2. Regime II (Full Insurance of Loans): Under this regime the terms of repayment of a loan are linked to the realization of an individual's future income (hence, linked to the realization of his human capital). This insurance arrangement pools the risks of *all* young agents who choose to invest in education. The governmental intervention includes releasing information about individual incomes, as well as guaranteeing the collection of debt.

3. Regime III (Restricted Insurance of Loans): Again, the terms of repayment are linked to random individual future incomes. Yet, the insurance arrangement pools the risks within each signal group (group of agents who have received the same signal) separately.

We shall study these three financing regimes separately, assuming that the *same* regime applies to all agents. In particular, students cannot choose between a loan in the credit market with uncontingent terms of repayment (Regime I) and a loan with contingent repayment (Regimes II or III). This assumption seems reasonable because the implementation of any regime requires some government intervention.[3]

[3]This holds true even for Regime I. Given the evidence about borrowing constraints that students face in the financial markets (see, e.g., Galor and Zeira (1993)), some intervention by the government is needed to implement the regime.

Hence, the regimes do not emerge, and compete against each other, endogenously. Rather, they should be viewed as political choice variables. The implications of those political choice variables for the time path of aggregate human capital and for the intragenerational income distribution will be analyzed below.

The agents are expected utility maximizers with von-Neumann Morgenstern lifetime utility function

$$U(c_1, c_2) = u_1(c_1) + u_2(c_2). \tag{3}$$

c_1 and c_2 denote consumption in the second and third period of life, respectively. In his first period of life each agent makes a capital investment in education, but he does not consume. The utility functions $u_i : \mathbb{R}_+ \to \mathbb{R}$, $i = 1, 2$, are strictly increasing and strictly concave.

In each period, competitive firms produce a commodity that can be used for consumption. The firms use physical capital, K, and human capital, H, as production factors. Physical capital fully depreciates in the production process. We describe the production process by an aggregate production function $F(K, L)$, which exhibits constant returns to scale. In his 'working period' each agent i inelastically supplies l units of labor and, hence, his supply of human capital is lh^i. Without loss of generality, we set $l = 1$. The production function has the following properties:

Assumption 2. *$F(K, H)$ is concave, homogeneous of degree 1, and satisfies $F_K > 0$, $F_H > 0, F_{KK} < 0, F_{HH} < 0$.*

Physical capital is internationally mobile while human capital is assumed to be immobile.[4] This implies that the interest rate, $\bar{r}_t$, is exogenously given at each date

[4] This assumption is in line with some implications of the globalization process that we have witnessed in recent decades. While globalization has increased the international mobility of physical capital tremendously, movements of labor across international borders are still the exception rather than the rule.

(small country assumption). Having assumed full depreciation of physical capital in each period, marginal productivity of aggregate physical capital, K_t equals $1 + \bar{r}_t$. Thus, given the aggregate stock of human capital at date t, H_t, the stock of physical capital, K_t, adjusts such that

$$R_t := 1 + \bar{r}_t = F_K(K_t, H_t) \qquad t = 1, 2, 3, \cdots \tag{4}$$

is satisfied. This implies by Assumption 1, that K_t/H_t is determined by the international rate of interest $\bar{r}_t$. Hence the wage rate (price of one unit of human capital), $w_t = F_L(K_t/H_t, 1)$, is also determined once $\bar{r}_t$ is given.

2.1. Financing Regime I. Let us consider the decision problem that each $i \in G_t$ faces under Regime I, given $\bar{r}_t, w_t$, and H_{t-1}. At date $t-1$, when 'young', this individual chooses investment in education, x^i, while his ability is still unknown. The investment decision will be based on the noisy information about the agent's ability that is conveyed by the signal y^i. The investment, x^i, is financed through a standard loan contract which is signed at date $t-1$, and which involves the obligation to pay back $R_t x^i$ in period t.

An optimal decision is taken in two consecutive steps. At date $t-1$, after the signal y^i has been observed, our agent $i \in G_t$ chooses an optimal level of investment in education, x^i, and signs the associated loan contract. When choosing the investment level, the agent perceives his ability to be randomly distributed according to $\nu_{y^i}(\cdot)$. Optimal savings, s^i, are chosen at date t *after* ability, A^i, has been observed. At this time, x^i (which has been chosen at date $t-1$) is predetermined.

For given levels of h^i, x^i, w_t, R_t, and R_{t+1} the optimal saving decision is determined by

$$\max_{s^i} u_1(c_1^i) + u_2(c_2^i) \tag{5}$$

$$\text{s.t. } c_1^i = w_t h^i - R_t x^i - s^i \tag{6}$$

$$c_2^i = R_{t+1} s^i \tag{7}$$

and satisfies the necessary and sufficient first order condition

$$u_1'(w_t h^i - R_t x^i - s^i) = R_{t+1} u_2'(R_{t+1} s^i), \quad \forall A^i. \tag{8}$$

The optimal level of investment in education is determined by (we mark random variables by a $\tilde{}$).

$$\max_{x^i} E\Big[u_1(\tilde{c}_1^i) + u_2(\tilde{c}_2^i)\Big| y^i\Big] \tag{9}$$

$$\text{s.t. } \tilde{c}_1^i = w_t \tilde{h}^i - R_t x^i - \tilde{s}^i \tag{10}$$

$$\tilde{c}_2^i = R_{t+1} \tilde{s}^i, \tag{11}$$

where $\tilde{h}^i$ is given by equation (1) and $\tilde{s}^i$ satisfies equation (8). By the Envelope theorem and the strict concavity of the utility functions, this optimization problem has a unique solution determined by the first order condition

$$E\Big\{\big[w_t \varphi(\tilde{A}) g_1'(x^i, H_{t-1}) - R_t\big] u_1'(\tilde{c}_1^i)\Big| y^i\Big\} = 0. \tag{12}$$

At date $t-1$, the members of G_t differ only by the signals they have received. Therefore, all individuals in the same signal group, $G_t(y)$, choose the same investment level, denoted $x_t(y)$.[5] The *net* income (gross income net of repayment of the loan) in the working period of individuals in $G_t(y)$ is

$$I_t(A, y) = w_t \varphi(A) g\big(x_t(y), H_{t-1}\big) - R_t x_t(y). \tag{13}$$

[5] $x_t(y)$ depends on H_{t-1}. For notational convenience, we have chosen not to include H_{t-1} as an argument of the investment function. We shall apply this convention to all behavioral functions and maintain it when we turn to other financing regimes.

The aggregate stock of human capital at date t can be expressed as

$$H_t = \int_Y \bar{\varphi}(y) g\big(x_t(y), H_{t-1}\big) \mu(y) \, \mathrm{d}y. \tag{14}$$

Using (13) in (8), we may write optimal savings as $s_t\big(I_t(A,y)\big)$. Optimal consumption levels in the second and third periods of life are denoted by $c_t^1(A,y)$ and $c_t^2\big(I_t(A,y)\big)$, respectively. $s_t(I_t)$ and $c_t^2(I_t)$ are increasing functions, and $c_t^1(A,y)$ is increasing in A. Our economy starts at date 0 with given initial stocks of physical capital, K_0, and human capital, H_0 . The dynamic equilibrium describes the time path of factor prices, savings and consumption profiles as well as the evolution of the individual human capital stocks which depend on the investments in education of the young generations.

Definition 1. *Given the international interest rates* (r_t) *and the initial stocks of human and physical capital* H_0 *and* K_0*, a competitive equilibrium consists of a sequence* $\{(c_1^i, c_2^i, x^i, s^i)_{i \in G_t}\}_{t=1}^{\infty}$*, and a sequence of wages* $(w_t)_{t=1}^{\infty}$*, such that:*

(i) *At each date t, given* r_t*,* H_{t-1}*, and* w_t*, the optimum for each* $i \in G_t$ *in problems (5)-(7) and (9)-(11) is given by* (c_1^i, c_2^i, x^i, s^i)*.*

(ii) *The aggregate stocks of human capital,* $H_t, t = 1, 2, \cdots$ *, satisfy (14).*

(iii) *Wage rates are determined by* $w_t = F_L(K_t/H_t, 1)$*,* $t = 1, 2 \cdots$*.*

Our comparative dynamics analysis assumes that competitive equilibria (under various regimes) start from the same initial stocks, K_0, H_0, and compares the allocations along these dynamic paths period by period. The above definition of equilibrium also applies (with minor and obvious modification), if the economy operates under one of the two financing schemes outlined below.

2.2. Financing Regime II. Next we analyze the behavior of young individuals when funds needed to finance investment in higher education take the form of 'insured loans'. Assume that the payback obligation of a loan is linked to an individual's future income: agents with higher incomes (i.e., higher abilities) have higher payback obligations.[6]. Clearly, such loan contracts provide insurance against uncertain income prospects which are due to random ability realizations. We shall consider a risk pooling program of education loans that includes all young individuals of a given generation and which requires no subsidization from the government. In particular, by assumption, the regular credit markets cannot be used for funding educational expenditures. Let $\bar{\varphi} := E_y \bar{\varphi}(y)$. An agent i in G_t who receives a loan to finance investment x^i is obliged to pay back $R_t x^i \frac{\varphi(A^i)}{\bar{\varphi}}$ in his working period, if his ability turns out to be A^i.

Proceeding as in Section 2.1, the necessary and sufficient conditions for optimal savings and investment decisions are

$$u_1'\left(w_t h^i - R_t x^i \frac{\varphi(A^i)}{\bar{\varphi}} - s^i\right) = R_{t+1} u_2'(R_{t+1} s^i), \quad \forall A^i \tag{15}$$

$$\bar{\varphi} g_1'(x^i, H_{t-1}) = \frac{R_t}{w_t}. \tag{16}$$

(16) implies that all individuals will invest the same amount, regardless of the signal they have received, i.e., $x^i = \hat{x}_t \; \forall i \in G_t$. Clearly, $\hat{x}_t$ depends on H_{t-1} and, by our assumptions, it is nondecreasing in H_{t-1}. Due to the 'fair insurance' arrangement provided under financing scheme II, coupled with the risk aversion assumption, the optimal investment in education $\hat{x}_t$ maximizes the expected lifetime income prior to the revelation of the signal; namely, $\hat{x}_t$ solves

$$\max_x E\left\{ w_t \varphi(\tilde{A}) g(x, H_{t-1}) - R_t x \frac{\varphi(\tilde{A})}{\bar{\varphi}} \right\} \tag{17}$$

[6] An example of income-dependent rate of interest on educational loans exists now at the US tax code: all interest payments related to student loans are tax deductible!

and, hence, it is independent of y.

Net income in the working period of an agent in G_t with ability A is given by

$$\hat{I}_t(A) = w_t\varphi(A)g(\hat{x}_t, H_{t-1}) - R_t\hat{x}_t\frac{\varphi(A)}{\bar{\varphi}}, \tag{18}$$

and the aggregate stock of human capital at date t is

$$\hat{H}_t = \bar{\varphi}g(\hat{x}_t, H_{t-1}). \tag{19}$$

Using (18) in (15), we may write optimal savings as $\hat{s}_t\big(\hat{I}_t(A)\big)$.

2.3. Financing Regime III. We finally consider a further class of 'insured' loan contracts which specify different terms of repayment for individuals in different signal groups. Again, the payback obligation of a loan is linked to an agent's future income and, hence, his random ability, but the implied risk pooling is restricted to individuals in a given signal group. An agent i in G_t with signal y^i who receives a loan to finance investment in education x^i is obliged to pay back $R_t x^i \frac{\varphi(A^i)}{\bar{\varphi}(y^i)}$ in his working period, if his income turns out to be determined by A^i. This program of education loans allows risk sharing on fair terms within each signal group, but does not provide risk sharing, or cross-subsidization, among different signal groups.[7] As before, this income-linked loan program does not require any funding from the government: The agency providing the loans pays a gross interest rate R_t in the capital market which is just equal to the rate realized on total loans within each signal group, i.e., $\int_{\mathcal{A}} R_t \frac{\varphi(A)}{\bar{\varphi}(y)}\nu_y(A)\,\mathrm{d}A = R_t$.

The necessary and sufficient conditions for optimal savings and investment deci-

[7]There exist real world examples where private fundings are based on grouping students either by universities (e.g., at Yale, Harvard, etc.) or by fields of career. Lleras (2004, p. 66) argues that such practice is justified because 'grouping students by fields reflects similarity in the risks and the expected returns within the same group'.

sions are

$$u_1'\left(w_t h^i - R_t x^i \frac{\varphi(A^i)}{\bar{\varphi}(y^i)} - s^i\right) = R_{t+1} u_2'(R_{t+1} s^i), \quad \forall A^i \tag{20}$$

$$\bar{\varphi}(y^i) g_1'(x^i, H_{t-1}) = \frac{R_t}{w_t}, \quad \forall y^i. \tag{21}$$

According to (21), optimal investment in education of agents in the signal group $G_t(y)$ depends on the signal only via the term $\bar{\varphi}(y)$. We may, therefore, express individual investment as $\check{x}_t\big(\bar{\varphi}(y)\big)$. Again, our notation suppresses the dependence of investment on H_{t-1}. Since $g(x, H)$ is concave in x and since $\bar{\varphi}(y)$ is increasing in y (due to MLRP), equation (21) implies

Lemma 1. *Optimal investment in education under Financing Regime III, $\check{x}_t(\cdot)$, is increasing in the signal y, and non-decreasing in H_{t-1}.*

Thus, good news (higher signal) stimulates investment in education. Net income in the working period of an agent in G_t with ability A is given by

$$\check{I}_t\big(A, \bar{\varphi}(y)\big) = w_t \varphi(A) g\big(\check{x}_t(\bar{\varphi}(y)), H_{t-1}\big) - R_t \check{x}_t(\bar{\varphi}(y)) \frac{\varphi(A)}{\bar{\varphi}(y)}, \tag{22}$$

and the aggregate stock of human capital at date t is

$$\check{H}_t = \int_Y \bar{\varphi}(y) g\big(\check{x}_t(\bar{\varphi}(y)), H_{t-1}\big) \mu(y)\, \mathrm{d}y. \tag{23}$$

Using (22) in (20), we may write optimal savings as $\check{s}_t\big(\check{I}_t(A, \bar{\varphi}(y))\big)$.

3. Human Capital Accumulation

In this section we compare the three financing schemes of educational investment with regard to their implications for the equilibrium accumulation of human capital. The financing schemes involve different degrees of risk sharing in the economy. It is a well known result from the literature on decisions under uncertainty that an investor may be inclined to invest more funds into a risky project, if, due to effective risk sharing arrangements, he can insure part of the project risk on easy terms. On the other hand, more effective insurance mechanisms also have the potential of destroying incentives for some agents to properly invest in education. The role of the various financing schemes for investment in education and human capital accumulation therefore deserves close scrutiny.

Proposition 1. *In equilibrium,*

(i) *each agent chooses higher investment in education under Financing Regime III compared to Financing Regime I:* $x_t(y) \leq \check{x}_t\big(\bar{\varphi}(y)\big)$ *for all signals* y*;*

(ii) *the stock of human capital under Financing Regime III is larger than that under Financing Regime I:* $\check{H}_t \geq H_t \quad$ *for* $t = 1, 2, \ldots$

This result demonstrates that the risk pooling, which is restricted to signal groups, enhances investment in education and the formation of human capital, compared to non-insured funding via credit markets.

Proof: (i) Under scheme I, individuals have access to loans provided by the banks at the market interest rates R_t. For each given y and fixed H_{t-1} we have,

$$\text{Cov}\left[\big(\varphi(\tilde{A})\big|y\big), u_1'\big(c_t^1(\tilde{A}, y)\big)\right] \leq 0. \tag{24}$$

The covariance in (24) is negative, because $c_t^1(A, y)$ and $\varphi(A)$ are both increasing in A. From equation (12) and equation (24) we derive $E\big[w_t g_1'\big(x_t(y), H_{t-1}\big)\varphi(\tilde{A}) - R_t\big|y\big] \geq 0$ which implies

$$g_1'\big(x_t(y), H_{t-1}\big)\bar{\varphi}(y) \geq \frac{R_t}{w_t}\,. \tag{25}$$

Combining (21) and (25), and making use of the concavity of $g(x, H)$ in x, we conclude that $x_t(y) \leq \check{x}_t\big(\bar{\varphi}(y)\big)$.

(ii) The proof is by induction over time periods $t = 1, 2, \cdots$. Since K_0, H_0 are given at the outset, part (i) implies $\check{H}_1 \geq H_1$. Assume $\check{H}_{t'} \geq H_{t'}$ for all $t' \leq t$. Since, by assumption, $g_1'(x, H)$ is non-decreasing in H, $g_1'\big(x_{t+1}(y), \check{H}_t\big)\bar{\varphi}(y) \geq g_1'\big(x_{t+1}(y), H_t\big)\bar{\varphi}(y) \geq \frac{R_{t+1}}{w_{t+1}}$ and $g_1'\big(\check{x}_{t+1}\big(\bar{\varphi}(y)\big), \check{H}_t\big)\bar{\varphi}(y) = \frac{R_{t+1}}{w_{t+1}}$ are satisfied. Thus, $x_{t+1}(y) \leq \check{x}_{t+1}\big(\bar{\varphi}(y)\big)$ holds for each individual in generation G_{t+1} with signal y. Integrating over all signals yields $H_{t+1} \leq \check{H}_{t+1}$. ∎

According to Proposition 1, risk sharing on conditionally fair terms stimulates individual investments in education in all signal groups and, thereby, raises the stocks of human capital at all dates. Based on this finding, we are led to speculate that risk sharing on *unconditionally* fair terms may provide even stronger incentives for investment in education and human capital formation. As it turns out, this presumption is misleading. While average investment in education may (but need not) be higher under Regime II than under Regime III, the latter regime always generates higher levels of aggregate human capital. To derive these results, we introduce the concepts of 'moderately decreasing concavity' and 'strongly decreasing concavity'. Let

$$\hat{K}(x, H) := -\frac{g_{11}''(x, H)}{\big(g_1'(x, H)\big)^2} \qquad \big[= K(x, H)/g_1'(x, H)\big]. \tag{26}$$

$K(\cdot)$ and $\hat{K}(\cdot)$ are both (different) measures of concavity w.r.t. to x for the accumulation function $g(\cdot)$.

Definition 2. *Given the restrictions formulated in Assumption 1, the accumulation function $g(x, H)$ exhibits*

(i) *moderately decreasing concavity w.r.t. x, if $\hat{K}(x, H)$ is increasing in x.*

(ii) *strongly decreasing concavity w.r.t. x, if $\hat{K}(x,H)$ is decreasing in x.*

Note that 'moderately decreasing concavity' and 'strongly decreasing concavity' are mutually exclusive properties and that by the definition of $K(x,H)$ and $\hat{K}(x,H)$ the former property is implied by the latter one.

Let aggregate investment in education at time t under Regime I be $X_t := E\big[x_t(\tilde{y})\big]$. For regimes II and III, aggregate investments $\hat{X}_t$ and $\check{X}_t$ are defined analogously.

Proposition 2. *Aggregate investment in education is higher (lower) under Regime II than under Regime III, if the human capital accumulation function $g(\cdot)$ exhibits moderately (strongly) decreasing concavity.*

Proof: Differentiating (21), we obtain

$$\frac{\partial \check{x}_t\big(\bar{\varphi}(y)\big)}{\partial \bar{\varphi}(y)} = \frac{w_t}{\hat{K}\big(\check{x}(\cdot),H\big)R_t} \, . \tag{27}$$

Since $\check{x}_t(\cdot)$ is increasing in $\bar{\varphi}(y)$ according to (21), $\check{x}_t(\cdot)$ is concave (convex) in $\bar{\varphi}(y)$ if $g(\cdot)$ exhibits moderately (strongly) decreasing concavity. Now, (16) and (21) imply $\hat{x}_t = \check{x}_t(\bar{\varphi})$. Under moderately (strongly) decreasing concavity we may therefore conclude

$$\check{X}_t = E\big[\check{x}_t\big(\bar{\varphi}(\tilde{y})\big)\big] \overset{(\geq)}{\leq} \check{x}_t(\bar{\varphi}) = \hat{x}_t = \hat{X}_t. \qquad \blacksquare$$

Warning: This proof implicitly assumes that $g(\cdot)$ (and hence $x(\cdot)$) does not depend on H.

This result is quite surprising because the better talented agents subsidize the less talented ones more heavily under Regime II, where *all* risks are pooled, than under Regime III, where risks are pooled conditional on the signals. One might conjecture, therefore, that the incentives to invest in education are always stronger under the latter regime. In view of Proposition 2, this intuition is misleading, if the accumulation function exhibits moderately decreasing concavity. Such curvature implies (as compared to the case of strongly decreasing concavity) that the marginal return to investment decreases more rapidly. As a consequence, investment is concave, i.e., $x(\cdot)$ responds increasingly less sensitive to higher signals. In other words, if the marginal return to investment declines very fast, then it is not worthwhile to increase x a lot in response to a higher signal. Yet, if the investment function is concave, then investment chosen at the average signal (which is the investment level under Regime II) is higher than average investment under Regime III.

Surprisingly, according to our next proposition, the financing regimes II and III can unambiguously be ranked with regard to their impact on human capital formation. Thus, in general, higher investment in education is neither necessary nor sufficient for higher economic growth.

Proposition 3. *Assume that (A1)-(A2) hold. The equilibrium aggregate human capital levels under Regime III are higher than those under Regime II at all dates:* $\check{H}_t \geq \hat{H}_t$*, for all* t*.*

Proof: The proof consists of two steps.

(i) Let $\bar{h}(z, H_{t-1}) := zg\big(\check{x}_t(z), H_{t-1}\big)$. In a first step we show that $\bar{h}(z, H_{t-1})$ is convex in z. Differentiating $\bar{h}(\cdot)$ with respect to z and using equation (21), we get

$$\bar{h}''_{11}(z, H_{t-1}) = \frac{R_t \check{x}'_t(z)}{w_t z} \left[1 + \frac{\check{x}''_t(z) z}{\check{x}'_t(z)}\right]. \tag{28}$$

From (21) we calculate the elasticity of the investment function as

$$\frac{\check{x}''_t(z) z}{\check{x}'_t(z)} = -\left(1 + \frac{K'_1\big(\check{x}_t(z), H_{t-1}\big)}{\big[K\big(\check{x}_t(z), H_{t-1}\big)\big]^2}\right). \tag{29}$$

Combining (28) and (29) we obtain

$$\bar{h}''_{11}(z, H_{t-1}) = -\frac{K'_1\big(\check{x}_t(z), H_{t-1}\big)/z}{\big[K\big(\check{x}_t(z), H_{t-1}\big)\big]^2 \hat{K}\big(\check{x}_t(z), H_{t-1}\big)}\,.$$

By Assumption 1, $K'_1(\cdot)$ is non-positive and, hence, $\bar{h}(\cdot)$ is convex in z.

(ii) Now we can prove the claim of the proposition by an induction argument. Assume $\check{H}_{t'-1} \geq \hat{H}_{t'-1}$ for $t' \leq t$. We conclude that

$$\check{H}_t = E\big[\bar{h}\big(\bar{\varphi}(\tilde{y}), \check{H}_{t-1}\big)\big] \geq \bar{h}(\bar{\varphi}, \check{H}_{t-1}) = \bar{\varphi} g\big(\check{x}_t(\bar{\varphi}), \check{H}_{t-1}\big) \geq \bar{\varphi} g(\hat{x}_t, \hat{H}_{t-1}) = \hat{H}_t\,,$$

where the first inequality follows from step (i), and the second inequality follows from the induction hypothesis in conjunction with Lemma 1. ■

Thus, the Financing Regime III is more efficient than Regime II in terms of generating economic growth. Propositions 2 and 3 together imply that higher human capital formation does not necessarily require higher aggregate investment in education. Since marginal returns to investment depend on individual abilities, the distribution of individual investments across agents with different abilities affects the formation of human capital in the economy. In particular, a financing regime that encourages investments of highly talented agents and discourages investments of poorly talented agents may achieve high levels of aggregate human capital with relatively low levels of aggregate investment in education. In fact, this happens under moderately decreasing concavity of the accumulation function, when we switch from Regime II to Regime III. Under Regime II, investment in education is high but uncorrelated to individual ability. Under Regime III, by contrast, the better talented agents tend to invest more aggressively than the poorly talented agents. Since individual investments and abilities are better aligned under Regime III than under Regime II, aggregate human capital levels are higher even though the economy as a whole invests less in education.

3.1. Special Cases. In order to illustrate the results in this section we focus on two classes of accumulation functions. The first class is the family of CRRA functions, and the second class is the family of CARA functions.

Case 1: Let $g(x, H) : \mathbb{R}^2_+ \to \mathbb{R}$ belong to the CRRA family, i.e.,

$$g(x, H) = \frac{x^{1-\gamma}}{1-\gamma} H \ , \quad 0 < \gamma < 1. \tag{30}$$

Straightforward calculation shows that $K_1'(x, H) \leq 0, \ \forall x, H$, i.e., the accumulation functions exhibit decreasing concavity. Furthermore, $\hat{K}(x, H) = \gamma x^{\gamma-1}/H$ is strictly decreasing in x, hence, the accumulation function exhibits strongly decreasing concavity. Based on our earlier results we conclude:

Corollary 1. *If $g(x, H)$ belongs to the class of CRRA functions in (30), then*

(i) *aggregate investment is higher under Regime III than under Regime II: $\check{X}_t \geq \hat{X}_t, \ \forall t$;*

(ii) *the stock of human capital is higher under Regime III than under Regime II: $\check{H}_t \geq \hat{H}_t, \ \forall t$.*

Case 2: Let $g(x, H)$ belong to the CARA family, i.e.,

$$g(x, H) = (1 - e^{-\gamma x})H, \quad \gamma > 0. \tag{31}$$

In this case, $\hat{K}(x, H) = e^{\gamma x}/H$, $K(x) = \gamma$ and, hence, $g(x, H)$ exhibits moderately decreasing concavity. Our earlier results then imply

Corollary 2. *If $g(x, H)$ belongs to the class of CARA functions in (31), then*

(i) *aggregate investment is lower under Regime III than under Regime II: $\check{X}_t \leq \hat{X}_t$, $\forall t$;*

(ii) *the stock of human capital is higher under Regime III than under Regime II: $\check{H}_t \geq \hat{H}_t$, $\forall t$.*

4. Welfare Implications

Our welfare analysis of the various financing regimes will be based on an ex-ante welfare concept. Note that all agents of the same generation are identical *ex ante*, i.e., before their signals have realized. We therefore define economic welfare, W_t, of generation G_t as the ex-ante expected utility of members of G_t. A financing regime j will be ranked higher than a financing regime k ($j, k = I, II, III$) if *all* generations attain higher welfare under Regime j than under Regime k.

Welfare of generation G_t under Regime I is defined by

$$W_t := \int_Y V_t(y, H_{t-1})\mu(y)\,\mathrm{d}y \tag{32}$$

where

$$V_t(y, H_{t-1}) := E\Big[u_1\big(c_t^1(\tilde{A}, y)\big) + u_2\big(c_t^2\big(I_t(\tilde{A}, y)\big)\big)\Big]\,. \tag{33}$$

$V_t(y, H_{t-1})$, the value function for generation G_t, represents the conditional expected utility of a member of G_t with signal y. Since $g(x, H_{t-1})$ is increasing in H_{t-1}, the value function is also increasing in H_{t-1}. The value functions and welfare levels of generation G_t under regimes II and III, $\hat{V}_t(\bar{\varphi}, \hat{H}_{t-1}), \check{V}_t\big(\bar{\varphi}(y), \check{H}_{t-1}\big)$ and $\hat{W}_t, \check{W}_t$, are defined symmetrically. We say that, e.g., welfare is higher under Regime III than under Regime II, if $\check{W}_t \geq \hat{W}_t$ holds for all $t \geq 1$.

Proposition 4. *In equilibrium, economic welfare is higher under Regime III than under Regime I.*

Thus, under any political voting process, if it were to be conducted prior to the revelation of signals, the arrangement of Regime III, which provides conditionally insured financing of private investment in education, will prevail against a regime of pure credit markets.

Proof: We show that $\check{V}_t\big(\bar{\varphi}(y), \check{H}_{t-1}\big) \geq V_t\big(y, H_{t-1}\big)$ holds for all y and any fixed t, from which the claim in the proposition follows immediately. From Proposition 1 we know that $\check{H}_{t-1} \geq H_{t-1}$. Therefore, since $\check{V}(\cdot)$ is increasing in the second argument, it is sufficient to show that $\check{V}_t\big(\bar{\varphi}(y), H_{t-1}\big) \geq V_t(y, H_{t-1})$ $\forall y$ is satisfied. Optimal consumption decisions under Regime I are given by

$$c_t^1(A, y) = \big[w_t g\big(x_t(y), H_{t-1}\big)\varphi(A) - s_t\big(I_t(A, y)\big)\big] - R_t x_t(y) \tag{34}$$

$$c_t^2\big(I_t(A, y)\big) = R_{t+1} s_t\big(I_t(A, y)\big), \tag{35}$$

where net income $I_t(\cdot)$ has been defined in (13). The value function is

$$V_t(y, H_{t-1}) = E\Big\{u_1\big(c_t^1(\tilde{A}, y)\big) + u_2\big(c_t^2\big(I_t(\tilde{A}, y)\big)\big)\big|y\Big\}.$$

If we set $\check{H}_{t-1} = H_{t-1}$ (as argued above) and denote by $\bar{s}_t(y) := E\big[s_t\big(I_t(\tilde{A}, y)\big)\big]$ average savings conditional on the signal y, then under Regime III the following -allocation is admissable (but not necessarily optimal):

$$
\begin{aligned}
\breve{x}_t(y) &= x_t(y) \\
\breve{s}_t(A,y) &= s_t\big(I_t(A,y)\big)\left[1-\frac{R_t x_t(y)}{w_t\bar{\varphi}(y)g\big(x_t(y),H_{t-1}\big)}\right]+R_t x_t(y)\frac{\bar{s}_t(y)}{w_t\bar{\varphi}(y)g\big(x_t(y),H_{t-1}\big)} \\
\breve{c}_t^1(A,y) &= \left[1-\frac{R_t x_t(y)}{w_t\bar{\varphi}(y)g\big(x_t(y),H_{t-1}\big)}\right]\big[w_t g\big(x_t(y),H_{t-1}\big)\varphi(A)-s_t\big(I_t(A,y)\big)\big] \\
&- R_t x_t(y)\frac{\bar{s}_t(y)}{w_t g\big(x_t(y),H_{t-1}\big)\bar{\varphi}(y)} \qquad (36) \\
\breve{c}_t^2(A,y) &= R_{t+1}\breve{s}_t(A,y)
\end{aligned}
$$

To complete the proof we show that the $\breve{}$-decision leads to higher expected utility conditional on y than the optimal decision under Regime 1. From (36) and (34) it is immediate that $E\{\breve{c}_t^1(\tilde{A},y)|y\} = E\{c_t^1(\tilde{A},y)|y\}$. Also, $\big[w_t g\big(x_t(y),H_{t-1}\big)\varphi(A) - s_t\big(I_t(A,y)\big)\big]$ is increasing in A (see equation (8)). Thus, $c_t^1(\tilde{A},y)$ differs from $\breve{c}_t^1(\tilde{A},y)$ by a mean preserving spread which implies $E\big\{u_1\big(\breve{c}_t^1(\tilde{A},y)\big)\big|y\big\} \geq E\big\{u_1\big(c_t^1(\tilde{A},y)\big)\big|y\big\}$. Similarly, $E\big\{u_2\big(\breve{c}_t^2(\tilde{A},y)\big)\big|y\big\} \geq E\big\{u_2\big(c_t^2(I_t\big(\tilde{A},y)\big)\big|y\big\}$ because $s_t\big(I_t(\tilde{A},y)\big)$ is a mean preserving spread of $\breve{s}_t(\tilde{A},y)$. Thus we have shown that $\check{V}_t\big(\bar{\varphi}(y),H_{t-1}\big) \geq V_t(y,H_{t-1})$. ■

To ease notation and in order to facilitate some technical matters, the subsequent analysis will be based on a more restricted form of the accumulation function $g(\cdot)$. Specifically, we assume that $g(\cdot)$ depends only on investment in education, but not on the human capital stock of the previous generation. Thus, we write $g(x)$ instead of $g(x,H)$. The main implication of this simplification is that optimal individual investment in education no longer depends on the human capital stock of the earlier generation.

The next proposition compares economic welfare under regimes II and III. As it turns out, the welfare properties of these two regimes depend critically on the nature of the accumulation process of human capital.

Proposition 5. *If the human capital accumulation function $g(x)$ exhibits moderately decreasing concavity, the economy attains higher welfare under Regime II than under Regime III.*

Proof: The value functions under regimes III and II are given by:

$$\check{V}_t\big(\bar{\varphi}(y)\big) \;=\; E\left\{u_1\left(w_t\varphi(\tilde{A})g\big(\check{x}_t\big(\bar{\varphi}(y)\big)\big) - R_t\frac{\check{x}_t\big(\bar{\varphi}(y)\big)}{\bar{\varphi}(y)}\varphi(\tilde{A}) - \check{s}_t(\cdot)\right) + u_2\big(R_{t+1}\check{s}_t(\cdot)\big)\right\}$$

$$\hat{V}(\bar{\varphi}) \;=\; E\left\{u_1\left(w_t\varphi(\tilde{A})g\big(\hat{x}_t(\bar{\varphi})\big) - R_t\frac{\hat{x}_t(\bar{\varphi})}{\bar{\varphi}}\varphi(\tilde{A}) - \hat{s}_t(\cdot)\right) + u_2\big(R_{t+1}\hat{s}_t(\cdot)\big)\right\}.$$

In addition, $\check{x}_t(\bar{\varphi}) = \hat{x}_t(\bar{\varphi})$ and $\check{s}_t(\bar{\varphi}, A) = \hat{s}_t(\bar{\varphi}, A)$, for all A, follow from the first order conditions (15),(16),(20),(21). Therefore, $\check{V}_t(\bar{\varphi}) = \hat{V}_t(\bar{\varphi})$ holds. Also, by definition we have $\bar{\varphi} = E[\bar{\varphi}(\tilde{y})]$. Below we show that under the assumptions of the proposition $\check{V}_t(\cdot)$ is a concave function of $\bar{\varphi}(y)$. Hence, the claim in the proposition follows from

$$\hat{W}_t = \hat{V}_t(\bar{\varphi}) = \check{V}_t(\bar{\varphi}) = \check{V}_t\big(E\big[\bar{\varphi}(\tilde{y})\big]\big) \geq E\big[\check{V}_t\big(\bar{\varphi}(\tilde{y})\big)\big] = \check{W}_t\,.$$

Thus, it remains to verify that $\check{V}_t\big(\bar{\varphi}(y)\big)$ is a concave function of $\bar{\varphi}(y)$. Differentiating $\check{V}_t\big(\bar{\varphi}(y)\big)$ with respect to $\bar{\varphi}(y)$ and using the Envelope theorem we obtain

$$\check{V}_t'\big(\bar{\varphi}(y)\big) = E\left\{u_1'\big(\check{c}_t^1\big(\tilde{A}, \bar{\varphi}(y)\big)\big)R_t\frac{\check{x}_t\big(\bar{\varphi}(y)\big)}{\big(\bar{\varphi}(y)\big)^2}\varphi(\tilde{A})\right\}. \tag{37}$$

Optimal consumption in the first period $\check{c}_t^1$ is increasing in $\bar{\varphi}(y)$, and so $u_1'\big((\check{c}_t^1(\cdot)\big)$ is decreasing in $\bar{\varphi}(y)$. Furthermore, since $g(x)$ exhibits moderately decreasing concavity,

$\check{x}_t\big(\bar{\varphi}(y)\big)$ is concave (see the proof of Prop. 2) and, hence, $\check{x}_t\big(\bar{\varphi}(y)\big)/\bar{\varphi}(y)$ is decreasing [note from equation (21) that $\lim_{\varphi\to 0}\check{x}_t(\varphi) = 0$]. This implies that $\check{V}_t'\big(\bar{\varphi}(y)\big)$ is decreasing and, hence, the value function under Regime III is concave. ■

The result in Proposition 5 is puzzling at first sight. In Section 3, we have seen that the allocation of investment in education is less efficient under Regime II than under Regime III, if $g(\cdot)$ exhibits moderately decreasing concavity: the stocks of human capital are lower while, at the same time, aggregate investment in education is higher under Regime II compared with Regime III. Nevertheless, according to Proposition 5 all agents are better off under Regime II. This puzzling result becomes less paradoxical once we realize that economic welfare depends not only on the efficiency of the human capital accumulation process, but also on the equilibrium risk allocation. Under Regime II individual ability risks are better (in fact, fully) insured, while under Regime III investment is more efficiently transformed into human capital. Proposition 5 tells us that the former effect is dominant in terms of economic welfare, if $g(\cdot)$ exhibits moderately decreasing concavity.

If $g(\cdot)$ exhibits *strongly* decreasing concavity, then economic welfare may be higher under Regime III than under Regime II. We illustrate this fact for the class of CRRA accumulation functions that has been discussed in Section 3.1.

Proposition 6. *Let* $g(x) := x^{1-\gamma}/(1-\gamma)$, $\gamma \in (0,1)$, *and assume that* u_1 *is from the CRRA family,* $u_1(c) = c^{1-\beta}/(1-\beta)$, $\beta \in (0,1)$. *Then* $\check{W}_t \overset{(\leq)}{\geq} \hat{W}_t\ \forall t$, *whenever* $2-\beta \overset{(\leq)}{\geq} 1/(1-\gamma)$.

Proof: Following the same line of reasoning as in the proof of Proposition 5, we need to show that $\check{V}_t'\big(\bar{\varphi}(y)\big)$ is an increasing (decreasing) function of $\bar{\varphi}(y)$ whenever $2-\beta \overset{(\leq)}{\geq} 1/(1-\gamma)$. Using the assumed functional forms of $g(\cdot)$ and $u_1(\cdot)$ in (37), we get

$$\check{V}_t'\big(\bar{\varphi}(y)\big) = B\big(\bar{\varphi}(y)\big)^{[(2-\beta)(1-\gamma)-1]/\beta} \tag{38}$$

for suitably chosen $B > 0$. Obviously, the RHS of equation (38) is increasing (decreasing) in $\bar{\varphi}(y)$, if $2 - \beta \overset{(\leq)}{\geq} 1/(1-\gamma)$ is satisfied. ■

In Section 3.1 we have shown that all functions from the CRRA family exhibit strongly decreasing concavity. If g is chosen from this family, then the economy attains higher welfare under Regime III than under Regime II, if relative risk aversion, β, is sufficiently small. In particular, for $\beta = 0$ (risk-neutrality) the inequality $2 - \beta \geq 1/(1-\gamma)$ in Proposition 6 is always satisfied, regardless of the production technology. Recall that Regime III leads to a more efficient capital accumulation process while Regime II generates a better risk allocation. Yet, with low risk aversion the utility costs that arise from an unfavorable risk allocation are small and, therefore, Regime II becomes less favorable compared with Regime III from the viewpoint of economic welfare.