Voluntary disclosure, manipulation and real effects*

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Abstract

We study a model in which a manager, who is privately informed about the value of his firm's assets in place, may issue equity to finance a profitable investment opportunity. In contrast to Myers and Majluf (1984) who do not consider any disclosure of information by the firm, we assume that the manager may voluntarily disclose his private information. If he chooses to do so, the manager can manipulate his report at a cost, either to him personally (accrual manipulation) or to the firm (real manipulation).

The model shows that treating managers' disclosure and investment decisions both as endogenous and allowing managers to manipulate their voluntary reports yields qualitatively different predictions than when the disclosure and investment decisions are considered separately and disclosures are assumed to be truthful. The model predicts that the manager's disclosure strategy is sometimes characterized by two distinct non-disclosure intervals (contrary to traditional threshold equilibria of voluntary disclosure models) and that it is the managers with intermediate news who sometimes forego the profitable investment opportunity (in contrast to Myers and Majluf 1984). The model also predicts that (*i*) the underinvestment problem is more prevalent if the return on investment is low; and (*ii*) low-performing firms have (weakly) higher cost of capital than high-performing firms. As such, the paper highlights the importance of considering the interdependencies between firms' disclosure and investment decisions and provides new empirical predictions.

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1 Introduction

Firms' real decisions and disclosure decisions are closely linked. The theoretical literature that studies the relationship between investment and disclosure decisions focuses on mandatory disclosure settings (e.g., Leland and Pyle 1977; Stein 1989). Empirical evidence, however, suggests that a substantial part of public information reaches markets through firms' voluntary disclosures (e.g., Beyer, Cohen, Lys and Walther 2010). In particular, prior to equity offerings, firms tend to increase both the quantity and the quality of their voluntary disclosures (e.g., Lang and Lundholm 1993, 2000; Marquardt and Weidman 1998).

When managers disclose information they can, and often do, manipulate their disclosures at some cost. Moreover, managers decide whether to disclose information and, if so, whether and to what extent to manipulate their disclosure, jointly with their firms' investment strategy. Therefore, it is important to consider management's incentives to issue voluntary reports and to manipulate such reports in a setting in which the firm's investment strategy is also endogenous. However, costly reporting distortions and real effects have been largely ignored in the voluntary disclosure literature.[1] Instead, the voluntary disclosure literature mostly focuses on settings in which disclosures have no real effects and reports are assumed to be truthful/verifiable.[2] To the best of our knowledge, there exists no model of a manager's decision whether to issue a report (voluntary disclosure decision) and, if so, what report to issue (manipulation decision) when the firm's investment strategy is also endogenous (investment decision).

Our model studies the interdependencies between firms' voluntary disclosure decisions and their investment decisions in a setting where managers can manipulate their reports at a cost. Most important, the model shows that the equilibrium characteristics of corporate investment and disclosure strategies are qualitatively different when studied jointly than when studied separately.

We build upon the model of Myers and Majluf (1984) in which an entrepreneur, who is privately informed about the value of his firm's assets in place, may issue equity to finance a profitable investment opportunity. In contrast to Myers and Majluf (1984), where firms lack the ability to communicate any private information to potential investors, we assume that the manager may

[1] As far as we are aware, the only papers that study costly misreporting in a voluntary disclosure setting are Korn (2004) and Einhorn and Ziv (2010). Neither of these papers considers real effects.

[2] There are a few papers that study product market competition in which voluntary disclosure has real effects (Vives 1984, Darrough 1993, Kanodia, Mukherji, Sapra and Venugopalan 2000, Hughes, Kao and Williams 2002, Fischer and Verrecchia 2004). These papers, however, do not consider investment decisions and restrict disclosures to be truthful. As such, the economic trade-offs considered are very different from the ones in our model.

voluntarily disclose his private information. When the manager chooses to voluntarily disclose information, he faces an incentive to manipulate the report in order to boost investors' beliefs about the value of the firm. If investors believe the firm's value is high, the firm's cost of equity capital decreases. Managers can manipulate their reports at a cost, either via accounting manipulation, or via real manipulation. Accounting manipulation is often feasible due to the forward-looking nature of many voluntary disclosures and the inherent flexibility in Generally Accepted Accounting Principles (GAAP).[3] Real manipulation involves making suboptimal operating or investment decisions that improve investors' perception of firm value. For example, managers might: (*i*) overinvest in assets in order to increase investors' perception of profitability; (*ii*) overproduce inventory in order to increase investors' perception of future demand; (*iii*) pay out too high a dividend in order to increase investors' perception of the firm's free cash flow; or (*iv*) "oversell" by stuffing the channels and infringe on future sales in order to increase investors' perception of customer demand for the firm's products.[4] The predictions of our model hold independent of whether the manager uses accounting or real manipulation to influence investors' perception. In the remainder of the introduction, we therefore do not distinguish between the two forms of manipulation.

Our model shows that the manager sometimes withholds his private information, even though there are no costs associated with making a disclosure per se. That is, a partial disclosure equilibrium evolves even though the manager can always issue a report without incurring any costs. The reason partial disclosure occurs is that the manager incurs no costs only if he does not manipulate the report. However, as common in costly signaling models, the manager manipulates his report upwards whenever he makes a disclosure in order to increase investors' perception of the value of his firm's assets in place. This manipulation gives rise to endogenous disclosure costs. When the endogenous disclosure costs become too high, the manager withholds his private information, resulting in a partial disclosure equilibrium.

The model further illustrates that, due to the interaction of investment and disclosure decisions, the manager's investment strategy differs from that discussed in Myers and Majluf (1984). In the Myers and Majluf (1984) model, there is a threshold below which the manager raises capital and

[3] Empirical evidence suggests that managers indeed bias their reports (see for example, Burgstahler and Dichev 1997; Teoh, Welch and Wong 1998a,b; and Ajinkya, Bhojraj and Sengupta 2005).

[4] While most voluntary disclosure models assume that disclosure may not be distorted (e.g., Verrecchia 1983, Dye 1985, Jung and Kwon 1988), some disclosure models take the opposite viewpoint and assume that misreporting is costless (cheap-talk models in Newman and Sansing 1993, Gigler 1994, Stocken 2000 and Fischer and Stocken 2001). In this paper, we cover the "middle ground" that we believe is representative of the environment where corporate disclosures take place.

invests and above which he foregoes the profitable investment opportunity. Instead, in our model the manager pursues the profitable investment opportunity when the value of the firm's assets in place is either sufficiently low or high but he may forego the investment opportunity for intermediate values of assets in place. Also, in our model, the disclosure strategy of the manager may differ from the common form of a "threshold" strategy. In our model, the manager sometimes discloses low and high values of assets in place but withholds intermediate and very low values of assets in place. This is due to the interdependencies between the manager's disclosure and investment decision. [5]

Next, we elaborate on the model's predictions about the manager's disclosure and investment strategies. First, the model predicts that if the investment opportunity is sufficiently profitable the manager always (i.e., for any value of the firm's assets in place) raises capital and pursues the investment opportunity. This is intuitive. If the profitability of the investment opportunity is sufficiently high, the expected return on investment outweighs the costs of undervaluation by investors or the costs from manipulating the report. Since sufficiently high profitability leads to the straight-forward case of efficient investment, we focus in the following discussion on the case of less profitable investment opportunities. For such investment opportunities, our model's predictions differ from Myers and Majluf (1984). In particular, our model predicts that a manager does not pursue the investment opportunity when the value of the firm's assets in place is in an intermediate range while the manager pursues the investment opportunity when the value of the firm's assets in place is either low or high.

Second, the model predicts that, for a subset of parameter values, there exist two distinct non-disclosure intervals such that firms with intermediate asset values and firms with sufficiently low asset values do not issue a report. These two non-disclosure intervals, which are separated by an interval of values of assets in place for which firms disclose, raise equity capital and invest, are distinct in terms of investment strategy. Firms with sufficiently low values of assets in place (left non-disclosure interval) raise capital and invest without issuing a report, while firms with intermediate values of assets in place (right non-disclosure interval) do not raise capital, because their costs from being pooled with lower types or from manipulating their report would exceed the expected return on investment. All firms that issue a report also raise capital and invest. For other parameter values, there is only one non-disclosure interval, and the disclosure strategy takes the standard form of a threshold below which the manager withholds his information and above which

[5] As common in the literature (e.g., Riley 1979, Miller and Rock 1985), we study equilibria in which – whenever a manager issues a report – the report fully reveals the manager's private information to investors.

the manager issues a report. For the latter subset of parameter values, the investment strategy takes one of the following two forms: Either (*i*) the firm raises capital and invests for all asset values; or (*ii*) firms with low and high assets values raise capital and invest while firms with intermediate asset values do not raise capital and consequently do not invest.

This shows that our model predicts a different equilibrium disclosure strategy than most standard disclosure threshold equilibria. The reason for the difference lies in the fact that in our model, the endogenous disclosure costs differ from the commonly assumed constant (exogenous) disclosure costs. In equilibrium, the endogenous manipulation and the manipulation costs turn out to be highest for intermediate values of assets in place. The manipulation costs of firms with intermediate values of assets in place would therefore exceed the expected return on investment if the investment opportunity is not very profitable. As a result, the manager might opt to withhold his information and to forego the investment opportunity.

At first, it might seem surprising that in equilibrium the manager biases his report upwards more when the value of assets in place is intermediate than when the value of assets in place is high. Instead, one might expect the manipulation by the manager to increase monotonically in his type – similar to standard signaling models in which the sender's payoff depends linearly on the receiver's perception of his type (e.g., Riley 1979, Miller and Rock 1985). In contrast, when equity is issued, the owner/manager's payoff is linear in the ownership fraction he must give up in exchange for the capital that outside investors provide. The ownership fraction the manager is required to give up depends on investors' perception about the value of the firm. The manager's benefit from marginally increasing investors' perception of the value of his firm's assets in place is higher when the value of those assets is lower. The reason is that the manager's benefit from making investors believe that the assets in place are marginally more valuable is greater when the "size of the pie" is smaller.[6] This, together with the standard result that the manager does not manipulate his report when the value of assets in place is lowest (zero), yields the prediction that the manipulation and the manipulation costs are initially increasing and then decreasing in the firm's value of assets in place. Hence, the manipulation and manipulation costs are highest for

[6] To see this, consider a manager whose firm's assets in place are worth x and who issues a report such that investors perceive the value of his firm's assets to be $x+\varepsilon$. If investors perceive the value of the firm's assets in place to be $x+\varepsilon$ they require fraction $\alpha = \frac{I}{x+\varepsilon+I+\mu_r}$ of the firm's ownership shares in exchange for providing capital I where μ_r denotes the expected return on investment. Since the assets are in fact worth x and not $x+\varepsilon$, the actual value of the shares investors obtain is $I\frac{x+I+\mu_r}{x+\varepsilon+I+\mu_r}$ rendering the manager's benefit from issuing a report that mimics a firm with assets worth $x+\varepsilon$ to be $I - I\frac{x+I+\mu_r}{x+\varepsilon+I+\mu_r} = I\frac{\varepsilon}{x+\varepsilon+I+\mu_r}$. Hence, the manager benefits less from mimicking a firm whose assets are marginally more valuable when the actual value of his firm's assets is higher.

intermediate values of assets in place. The fact that the manipulation function that emerges in this paper is different from the manipulation function in standard signaling models illustrates that modeling specific signaling settings and incorporating institutional details – such as financing needs and investment opportunities – can qualitatively alter predictions about properties of the sender's report and signaling costs.

Third, the model predicts that the underinvestment problem is more prevalent, in the sense that the manager foregoes the profitable investment opportunity more often, when the expected return on investment is lower. This is intuitive, since it is less attractive for the manager to raise capital and invest and therefore the manager is less willing to incur manipulation costs from issuing a report.

Fourth, the model predicts that, if investors are risk-averse, firms that voluntarily disclose information prior to raising capital have a lower cost of capital than firms that do not make such disclosures. The reason is that investors face greater uncertainty about the value of the firm's assets in place, and hence, require a higher expected return on their equity investment when the firm does not issue a report. Finally, the model predicts a negative association between firm performance and cost of capital. This association is driven by the fact that in equilibrium, only firms with low values of assets in place raise capital without issuing a report, while firms with higher values of assets in place always issue a report prior to raising capital.

In the equilibrium described above, the manager raised capital without issuing a report for some realizations of asset values. In an extension to the above analysis, we show that there exists an additional equilibrium in which raising capital without issuing a report is never part of the equilibrium strategy. However, such an equilibrium exists only if investors' off-equilibrium beliefs are restricted in a way that they infer that any firm that raises capital without issuing a report has the lowest possible value of assets in place. Under such beliefs, in equilibrium no firm chooses to raise capital without issuing a report. Nevertheless, firms' disclosure strategies differ form the one in the equilibrium described earlier mainly insofar that firms with low values of assets in place issue a report, instead of remaining silent. The investment strategy is qualitatively the same in both equilibria.

The remainder of the paper proceeds as follows: Section 2 provides a brief review of the literature. Section 3 outlines the setting of the model. Section 4 derives and analyzes the equilibrium. Section 5 demonstrates that the equilibrium properties of the manager's disclosure and investment strategy are robust to endogenous real manipulation costs. Section 5 also shows that under certain off-equilibrium beliefs, an equilibrium exists in which bad news is disclosed. Section 6 provides concluding remarks. All proofs are delegated to the Appendix.

2 Literature Review

Our paper studies a model that jointly considers a firm's voluntary disclosure strategy and the underinvestment problem in Myers and Majluf (1984). In Myers and Majluf (1984), a manager who acts on behalf of the existing shareholders decides whether to implement a new profitable investment opportunity. The firm does not have the internal funds; hence, it must raise equity capital to implement the investment. In particular, the manager decides whether to issue a fraction α of the firm's shares to outside investors in exchange for the required investment capital. The actual value of the shares offered to outside investors varies across firms, because firms differ with respect to the value of their assets in place. Whether the manager chooses to issue equity depends on the value of the shares demanded by outside investors relative to the return he expects the new investment to generate. As a result, the manager only issues shares if the value of those shares – or equivalently the value of the firm's assets in place – is relatively low. If the value of his firm's assets is high, the undervaluation of the shares he would have to issue in order to raise the required capital is so severe so that the manager prefers foregoing the investment opportunity instead. As a result, in Myers and Majluf (1984), the manager's equilibrium investment strategy is characterized by a threshold of values of assets in place up to which the manager pursues the investment opportunity and beyond which the manager foregoes it.

We extend the setting in Myers and Majluf (1984) along two dimensions. First, we allow for communication between the manager and outside investors. In particular, we allow the manager to issue a (potentially manipulated) report prior to raising capital. Incorporating a disclosure decision into the Myers and Majluf-setting enables us to study real effects of voluntary disclosure. Prior literature on real effects of voluntary disclosure has focused almost exclusively on the decision of firms to disclose private information about market conditions to their competitors (e.g., Vives 1984, Darrough and Stoughton 1990, Wagenhofer 1990, Feltham and Xie 1992, Darrough 1993, Newman and Sansing 1993, Gigler 1994, Kanodia et al. 2000, Hughes et al. 2002, Fischer and Verrecchia 2004).[7] To the best of our knowledge, the only voluntary disclosure model that considers firms'

[7] For a survey of the literature on accounting disclosure and real effects see Kanodia (2006).

investment decisions is Goex and Wagenhofer (2009), who derive the optimal disclosure rule that firms commit to when they want to raise debt capital for investment purposes. Our model differs from Goex and Wagenhofer (2009) insofar as we do not assume that firms can credibly commit ex-ante to a certain disclosure policy. Second, we allow the manager to make not only an investment decision after the opening of the equity market, but also to make a real decision prior to the opening of the equity market. This allows the manager to manipulate investors' inferences about the firm's value through sub-optimal real decisions.

Moreover, with the exception of Newman and Sansing (1993) and Gigler (1994), who both consider cheap-talk settings, none of the above-referenced papers allows managers to bias their reports. In order to study the empirical phenomenon of bias in corporate disclosures,[8] We relax the assumption of truthful reporting and study managers' propensity to manipulate voluntary reports prior to raising equity capital. Since empirical evidence suggests that managers bias their reports to a lesser extent if monitoring mechanisms are more effective, we assume that biasing reports is costly to the manager.[9] Costly misreporting has been widely studied in mandatory disclosure models.[10] However, in voluntary disclosure settings, costly misreporting has gotten little attention in the literature. To the best of our knowledge, the only papers that study costly misreporting in a voluntary disclosure setting are Korn (2004) and a recent working paper by Einhorn and Ziv (2010). Korn (2004) and Einhorn and Ziv (2010) assume that the manager maximizes the firm's share price net of his costs from biasing the report and do not consider any real effects. Both papers predict that, in equilibrium, the manager issues a voluntary report when his private information is sufficiently favorable and does not make a disclosure otherwise. In that sense, these models yield predictions similar to voluntary disclosure models in which managers are assumed to report truthfully and disclosure costs are fixed and exogenous (e.g., Jovanovic 1982; Verrecchia 1983). Our model differs from Korn (2004) and Einhorn and Ziv (2010) in that we consider the manager's voluntary disclosure decision jointly with the firm's investment decision. In contrast to Korn (2004) and Einhorn and Ziv (2010), we find that the manager's disclosure strategy is not necessarily characterized by a single threshold but may feature two distinct non-disclosure intervals. Overall,

[8] For a review of the earnings management literature see Dechow, Ge and Schrand (2010).

[9] For instance, Ajinkya, Bhojrai and Sengupta (2005) find that managers issue less optimistic earnings forecasts in firms with more outside directors and greater institutional ownership, suggesting that more effective monitoring will limit managers' propensity to bias their reports.

[10] Models of costly reporting distortions include models of earnings management by Stein (1989); Fischer and Verrecchia (2000); Sankar and Subramanyam (2001); Dye and Sridhar (2004); Guttman, Kadan and Kandel (2006) and Beyer (2009).

we believe that the paper contributes to the literature by characterizing the interdependencies between firms' joint decisions (1) whether to disclose, (2) given disclosure, whether and to what extent to manipulate the report and (3) whether to raise equity capital for investment purposes.

3 Model setup

An individual (called the "manager" in what follows) owns a firm with assets in place and with a new investment opportunity that requires external financing of $\$I$.[11] The net return of this investment opportunity is the realization of a random variable $\tilde{r}$ with expected value $\mu_r > 0$. The manager privately learns the value of his firm's assets in place, x.[12] Next, the manager simultaneously decides whether to voluntarily issue a report on his firm's asset value and whether to raise equity capital from outside investors to finance the new investment opportunity. Both the current assets in place and the new investment opportunity (if carried out) generate their final cash flows at the end of the game. If the manager chooses to issue a report, he may manipulate the reported number. Manipulation of the report can be carried out by either: (*i*) "real manipulation" – making suboptimal investment or operating decisions prior to the issuance of the report and truthfully reporting the resulting numbers; or (*ii*) "accounting manipulation" – exploiting of the forward looking nature of many voluntary disclosures to manipulate the reported number. When the manager engages in real manipulation, the value of the firm decreases due to suboptimal real decisions, resulting in endogenous manipulation costs. When the manager engages in accounting manipulation he spends additional time and effort generating supportive evidence for the report. This imposes a personal cost on the manager. Moreover, the manager may be exposed to psychic costs associated with manipulating the report (e.g., Fischer and Verrecchia, 2000; Ewert and Wagenhofer, 2005; Guttman et al., 2006). For simplicity of disposition, we start by analyzing the case of accounting manipulation and defer the analysis of the case of real manipulation to Section 5.1. We show that the model's predictions about the manager's disclosure and investment strategy are

[11] Equivalently, we may assume that rather than owning the firm, the manager is hired by the firm's current owners and that there are no moral hazard problems between the manager and the current owners.

In the conclusion section, we discuss our assumption that the new investment opportunity is carried out within the legal structure of the existent firm and that it is financed with equity capital.

[12] We consider asymmetric information with respect to the value of the firm's assets in place but not with respect to the expected return of the new investment opportunity. We make this assumption, since information asymmetry with respect to the value of the firm's assets in place is necessary and sufficient to obtain the underinvestment problem described in Myers and Majluf (1984). While the underinvestment problem in Myers and Majluf (1984) is robust to the additional information asymmetry with respect to the return on investment, it would add significant complexity in our model. To maintain tractability, we therefore assume that the manager and outside investors are symmetrically informed about the expected return on investment.

qualitatively the same in both settings. We next describe in detail the setting of the model.

The value of the firm's assets in place is a realization of the random variable $\tilde{x}$ which is distributed over $[0, \infty)$ according to the probability density function $f(x) > 0$ for all x.[13] We restrict x to non-negative values based on the rationale that the assets in place have an abandonment option.

At $t = 1$, the manager privately learns the realization of the value of his firm's assets in place. (In the following, we sometimes refer to the value of the firm's assets in place as the manager's "type.")

At $t = 2$, the manager simultaneously decides whether to issue a report to investors and whether to raise capital in the equity market. The investors observe the manager's report, if one has been issued, prior to the opening of the equity market. If the manager decides to issue a report on his firm's asset value, $x_R \in [0, \infty)$, he is not confined to tell the truth and may bias his report. We denote the manager's reporting bias by $b(x) = x_R(x) - x$. If the report differs from the true value of the assets in place, the manager incurs a personal cost of manipulating the report. This cost increases in the difference between the report and the true value of the firm's assets in place, x.[14] In particular, we assume that the manager incurs the cost $g(x_R - x)$ where the cost function $g(\cdot)$ is a well behaved U-shaped function, i.e., it is convex with $g(0) = 0$ and $g'(0) = 0$. The manager may raise equity capital whether or not he issues a report x_R. If the manager decides to raise equity capital, he offers a fraction α of the firm's ownership to investors in return for their investment of capital I with the firm. Investors may accept or reject the offer. We assume that investors are risk-neutral and that they accept the offer when they break even on average.[15] If investors accept the offer, they contribute capital I and the manager pursues the investment opportunity. If investors do not contribute capital, the manager lacks the necessary funds to pursue the investment opportunity.

At $t = 3$, the firm's final cash flows are realized. If the manager did not pursue the investment opportunity, the firm's final cash flows equal x and the manager retains all of it. If the manager raised capital and pursued the investment opportunity, the firm's final cash flows are $x + I + r$, the manager retains fraction $(1 - \alpha)$ and investors are paid fraction α of the final cash flows.

[13] x does not have to refer to the actual future cash flows that the firm's assets in place will generate, but rather may denote the expected value of the firm's assets in place conditional on the manager's private information.

[14] This is a standard assumption in the earnings management literature (e.g., Riley 1979; Stein 1989; Fischer and Verrecchia 2000; Guttman, Kadan, and Kandel 2006; Beyer 2009).

[15] We assume that investors are risk-neutral for simplicity only. If investors were risk-averse the model's predictions would remain qualitatively the same. In Corollaries 3 and 4, we discuss the implications of investors being risk-averse.

In equilibrium, the manager simultaneously decides whether to issue a report, and if so, to what extent to manipulate the report, and whether to raise capital. If the manager decides to raise capital, he rationally anticipates the investor response and chooses the fraction α of the firm's ownership offered to investors in exchange for their investment such that investors break even on average. The fraction α is determined by

$$I = \alpha \left(E \left[\widetilde{x} | \Omega \right] + I + \mu_r \right).$$

where Ω denotes the public information that is available to investors at $t = 2$. All parameters of the model, i.e., the required capital I, the expected return on investment μ_r, the cost function $g(\cdot)$ and the prior distribution of value of assets in place $f(\cdot)$ are common knowledge. Figure 1 summarizes the sequence of events of the model.

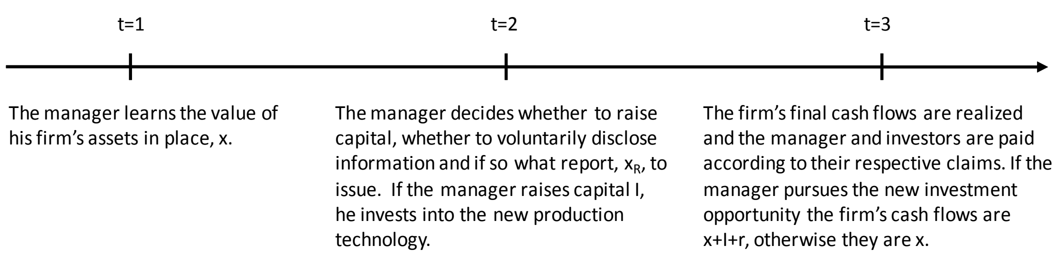

Figure 1: Timeline

In the model, the manager jointly considers his disclosure and investment decisions. The reason is that the manager's disclosure decision depends on his investment decision and vice versa. On the one hand, voluntary disclosure can only be beneficial to the manager should he decide to raise capital. In the absence of equity issuance, investors' perception of the firm's value is irrelevant and, hence, the manager cannot benefit from influencing investors' beliefs. On the other hand, the return of the new investment opportunity to the manager depends on his voluntary disclosure decision, even though the expected profitability of the new investment opportunity is common knowledge. The reason is that the manager's report, or its absence, affects investor perception of the value of the firm's assets in place. Furthermore, it determines the fraction of shares investors require in return for the capital they provide. Because of these interdependencies, it is important to jointly consider firms' voluntary disclosure and investment decisions. The following analysis studies these interdependencies and shows that the manager's decision whether to issue a voluntary report – and if so, how much to manipulate the report – affects and is affected by the manager's investment decision.

4 Analysis

The manager has to decide simultaneously (1) whether to raise capital and (2) whether to issue a report, and if so, to what extent to manipulate the report. Before solving for the equilibrium, in which the manager jointly determines his investment and disclosure decision, we provide two preliminary steps. In Section 4.1, we study the manager's manipulation strategy in a setting where we assume exogenously that the manager issues a report and raises capital. In Section 4.2, we take the manager's disclosure strategy as given and analyze his decision to raise capital and invest. Building on these preliminary results, in Section 4.3, we derive the equilibrium of our model and analyze the manager's joint disclosure and investment decision.

4.1 Mandatory Disclosure and Investment

In this section, we study the extent to which the manager biases his report under the assumption that he always issues a report, raises capital and invests. As is often the case in disclosure games with continuous support, multiple equilibria with pooling reports may evolve (e.g., Guttman et al. 2006). We limit our analysis to a subset of equilibria in which the manager's report allows investors to perfectly infer the manager's private information.

The report issued by the manager affects his payoff in two ways. On the one hand, the report affects investors' beliefs about the value of the firm's assets in place, and those beliefs determine the fraction α of equity that investors require in exchange for providing capital I. On the other hand, the manager incurs disclosure costs whenever his report differs from the true value of assets in place. The manager's biasing costs increase as the magnitude of the bias increases. The trade-off between these two factors is reflected in the first order condition of the manager's optimization problem and determines the extent to which the manager biases his report. The following Lemma characterizes the manager's equilibrium bias strategy, $b(x)$.

Lemma 1 *The equilibrium bias in the manager's report is given by the solution to the differential equation*

$$b'(x) = \frac{I}{g'(b(x))(x + I + \mu_r)} - 1, \tag{1}$$

with the boundary condition $b(0) = 0$.

The equilibrium bias $b(x)$ has the following properties: it is continuous, always positive, initially *increasing, obtains a unique maximum and converges to zero as the value of the firm's assets in place goes to infinity.*

Figure 2 illustrates the equilibrium bias, $b(x)$, and the manager's equilibrium report, $x_R(x) = x + b(x)$. The figure is based on a quadratic cost function, $g(b) = \frac{1}{2}b^2$, and the parameter values $I = 1$ and $\mu_r = 0.25$.

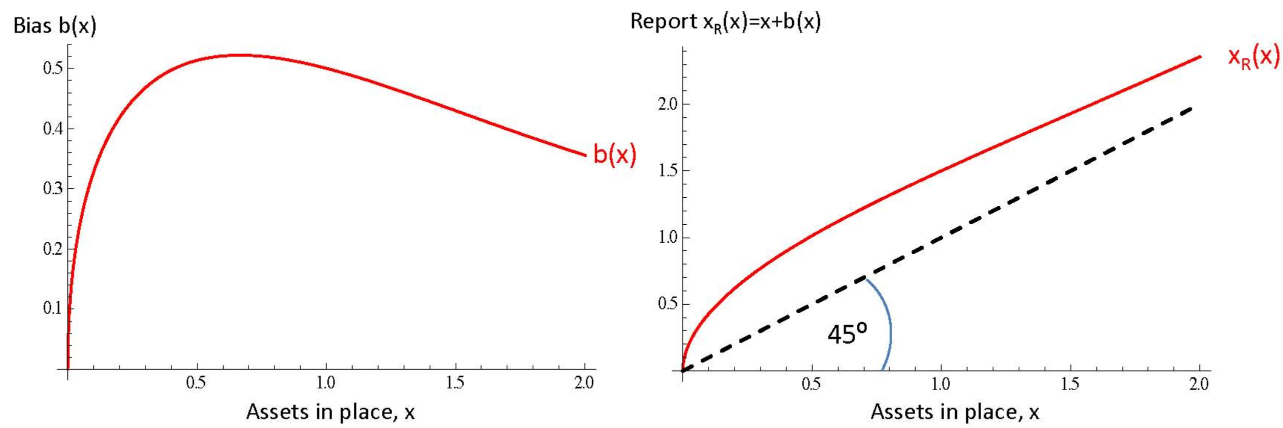

Figure 2: Bias function $b(x)$ and reporting function $x_R(x)$

In equilibrium, the manager biases his report upwards when making a disclosure. The reason is that investors associate higher reports with the firm's assets in place being more valuable. When investors perceive the firm's assets in place to be more valuable, investors require a smaller fraction of the firm's equity in exchange for investing capital I. In turn, the manager can keep a larger fraction of the firm's equity to himself. The extent to which the manager biases his report upwards depends on the benefits and costs associated with reporting a higher value for the assets in place. Three observations jointly explain the shape of the equilibrium bias function as shown in Figure 2.

First, a manager whose firm's assets in place are worth zero does not bias his report. This is intuitive since in equilibrium investors identify him as the lowest type and he is therefore not willing to bear any signaling costs.

Second, the manager's benefit from investors' perception that his firm's assets in place are marginally more valuable than their actual value depends on the actual value of those assets. When the value of the firm's assets in place is lower, the manager benefits more from mimicking a firm whose assets in place are marginally more valuable. The reason is that the effect of making investors believe that the firm's assets in place are marginally more valuable is greater when the "size of the pie" is smaller. To illustrate this further, we consider a manager whose firm's assets in place are worth x and who issues a report such that investors perceive the value of his firm's assets in place to be $\hat{x}$. If investors perceive the value of the firm's assets in place to be $\hat{x}$ they require a fraction $\alpha = \frac{I}{\hat{x}+I+\mu_r}$ of the firm's shares in exchange for providing capital I. Hence, the actual value of the shares investors obtain is $I\frac{x+I+\mu_r}{\hat{x}+I+\mu_r}$. Since investors provide capital I in exchange for those shares, investors overpay by $I - I\frac{x+I+\mu_r}{\hat{x}+I+\mu_r} = I\frac{\hat{x}-x}{\hat{x}+I+\mu_r}$.[16] In turn, the manager's benefit from issuing a report that mimics a firm with assets worth $\hat{x}$ is also $I - I\frac{x+I+\mu_r}{\hat{x}+I+\mu_r}$. The manager's benefit from inducing investors to believe that the firm's assets in place are worth marginally more than their actual value is

$$\lim_{\hat{x}\to x} \frac{\partial}{\partial \hat{x}} \left(I - I\frac{x+I+\mu_r}{\hat{x}+I+\mu_r} \right) = \frac{I}{x+I+\mu_r},$$

which decreases in the actual value of the firm's assets in place, x. Hence, the manager benefits less when he mimics a firm whose assets are marginally more valuable if the actual value of his firm's assets in place is higher.

Third, the costs that the manager incurs from marginally increasing investors' beliefs about the value of the firm's assets in place depends on (*i*) the overall magnitude of the bias (due to the convexity of the cost function $g(b)$) and (*ii*) the sensitivity of investors' inferences to changes in the manager's report (which determines the additional bias necessary to make investors believe that the value of the firm's assets in place is marginally higher). Letting $\hat{x}$ once again denote investors' perception of the value of the firm's assets in place, $\frac{\partial x_R}{\partial \hat{x}}$ measures the additional bias necessary to marginally increase investors' beliefs about the value of the firm's assets in place and $\frac{\partial g(x_R - x)}{\partial \hat{x}} = g'(b(x)) \frac{\partial x_R}{\partial \hat{x}}$ measures the additional costs the manager incurs from marginally increasing investors' beliefs. In equilibrium, investors' inferences have to be consistent with the manager's reporting strategy, i.e., $\frac{\partial x_R}{\partial \hat{x}} = \frac{\partial x_R}{\partial x}$ or $\frac{\partial x_R}{\partial \hat{x}} = 1 + b'(x)$. So, in equilibrium the manager's cost of marginally increasing investors' beliefs is $\frac{\partial g(x_R - x)}{\partial \hat{x}} = g'(b(x))(1 + b'(x))$.[17]

These three observations jointly explain the shape of the equilibrium bias function: Initially, the bias is zero due to the fact that a manager with assets in place worth zero does not bias his report. Managers with assets slightly more valuable are willing to incur signaling costs and therefore bias

[16] See also Footnote 6 in which ε denotes the difference beteween $\hat{x}$ and x.

[17] Note that these three observations are equivalent to Lemma 1. The first observation provides the boundary condition $b(0) = 0$. The second observation gives the marginal benefit from inducing investors to believe that the value of the firm's assets in place are worth marginally more than their actual value, $\frac{I}{x+I+\mu_r}$, while the third observation provides the marginal costs, $\frac{\partial g(x_R - x)}{\partial \hat{x}} = g'(b(x))(1 + b'(x))$. Equating the marginal benefit and costs and rearranging terms yields the differential equation in Lemma 1.

their reports upwards. Since the bias is still relatively small, the marginal costs of biasing the report are also relatively small. In equilibrium, the manager's marginal costs of biasing his report must equal his marginal benefit from biasing his report. This implies that the marginal benefit to a manager with low asset values must also be relatively small. Since, for low asset values, the marginal benefit to a manager from increasing investors' beliefs is high, it must be that investors' beliefs are relatively insensitive to the manager's report. This is the case when investors attribute most of an increase in the report to an increase in the manager's bias and only a small part to an increase in the asset value. As a result, the equilibrium bias function must increase at a high rate when asset values are relatively low. As the bias continues to increase, the marginal costs of biasing the report increase as well. At the same time, the manager's marginal benefit from increasing investors' beliefs decreases. In order for the marginal costs to equal the marginal benefit from biasing the report, investors' beliefs must become more sensitive to the report. This implies that the rate of increase in the manager's bias must decrease. Since the marginal benefit from increasing investors' beliefs eventually approaches zero, the equilibrium bias decreases once assets in place reach a certain value.[18] In the limit, when the value of the firm's assets in place is very high the manager's benefit from changing investors' beliefs goes to zero, and therefore, the manager biases his report upward by a vanishing amount, i.e., $\lim_{x\to\infty} b(x) = 0$.

When we characterize the manager's equilibrium disclosure and investment strategy in the following section, it will prove useful to establish how the bias function $b(x)$ characterized in Lemma 1 varies with the expected profitability of the investment opportunity μ_r.

Lemma 2 *The bias function $b(x)$ characterized in Lemma 1 is decreasing in the expected profitability of the investment opportunity, μ_r. That is, $\frac{\partial b(x)}{\partial \mu_r} < 0$ for all $x > 0$.*

Lemma 2 establishes that the effect of μ_r on the equilibrium bias $b(x)$ is such that a higher expected return on investment causes the equilibrium bias to be lower for any given $x > 0$. The intuition is that as μ_r – which is common across all firms – increases, the difference in the value

[18] Assume that at $x = x^*$, the manager's bias is (weakly) decreasing. To see that for any $x > x^*$ the manager's bias is monotonically decreasing, suppose to the contrary that at some $x' > x^*$ the bias function starts to increase in x. The fact that the bias function starts to increase at x' has two implications: First, the manager's marginal cost from biasing the report is increasing in x at x'. Second, the bias function is increasing and convex in x at x', which implies that the sensitivity of investors' beliefs, to the report decreases at x'. The decreased sensitivity of investors' beliefs combined with the fact that the benefit from marginally increasing investors' beliefs is decreasing in x, implies that the manager's marginal benefit from increasing his bias is decreasing at x'. This leads to a contradiction, since in equilibrium the manager's marginal benefit from increasing the bias in his report must equal his marginal costs from doing so for any x.

of assets in place across firms becomes relatively less important. As a result, the manager is less willing to bear signaling costs and biases his report less in equilibrium.

The equilibrium bias described in Lemma 1 shows that – as is standard in costly signaling settings – truth-telling is not an equilibrium and the manager ends up paying signaling costs, even though he does not mislead investors in equilibrium. As a result, the manager always bears some costs when he makes a disclosure (except when his firm's assets in place are worth zero). The signaling costs incurred by the manager differ from the signaling costs in standard signaling settings where the sender (manager) maximizes his perceived type net of his signaling costs (e.g., Riley 1979, Miller and Rock 1985). In the standard signaling models, that consider only the disclosure decision of the manager, the marginal benefit to the manager from increasing investors' beliefs about his type is assumed to be constant. This property combined with a convex cost function, yields increasing signaling costs that converge to a finite upper bound as the sender's type goes to infinity. Our model studies a different setting, in which a manager who considers raising capital to finance an investment opportunity makes an investment decision in addition to the disclosure decision. The differences in the setting give rise to qualitatively different disclosure behaviors. This shows that modeling specific signaling settings and incorporating institutional details into the model can qualitatively alter predictions about equilibrium properties of the sender's message and signaling costs.

In contrast to the analysis so far, the manager is not required to disclose and raise capital. As an additional building block for solving for the equilibrium, we next analyze the manager's decision to raise capital and invest for an exogenously assumed disclosure strategy.

4.2 Investment Decision for an Exogenous Disclosure Strategy

In this section, we discuss the manager's investment decision while taking his disclosure strategy as given. In the equilibrium we study, investors can infer the value of the firm's assets in place whenever the manager voluntarily discloses information. In such equilibria, a manager who voluntarily issues a report always raises capital and invests. However, for some values of assets in place the manager may choose not to issue a voluntary report. We denote the set of values of assets in place for which the manager does not issue a voluntary report by X_{nd}. In this section, we take X_{nd} as given and solve for the manager's investment strategy when he remains silent. This analysis is similar to the analysis in Myers and Majluf (1984).

When the manager does not disclose information, the information asymmetry between the manager and investors is not fully resolved. While the manager knows the precise value of the firm's assets in place, investors can only make inferences about the average value of the assets in place for which the manager does not issue a report. Based on their inferences, investors will require a share α_{nd} of the firm's equity in return for providing capital I when the firm raises capital without issuing a report. In equilibrium, the equity share α_{nd} will guarantee that the investors break even on average.

Although investors on average correctly value the equity capital they invest in, they sometimes overvalue or undervalue the value of the equity issued. When investors undervalue the equity issued in exchange for their investment, the firm's manager must give up a larger fraction of equity ownership to new investors than he feels is necessary according to his private information. As a result, when the actual value of the firm's assets in place is sufficiently higher than investors' inferences, the manager chooses not to raise capital, even though this means foregoing the profitable investment opportunity. Hence, whenever the value of the firm's assets in place exceed a certain threshold, the manager will prefer passing up the profitable investment opportunity over being pooled with firms that also invest without disclosing information but whose assets in place are worth less. We let x_I denote this threshold. Of course, investors anticipate that a manager will only raise equity capital when the firm's assets in place are not particularly valuable and will price new equity issues (as reflected in α_{nd}) accordingly. In equilibrium, the manager's investment and disclosure strategy and the share α_{nd} investors demand in exchange for their investment I when the manager does not issue a voluntary report satisfy the following condition:

$$I = \alpha_{nd} E\left[\widetilde{x} + I + \mu_r | \widetilde{x} < x_I, x \in X_{nd}\right]. \tag{2}$$

In equilibrium, a manager whose assets in place are worth x_I must be indifferent between investing without disclosing information and his next best option. When the manager invests without disclosing information, he retains an equity stake worth $(1 - \alpha_{nd})(x_I + I + \mu_r)$. The manager's next best option is either to voluntarily disclose information and invest, yielding an expected payoff of $x_I + \mu_r - g(b(x_I))$, or to not raise capital and forego the profitable investment opportunity. In this case the manager simply keeps x_I. In equilibrium, x_I must satisfy

$$(1 - \alpha_{nd})(x_I + I + \mu_r) = \max\left\{x_I + \mu_r - g(b(x_I)), x_I\right\} \tag{3}$$

Only firms whose assets in place are worth less than x_I will raise capital and invest if they choose not to issue a report. Lemma 3 summarizes firms' investment behavior for a given disclosure strategy:

Lemma 3

(*a*) *In equilibrium, if a manager issues a report* $x_R(x)$ *he also raises capital and invests. The manager's expected payoff is* $x + \mu_r - g(b(x))$.

(*b*) *For any set of asset values,* X_{nd}, *for which investors anticipate that the manager does not issue a report, there exists* $\alpha_{nd} \in (0,1)$ *and* $x_I \in (0,\infty)$ *that jointly satisfy equations* (2) *and* (3). *In equilibrium,*

- *for* $x \geq x_I$, $x \in X_{nd}$, *the manager does not raise capital and consequently does not invest. The manager's payoff is* x.
- *for* $x < x_I$, $x \in X_{nd}$, *the manager raises capital and invests. The manager's payoff is* $(1-\alpha_{nd})(x + I + \mu_r)$.

The following section analyzes the manager's equilibrium disclosure and investment strategy.

4.3 Equilibrium

Since the manager makes both an investment and a disclosure decision, he can implement four qualitatively distinct strategies, depending on 1) whether he discloses, and 2) whether he invests. As we have argued before, the manager never discloses without raising equity capital, because disclosure is (weakly) costly. Furthermore, the sole purpose of making a disclosure is to gain access to cheaper capital by convincing investors of the value of the firm's assets in place. Hence, for any value of the assets in place, the manager pursues one of the following three options: disclose/invest; not disclose/invest; not disclose/not invest. As a first step, we analyze the manager's choice between two of the three options. In particular, we study whether the manager prefers to disclose and invest over not disclosing and not investing.

Section 4.1 characterized the properties of the manager's bias and the resulting endogenous disclosure costs under the assumption that he issues a report prior to raising equity capital. However, the manager is not required to issue a report. Since the sole purpose of making a disclosure is to convince outside investors of the value of the firm's assets in place, the manager would prefer not to issue a report if the biasing costs outweigh his expected return from the investment. That is, the manager is better off withholding his information and foregoing the investment opportunity if the bias imposed costs of $g(b(x))$ that exceed the expected investment return, μ_r, i.e., if

$b(x) > g^{-1}(\mu_r)$. Section 4.1 established that the bias and, as a result, the biasing costs are low for both small and large values of assets in place (see Figure 2). Hence, no disclosure and no investment may occur only for intermediate values of assets in place. The manager's decision not to disclose and not to invest for intermediate values depends on the expected return on the investment opportunity, μ_r. For more profitable investment opportunities (i.e., for higher values of μ_r), the manager is willing to bear higher biasing costs. These costs are reflected in a higher threshold of bias, $g^{-1}(\mu_r)$, up to which the manager is willing to bias his report. If the bias exceeds $g^{-1}(\mu_r)$ then there exists an interval of intermediate values of assets in place for which the manager prefers not to disclose and thus chooses to forego the profitable investment opportunity.[19] We denote this non-disclosure interval of intermediate values of assets in place by $\left(x_1^D, x_2^D\right)$, where the boundaries x_1^D and x_2^D are uniquely defined by the cost of disclosure such that they exactly offset the expected return on investment, i.e., $g\left(b\left(x_1^D\right)\right) = g\left(b\left(x_2^D\right)\right) = \mu_r$.

Next, we consider the manager's third option: to raise capital and invest without making a disclosure. The following lemma shows that if it is optimal for the manager with asset value x' to raise capital and invest without making a disclosure, then it is also optimal for him to raise capital and invest without making a disclosure for asset values lower than x'.

Lemma 4 *In equilibrium, if the manager invests without issuing a report when the firm's assets in place are worth x', then the manager also invests without issuing a report when the firm's assets in place are worth less, i.e., when $x < x'$.*

Recall that x_I denotes the highest value of assets in place for which the manager (weakly) prefers investing without issuing a report over issuing a report. Lemma 4 establishes that the manager strictly prefers raising capital without making a disclosure for all $x < x_I$.[20] The intuition is as follows: In equilibrium, investors rationally infer the average value of assets in place for which the manager raises capital without issuing a report. While they price the firm's assets correctly on average, investors overvalue assets that are worth little and undervalue assets that are worth more. When his assets are undervalued, it is conceivable that, rather than raise capital without issuing a report, the manager may prefer to either (*i*) forego the profitable investment opportunity or (*ii*) to issue a (costly) report that allows investors to infer the actual value of his firm's assets.

[19] Recall that Lemma 2 established that the bias function $b(x)$ is lower for higher values of μ_r. This, together with the fact that the threshold $g^{-1}(\mu_r)$ is increasing in μ_r, implies that the manager is less likely to withhold information when the expected return on investment is higher.

[20] Lemma 8 and Lemma 9 in the Appendix shows that $x_I > 0$.

However, Lemma 4 establishes that this is never the case when the manager prefers raising capital without issuing a report for some higher values of assets in place. The footnote below provides the intuition for this result.[21]

What Lemma 4 does not specify is the highest asset value for which the manager prefers to raise capital without making a disclosure, i.e., x_I. Since, by definition, x_I is the highest value of assets in place for which the manager prefers to raise capital without making a disclosure, the firm's assets will be undervalued by investors when $x = x_I$. Depending on the extent of the undervaluation, the manager is willing to incur disclosure costs to communicate the true value of his firm's assets in place to investors. For any value of assets in place, x, we let $b_I(x)$ denote the maximum amount by which a manager is willing to manipulate his report in order to separate himself from firms with lower asset values. The function $b_I(x)$ will prove useful in determining x_I. The following provides a formal definition of the function $b_I(x)$.

Definition 1 *$b_I(x)$ denotes the bias that renders a manager whose assets in place worth x indifferent between the following:* (1) *issuing a report with a bias $b_I(x)$ which reveals the value of his firm's assets in place and pursuing the investment opportunity; and* (2) *not issuing a report but investing when investors expect the firm's assets in place to be worth $E(\tilde{x}|\tilde{x} < x)$. Formally, $b_I(x)$ is defined by*

$$\left(1 - \frac{I}{x + I + \mu_r}\right)(x + I + \mu_r) - g(b_I(x)) = \left(1 - \frac{I}{E(\tilde{x}|\tilde{x} < x) + I + \mu_r}\right)(x + I + \mu_r)$$

In equilibrium, if the value of the assets in place is x_I, the manager must be indifferent between raising capital without disclosing information and the next best option. The next best option is

[21] It turns out that the manager never prefers option (i), i.e., he never prefers to forego the profitable investment opportunity for $x < x_I$. The reason is as follows: When the manager raises capital without issuing a report, the undervaluation of the firm when its assets are worth x is less severe than when its assets are worth x_I. Since the expected return on investment is sufficient to compensate the manager for the undervaluation when the assets are worth x_I, it is more than sufficient if the assets are worth $x < x_I$. In addition, the manager never prefers option (ii), i.e., he never prefers to issue a report for any $x < x_I$, even though he would avoid the discount from being pooled with less valuable asset realizations. To illustrate this point, suppose instead that the highest asset value (lower than x_I) for which the manager preferred to issue a report were $x' < x_I$. Then, for asset values worth slightly more than x' the manager would prefer to issue the same report $x_R(x')$ over raising capital without making a disclosure – contradicting the assumption that x' is the highest asset value for which the manager prefers to issue a report. The intuition is as follows. Suppose the manager preferred to issue a report $x_R(x')$ when the assets are worth x'. In this case, his benefit from retaining a larger fraction of the firm must outweigh his disclosure costs. The costs of issuing a report $x_R(x')$ would be lower for asset values slightly higher than x' than they are for x' because the manager would have to bias his report upwards by less if his assets are worth slightly more. Moreover, the manager would benefit more from retaining a larger fraction of ownership (as a result of reporting $x_R(x')$ instead of issuing no report) when his firm's assets in place are worth more. Hence, for asset values slightly higher than x', the manager would strictly prefer to issue the report $x_R(x')$ over raising capital without issuing a report – contradicting the assumption that x' is the highest asset value for which the manager prefers to issue a report.

either to issue a report $x_R(x_I)$, which provides the manager with an expected payoff of $x_I + \mu_r - g(b(x_I))$, or not to disclose and forego the investment opportunity, which provides the manager with a payoff of x_I. Hence, in equilibrium, for $x = x_I$ the maximum costs that the manager is willing to bear to separate himself from managers with lower realizations of asset values is $g(b(x_I))$ or μ_r, whichever is lower. In other words, in equilibrium, $b_I(x_I)$ has to equal the lower of the equilibrium bias $b(x_I)$ and $g^{-1}(\mu_r)$. This guarantees that for $x = x_I$ the manager is indifferent between raising capital without disclosing information and the next best option where for $x < x_I$ he strictly prefers to raise capital without disclosing information.

Hence, in equilibrium, the manager always raises capital without making a disclosure when the value of the firm's assets in place is less than x_I. For higher values of assets in place, there exist up to three distinct intervals, depending on how x_I compares to x_1^D and x_2^D. The three distinct regions exist if $x_I < x_1^D$. In that case, for $x \in (x_I, x_1^D)$, the manager raises capital but makes a disclosure to distinguish himself from firms with lower values of assets in place. For $x \in (x_1^D, x_2^D)$, the manager does not issue a report and foregoes the profitable investment opportunity because (1) the disclosure costs are prohibitively high and (2) the undervaluation is too severe if he raises capital without issuing a report. For even higher values of assets in place, disclosure costs are lower and the manager again makes a disclosure, raises capital and invests. Proposition 1 provides a full characterization of the equilibrium.

Proposition 1 *There exists an equilibrium which is characterized as follows. For $x \in [0, x_I)$, where the threshold $x_I > 0$ is uniquely defined by $x_I = \min\{x | b_I(x) = \min\{g^{-1}(\mu_r), b(x)\}\}$, the manager raises capital and invests without issuing a report. For $x \in [x_I, \infty)$, the manager's equilibrium strategy is given by one of the following:*

(i) For $\mu_r \geq \mu_r^*$ the manager issues a report, raises capital and invests for all $x \geq x_I$ ("Full investment" in Figure 3).[22]

(ii) For $\mu_r < \mu_r^*$, there exist two thresholds x_1^D and x_2^D, which are uniquely defined by $b(x_1^D) = b(x_2^D) = g^{-1}(\mu_r)$ and $0 < x_1^D < x_2^D$.

a. If $x_2^D < x_I$, the manager issues a report, raises capital and invests for all $x \geq x_I$

[22] $\mu_r^* = g(b(x^*))$ where x^* is the value of assets in place for which $b'(x^*) = 0$ and $b(\cdot)$ is given in Lemma 1. That is, μ_r^* is the expected return of the investment opportunity for which the bias function in Lemma 1 is tangential to the horizontal line $g^{-1}(\mu_r^*)$.

("Full investment" in Figure 4).

b. If $x_1^D < x_I < x_2^D$, the manager does not issue a report, does not raise capital and does not invest for $x \in [x_I, x_2^D)$; he issues a report, raises capital and invests for $x \in [x_2^D, \infty)$ ("Partial investment" in Figure 5).

c. If $x_I < x_1^D$, the manager issues a report, raises capital and invests for $x \in \left[x_I, x_1^D\right]$; he does not issue a report, does not raise capital and does not invest for $x \in \left(x_1^D, x_2^D\right)$; he issues a report, raises capital and invests for $x \in [x_2^D, \infty)$

("Partial investment with two non-disclosure intervals" in Figure 6).

When a manager issues a report, he biases it according to Lemma 1. If investors observe an off-equilibrium report, they believe that the manager biased his report according to Lemma 1.[23] Which part of the proposition describes the equilibrium depends on the parameters of the model.[24]

Proposition 1 part (i) shows that, if the expected return on investment is sufficiently high, the manager raises capital and invests for any value of the assets in place. For asset values lower than x_I, the manager does not issue a report, while for higher asset values he does issue a report. Figure 3 illustrates such an equilibrium. This and the following figures use again the cost function $g(b) = \frac{1}{2}b^2$ and the parameter values $I = 1$, $\mu_r = 0.12$ or 0.25 and assume that the value of assets in place is distributed according to a Gamma distribution with shape $k = 3$ and scale $\theta = 2$.

If the expected return on investment is lower, the characteristics of the equilibrium are determined by how the thresholds x_I, x_1^D and x_2^D compare. If $x_I > x_2^D$ then $[0, x_I)$ is the unique non-disclosure interval because managers whose asset values exceed x_2^D prefer to disclose and invest. This case is characterized in part $(ii)\,a$ of Proposition 1 and is illustrated in Figure 4. If the intersection of $b(x)$ and $b_I(x)$ is such that $x_I \in \left(x_1^D, x_2^D\right)$ there is also a single non-disclosure interval but it spans $[0, x_2^D)$ rather than $[0, x_I)$. Firms with $x \in [0, x_I)$ invest while firms with $x \in \left(x_I, x_2^D\right)$

[23] There exist other equilibria that rely on different off-equilibrium beliefs. All these equilibria share the same qualitative characteristics as the equilibrium described in Proposition 1. In particular, the equilibrium has either one or two non-disclosure intervals. When the equilibrium has a single non-disclosure interval, the boundary condition is given by $b(x_I) = b_I(x_I)$. When the equilibrium obtains two non-disclosure intervals, the boundary condition for the bias in the first disclosure interval is given by $b(x_I) = b_I(x_I)$ and for the second non-disclosure interval it is given by $b\left(x_2^D\right) = g^{-1}(\mu_r)$ where x_2^D is greater than the solution to

$$\left\{b'(x) = \frac{I}{g'(b(x))(x + I + \mu_r)} - 1,\ b'\left(x_2^{D\prime}\right) = 0,\ b\left(x_2^{D\prime}\right) = g^{-1}(\mu_r)\right\}.$$

[24] Below and in Corollary 2, we characterize the effect of the model's parameters on the prevailing form of the equilibrium.

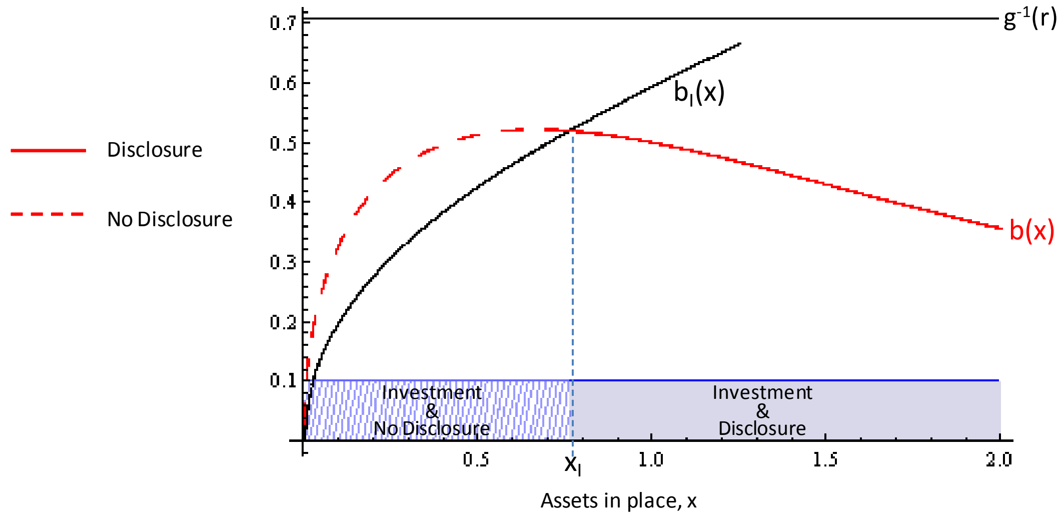

Figure 3: Full investment ($\mu_r = 0.25, k = 3, \theta = 2$)

do not invest. This case is characterized in part $(ii)\, b$ of Proposition 1 and illustrated in Figure 5. Finally, if $x_I < x_1^D$, there are two distinct non-disclosure intervals. For $x < x_I$ the manager does not make a disclosure but raises capital and invests. For $x \in \left(x_1^D, x_2^D\right)$ the manager does not make a disclosure but also does not raise capital and does not invest. This case is characterized in part $(ii)\, c$ of Proposition 1 and is illustrated in Figure 6.

We have so far not specified which of the three forms illustrated in Figures 4-6 (or, equivalently in parts (ii) a-c in Proposition 1) prevails. The form of the equilibrium depends on how the threshold x_I compares to x_1^D and x_2^D. The location of x_I is determined by the properties of the prior distribution of $\tilde{x}$. When the prior distribution of $\tilde{x}$ assigns a relatively low weight to small asset values, the manager is willing to pay less in order to separate himself from lower types. This is reflected in a downward shift of $b_I(x)$. As a result, x_I is farther to the right. In contrast, x_1^D and x_2^D are independent of the prior distribution of the value of assets in place, x, because both $g^{-1}(\mu_r)$ and $b(x)$ do not vary with the prior distribution of $\tilde{x}$.

Corollary 1 formalizes this intuition and establishes that x_I is higher for distribution h than for distribution f if h is a convex transformation of f such that $E_h[\tilde{x}|\tilde{x} < x'] > E_f[\tilde{x}|\tilde{x} < x']$ for all $x' > 0$. As a result, a firm with asset value x' is undervalued by less if investors believe that firms' asset values are distributed according to h rather than f. Then, a manager with asset value x' is willing to pay less for separating his firm from firms with lower asset values. This is reflected in a lower function $b_I(x)$ (which indicates the maximum bias for which a firm is willing to bear the

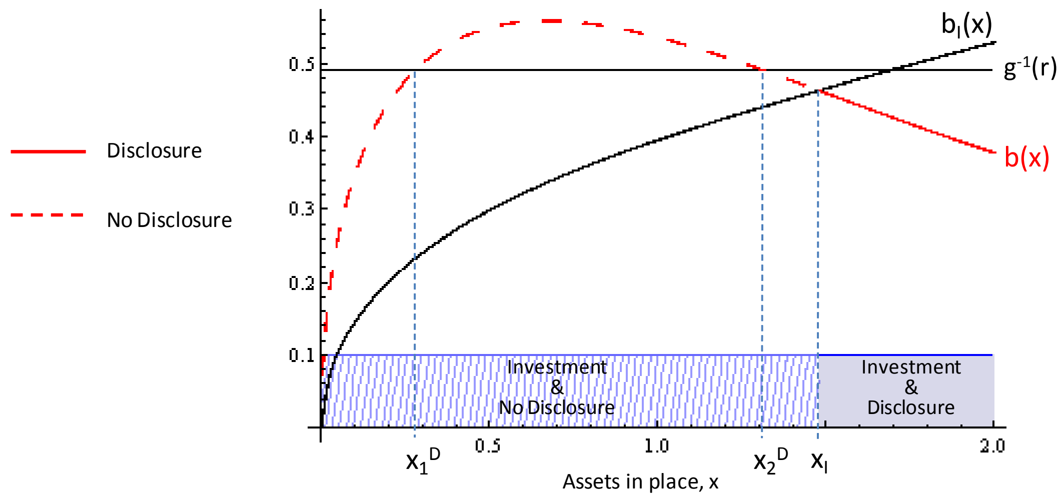

Figure 4: Full investment ($\mu_r = 0.12, k = 7, \theta = 2$)

costs to separate itself from firms with lower asset values), which maps into a larger value of x_I.

Corollary 1 *Let the probability density function $h(\cdot)$ be a convex transformation of $f(\cdot)$ in the sense that $h(x) = \frac{\pi(x)f(x)}{\int_0^\infty xf(x))dx}$ where $\pi(\cdot)$ is an increasing function of x. Then,*

$$x_I^h > x_I^f$$

where x_I^j denotes the highest value of assets in place for which the manager raises capital without issuing a report when the firm's asset values are distributed according to $j = f, h$.

Proposition 1 shows that inefficient investment behavior occurs when the investment opportunity is not too profitable. In contrast to Myers and Majluf (1984), the firms with intermediate asset values forego the profitable investment opportunity and not the firms with high asset values. The reason is that, for firms with high values of assets in place, disclosure costs are sufficiently low such that they are outweighed by the expected return on investment. While the investment behavior characterized in Proposition 1 differs from the investment behavior in Myers and Majluf (1984), the underinvestment problem in our model is also less prevalent (in the sense that the manager foregoes the profitable investment opportunity less often) when the expected return on investment is higher. This is intuitive. As the expected return on investment increases, the manager is willing to incur higher costs to raise capital and invest. The following corollary formalizes this observation.

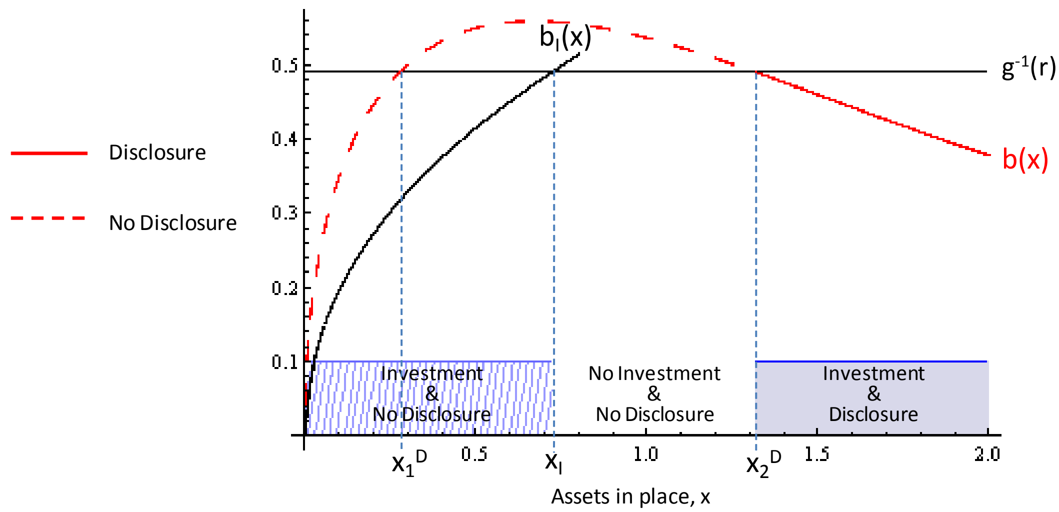

Figure 5: Partial investment ($\mu_r = 0.12, k = 3, \theta = 2$)

Corollary 2 *For $\mu_r < \mu_r^*$, x_1^D is increasing in μ_r and x_2^D is decreasing in μ_r. Hence, in the equilibrium characterized in Proposition 1, for higher expected returns on investment the underinvestment problem is less prevalent (in the sense that the manager foregoes the profitable investment opportunity less often).*

In the equilibrium of Proposition 1, low types raise capital and invest without disclosing information. When the firm raises capital without disclosing information, investors can infer the average value of the firm's assets in place but investors are not able to infer the exact value of the firm's assets in place. In contrast, when the firm makes a disclosure before raising capital, investors are able to infer the exact value of the firm's assets in place and therefore obtain a more precise estimate of the return on their investment in the firm's shares. So far, we have assumed that investors are risk-neutral. Risk-neutral investors do not price this additional uncertainty. If investors are risk-averse and the variation in the return on their equity investment contributes to their exposure to systematic risk, then equity issuance that are not accompanied by a voluntary disclosure will be priced at a discount. Due to the discount associated with non-disclosure, the firm is (weakly) less likely to withhold information when investors are risk-averse.[25]

Corollary 3 *If investors are risk-averse, the firm's cost of capital is on average lower when it*

[25] Equity offerings priced at a discount in the absence of a voluntary report causes the $b_I(x)$ function to be higher. The reason is that, when investors are risk-averse, the manager is willing to bear even higher costs to separate himself from the pool of firms with lower assets in place and avoid the risk-premium.

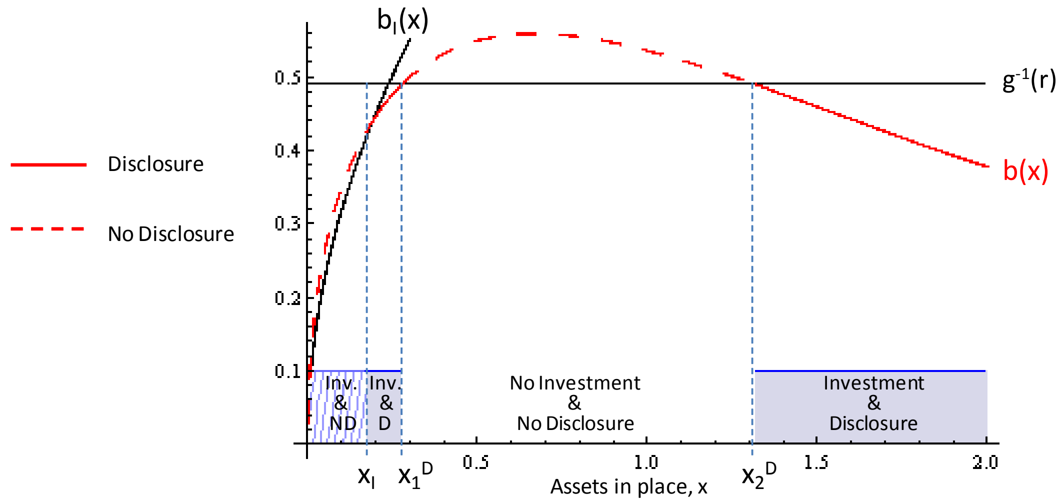

Figure 6: Partial investment with two non-disclosure intervals ($\mu_r = 0.12, k = 0.75, \theta = 2$)

makes a voluntary disclosure than when it does not. As investors' risk-aversion increases, the likelihood of the firm raising capital without disclosing information decreases.

This prediction is consistent with several empirical studies that analyze the association between disclosures, financial reporting quality, and the cost of capital. Recent research demonstrates that more extensive pre-IPO disclosures are associated with less severe under-pricing. They do so by using several proxies for the firm's level of disclosure prior to its IPO. In a sample of Canadian IPOs, Jog and McConomy (2003) use voluntary earnings forecasts that the Canadian securities commission allows, but does not require, issuers of IPOs to include in their prospectus as their measure of voluntary disclosure. Jog and McConomy (2003) find that firms that provide voluntary earnings forecasts experience less severe underpricing. Schrand and Verrecchia (2005) use the announcements contained in firms' press releases as a measure of disclosure level and find evidence consistent with less severe underpricing of firms that voluntarily provide more information about their operating, investing and financing activities to the public in press releases prior to their IPO. Leone, Rock and Willenborg (2007) measure the detail an IPO issuer provides regarding its intended use of cash proceeds and first-day under-pricing. They also find evidence of a negative association between the level of detail provided and first-day underpricing. Together, the evidence in these studies indicates that voluntary disclosure has a favorable and noticeable impact on the degree of underpricing and the post-issue return performance, despite the comprehensive disclosures required by securities commissions prior to an IPO. This suggests that adverse selection is, at least in part, a determinant of cross-sectional variation in underpricing and that voluntary disclosure can reduce the adverse selection effect consistent with the prediction of our model.[26]

Our model also predicts that cost of capital and firm performance are related. Corollary 4 describes this relation.

Corollary 4 *If investors are risk-averse, the firm's cost of capital is on average higher for low-performing firms than for high-performing firms. As investors' risk-aversion increases, the difference in cost of capital between low- and high-performing firms increases.*

The intuition for Corollary 4 is as follows. In the model, better performing firms, i.e., firms whose assets in place turned out to be higher, voluntarily issue a report while firms with low values of assets in place do not issue a report. This is consistent with empirical evidence provided by Miller (2002), who finds that high-performing firms tend to issue voluntary reports more frequently than low-performing firms. Since issuing a voluntary report reduces investors' risk-exposure, our model predicts that a firm's cost of capital is lower when the firm issued a voluntary report (Corollary 3). Consequently, the model predicts that better-performing firms have a lower cost of capital.

5 Extensions

5.1 Real Manipulation

So far, we have considered the case in which the manager's decision to manipulate his report results in personal costs for the manager but does not reduce the value of the firm's assets in place. In this section, we consider an alternative setting in which the value of the firm's assets in place is affected by a (real) decision the manager makes based on his private information. In this case, the manipulation is best thought of not as a reporting bias, but rather as a deviation from first-best investment, operating or financing behavior. For instance, the manager might (*i*) overinvest in assets to increase investors' perception of their profitability; (*ii*) overproduce inventory to increase investors' perception of future demand; (*iii*) pay out a too high dividend to increase investors' perception of the firm's free cash flow; or (*iv*) "oversell" by stuffing the channels and infringe on future sales to increase investors' perception of customer demand for the firm's products. In these examples, if the manager decides to make a disclosure, he must do so truthfully and not bias his

[26] The association between firms' cost of capital and disclosure has been studied both empirically (e.g., Botosan 1997; and Botosan and Plumlee 2002) and theoretically (e.g., Lambert, Leuz and Verrecchia 2007; and Christensen, De La Rosa and Feltham 2010). The survey by Beyer et al. (2010) further discusses this literature.

report. Similar to Verrecchia (1983) and Dye (1985) and the voluntary disclosure literature that followed, this assumes that the manager's decisions are verifiable and that litigation concerns etc. are sufficiently strong to deter any kind of misreporting.

As mentioned earlier, the characteristics of the equilibrium in the previous section are robust to manipulation costs that reduce firm value. In the following, we endogenize these manipulation costs by modeling scenario (i) above, in which the manipulation costs result from a (potentially suboptimal) investment decision made by the manager prior to raising equity. In Section 5.1.2, we briefly discuss scenarios $(ii) - (iv)$.

As part of the analysis, we also show that the model does not require that the expected firm value is necessarily linear in the manager's private information (as it was the case in the previous section where the expected firm value $V = x + I + \mu_r$ was linear in the manager's private information, x). In particular, we show that the model's predictions remain qualitatively the same if the firm value is

$$V(\theta, b) = x(\theta) + I + \mu_r - g(b), \tag{4}$$

where b is the deviation of the manager's decision from the first-best level, $g(b)$ captures the reduction in firm value due to the manager's suboptimal decision and $x(\cdot)$ may be either linear or quadratic in the manager's private information, θ.[27] In that sense, this section extends the model of the previous section along three dimensions: first, it allows the manipulation to capture real effects rather than accounting manipulation, second, the manipulation costs arise endogenously from an earlier, real decision and, third, it allows for a more general relation between the manager's private information and firm value.

5.1.1 Overinvestment in the firm's assets in place

This section describes a parsimonious model of scenario (i) in which the manager may overinvest in the firm's assets in place to increase investors' perception of the firm's profitability. In the model, there are three points in time. At $t = 1$, the manager privately observes the profitability of the firm's production technology, θ. Based on this information, the manager chooses the amount k to be invested into the firm's production technology. At $t = 2$, the manager simultaneously decides whether to voluntarily disclose $x_R = k$ and whether to raise equity capital from outside investors to finance a new investment opportunity. Both the firm's current production technology and the new

[27] We identify more general conditions for $x(\cdot)$ as part of the proof of Lemma 5.

project (if carried out) will generate their final cash flows at $t = 3$. In the following, we provide more details on the three stages of the model and highlight the differences compared to the model outlined in Section 3.

The value of the firm's assets in place is a function of the profitability parameter θ and the amount k invested into the firm's production technology. In particular, suppose that the value of the firm's assets in place is given by[28]

$$\theta k - \frac{1}{2}k^2. \tag{5}$$

The profitability parameter θ is privately observed by the manager prior to choosing k and is distributed over $[0, \infty)$.[29] The distribution of θ is denoted by $f(\theta)$ with $f(\theta) > 0$ for all $\theta \in [0, \infty)$. Hence, in contrast to the model in Section 3, the value of the firm's assets in place is not determined exogenously, but rather is a function of the manager's choice of investment level k. After the manager chooses k, he can decide whether to disclose it. If he chooses to disclose the investment level, he must do so truthfully, i.e., $x_R = k$. This assumption reflects that in this setting, and in contrast to the setting described in Section 3, the disclosure is about the amount of capital invested into the firm's production technology. The amount of capital invested represents a historical cost accounting measure and is therefore more likely to be verifiable such that, in conjunction with sufficiently strong anti-fraud rules, disclosures can be assumed to be truthful. (For similar assumptions see, e.g., Verrecchia 1983; Vives 1984; Dye 1985; Jung and Kwon 1988; Penno 1997; Kanodia et al. 2000). In contrast, in Section 3 the disclosure is about the fair value of the firm's assets in place, and this requires estimates of future cash flows generated by the assets. Forward-looking measures are by their nature more subjective, and manipulation in the manager's report of estimated future cash flows may not be verifiable even ex-post. Therefore, prohibitively costly penalties to deter bias in the manager's report may not be effective. Accordingly, Section 3, in contrast to this section, assumes that the manager can bias his report.

When the manager privately observes the profitability $\widetilde{\theta} = \theta$, he can maximize the value of the firm's assets in place in (5) by choosing $k^*(\theta) = \theta$. This first-best level of investment yields assets in place worth $\frac{1}{2}\theta^2$. If the manager deviates from the first-best level of investment $k^*(\theta) = \theta$ and chooses $k = \theta + b$ where b denotes the manipulation or distortion in the manager's investment

[28] A similar production function has been used in, e.g., Kanodia, Singh and Spero (2005) who model a firm's short-term return as $k\theta - c(k)$ where $c(\cdot)$ is increasing and convex.

[29] The analysis can easily be extended to allow for $\theta \in [\underline{\theta}, \infty)$. For simplicity, we set $\underline{\theta} = 0$.

decision, the value of the firm's assets in place is

$$\frac{1}{2}\theta^2 - \frac{1}{2}b^2.$$

If the manager raises equity from outside investors to finance the new investment opportunity, the firm value is

$$V(\theta, b) = \frac{1}{2}\theta^2 + I + \mu_r - \frac{1}{2}b^2$$

which is equivalent to equation (4) where $x(\theta) = \frac{1}{2}\theta^2$ reflects the value of the firm's assets in place if the manager chooses the first-best level of k and where $g(b) = \frac{1}{2}b^2$ reflects the loss in value due to distortions in the manager's pre-equity issuance decisions.

When deciding about the level of k at $t = 1$, the manager anticipates the profitable investment opportunity he will face at $t = 2$ and the need to raise equity capital if he chooses to pursue this investment opportunity. If he plans not to raise capital at $t = 2$ or to raise capital without disclosing the level of k, the manager has no incentive to deviate from the first-best level of k at $t = 1$. However, if the manager plans to disclose $x_R = k$ and to raise capital at $t = 2$ he attempts to decrease the cost of the firm's equity capital by increasing investors' beliefs about the value of the firm's assets in place. The manager benefits if investors' beliefs about the value of the firm's assets in place are higher because investors will require a smaller fraction of equity in exchange for providing capital I. In order to increase investors' beliefs about the value of the firm's assets in place, in equilibrium the manager chooses k greater than the first best investment level, $k^*(\theta) = \theta$.

As in Section 3, if the manager decides to raise capital at $t = 2$ he offers a fraction α of the firm's ownership such that investors break even on average:

$$I = \alpha E\left[\widetilde{V}\middle|\Omega\right]$$

where Ω again denotes the public information that is available to investors at $t = 2$. The publicly available information depends on whether the manager disclosed $x_R = k$. If the manager does not disclose $x_R = k$ at $t = 2$, he optimally chooses the first-best level $k^*(\theta) = \theta$. Investors anticipate this behavior and when x_R is not disclosed they require a fraction

$$\alpha_{nd} = \frac{I}{E\left[x(\widetilde{\theta})|\theta \in \widehat{\Theta}_{nd}\right] + I + \mu_r}$$

where $x(\theta) = \frac{1}{2}\theta^2$ is the first-best value of assets in place and $\widehat{\Theta}_{nd}$ denotes investors' beliefs about the set of profitability parameters for which the manager raises equity capital without making a disclosure. Finally, if the manager discloses $x_R = k$ at $t = 2$, investors infer that the firm's profitability is $\widehat{\theta}(x_R)$. Hence, investors require fraction

$$\alpha = \frac{I}{V\left(\widehat{\theta}(x_R), \widehat{b}(x_R)\right)}$$

of equity in exchange for their capital contribution where $\widehat{b}(x_R) = x_R - \widehat{\theta}(x_R)$ denotes investors' inferences about the distortions in the manager's choice of k relative to the first-best level. Investors receive final cash flows of $\alpha V(\theta, b)$ while the manager retains fraction $(1-\alpha)$, of the ownership and receives final cash flows of $(1-\alpha) V(\theta, b)$.

As before, we focus our analysis on fully separating equilibria such that investors' inferences about the profitability parameter $\widehat{\theta}(x_R)$ are accurate and equal θ in equilibrium. In such equilibria, investors break even and receive expected cash flows equal to their initial investment, while the manager bears the entire costs of distorting the choice of k. Hence, even though the costs are due to real effects and reduce firm value, in equilibrium the costs are not shared between investors and the manager, but borne by the manager alone. The following lemma establishes this result.

Lemma 5 *Suppose the manager discloses $x_R = k$ prior to raising equity capital. Then, in equilibrium, he chooses $k(\theta) = k^*(\theta) + b(\theta)$ where the distortion $b(\theta)$ relative to first-best is given by the solution to the differential equation*

$$b'(\theta) = \frac{x'(\theta) + g'(b(\theta))}{g'(b(\theta))} \frac{I}{x(\theta) + I + \mu_r - g(b(\theta))} - 1, \tag{6}$$

with the boundary condition $b(0) = 0$ where $x(\theta) = \frac{1}{2}\theta^2$ and $g(b) = \frac{1}{2}b^2$.[30]

The equilibrium distortion $b(x)$ has the following properties: it is continuous, always positive, initially increasing, obtains a unique maximum, and converges to zero as the firm's profitability, θ, goes to infinity.

[30] Notice the similarity to the differential equation in (1). Rewriting the differential equation in (1) in terms of θ where $x(\theta) = \theta$ yields

$$b'(\theta) = \frac{x'(\theta)}{g'(b(\theta))} \frac{I}{x(\theta) + I + \mu_r} - 1.$$

The numerator of the first ratio captures the increase in firm value as perceived by investors if investors attribute an additional marginal unit of the report x_R to θ rather than to the bias. In the model of Section 4.1, if – for a given report x_R – investors perceive the bias to be marginally lower (i.e., lower by db), they value the firm higher by the same amount, i.e., the firm value as perceived by investors increases by $x'(\theta)\, db = db$. In contrast, in the model of this section, if – for a given report $x_R = k$ – investors perceive the bias to be marginally lower, they value the firm higher for two reasons: first, they perceive the first-best value of assets in place to be higher by $x'(\theta)\, db$, and second, they perceive the costs from distortions to be lower by $g'(b)\, db$. Hence, their perception of the firm value increases by $(x'(\theta) + g'(b))\, db$. The denominator of the second ratio captures the equilibrium firm value. In Section 4.1, the firm value equals $x(\theta) + I + \mu_r$ and excludes the biasing costs $g(b)$, because the costs represent personal costs to the manager. In this section, the costs of distortion $g(b)$ are due to real effects and reduce firm value to $x(\theta) + I + \mu_r - g(b)$.

Lemma 5 establishes that the solution to the differential equation that describes the distortions in the manager's real decisions has qualitatively similar properties as the manager's reporting bias characterized in Lemma 1 in Section 4.1. In particular, the solution to the differential equation in (6) has the same "hump-shaped" form illustrated in Figure 2. As a result, whether the manager can influence investors' beliefs through overinvestment or accounting manipulation, the properties of the manager's decision to disclose, raise equity and pursue the new investment opportunity are qualitatively similar. This result is formally established in Proposition 2. To characterize the equilibrium, we first define the maximum distortion, $b_I(\theta)$, that a manager is willing to take in order to separate himself from less profitable firms.

Definition 2 *Let $b_I(\theta)$ denote the distortion that renders a manager with profitability θ indifferent between the following:* (1) *distorting the initial investment by $b_I(\theta)$, issuing a report which reveals the firm's profitability and pursuing the new investment opportunity; and* (2) *not distorting the initial investment, not issuing a report but raising equity capital and pursuing the new investment opportunity when investors expect the firm's value to be $E(V(\tilde{\theta},0)|\tilde{\theta}<\theta)$. That is, $b_I(\theta)$ is given by*

$$\left(1-\frac{I}{V(\theta,b_I(\theta))}\right)V(\theta,b_I(\theta))=\left(1-\frac{I}{E(V(\tilde{\theta},0)|\tilde{\theta}<\theta)}\right)V(\theta,0).$$

Proposition 2 *Let $x_1^D=k\left(\theta_1^D\right)=\theta_1^D+b\left(\theta_1^D\right)$ and $x_2^D=k\left(\theta_2^D\right)=\theta_2^D+b\left(\theta_2^D\right)$ where θ_1^D and θ_2^D are given by $g\left(b\left(\theta_1^D\right)\right)=g\left(b\left(\theta_2^D\right)\right)=\mu_r$ with $\theta_1^D<\theta_2^D$ and $b(\cdot)$ is given in Lemma 5. Further, let $x_I>0$ be $x_I=\min\{k(\theta)=\theta+b(\theta)\,|b_I(\theta)=\min\left\{g^{-1}(\mu_r),b(\theta)\right\}\}$ where $b(\cdot)$ is again given in Lemma 5 and $b_I(\cdot)$ is given in Definition 2.*

Then, there exists an equilibrium as described in Proposition 1.

Proposition 2 establishes that the equilibrium in Proposition 1 continues to hold when the manager influences investors' perception of firm value by overinvesting in the firm's current assets rather than by accounting manipulation for which he bears a personal cost. In particular, in the setting described in this section, the manager's choice between (1) disclosing (a suboptimal) k and raising equity to pursue the investment opportunity at $t=2$, (2) raising equity to pursue the new investment opportunity at $t=2$ without making a disclosure, and (3) foregoing the investment opportunity is qualitatively similar to his disclosure and investment decision described in Proposition 1. The only difference between the equilibrium in the two settings is due to the fact that the distortions, b, differ somewhat in their functional form even though they share the same qualitative characteristics.

This section outlined one particular scenario of real earnings management. The following section briefly describes three other real decisions the manager can make to influence investors' perception of firm value. We demonstrate that the manager's disclosure strategy, and his decision to raise equity capital and pursue the new investment opportunity, are qualitatively the same for any of the four scenarios of real manipulation.

5.1.2 Other real earnings management scenarios

In this section, we briefly describe scenarios $(ii) - (iv)$ introduced in Section 5.1. In particular, we specify the manager's private information, θ, the decision he makes and the reduction in firm value if the manager makes a suboptimal decision. In each of the scenarios, if the manager raises equity capital and pursues the new investment opportunity, the firm value is given by $V(\theta, b)$ in equation (4), where the firm value if the manager chooses the first best action, $x(\theta)$, is either linear or quadratic in θ; and the reduction in firm value due to the manager's suboptimal decision, $g(b)$, is convex and satisfies $g(0) = g'(0) = 0$. Here, θ again denotes the manager's private information and b captures the deviation of the manager's action from the first-best action. From this it follows that Lemma 5 and Proposition 2 continue to hold. For brevity, we omit any formal model setup and proofs. Both are available upon request.

Scenario *(ii)*: Excessive inventory levels. When firms are privately informed about future demand, investors can use inventory levels to infer information about the firms' private information and future prospects. We analyze the extent to which a manager overproduces inventory to signal to potential investors that the demand of the firm's products and, as a consequence, the value of the firm's current business (i.e., the value of the firm's "assets in place") is high. If investors' inferences about the firm's future demand are favorable they will be satisfied with a relatively small fraction of the firm's equity in exchange for providing capital I, allowing the manager to retain a larger fraction of the firm's ownership.

We consider a monopolist who is privately informed about the intercept of his inverse demand function, θ.[31] The manager chooses the quantity of inventory produced. Inventory production

[31] Similar assumptions about private information in product markets have been used in, e.g., Vives 1984, Gal-Or 1985, Darrough 1993, Hughes et al. 2002, Fischer and Verrecchia 2004.

takes place prior to the opening of the market for the firm's equity. Inventory is sold (and the inventory's market price observed) only after the new investors obtain equity in the firm. Prior to raising equity, the manager may report the cost of the firm's inventory, x_R. If he chooses to issue a report, he must do so truthfully. The firm's first-best inventory level is such that the quantity produced maximizes the firm's profits in the next period given the manager's private information about the demand, θ. If the firm wants to report inventory x_R that exceeds the cost of first-best inventory, it must produce more inventory than it would in a first-best case. The excessive inventory levels reduce firm value by $g(\cdot)$ relative to first-best inventory level because the excessive supply reduces the market price for the firm's goods.

Scenario *(iii)*: Overpaying dividends. While some business activities that yield future returns are observable, others are not. For instance, financial statements might only provide imprecise information about firms' investment in human capital, maintenance activities for productive assets or certain research and development activities. This may especially hold true when firms are private and don't have to adhere to the extensive disclosure requirements set forth by the SEC. In particular, such activities can reduce current reported earnings without being observable to investors. Managers might underinvest in such productive activities in order to report higher current earnings. In our setting, managers, who plan to raise equity capital in the future to pursue new investment opportunities, seek to increase potential investors' valuation of the firm's current assets, and therefore may report higher earnings by underinvesting in such unobservable productive activities.

Similar signaling settings have been considered by, for example, Miller and Rock (1985), Hausch and Seward (1993) and Guttman, Kadan and Kandel (2010). We follow Miller and Rock (1985) and label the disclosed amount which the firm does not invest in the aforementioned productive activities as "dividends." We refer to the firm's resources that are privately known to the manager, θ, as "earnings." These resources are available to be invested in either productive activities or paid out as dividends. Prior to raising equity capital, the manager may announce dividends, x_R. If he announces dividends he must pay out exactly the amount announced. If the manager wants to report dividends x_R that exceed the firm's resources net of first-best investment, he must underinvest in productive activities. The excess dividends reflect the distortion in the manager's decision relative to first-best dividend levels, which leads to a reduction in firm value, $g(\cdot)$. If the manager does not announce dividends, investors learn about the amount of dividends only after the market for the firm's equity closes.

Scenario *(iv)*: Channel stuffing. Channel stuffing, also known as trade loading, is an attempt by companies to inflate their current sales figures. Because firms inflate current-period sales to a level exceeding sustainable demand, future sales are likely to suffer, or excess products may be returned in subsequent periods. Moreover, discounts used to drive channel stuffing or restocking costs reduce the firms' overall profits. For instance, if a firm incentivizes retailers to purchase more inventory than they can possibly sell in the current period, retailers will have a positive inventory balance left at the end of the period and order less in the future. In addition, retailers are likely to demand a discount that compensates them for storage costs and the risk of inventory becoming obsolete.

We model this scenario as follows. A firm sells its services to customers over two periods. The firms' customers differ in their preferences for when to purchase the service of the firm. Some of the firm's customers prefer to purchase the service in the first period, and the rest of the customers prefer to purchase the service in the second period. In the period in which they do not prefer the service, customers will purchase the service only of they receive a discount. The manager observes the customers' types and can price discriminate. The firm is privately informed about the demand for its services in the first period. In particular, the firm learns, prior to the market opening, that the measure of customers who prefer its services in the first period is θ. If the firm wants to report earnings x_R that exceed first-best earnings, it must make first-period sales to customers who prefer the services in the second period. At the end of the first period, the firm can choose to report its first-period profits to potential investors prior to raising equity capital for the new investment opportunity. If the firm decides to issue a report, it must report its profits truthfully.

In the three scenarios outlined above, Lemma 5 characterizes the distortion in the manager's real decision and Proposition 2 characterizes the manager's disclosure strategy, along with his decision to raise equity capital and pursue the new investment opportunity. This shows that the model's predictions about the manager's disclosure strategy and his decision to raise equity capital are robust to the ways the manager attempts to influence investors' beliefs about firm value.

5.2 Equilibrium with Bad News Disclosed

If investors' beliefs are such that when a firm raises capital without issuing a report they infer that the value of its assets in place must be the lowest possible value, then there exists another equilibrium where the manager only raises capital if he issues a report.

In the equilibrium described in Section 4, the manager sometimes raises equity capital without issuing a report. In this case, investors update their beliefs about the value of assets in place using Bayes' Rule. In contrast, suppose investors' beliefs are such that, when a firm raises capital without issuing a report they infer that the value of its assets in place must be the lowest possible value. Then, there exists another equilibrium where the manager only raises capital if he issues a report. The equilibrium that evolves under these off-equilibrium beliefs differs from the equilibrium we described in Section 4 mainly in that firms with the lowest values of assets in place issue a report, whereas in the equilibrium of Section 4 they remained silent. The investment strategy is qualitatively similar in both equilibria. Next, we briefly discuss the equilibrium in which the manager discloses low values of assets in place.

If investors infer that the firm's assets in place are of the lowest possible value when the manager raises capital without issuing a report, the manager never implements such a strategy. The reason is that he would always prefer to truthfully disclose the value of his firm's assets in place (which is costless) over raising equity capital without issuing a report. This is due to the fact that – even if investors underestimate the value of his firm's assets in place when the manager reports truthfully – they always value the firm's assets in place higher than 0. Conversely, investors value the firm's assets in place at 0 if he raises capital without issuing a report.

As a result, the manager chooses between the two remaining options: (1) whether to issue a report and raise equity capital, or (2) to forego the profitable investment opportunity and remain silent. As before, the manager prefers to remain silent if the biasing costs outweigh the manager's expected return from the investment. That is, the manager is better off withholding his information and foregoing the investment opportunity if the bias imposes costs of $g(b(x))$ that exceed the expected investment return, μ_r.

Section 4.1 established that the bias and, as a result, the biasing costs are low for both small and large values of assets in place (see Figure 2). Hence, there exists an equilibrium in which the manager either (1) issues a report for all realizations of values of assets in place or (2) issues a report for low and high values of assets in place but not for intermediate values of assets in place.

The form of the equilibrium depends on the expected return on the investment opportunity, μ_r. For more profitable investment opportunities (i.e., for higher values of μ_r), the manager is willing to bear higher biasing costs in order to realize the expected return on investment. The manager's willingness to bear higher biasing costs is reflected in a higher threshold of bias, $g^{-1}(\mu_r)$, up to which the manager is willing to bias his report to communicate the value of the assets in place to investors. This, together with the fact that the bias function $b(x)$ is lower for higher values of μ_r (see Lemma 2), implies that the manager is less likely to withhold information when the expected return on investment is higher.

Proposition 3 formalizes the manager's equilibrium disclosure and investment strategy when the off-equilibrium beliefs are such that investors perceive the value of a firm's assets in place to be zero if it raises capital without issuing a report.

Proposition 3 *There exists an equilibrium which is characterized as follows.*

(i) For $\mu_r \geq \mu_r^*$ the manager issues a report, raises capital and invests for all $x \geq 0$ ("Full disclosure and full investment" in Figure 7).[32]

(ii) For $\mu_r < \mu_r^*$ there exist two thresholds x_1^D and x_2^D, which are uniquely defined by $b\left(x_1^D\right) = b\left(x_2^D\right) = g^{-1}(\mu_r)$ and $0 < x_1^D < x_2^D$. In equilibrium, the manager:

issues a report, raises capital and invests for $x \in [0, x_1^D] \cup [x_2^D, \infty)$; and

does not issue a report, does not raise capital and does not invest for $x \in \left(x_1^D, x_2^D\right)$

("Non-disclosure of intermediate news and partial investment" in Figure 8).

When the manager issues a report, he biases it according to Lemma 1. Investors' off-equilibrium beliefs are as follows. If they observe an off-equilibrium report, investors believe that the manager biased his report according to Lemma 1. If they observe the manager raising capital without issuing a report, investors believe that the value of the firm's assets in place is zero.

The key difference between the equilibrium in Proposition 3 and the equilibrium in Proposition 1 is that, in Proposition 3, the manager issues a report for low asset values. In Proposition 3, the off-equilibrium beliefs guarantee that, even for low values of assets in place, the manager is better off pursuing the investment opportunity with, rather than without, issuing a report. To

[32] μ_r^* and the thresholds x_1^D and x_2^D below are the same as in Proposition 1.

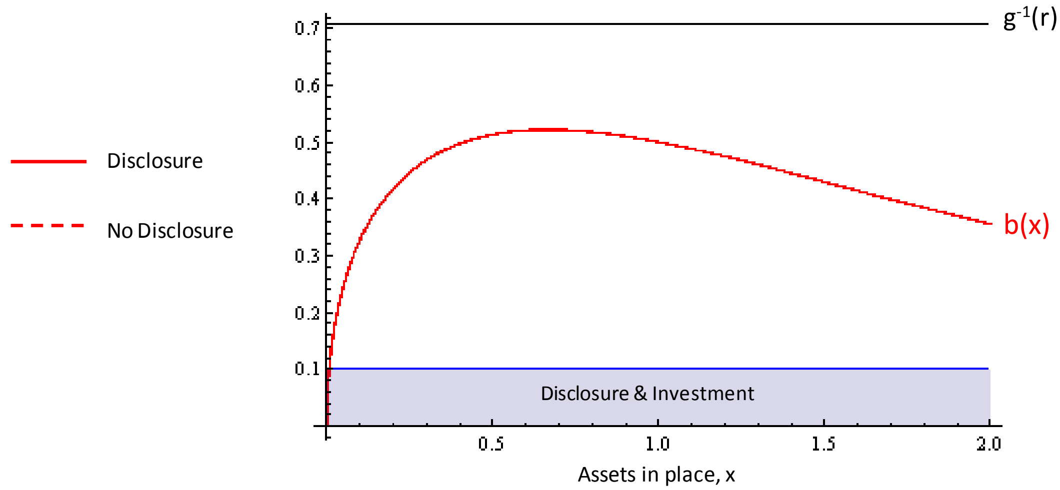

Figure 7: Disclosure and investment strategy for highly profitable investments ($\mu_r = 0.25$): Full disclosure and full investment

see that these off-equilibrium beliefs are necessary for the equilibrium to exist, suppose investors' off-equilibrium beliefs are such that they infer the firm's assets to be worth $x' > 0$ if the manager raises capital without making a disclosure. Then, a manager with assets worth less than x' can not only save on the disclosure costs by raising capital without making a disclosure but his firm's assets will also be perceived as more valuable than in fact they are. As a result, the manager would prefer to deviate and raise capital without making a disclosure – which is inconsistent with investors' assumed beliefs.

6 Conclusion

This paper models managers' joint investment and voluntary disclosure strategies along with their propensity to engage in costly reporting manipulation. We contrast the predictions of the model with firms' optimal investment strategies when managers lack the ability to communicate private information (Myers and Majluf 1984) and with their optimal voluntary disclosure strategies when investment is exogenous.

As in Myers and Majluf (1984), we assume in the model that the new investment opportunity is (1) carried out within the legal structure of the existent firm and (2) financed with equity capital.[33] We briefly discuss these assumptions next.

While based on the features captured in the model, it would be preferable to incorporate the

[33] Myers and Majluf (1984) also analyze a case in which debt financing is assumed exogenously.

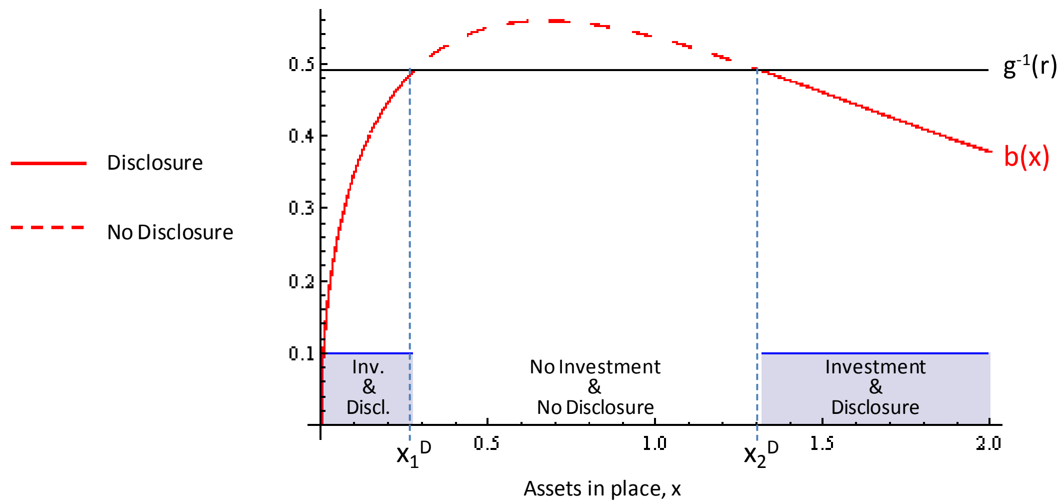

Figure 8: Disclosure and investment strategy for less profitable investments ($\mu_r = 0.12$): non-disclosure of intermediate news and partial investment

new project as a separate entity with a distinct ownership structure there are numerous reasons why such an organizational design might be infeasible or undesirable in practice. For instance, the new project might require (informal) resources such as know-how transfer from the existing operations. Such informal resources may be difficult to measure and therefore these performance measures are non-contractible. As a result, it will be difficult if not impossible to transfer such resources across legal entities with different ownership base. Similar arguments that explain the boundaries of firms have also been discussed in the literature on transaction costs (Coase 1937), asset specificity (Williamson 1975), and knowledge as the most significant resource of the firm (Penrose 1959, Wernerfeldt 1984).

In addition to the organizational structure, we assume in the model that the firm raises capital in the form of equity rather than in the form of, e.g., debt. There are several reasons for firms to prefer issuing equity over debt capital. Agency or bankruptcy costs of debt can render equity preferable overall (e.g., Kraus and Litzenberger 1973, Jensen and Meckling 1976, Myers 1977, and Smith and Warner 1979). For instance, issuing debt may provide the manager with incentives to make choices that increase cash flow volatility at the expense of reducing overall firm value (e.g., assets substitution).

Moreover, while Myers and Majluf (1984) established that managers prefer a regime in which only debt financing is available over a regime in which only equity financing is available, this "pecking order" result relies on the assumption that whether debt or equity financing is available is exogenously given, i.e., not the choice of the manager. In practice, firms may choose whether they raise external capital in the form of debt or equity. Noe (1988) has shown that when managers may choose between debt and equity to raise external capital in a sequential signaling game, debt financing does not necessarily prevail. That is the "pecking order" result may break down. This illustrates that equity capital may sometimes be preferred over debt capital. Our paper does not take an optimal contracting perspective and we do not analyze whether, and under what conditions, equity dominates debt. Instead, our model can be viewed as an analysis of investment and disclosure strategies for the subset of those firms in the "bigger game" which choose to issue equity rather than debt to finance their new investment opportunity.

Unlike most analytical models of voluntary disclosure, our model allows managers to bias their disclosures and to jointly optimize their investment and disclosure strategies. This seems to be the realistic case for two reasons: First, in practice, firms are often unable to issue perfectly credible or verifiable reports. Rather, managers can bias their reports because of the reporting discretion they enjoy due to the forward-looking nature of many voluntary disclosures and the inherent flexibility in GAAP. Managers can also influence investors' perception of firm value by making suboptimal real decisions. Second, in practice, managers often voluntarily disclose information because it alters the information asymmetry between firms and investors allowing firms to raise equity capital at favorable rates.

The model shows that when managers' disclosure and investment decisions are both endogenous, and the manager can bias his voluntary report, the model yields qualitatively different predictions than when the disclosure and investment decisions are considered separately and truthful disclosure is assumed. In particular, the optimal investment and disclosure strategies are no longer characterized by a single threshold (in contrast to traditional voluntary disclosure models and the investment model in Myers and Majluf, 1984). For example, the model predicts that there may be two distinct non-disclosure intervals and that firms with low and high values of assets in place always pursue the new investment opportunity. In contrast, firms with intermediate values of assets in place may forego the profitable investment opportunity. The model also predicts that (*i*) the underinvestment problem is more prevalent if the return on investment is low; and (*ii*) low-performing firms have a (weakly) higher cost of capital than high-performing firms. As such, the paper illustrates the importance of considering the interdependencies between firms' disclosure and investment decisions and suggests that management's ability to bias its reports significantly affects firms' voluntary disclosure strategies. Future research that investigates the effect of bias in voluntary disclosures on firms' disclosure policies, investment strategies and other real decisions has the potential to contribute to our understanding of corporate disclosure policies.

Appendix

Proof of Lemma 1

In equilibrium, the manager's marginal benefit from biasing the report equals his marginal cost from biasing the report. The marginal benefit is

$$\frac{\partial}{\partial x_R}\left(1-\frac{I}{x_R-\widehat{b}\left(x_R\right)+I+\mu_r}\right)\left(x+I+\mu_r\right)=\frac{I\left(x+I+\mu_r\right)\left(1-\frac{\partial \widehat{b}\left(x_R\right)}{\partial x_R}\right)}{\left(x_R-\widehat{b}\left(x_R\right)+I+\mu_r\right)^2}.$$

On the equilibrium path, $x_R-\widehat{b}\left(x_R\right)=x$. This, together with the marginal cost, yields the FOC (in terms of $\widehat{b}\left(x_R\right)$)

$$\frac{I}{x_R-\widehat{b}\left(x_R\right)+I+\mu_r}\left(1-\frac{\partial \widehat{b}\left(x_R\right)}{\partial x_R}\right)-g'\left(\widehat{b}\left(x_R\right)\right)=0. \tag{7}$$

In equilibrium, $\frac{\partial \widehat{b}\left(x_R\right)}{\partial x}=b'\left(x\right)$. Therefore, $\frac{\partial \widehat{b}\left(x_R\right)}{\partial x}\frac{1}{\partial x_R/\partial x}=b'\left(x\right)\frac{1}{1+b'\left(x\right)}$. In addition, $\widehat{b}\left(x_R\left(x\right)\right)=b\left(x\right)$ and $x_R-\widehat{b}\left(x_R\right)=x$ on the equilibrium path. This yields the FOC (in terms of $b\left(x\right)$)

$$\frac{I}{x+I+\mu_r}\left(1-\frac{b'\left(x\right)}{1+b'\left(x\right)}\right)-g'\left(b\left(x\right)\right)=0. \tag{8}$$

Rearranging yields (1).

The equilibrium bias function is given by the solution to (1) with the boundary condition $b\left(0\right)=0$. We next want to show that there exists a solution to this initial value problem. We cannot invoke the Fundamental Theorem of Differential Equations in order to show that the solution to (1) exists and is unique because the RHS of (1) is not finite at $b\left(0\right)=0$. In order to show that the solution exists, we substitute $c\left(x\right)$ for $\frac{1}{2}\left(b\left(x\right)\right)^2$. This implies that $b\left(x\right)=\sqrt{2\left|c\left(x\right)\right|}$ and $b'\left(x\right)=\frac{c'\left(x\right)}{\sqrt{2\left|c\left(x\right)\right|}}$. Rewriting the differential equation in (1) in terms of $c\left(\cdot\right)$ yields

$$c'\left(x\right)=\frac{I}{x+I+\mu_r}\frac{\sqrt{2\left|c\left(x\right)\right|}}{g'\left(\sqrt{2\left|c\left(x\right)\right|}\right)}-\sqrt{2\left|c\left(x\right)\right|} \tag{9}$$

with the boundary condition $c\left(0\right)=0$. Let $F\left(x,c\right)=\frac{\sqrt{2\left|c\right|}}{g'\left(\sqrt{2\left|c\right|}\right)}\frac{I}{x+I+\mu_r}-\sqrt{2\left|c\right|}$ denote the RHS of (9). We want to show that $F\left(x,c\right)$ is continuous in x and c at $\left(0,0\right)$. While $F\left(x,c\right)$ is clearly continuous in x, it is only continuous in c if $\lim_{c\to 0}F\left(x,c\right)$ is finite:

$$\lim_{c\to 0}F\left(x,c\right)=\lim_{c\to 0}\frac{\frac{\partial}{\partial c}\sqrt{2\left|c\right|}}{\frac{\partial}{\partial c}g'\left(\sqrt{2\left|c\right|}\right)}\frac{I}{x+I+\mu_r}-0=\lim_{c\to 0}\frac{\frac{1}{\sqrt{2\left|c\right|}}}{g''\left(\sqrt{2\left|c\right|}\right)\frac{1}{\sqrt{2\left|c\right|}}}\frac{I}{x+I+\mu_r}=\frac{1}{g''\left(0\right)}\frac{I}{x+I+\mu_r}.$$

This is finite because $g(\cdot)$ is strictly convex everywhere. Hence, there exists a continuous and differentiable solution to (9) with the boundary condition $c(0) = 0$. Next, we show that $c(x)$ provides a solution to the manager's disclosure problem. That requires $c(x) \geq 0$ such that $b(x)$ is real. Note that $c(x) \geq 0$ for $[0, \varepsilon]$ and ε sufficiently small because $c'(0) = F(0,0) = \frac{1}{g''(0)} \frac{I}{I+\mu_r} > 0$. Suppose there existed x' such that $c(x') < 0$. Since $c(x)$ is continuous, there must exist $x'' < x'$ such that $c(x'') = 0$ and $c'(x'') < 0$. However, $c(x'') = 0$ implies that $c'(x'') > 0$. Hence, $c(x) \geq 0$ for all $x \in [0, \infty)$. As a result, $b(x) = \sqrt{2c(x)}$ is real and provides a solution to the manager's disclosure problem. Moreover, the solution is unique. Suppose it were not unique and there existed two solutions, $b_1(x)$ and $b_2(x)$ which both satisfy the differential equation in (8) and $b_1(0) = b_2(0) = 0$. Further, since $b_1(x)$ and $b_2(x)$ differ and are differentiable there must exist an interval (x', x'') for which $b_1(x) > b_2(x)$ and $b_1'(x) > b_2'(x)$. However, for a given x, b' is lower for higher values of b. As a result, $b_1(x) > b_2(x)$ and $b_1'(x) > b_2'(x)$ cannot hold for any x and the solution to the initial value problem is unique.

We next want to show that the equilibrium bias $b(x)$ has the following properties: it is continuous, always positive, initially increasing, obtains a unique maximum and converges to zero as the value of the firm's assets in place, x, goes to infinity.

We have already shown that the equilibrium bias is continuous and strictly positive for $x > 0$. From this and the boundary condition $b(0) = 0$, it also follows that the bias is initially increasing. However, $b(x)$ cannot always be increasing. Suppose it were the case that $b'(x) > 0$ for all x. Then, $1 - \frac{b'(x)}{1+b'(x)} \in (0,1)$. Hence, the marginal benefit, $\frac{I}{x+I+\mu_r}\left(1 - \frac{b'(x)}{1+b'(x)}\right)$, converges to 0 for $x \to \infty$. In equilibrium, the marginal cost must then also converge to zero. This cannot be the case if $b(x)$ is positive and always increasing for any x. As a result, for x sufficiently large $b(x)$ is decreasing $(-1 < b'(x) < 0)$ and hence $1 - \frac{b'(x)}{1+b'(x)} > 1$. Further, since $b(x) > 0$ for all x, $b'(x)$ has to converge to 0 for $x \to \infty$. As a result, $\frac{I}{x+I+\mu_r}\left(1 - \frac{b'(x)}{1+b'(x)}\right)$ converges to 0 for $x \to \infty$. Hence, the marginal cost must also converge to zero and therefore $b(x) \to 0$ for $x \to \infty$.

Finally, we want to show that $b(x)$ does not have a local minimum. This follows from the fact that there exists no x for which $b(x)$ is weakly increasing and weakly convex. Suppose it were the case that $b'(x) \geq 0$ and $b''(x) \geq 0$ for a given x. As x increases, $b(x)$ weakly increases and hence $g'(b)$ weakly increases (due to $b(x) \geq 0$ and $b'(x) \geq 0$). Moreover, as x increases, $b'(x)$ weakly increases and hence $\frac{b'(x)}{1+b'(x)}$ weakly increases (due to $b''(x) \geq 0$). The latter implies that the marginal benefit, $\frac{I}{x+I+\mu_r}\left(1 - \frac{b'(x)}{1+b'(x)}\right)$, strictly decreases in x. Since the marginal cost, $g'(b)$, weakly increases, this yields a contradiction. ■

Proof of Lemma 2

From the proof of Lemma 1, we know that $c'(0) = \frac{1}{g''(0)} \frac{I}{0+I+\mu_r}$ when the boundary condition is $c(0) = 0$. Hence, the slope of $c(x)$ is higher at $x = 0$ when μ_r is lower. Since the solution $c(x)$ is differentiable, there exists $\varepsilon > 0$ such that for all $x \in (0, \varepsilon)$ the slope of $c(x)$ is higher when μ_r is lower. Let $\mu_{r1} < \mu_{r2}$. From continuity it follows that $c(x; \mu_{r1}) > c(x; \mu_{r2})$ and equivalently $b(x; \mu_{r1}) > b(x; \mu_{r2})$ for some neighborhood $(0, \delta)$. Next, we want to show that $b(x; \mu_{r1}) > b(x; \mu_{r2})$ for all $x \in (0, \infty)$. Suppose there existed $x' > 0$ such that $b(x'; \mu_{r1}) = b(x'; \mu_{r2})$. From (1), we know that at this point $b'(x'; \mu_{r1}) > b'(x'; \mu_{r2})$. Hence, the slope of the lower bias curve (for $\mu_r = \mu_{r2}$) is lower than the slope of the higher bias curve (for $\mu_r = \mu_{r1}$) and hence $b(x; \mu_{r1}) > b(x; \mu_{r2})$ for all $x \in (0, \infty)$. ■

Proof of Lemma 3

Suppose, for a given X_{nd}, $E[\widetilde{x} + I + \mu_r | x \in X_{nd}] = \bar{x}_{nd}$. Further, if $\{x | x < x_I\} \cap X_{nd} = \emptyset$ then $E[\widetilde{x} | \widetilde{x} < x_I, x \in X_{nd}] \equiv 0$. It follows that: (i) $\alpha_{nd} = I / E[\widetilde{x} + I + \mu_r | \widetilde{x} < x_I, x \in X_{nd}]$ decreases weakly in x_I, (ii) $\alpha_{nd}(x_I = 0) = \frac{I}{I+\mu_r}$ and (iii) $\lim_{x_I \to \infty} \alpha_{nd}(x_I) = \frac{I}{\bar{x}_{nd}+I+\mu_r}$. Hence, $\alpha \in \left(\frac{I}{I+\mu_r}, \frac{I}{\bar{x}_{nd}+I+\mu_r} \right) \subset (0,1)$. Substituting equation (2) into equation (3) and rearranging yields

$$\frac{I}{E[\widetilde{x} | \widetilde{x} < x_I, x \in X_{nd}] + I + \mu_r} x_I + \max\{0, \mu_r - g(b(x_I))\} = \left(1 - \frac{I}{E[\widetilde{x} | \widetilde{x} < x_I, x \in X_{nd}] + I + \mu_r}\right)(I + \mu_r).$$

Both the LHS and the RHS are continuous in x_I. Moreover, for $x_I = 0$ the LHS equals 0 and the RHS equals μ_r while for $x_I \to \infty$ the LHS approaches ∞ and the RHS equals $\frac{\bar{x}_{nd}+\mu_r}{\bar{x}_{nd}+I+\mu_r}(I + \mu_r)$ which is finite. Hence, there exists $x_I \in (0, \infty)$ such that equation (3) holds when α_{nd} is given by $\alpha_{nd} = I / E[\widetilde{x} + I + \mu_r | \widetilde{x} < x_I, x \in X_{nd}]$. ■

Proof of Lemma 4

As an intermediate step, we prove the following lemma.

Lemma 6 *If there exists a non-disclosure interval,* (x_1^D, x_2^D) *such that there exists* x' *which is* ε *to the left of* x_1^D *for which the manager issues a report and invests then for any* $x \in (x_1^D, x_2^D)$ *the manager does not invest.*

Proof. Let $y \in \{0,1\}$ denote the investment decision where $y=1$ indicates that the firm pursues the investment opportunity and $y=0$ otherwise. Suppose type x_1^D were indifferent between issuing a report $x_R\left(x_1^D\right)$ and investing and between not issuing a report and investing, i.e.,

$$\begin{aligned}&\left(1-\frac{I}{E\left(\tilde{x}|x_R\left(x_1^D\right)\right)+I+\mu_r}\right)\left(x_1^D+I+\mu_r\right)-g\left(x_R\left(x_1^D\right)-x_1^D\right) \\ &=\left(1-\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}\right)\left(x_1^D+I+\mu_r\right)\end{aligned} \tag{10}$$

For x_1^D to be indifferent, investors' beliefs have to be such that $E\left(\tilde{x}|x_R\left(x_1^D\right)\right) > E\left(\tilde{x}|nd,y=1\right)$ because the disclosure costs are strictly positive (due to $b\left(x_1^D\right) > 0$). We also know that type x' prefers to invest and disclose over non-disclosure and investment, i.e.,

$$\begin{aligned}A \equiv &\left(1-\frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right)+I+\mu_r}\right)\left(x'+I+\mu_r\right)-g\left(x_R\left(x'\right)-x'\right) \\ &-\left(1-\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}\right)\left(x'+I+\mu_r\right)>0\end{aligned} \tag{11}$$

Since x' is sufficiently close to x_1^D it is the case that $E\left(\tilde{x}|x_R\left(x_1^D\right)\right) > E\left(\tilde{x}|x_R\left(x'\right)\right) > E\left(\tilde{x}|nd,y=1\right)$. The difference between the payoff of type x_1^D from mimicking x' and his payoff from investing without disclosing is given by

$$\begin{aligned}&\left(1-\frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right)+I+\mu_r}\right)\left(x_1^D+I+\mu_r\right)-g\left(x_R\left(x'\right)-x_1^D\right) \\ &-\left(1-\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}\right)\left(x_1^D+I+\mu_r\right).\end{aligned} \tag{12}$$

In order to arrive at a contradiction, we want to show that the expression in (12) is positive (which implies that type x_1^D wants to deviate and mimic x'). We can rewrite the expression in (12) as:

$$\begin{aligned}&\left(1-\frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right)+I+\mu_r}\right)\left(x'+\varepsilon+I+\mu_r\right)-g\left(x_R\left(x'\right)-x'-\varepsilon\right) \\ &-\left(1-\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}\right)\left(x'+\varepsilon+I+\mu_r\right),\end{aligned}$$

which yields

$$A+\left(\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}-\frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right)+I+\mu_r}\right)\varepsilon+g\left(x_R\left(x'\right)-x'\right)-g\left(x_R\left(x'\right)-x'-\varepsilon\right) \tag{13}$$

where A is defined in (11). Note that since $\left(\tilde{x}|x_R\left(x'\right)\right) > E\left(\tilde{x}|nd,y=1\right)$ we have

$$\left(\frac{I}{E\left(\tilde{x}|nd,y=1\right)+I+\mu_r}-\frac{I}{E\left(\tilde{x}|x_R\left(x'\right)\right)+I+\mu_r}\right)\varepsilon>0.$$

Also, since the bias is positive we have

$$g\left(x_R\left(x'\right)-x'\right)-g\left(x_R\left(x'\right)-x^{'}-\varepsilon\right)>0.$$

This and the fact that $A>0$ imply that the expression in (13) is positive. As a result, type x_1^D prefers mimicking the report of x' which contradicts the assumed equilibrium behavior. ■

To complete the proof of Lemma 4, we show that if there exists a type x that prefers to invest without issuing a report then all types to the left of x also prefer to invest without issuing a report. Based on Lemma 6, type x belongs to the left-most non-disclosure interval. Hence, all types to the left of x do not issue a report. We need to show that if x prefers investing without issuing a report to non-investing without issuing a report then all types to the left of x also do. This follows from the fact that lower types give up the same fraction in exchange for the same expected return on investment μ_r but that (the fraction of) their firm is worth less. ■

Proof of Proposition 1

We start out by showing that $g\left(b\left(x\right)\right)-\mu_r$ is monotonically decreasing in μ_r. This implies that there exists $\mu_r^*>0$ such that for $\mu_r\geq\mu_r^*$ the disclosure costs $g\left(b\left(x\right)\right)$ are (weakly) less than the expected return on investment μ_r for all x and that for $\mu_r<\mu_r^*$ there are some values of x for which the disclosure costs $g\left(b\left(x\right)\right)$ strictly exceed the expected return on investment μ_r.

Lemma 7 *For any given x, $g\left(b\left(x\right)\right)-\mu_r$ is monotonically decreasing in μ_r where $b\left(x\right)$ is given by (1) with $b\left(0\right)=0$ as a boundary condition.*

Proof. We know from Lemma 2 that $b\left(x\right)$ is decreasing in μ_r. It follows that $g\left(b\left(x\right)\right)-\mu_r$ is decreasing in μ_r. ■

Next, we show that for sufficiently small x, $b\left(x\right)$ is strictly greater than $b_I\left(x\right)$. From this it follows that $b\left(x\right)$ and $b_I\left(x\right)$ intersect at least once.

Lemma 8 *There exists $\varepsilon>0$ such that $b\left(x\right)>b_I\left(x\right)$ for $x\in\left(0,\varepsilon\right)$.*

Proof. From Definition 1, it follows that

$$b_I\left(x\right)=g^{-1}\left(I\frac{x-E\left(\tilde{x}|\tilde{x}<x\right)}{E\left(\tilde{x}|\tilde{x}<x\right)+I+\mu_r}\right)$$

and

$$b_I'\left(x\right)=\frac{1}{g'\left(b_I\left(x\right)\right)}\frac{I}{E\left(\tilde{x}|\tilde{x}<x\right)+I+\mu_r}\left(1-\frac{x+I+\mu_r}{E\left(\tilde{x}|\tilde{x}<x\right)+I+\mu_r}\left(x-E\left(\tilde{x}|\tilde{x}<x\right)\right)\frac{f\left(x\right)}{F\left(x\right)}\right),$$

where $F(x)$ is the cumulative distribution function of the value of the firm's assets in place. We first compute $\lim_{x\to 0}\frac{b'(x)}{b_I'(x)} > 1$ where $b'(x)$ is given by equation (1).

$$\begin{aligned}
\lim_{x\to 0}\frac{b'(x)}{b_I'(x)} &= \lim_{x\to 0}\frac{\frac{I}{g'(b(x))(x+I+\mu_r)}-1}{\frac{1}{g'(b_I(x))}\frac{I}{E(\tilde{x}|\tilde{x}<x)+I+\mu_r}\left(1-\frac{x+I+\mu_r}{E(\tilde{x}|\tilde{x}<x)+I+\mu_r}\left(x-E\left(\tilde{x}|\tilde{x}<x\right)\right)\frac{f(x)}{F(x)}\right)} \\
&= \lim_{x\to 0}\frac{\frac{I}{(x+I+\mu_r)}-g'(b(x))}{\frac{I}{E(\tilde{x}|\tilde{x}<x)+I+\mu_r}\left(1-\frac{x+I+\mu_r}{E(\tilde{x}|\tilde{x}<x)+I+\mu_r}\left(x-E\left(\tilde{x}|\tilde{x}<x\right)\right)\frac{f(x)}{F(x)}\right)}\lim_{x\to 0}\frac{g'(b_I(x))}{g'(b(x))} \\
&= \frac{\frac{I}{I+\mu_r}-0}{\frac{I}{I+\mu_r}\left(1-\lim_{x\to 0}\left(x-E\left(\tilde{x}|\tilde{x}<x\right)\right)\frac{f(x)}{F(x)}\right)}\lim_{x\to 0}\frac{g'(b_I(x))}{g'(b(x))} \\
&= 2\lim_{x\to 0}\frac{g'(b_I(x))}{g'(b(x))}
\end{aligned}$$

where the last equality follows from

$$\lim_{x\to 0}x\frac{f(x)}{F(x)} = \lim_{x\to 0}\frac{f(x)+xf'(x)}{f(x)} = 1+\lim_{x\to 0}\frac{xf'(x)}{f(x)} = 1 \text{ for } f(0)>0 \text{ and } \left|f'(0)\right|<\infty$$

$$\begin{aligned}
\lim_{x\to 0}E\left(\tilde{x}|\tilde{x}<x\right)\frac{f(x)}{F(x)} &= \lim_{x\to 0}\frac{f(x)}{F^2(x)}\int_0^x zf(z)\,dz = \lim_{x\to 0}\frac{f'(x)\int_0^x zf(z)\,dz+f(x)\,xf(x)}{2F(x)\,f(x)} \\
&= \lim_{x\to 0}\frac{f'(x)}{2f(x)}E\left(\tilde{x}|\tilde{x}<x\right)+\lim_{x\to 0}\frac{xf(x)}{2F(x)} = \frac{1}{2}.
\end{aligned}$$

We have just shown that

$$\lim_{x\to 0}\frac{b'(x)}{b_I'(x)}\frac{g'(b(x))}{g'(b_I(x))} = 2.$$

First, consider the case when $\lim_{x\to 0}\frac{b'(x)}{b_I'(x)} \leq 1$. Then, $\lim_{x\to 0}\frac{g'(b(x))}{g'(b_I(x))} \geq 2$. Which implies that there exists an interval $(0,\varepsilon)$ for which $b(x) > b_I(x)$ (which can only hold when $\lim_{x\to 0}\frac{b'(x)}{b_I'(x)} = 1$; if $\lim_{x\to 0}\frac{b'(x)}{b_I'(x)} < 1$ we arrive at a contradiction). Next, consider the case when $\lim_{x\to 0}\frac{b'(x)}{b_I'(x)} > 1$. Then, there must be an interval for which $b(\cdot)$ is steeper than $b_I(\cdot)$ because both functions are differentiable for $x > 0$, and as a result there exists an interval $(0,\varepsilon)$ for which $b(x) > b_I(x)$. Hence, there always exists an interval $(0,\varepsilon)$ for which $b(x) > b_I(x)$. ■

Next, we show that there always exists an equilibrium in which low types do not disclose but invest.

Lemma 9 *There always exists $x_I > 0$ s.t. firms with $x < x_I$ prefer raising capital and investing without issuing a report while firms with $x > x_I$ never prefer raising capital and investing without issuing a report over (i) not issuing a report and not raising capital, and (ii) issuing a report, raising capital and investing when investors' beliefs are characterized by Lemma 1.*

Proof. First, we consider the case in which the parameters are such that $b(x) \le g^{-1}(\mu_r)$ for all x (i.e., $\mu_r \ge \mu_r^*$). Let x' be the highest value of the firm's assets in place for which the manager raises capital without issuing a report. Define $u_1(x')$ as the difference between the payoff of type x' if he raises capital without issuing a report and the payoff if he raises capital and issues a report $x_R(x') = x' + b(x')$, i.e.,

$$u_1(x') = \frac{E[\tilde{x}|x < x'] + \mu_r}{E[\tilde{x}|x < x'] + I + \mu_r}\left(x' + \mu_r + I\right) - \left(x' + \mu_r - g\left(b\left(x'\right)\right)\right).$$

We want to show that there exists $x' > 0$ such that $u_1(x') = 0$. From the definition of $u_1(x')$ it follows that $\lim_{x'\to 0} u_1(x') = 0$ and $\lim_{x'\to\infty} u_1(x') = -\infty$. Moreover,

$$\begin{aligned}
\left.\frac{\partial u_1(x')}{\partial x'}\right|_{x'=0} &= \left.\frac{\left(E[\tilde{x}|x < x'] + I + \mu_r\right)\frac{\partial E[\tilde{x}|x<x']}{\partial x'} - \left(E[\tilde{x}|x < x'] + \mu_r\right)\frac{\partial E[\tilde{x}|x<x']}{\partial x'}}{\left(E[\tilde{x}|x < x'] + I + \mu_r\right)^2}\left(x' + \mu_r + I\right)\right|_{x'=0} \\
&+ \left.\left(\frac{E[\tilde{x}|x < x'] + \mu_r}{E[\tilde{x}|x < x'] + I + \mu_r} - \left(1 - g'\left(b\left(x\right)\right)b'\left(x'\right)\right)\right)\right|_{x'=0} \\
&= \left.\left(\frac{I\left(x' + \mu_r + I\right)}{\left(E[\tilde{x}|x < x'] + I + \mu_r\right)^2}\frac{\partial E[\tilde{x}|x < x']}{\partial x'}\right)\right|_{x'=0} \\
&+ \left.\left(\frac{E[\tilde{x}|x < x'] + \mu_r}{E[\tilde{x}|x < x'] + I + \mu_r} - \left(1 - \frac{I}{x' + I + \mu_r}\frac{b'(x')}{1 + b'(x')}\right)\right)\right| \\
&= \frac{I}{I + \mu_r}\frac{\partial E[\tilde{x}|x < x']}{\partial x'} + \frac{\mu_r}{I + \mu_r} - \left(1 - \frac{I}{I + \mu_r}\right) \\
&= \frac{I}{I + \mu_r}\frac{\partial E[\tilde{x}|x < x']}{\partial x'} > 0.
\end{aligned}$$

Hence, continuity of $u(x')$ implies that there exists at least one $x' > 0$ such that $u_1(x') = 0$. Let

$$x_I = \min\left\{x' \left| u_1\left(x'\right) = 0, x' > 0\right.\right\}.$$

We have shown that $\lim_{x'\to 0} u_1(x') = 0$, $\lim_{x'\to\infty} u_1(x') = -\infty$, and that $u_1(x')$ is increasing in x'. By construction of x_I, for all $x' \in [0, x_I)$ we have $u_I(x') \ge 0$. Since $\frac{E[\tilde{x}|x<x'] + \mu_r}{E[\tilde{x}|x<x'] + I + \mu_r}$ is increasing in x' it follows that

$$\frac{E[\tilde{x}|x < x_I] + \mu_r}{E[\tilde{x}|x < x_I] + I + \mu_r}\left(x' + \mu_r + I\right) - \left(x' + \mu_r - g\left(b\left(x'\right)\right)\right) \ge 0.$$

Hence, for all $x' \in [0, x_I)$, the manager prefers investment without disclosure to investment with disclosure. We further need to show that no type $x' > x_I$ wants to deviate and invest without disclosure. We know that $x' > x_I$ does not mimic x_I by issuing the report $x^R(x_I)$ and investing. That is,

$$x' + \mu_r - g\left(b\left(x'\right)\right) \ge \frac{x_I + \mu_r}{x_I + I + \mu_r}\left(x' + \mu_r + I\right) - g\left(b\left(x_I\right) - \left(x' - x_I\right)\right).$$

We want to show that type $x' > x_I$ prefers issuing the report $x^R(x_I)$ to investment without disclosure, i.e., we want to show that

$$\frac{x_I + \mu_r}{x_I + I + \mu_r}\left(x' + \mu_r + I\right) - g\left(b(x_I) - \left(x' - x_I\right)\right) > \frac{E[\tilde{x}|x < x_I] + \mu_r}{E[\tilde{x}|x < x_I] + I + \mu_r}\left(x' + \mu_r + I\right). \quad (14)$$

Note that we can restrict the analysis to types x' that do not need to bias their report downwards in order to mimic x_I. The reason is that types x' that would have to bias its report downwards in order to mimic x_I can mimic a higher type without incurring any biasing costs. Hence, we restrict attention to $x' \in (x_I, x_I + b(x_I))$. Taking the derivative of the LHS of (14) net of the RHS of (14) with respect to x' yields

$$\frac{x_I + \mu_r}{x_I + I + \mu_r} + g'\left(b(x_I) - \left(x' - x_I\right)\right) - \frac{E[\tilde{x}|x < x_I] + \mu_r}{E[\tilde{x}|x < x_I] + I + \mu_r},$$

which is positive for $x' \in (x_I, x_I + b(x_I))$. We know that (14) holds with equality for x_I. Hence, the inequality in (14) holds for all $x' \in (x_I, x_I + b(x_I))$. This proves that no type $x' > x_I$ wants to deviate and invest without disclosure. Moreover, no manager who makes a disclosure wants to deviate to a bias other than the bias described in Lemma 1. The reason is that by construction of the bias function $b(x)$ in Lemma 1, any deviation to a report of another type is precluded. As a result, the manager's equilibrium strategy is as follows: $x \in [0, x_I)$ invest but do not disclose and $x \in [x_I, \infty)$ invest and disclose where the manager's reporting bias is described in Lemma 1.

Next, we consider the case in which $b(x) > g^{-1}(\mu_r)$ for some x. We define

$$u_1(x') = \frac{E[\tilde{x}|x < x'] + \mu_r}{E[\tilde{x}|x < x'] + I + \mu_r}\left(x' + \mu_r + I\right) - \left(x' + \max\left\{0, \mu_r - g\left(b\left(x'\right)\right)\right\}\right).$$

By the same argument as above, there exists at least one $x' > 0$ such that $u_1(x') = 0$. Let $x_I = \min\{x' \,|u_1(x') = 0, x' > 0\}$. Moreover, following the same line of argument we can show that no manager with assets in place valued at more than x_I wants to deviate and invest without issuing a report. As a result, the investing and reporting strategy described in part (*ii*) of Proposition 1 constitute an equilibrium when investors' beliefs given a report (including reports off the equilibrium path) are given by the bias function in Lemma 1. ■

Proof of Corollary 1

The cumulative distribution function of $h(x) = \frac{\pi(x)f(x)}{\int_0^\infty \pi(y)f(y)dy}$ is given by

$$H(x) = \int_0^x h(z)\,dz = \int_0^x \frac{\pi(z)\,f(z)}{\int_0^\infty \pi(y)\,f(y)\,dy}dz = \frac{F(x)\,E_f[\pi(\tilde{x})\,|\tilde{x} < x]}{\int_0^\infty \pi(y)\,f(y)\,dy}$$

We want to show that $E_f[\tilde{x}|\tilde{x}<x] < E_h[\tilde{x}|\tilde{x}<x]$. We can rewrite $E_h[\tilde{x}|\tilde{x}<x]$ as

$$E_h[\tilde{x}|\tilde{x}<x] = \frac{1}{H(x)}\int_0^x \frac{z\pi(z)f(z)}{\int_0^\infty \pi(y)f(y)\,dy}dz = \frac{1}{F(x)E_f[\pi(\tilde{x})|\tilde{x}<x]}\int_0^x z\pi(z)f(z)\,dz$$

Hence, $E_f[\tilde{x}|\tilde{x}<x] < E_h[\tilde{x}|\tilde{x}<x]$ is equivalent to

$$\begin{aligned} E_f[\tilde{x}|\tilde{x}<x] &< \frac{1}{F(x)E_f[\pi(\tilde{x})|\tilde{x}<x]}\int_0^x z\pi(z)f(z)\,dz \\ E_f[\tilde{x}|\tilde{x}<x]\,E_f[\pi(\tilde{x})|\tilde{x}<x] &< E_f[\tilde{x}\pi(\tilde{x})|\tilde{x}<x] \\ 0 &< Cov_f[\tilde{x},\pi(\tilde{x})|\tilde{x}<x] \end{aligned}$$

which always holds for increasing functions $\pi(x)$ because (we omit the condition $\tilde{x}<x$ and the subscript f for readability)

$$\begin{aligned} Cov[\tilde{x},\pi(\tilde{x})] &= E[(\tilde{x}-\mu_x)(\pi(\tilde{x})-E[\pi(\tilde{x})])] \\ &= E[(\tilde{x}-\mu_x)(\pi(\tilde{x})-\pi(\mu_x))] + E[\tilde{x}-\mu_x](\pi(\mu_x)-E[\pi(\tilde{x})]) \\ &= E[(\tilde{x}-\mu_x)(\pi(\tilde{x})-\pi(\mu_x))]\,. \end{aligned}$$

The last expression is non-negative for every x because $\pi(x)$ is increasing. ■

Proof of Corollary 2

There is no underinvestment in the equilibrium described in part (i) and in part $(ii)\,a$ of the proposition. In part $(ii)\,b$ of the proposition, the non-investment interval is $\left(x_I, x_2^D\right)$ and in part $(ii)\,c$ of the proposition, the non-investment interval is $\left(x_1^D, x_2^D\right)$. In the following, we show that x_I and x_1^D increase in μ_r while x_2^D decreases in μ_r.

Recall that x_1^D and x_2^D are given by $g\left(b\left(x_1^D\right)\right) = g\left(b\left(x_2^D\right)\right) = \mu_r$. Differentiating this condition with respect to x_i^D and μ_r for $i=1,2$ yields

$$g'\left(b\left(x_i^D\right)\right)b'\left(x_i^D\right)dx_i^D + \left(g'\left(b\left(x_i^D\right)\right)\frac{\partial b\left(x_i^D\right)}{\partial \mu_r} - 1\right)d\mu_r = 0.$$

Rearranging yields

$$\frac{dx_i^D}{d\mu_r} = -\frac{g'\left(b\left(x_i^D\right)\right)\frac{\partial b\left(x_i^D\right)}{\partial \mu_r} - 1}{g'\left(b\left(x_i^D\right)\right)b'\left(x_i^D\right)}.$$

$g'\left(b\left(x_i^D\right)\right)$ is positive and Lemma 2 implies that $\frac{\partial b\left(x_i^D\right)}{\partial \mu_r}$ is negative. Hence, the numerator is always negative and the denominator takes the sign of $b'\left(x_i^D\right)$. For x_1^D $\left(x_2^D\right)$ the bias is increasing and hence $\frac{dx_1^D}{d\mu_r} > 0$ $\left(\frac{dx_2^D}{d\mu_r} < 0\right)$.

In order to establish how x_I varies with μ_r, we first derive how $b_I(x)$ varies with μ_r. From (1) it follows that

$$\frac{\partial b_I(x)}{\partial \mu_r} = -\frac{1 - \frac{((x+I+\mu_r)+E(\tilde{x}|\tilde{x}<x)+\mu_r)(E(\tilde{x}|\tilde{x}<x)+I+\mu_r)-(E(\tilde{x}|\tilde{x}<x)+\mu_r)(x+I+\mu_r)}{(E(\tilde{x}|\tilde{x}<x)+I+\mu_r)^2}}{-g'(b_I(x))}$$
$$= \frac{E(\tilde{x}|\tilde{x}<x)-x}{g'(b_I(x))(E(\tilde{x}|\tilde{x}<x)+I+\mu_r)^2} I < 0.$$

In part *ii b* of the proposition, x_I is given by the intersection of $b_I(x)$ and $g^{-1}(\mu_r)$. Hence,

$$\frac{dx_I}{d\mu_r} = -\frac{\frac{\partial b_I(x)}{\partial \mu_r} - \frac{\partial g^{-1}(\mu_r)}{\partial \mu_r}}{\frac{\partial b_I(x)}{\partial x}} > 0. \blacksquare$$

Proof of Claim in Footnote ?? I WOULD TAKE IT OUT

Claim 1 *In any equilibrium in which the reporting strategy is given by Lemma 1, there are at most two non-disclosure intervals.*

We start by proving the following lemma.

Lemma 10 *If the highest type of a non-disclosure interval, x', does not invest then there does not exist an additional non-disclosure interval to the right of x'.*

Proof. x' must be indifferent between disclosing and investing and not disclosing and not investing. This requires $b(x') = g^{-1}(\mu_r)$. Suppose there exists an additional non-disclosure interval, $\left(x_1^D, x_2^D\right)$, where $x' < x_1^D < x_2^D$. Then, the bias must be decreasing over the interval $\left[x', x_1^D\right]$ because the bias can not exceed $g^{-1}(\mu_r)$ and the bias never increases in x once it has decreased. This implies that the bias at x_1^D is strictly lower than $g^{-1}(\mu_r)$. Since x' prefers not to invest if he does not disclose, x_1^D must strictly prefer not to invest if he does not disclose. x_1^D is indifferent between disclosing and investing and not disclosing and not investing only if $b\left(x_1^D\right) = g^{-1}(\mu_r)$. This yields a contradiction because we argued that $b\left(x_1^D\right) < b(x') = g^{-1}(\mu_r)$. $\blacksquare$

From Lemma 6 it follows that the highest type of the second non-disclosure interval, x_2^D, never invests. Following an argument similar to the proof of Lemma 10, it follows that there are at most two non-disclosure intervals. $\blacksquare$

Proof of Lemma 5

In the following, we presume that $x(\theta)$ is either linear or quadratic and that $x(0) = 0$. This subsumes all real manipulation scenarios outlined in Section 5.1. At the end of this proof, we briefly discuss more general conditions for $x(\theta)$ under which the characteristics of the manager's distortion $b(\theta)$ remain qualitatively the same.

Let $k^*(\theta) = \theta$ denote the first-best investment decision. To apply the proof to other scenarios of real manipulation, in the following, we slightly generalize the first-best decision and assume that $k^*(\theta) = \kappa\theta$ for $\kappa > 0$. When choosing the investment distortion b, the manager maximizes the value of his stake in the firm.

$$
\begin{aligned}
&\max_b (1-\alpha) V(\theta, b) \\
&s.t.\ \alpha = \frac{I}{V\left(\widehat{\theta}(x_R), \widehat{b}(x_R)\right)} \\
&\widehat{\theta}(x_R) = \frac{1}{\kappa}\left(x_R - \widehat{b}(x_R)\right) \\
&x_R(\theta) = \kappa\theta + b(\theta) \\
&V(\theta, b) = x(\theta) + I + \mu_r - g(b)
\end{aligned}
$$

where $\widehat{b}(x_R)$ denotes investors' inferences about the investment distortion b when the manager reports investment x_R. The first order condition to the manager's optimization problem is:

$$
\frac{I\left(x'\left(\widehat{\theta}(x_R)\right)\frac{1}{\kappa}\left(1-\frac{\partial\hat{b}(x_R)}{\partial x_R}\right) - g'\left(\widehat{b}(x_R)\right)\frac{\partial\hat{b}(x_R)}{\partial x_R}\right)V(\theta,b)}{V\left(\widehat{\theta}(x_R), \widehat{b}(x_R)\right)^2} + (1-\alpha)\left(-g'(b)\right) = 0.
$$

In a fully separating equilibrium, investors can perfectly infer the manager's bias such that $\hat{b}(x_R(\theta)) = b(\theta)$ which implies $\frac{\partial\hat{b}(x_R)}{\partial x_R}\frac{\partial x_R}{\partial\theta} = \frac{\partial b}{\partial\theta} \Leftrightarrow \frac{\partial\hat{b}(x_R)}{\partial x_R} = \frac{b'(\theta)}{\kappa + b'(\theta)}$ and $V\left(\widehat{\theta}(x_R), \widehat{b}(x_R)\right) = V(\theta, b)$. Simplifying yields

$$
\begin{gathered}
\frac{I\left(x'(\theta)\frac{1}{\kappa+b'(\theta)} - g'(b(\theta))\frac{b'(\theta)}{\kappa+b'(\theta)}\right)}{V(\theta, b(\theta))} - \frac{V(\theta, b(\theta)) - I}{V(\theta, b(\theta))}g'(b(\theta)) = 0 \\
I\left(x'(\theta) - g'(b(\theta))\,b'(\theta)\right) - \left(V(\theta, b(\theta)) - I\right)g'(b(\theta))\left(\kappa + b'(\theta)\right) = 0.
\end{gathered}
$$

Rearranging yields

$$
\begin{aligned}
b'(\theta) &= \frac{Ix'(\theta) - \left(V(\theta, b(\theta)) - I\right)g'(b(\theta))\,\kappa}{V(\theta, b(\theta))}\frac{1}{g'(b(\theta))} \\
b'(\theta) &= \frac{x'(\theta) + g'(b(\theta))\,\kappa}{g'(b(\theta))}\frac{I}{V(\theta, b(\theta))} - \kappa \qquad (15)
\end{aligned}
$$

which yields the equation in Lemma 5 for $\kappa = 1$. We want to show that there exists a unique solution to the above differential equation with the boundary condition $b(0) = 0$. As before, we cannot invoke the Fundamental Theorem of Differential Equations in order to show that the solution exists and is unique because the RHS of (15) is not finite at $b(0) = 0$. In order to show that the solution exists, we perform the same substitution as in the proof of Proposition 1, $c(\theta) = \frac{1}{2}(b(\theta))^2$. This implies that $b(\theta) = \sqrt{2|c(\theta)|}$ and $b'(\theta) = \frac{c'(\theta)}{\sqrt{2|c(\theta)|}}$. Rewriting the differential equation in (15) in terms of $c(\cdot)$ yields

$$c'(\theta) = \frac{x'(\theta) + g'\left(\sqrt{2|c(\theta)|}\right)\kappa}{g'\left(\sqrt{2|c(\theta)|}\right)} \frac{I\sqrt{2|c(\theta)|}}{V\left(\theta, \sqrt{2|c(\theta)|}\right)} - \kappa\sqrt{2|c(\theta)|} \tag{16}$$

with the boundary condition $c(0) = 0$. Let $F(\theta, c) = \frac{x'(\theta)+g'\left(\sqrt{2|c|}\right)\kappa}{g'\left(\sqrt{2|c|}\right)} \frac{I\sqrt{2|c|}}{V\left(\theta,\sqrt{2|c|}\right)} - \kappa\sqrt{2|c|}$ denote the RHS of (16). We want to show that $F(\theta, c)$ is continuous in θ and c at $(0, 0)$. This requires that $\lim_{\theta\to 0} F(\theta, c)$ and $\lim_{c\to 0} F(\theta, c)$ are finite. $\lim_{\theta\to 0} F(\theta, c)$ is finite because, by assumption, $\lim_{\theta\to 0} x'(\theta)$ is finite (for the setting outlined in Section 5.1.1 $x(\theta) = \frac{1}{2}\theta^2$ and hence $\lim_{\theta\to 0} x'(\theta) = 0$) and $x(0) \geq 0$. Hence $F(\theta, c)$ is continuous in θ at $\theta = 0$. Next, we show that $\lim_{c\to 0} F(\theta, c)$ is finite.

$$\lim_{c\to 0} F(\theta, c) = \lim_{c\to 0} \frac{0 + \frac{\partial}{\partial c} g'\left(\sqrt{2|c|}\right)}{\frac{\partial}{\partial c} g'\left(\sqrt{2|c|}\right)} \frac{I}{x(\theta) + I + \mu_r} - 0 = \frac{I}{x(\theta) + I + \mu_r}$$

which is finite. Hence, there exists a continuous and differentiable solution to (16) with the boundary condition $c(0) = 0$. Next, we need to show that $c(\theta)$ provides a solution to the manager's disclosure problem and that the solution is unique. The argument follows exactly the same steps as the corresponding part of the proof in Lemma 1 and is omitted for brevity.

We next want to show that the equilibrium distortion function $b(\theta)$ has the following properties: it is continuous, always positive, initially increasing, obtains a unique maximum and converges to zero as the profitability of the firm's assets in place, θ, goes to infinity.

From $b(0) = 0$ and $b(\theta) = \sqrt{2|c(\theta)|}$ where $c(\theta) \geq 0$ it follows that $b(\theta)$ is initially increasing. However, $b(\theta)$ cannot always be increasing. Suppose $b(\theta)$ were always increasing. Then, for $\theta \to \infty$, $b(\theta)$ and therefore $g'(b(\theta))$ would be strictly positive. Moreover, the manager's payoff is always higher than $\frac{x(0)+\mu_r}{x(0)+I+\mu_r}(x(\theta) + I + \mu_r)$. To see that, consider that the firm value if the manager does not distort the investment decision is $x(\theta)+I+\mu_r > 0$. He retains a positive fraction of the firm value which exceeds the fraction $\frac{x(0)+\mu_r}{x(0)+I+\mu_r}$ he would obtain if investors perceived the profitability of his firm's assets to be zero. For $\theta \to \infty$, this lower bound for the manager's payoff goes to infinity. Moreover, the firm value $V(\theta, b(\theta))$ always exceeds the manager's payoff and hence for $\theta \to \infty$, $V(\theta, b(\theta)) \to \infty$. This would imply that the RHS of (15)

$$\begin{aligned}\lim_{\theta\to\infty} \frac{x'(\theta) + g'(b(\theta))\kappa}{g'(b(\theta))} \frac{I}{V(\theta, b(\theta))} - \kappa &= \lim_{\theta\to\infty} \left(\frac{1}{g'(b(\theta))} \frac{x'(\theta)}{V(\theta, b(\theta))} + \frac{I\kappa}{V(\theta, b(\theta))} \right) - \kappa \\ &< \lim_{\theta\to\infty} \left(\frac{1}{g'(b(\theta))} \frac{x'(\theta)}{x(\theta)} + \frac{I\kappa}{V(\theta, b(\theta))} \right) - \kappa \\ &= -\kappa\end{aligned}$$

where the last equality follows from $\lim_{\theta\to\infty} \frac{x'(\theta)}{x(\theta)} = 0$. This yields a contradiction because $b'(\theta) > 0$ for all θ requires that the RHS of (15) is positive for all $\theta > 0$. As a result, $b(\theta)$ is eventually decreasing. Since $b(\theta) > 0$ for all $\theta > 0$, this implies that $\lim_{\theta\to\infty} b'(\theta) = 0$ which can only hold if $\lim_{\theta\to\infty} \frac{x'(\theta)}{g'(b(\theta))} \frac{I}{V(\theta,b(\theta))} = \kappa$. Since $\lim_{\theta\to\infty} \frac{x'(\theta)}{V(\theta,b(\theta))} = 0$, we need to have $\lim_{\theta\to\infty} g'(b(\theta)) = 0$ which implies $\lim_{\theta\to\infty} b(\theta) = 0$.

Next, we want to show that $b(\theta)$ does not have a local minimum. We have just established that $b(0) = 0$ and $\lim_{\theta\to\infty} b(\theta) = 0$. Suppose $b(\theta)$ would obtain a local minimum for $\theta = \underline{\theta}$. Then, there would also exist a second local maximum to the right of $\underline{\theta}$ and there would exist values of $b^* > b(\underline{\theta})$ s. t. there are four values of θ for which $b(\theta) = b^* > 0$. Let θ_i, $i = 1, 2, 3, 4$ denote the values of θ s.t. $b(\theta_i) = b^*$ and $0 < \theta_1 < \theta_2 < \theta_3 < \theta_4$. It must also be the case that $b'(\theta_1), b'(\theta_3) > 0$ and $b'(\theta_2), b'(\theta_4) < 0$ where $b'(\cdot)$ is given in equation (15). Since $b(\theta_i) = b^*$, the slope of $b(\cdot)$ at θ_i, $i = 1, 2, 3, 4$ equals

$$b'(\theta_i) = \frac{x'(\theta_i) + g'(b^*)\kappa}{g'(b^*)} \frac{I}{V(\theta_i, b^*)} - \kappa. \tag{17}$$

From $b'(\theta_1), b'(\theta_3) > 0$ and $b'(\theta_2), b'(\theta_4) < 0$ it follows that RHS of (17) must have at least three positive roots. Solving $RHS\,(17) = 0$ yields

$$x'(\theta_i) = \left(\frac{V(\theta_i, b^*)}{I} - 1 \right) g'(b^*)\kappa = \frac{x(\theta_i) - g(b^*) + \mu_r}{I} g'(b^*)\kappa \tag{18}$$

If $x(\cdot)$ is quadratic in θ_i the above equation is a second order polynomial and hence has at most two roots. This is a contradiction and hence the distortion function $b(\theta)$ does not have a local minimum.■

The proof continues to hold even if $x(\theta)$ is neither linear or quadratic as long as (1) $\lim_{\theta\to\infty} \frac{x'(\theta)}{x(\theta)} = 0$, and (2) equation (18) has at most two positive roots.

Proof of Proposition 2

We focus on equilibria in which investors can perfectly infer the manager's private information if he makes a disclosure. Hence, if the manager makes a disclosure, investors break even and receive expected cash flows equal to their initial investment while the manager bears the entire costs of distorting his pre-issuance investment decision. This is similar to Section 3 in which the costs of biasing the report are personal costs to the manager. As a result, the proof of Proposition 2 follows the same line of argument as the proof of Proposition 1.

As in the proof of Lemma 5, we allow the first-best value of the firm's existing business, $x(\theta)$, to be either linear or quadratic in the manager's private information, θ. We also allow the first-best pre-equity issuance investment decision to equal $k^*(\theta) = \kappa\theta$ where $\kappa = 1$ for the model outlined in Section 5.1.1.

We start again by showing that $g(b(\theta)) - \mu_r$ is monotonically decreasing in μ_r. This implies that there exists $\mu_r^* > 0$ such that for $\mu_r \geq \mu_r^*$ the reduction in firm value due to investment distortions, $g(b(\theta))$, are (weakly) less than the expected return on the new investment opportunity μ_r for all θ and that for $\mu_r < \mu_r^*$ there are some values of θ for which $g(b(\theta))$ strictly exceed the expected return on the new investment opportunity, μ_r.

Lemma 11 *For any given θ, $g(b(\theta)) - \mu_r$ is monotonically decreasing in μ_r where $b(\theta)$ is given by* (6) *with $b(0) = 0$ as a boundary condition.*

Proof. A sufficient condition for this result to hold is that $\frac{\partial b(\theta)}{\partial \mu_r} \leq 0$. The proof follows exactly the same argument as the Proof of Lemma 2. ■

Next, we show that for sufficiently small θ, $b(\theta)$ is strictly greater than $b_I(\theta)$. From this it follows that $b(\theta)$ and $b_I(\theta)$ intersect at least once.

Lemma 12 *There exists $\varepsilon > 0$ such that $b(\theta) > b_I(\theta)$ for $\theta \in (0, \varepsilon)$.*

Proof. From Definition 2, it follows that

$$b_I(\theta) = g^{-1}\left(I\frac{x(\theta) - E(x(\tilde{\theta})|\tilde{\theta} < \theta)}{E(x(\tilde{\theta})|\tilde{\theta} < \theta) + I + \mu_r}\right)$$

Taking the derivative of $b_I(\theta)$ with respect to θ and simplifying the expression yields

$$b_I'(\theta) = \frac{\frac{1}{g'(b_I(\theta))}I}{E(x(\tilde{\theta})|\tilde{\theta} < \theta) + I + \mu_r}\left(x'(\theta) - \left(x(\theta) - \frac{x(\theta) + I + \mu_r}{E(x(\tilde{\theta})|\tilde{\theta} < \theta) + I + \mu_r}E\left(x\left(\tilde{\theta}\right)|\tilde{\theta} < \theta\right)\right)\frac{f(\theta)}{F(\theta)}\right)$$

where $F(\theta)$ is the cumulative distribution function of the value of the profitability θ. We first compute $\lim_{\theta\to 0}\frac{b'(\theta)}{b'_I(\theta)} > 1$ where $b'(\theta)$ is given by equation (6).

$$\lim_{\theta\to 0}\frac{b'(\theta)}{b'_I(\theta)} = \lim_{\theta\to 0}\frac{\frac{x'(\theta)+\kappa g'(b(\theta))}{g'(b(\theta))}\frac{I}{x(\theta)+I+\mu_r-g(b(\theta))}-\kappa}{\frac{1}{g'(b_I(\theta))}\frac{I}{E(x(\tilde{\theta})|\tilde{\theta}<\theta)+I+\mu_r}\left(x'(\theta)-\left(x(\theta)-\frac{x(\theta)+I+\mu_r}{E(x(\tilde{\theta})|\tilde{\theta}<\theta)+I+\mu_r}E(x(\tilde{\theta})|\tilde{\theta}<\theta)\right)\frac{f(\theta)}{F(\theta)}\right)}$$

$$= \frac{\frac{I\lim_{\theta\to 0}x'(\theta)+0}{I+\mu_r}-0}{\frac{I}{I+\mu_r}\left(\lim_{\theta\to 0}x'(\theta)-\lim_{\theta\to 0}\left(x(\theta)-E(x(\tilde{\theta})|\tilde{\theta}<\theta)\right)\frac{f(\theta)}{F(\theta)}\right)}\lim_{\theta\to 0}\frac{g'(b_I(\theta))}{g'(b(\theta))}$$

$$= \frac{\frac{Ix'(0)}{I+\mu_r}}{\frac{I}{I+\mu_r}\left(x'(0)-\lim_{\theta\to 0}\left(x(\theta)-E(x(\tilde{\theta})|\tilde{\theta}<\theta)\right)\frac{f(\theta)}{F(\theta)}\right)}\lim_{\theta\to 0}\frac{g'(b_I(\theta))}{g'(b(\theta))}$$

$$= \frac{\frac{Ix'(0)}{I+\mu_r}}{\frac{I}{I+\mu_r}\left(x'(0)-\left(x'(0)-\frac{1}{2}x'(0)\right)\right)}\lim_{\theta\to 0}\frac{g'(b_I(\theta))}{g'(b(\theta))}$$

$$= 2\lim_{\theta\to 0}\frac{g'(b_I(\theta))}{g'(b(\theta))}$$

where the last two equalities follow from

$$\lim_{\theta\to 0}x(\theta)\frac{f(\theta)}{F(\theta)} = \lim_{\theta\to 0}\frac{x'(\theta)f(\theta)+x(\theta)f'(\theta)}{f(\theta)} = x'(0)+\lim_{\theta\to 0}\frac{x(\theta)f'(\theta)}{f(\theta)} = x'(0) \text{ for } f(0)>0 \text{ and } \left|f'(0)\right|<\infty$$

$$\lim_{\theta\to 0}E\left(x\left(\tilde{\theta}\right)|\tilde{\theta}<\theta\right)\frac{f(\theta)}{F(\theta)} = \lim_{\theta\to 0}\frac{f(\theta)}{F^2(\theta)}\int_0^\theta x\left(\tilde{\theta}\right)f\left(\tilde{\theta}\right)d\tilde{\theta} = \lim_{\theta\to 0}\frac{f'(\theta)\int_0^\theta x\left(\tilde{\theta}\right)f\left(\tilde{\theta}\right)d\tilde{\theta}+f(\theta)x(\theta)f(\theta)}{2F(\theta)f(\theta)}$$

$$= \lim_{\theta\to 0}\frac{f'(x)}{2f(x)}E\left(x\left(\tilde{\theta}\right)|\tilde{\theta}<\theta\right)+\lim_{\theta\to 0}\frac{x(\theta)f(\theta)}{2F(\theta)} = \lim_{\theta\to 0}\frac{x'(\theta)f(\theta)+x(\theta)f'(\theta)}{2f(\theta)} = \frac{1}{2}\lim_{\theta\to 0}x'(\theta) = \frac{1}{2}x'(0).$$

We have just shown that

$$\lim_{\theta\to 0}\frac{b'(\theta)}{b'_I(\theta)}\frac{g'(b(\theta))}{g'(b_I(\theta))} = 2.$$

First, consider the case when $\lim_{\theta\to 0}\frac{b'(\theta)}{b'_I(\theta)} \leq 1$. Then, $\lim_{\theta\to 0}\frac{g'(b(\theta))}{g'(b_I(\theta))} \geq 2$. Which implies that there exists an interval $(0,\varepsilon)$ for which $b(\theta) > b_I(\theta)$ (which can only hold when $\lim_{\theta\to 0}\frac{b'(\theta)}{b'_I(\theta)} = 1$,since if $\lim_{\theta\to 0}\frac{b'(\theta)}{b'_I(\theta)} < 1$ we arrive at a contradiction). Next, consider the case when $\lim_{\theta\to 0}\frac{b'(\theta)}{b'_I(\theta)} > 1$. Then, there must be an interval for which $b(\cdot)$ is steeper than $b_I(\cdot)$ because both functions are differentiable for $\theta > 0$, and hence there exists an interval $(0,\varepsilon)$ for which $b(\theta) > b_I(\theta)$. ■

Next, we show that there always exists an equilibrium in which low types do not disclose but raise equity capital and pursue the new investment opportunity.

Lemma 13 *There always exists $x_I > 0$ s.t. firms with $x(\theta) < x_I$ prefer raising capital and pursuing the new investment opportunity without issuing a report while firms with $x(\theta) > x_I$ never prefer raising capital and pursuing the new investment opportunity without issuing a report over (i)* *not issuing a report and not raising capital, and (ii) issuing a report, raising capital and pursuing the new investment opportunity when investors' beliefs are characterized by Lemma 5.*

Proof. First, we consider the case in which the parameters are such that $b(\theta) \leq g^{-1}(\mu_r)$ for all θ (i.e., $\mu_r \geq \mu_r^*$). Let θ' be the highest value of the firm's assets in place for which the manager raises capital without issuing a report. Define $u_1(\theta')$ as the difference between the payoff of type θ' if he raises capital without issuing a report and the payoff if he raises capital and issues a report $x_R(\theta') = \kappa\theta' + b(\theta')$, i.e.,

$$u_1(x') = \frac{E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + \mu_r}{E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + I + \mu_r}\left(x(\theta') + \mu_r + I\right) - \left(x(\theta') + \mu_r - g\left(b(\theta')\right)\right).$$

We want to show that there exists $\theta' > 0$ such that $u_1(\theta') = 0$. From the definition of $u_1(\theta')$ it follows that $\lim_{\theta' \to 0} u_1(\theta') = 0$ and $\lim_{\theta' \to \infty} u_1(\theta') = -\infty$. Moreover,

$$\begin{aligned}
&\left.\frac{\partial u_1(\theta')}{\partial \theta'}\right|_{\theta'=0} \\
&= \left.\frac{\left(E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + I + \mu_r\right)\frac{\partial E[x(\tilde{\theta})|\tilde{\theta}<\theta']}{\partial \theta'} - \left(E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + \mu_r\right)\frac{\partial E[x(\tilde{\theta})|\tilde{\theta}<\theta']}{\partial \theta'}}{\left(E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + I + \mu_r\right)^2}\left(\theta' + I + \mu_r\right)\right|_{\theta'=0} \\
&+ \left.\left(\frac{E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + \mu_r}{E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + I + \mu_r}x'(\theta') - \left(x'(\theta') - g'\left(b(\theta')\right)b'(\theta')\right)\right)\right|_{\theta'=0} \\
&= \left.\frac{I}{I + \mu_r}\frac{\partial E[x(\tilde{\theta})|\tilde{\theta} < \theta']}{\partial \theta'}\right|_{\theta'=0} \\
&+ \left.x'(\theta')\left(\frac{\mu_r}{I + \mu_r} - \left(1 - \frac{Ib'(\theta')}{b'(\theta)(I + \mu_r) + \kappa\mu_r}\right)\right)\right|_{\theta'=0} \\
&= \left.\frac{I}{I + \mu_r}\frac{\partial E[\tilde{x}|\tilde{x} < x']}{\partial x'}\right|_{x'=0} x'(\theta')\big|_{\theta'=0} + \left.x'(\theta')\left(\frac{\mu_r}{I + \mu_r} - \frac{b'(\theta)\mu_r + \kappa\mu_r}{b'(\theta)(I + \mu_r) + \kappa\mu_r}\right)\right|_{\theta'=0} \\
&= \left.x'(\theta')\left(\frac{\mu_r}{I + \mu_r} + \left(\frac{I}{I + \mu_r}\frac{\partial E[\tilde{x}|\tilde{x} < x']}{\partial x'} - \frac{b'(\theta)\mu_r + \kappa\mu_r}{b'(\theta)(I + \mu_r) + \kappa\mu_r}\right)\right)\right|_{x'=0,\theta'=0} \\
&= \left.x'(\theta')\left(\frac{\mu_r}{I + \mu_r} + \left(\frac{I}{I + \mu_r}\frac{\partial E[\tilde{x}|\tilde{x} < x']}{\partial x'} - \frac{\mu_r}{I + \mu_r}\right)\right)\right|_{x'=0} \\
&= \left.x'(\theta')\frac{I}{I + \mu_r}\frac{\partial E[\tilde{x}|\tilde{x} < x']}{\partial x'}\right|_{x'=0} \\
&> 0
\end{aligned}$$

where the equality before the last equality follow from $b'(\theta')|_{\theta'=0} = \infty$. Hence, continuity of $u_1(\theta')$ implies that there exists at least one $\theta' > 0$ such that $u_1(\theta') = 0$. Let

$$x_I = \min\left\{x = x(\theta')\,|u_1(\theta') = 0, \theta' > 0\right\}.$$

We have shown that $\lim_{\theta' \to 0} u_1(\theta') = 0$, $\lim_{\theta' \to \infty} u_1(\theta') = -\infty$, and that $u_1(\theta')$ is increasing in θ'. By construction of x_I, for all θ' s.t. $x(\theta') < x_I$ we have $u_I(\theta') \geq 0$. Since $\frac{E[x(\tilde{\theta})|\tilde{\theta}<\theta']+\mu_r}{E[x(\tilde{\theta})|\tilde{\theta}<\theta']+I+\mu_r}$ is increasing in θ' it follows that

$$\frac{E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + \mu_r}{E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + I + \mu_r} \left(x(\theta') + \mu_r + I\right) - \left(x(\theta') + \mu_r - g(b(\theta'))\right) \geq 0.$$

Hence, for all θ' s.t. $x(\theta') < x_I$, the manager prefers investment without disclosure to investment with disclosure. We further need to show that no type θ' s.t. $x(\theta') > x_I$ wants to deviate and invest without disclosure. We know that for θ' s.t. $x(\theta') > x_I$ manager with θ' does not mimic θ_I for which $x(\theta_I) = x_I$ by issuing the report $x^R(\theta_I)$ and investing. That is

$$x(\theta') + \mu_r - g\left(b(\theta')\right) \geq \frac{x_I + \mu_r}{x_I + I + \mu_r} \left(x(\theta') + \mu_r + I\right) - g\left(b(\theta_I) - \kappa\left(\theta' - \theta_I\right)\right).$$

We want to show that type $\theta' > \theta_I$ prefers issuing the report $x^R(\theta_I)$ to investment without disclosure, i.e., we want to show that

$$\frac{x_I + \mu_r}{x_I + I + \mu_r} \left(x(\theta') + \mu_r + I\right) - g\left(b(\theta_I) - \kappa\left(\theta' - \theta_I\right)\right) > \frac{E[x(\tilde{\theta})|\tilde{\theta} < \theta_I] + \mu_r}{E[x(\tilde{\theta})|\tilde{\theta} < \theta_I] + I + \mu_r} \left(x(\theta') + \mu_r + I\right). \tag{19}$$

As before, we can restrict the analysis to θ' that do not need to distort their investment level downwards in order to mimic θ_I, i.e., to $\theta' \in \left(\theta_I, \theta_I + \frac{1}{\kappa}b(x_I)\right)$. Taking the derivative of the LHS of (19) net of the RHS of (19) with respect to θ' yields

$$\frac{x_I + \mu_r}{x_I + I + \mu_r} x'(\theta') + \kappa g'\left(b(x_I) - \kappa\left(\theta' - \theta_I\right)\right) - \frac{E[x(\tilde{\theta})|\tilde{\theta} < \theta_I] + \mu_r}{E[x(\tilde{\theta})|\tilde{\theta} < \theta_I] + I + \mu_r} x'(\theta'),$$

which is positive for $\theta' \in \left(\theta_I, \theta_I + \frac{1}{\kappa}b(x_I)\right)$. We know that (19) holds with equality for θ_I. Hence, the inequality in (19) holds for all $\theta' \in \left(\theta_I, \theta_I + \frac{1}{\kappa}b(x_I)\right)$. This proves that no type $\theta' > \theta_I$ wants to deviate and invest without disclosure. Moreover, no manager who makes a disclosure wants to deviate to an investment level x_R other than the one described in Lemma 5. The reason is that by construction of the investment distortion function $b(\theta)$ in Lemma 5, any deviation to the investment level of another type decreases the manager's payoff. As a result, the manager's equilibrium strategy is as follows: $\theta \in [0, \theta_I)$ raise equity capital but do not disclose and $\theta \in [\theta_I, \infty)$ raise equity capital and disclose where the manager's investment distortion is described in Lemma 5.

Next, we consider the case in which $b(\theta) > g^{-1}(\mu_r)$ for some θ. We define

$$u_1(\theta') = \frac{E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + \mu_r}{E[x(\tilde{\theta})|\tilde{\theta} < \theta'] + I + \mu_r} \left(x(\theta) + \mu_r + I\right) - \left(x(\theta) + \max\left\{0, \mu_r - g\left(b\left(\theta'\right)\right)\right\}\right).$$

By the same argument as above, there exists at least one $\theta' > 0$ such that $u_1(\theta') = 0$. Let $x_I = \min\{x(\theta') | u_1(\theta') = 0, \theta' > 0\}$ and let θ_I be s.t. $x(\theta_I) = x_I$. Moreover, following the same line of argument we can show that no manager with profitability higher than θ_I wants to deviate and raise equity capital without issuing a report. As a result, the investing and reporting strategy described in Proposition 2 constitute an equilibrium when investors' beliefs given a report (including reports off the equilibrium path) are given by the bias function in Lemma 5. ■

Proof of Proposition 3 I WOULD TAKE IT OUT

From Lemma 7, we know that $g(b(x)) - \mu_r$ decreases in μ_r. Based on this result, we show that a necessary condition for an equilibrium with full disclosure to exist is that $\mu_r \leq \mu_r^*$ which implies $g(b(x)) \leq \mu_r$ for all x (Proposition 3, part (i)). Further, if $\mu_r > \mu_r^*$ then there exist some x for which $g(b(x)) > \mu_r$ which is a necessary condition for an equilibrium with intermediate news undisclosed to exist (Proposition 3, part (ii)). In addition, these conditions in conjunction with the off-equilibrium beliefs that managers that raise capital without issuing a report are of the lowest type ($x = 0$) are sufficient for the existence of the equilibrium in Proposition 3.

Lemma 14 *A necessary and sufficient condition for existence of a full disclosure equilibrium is that for any value of the assets in place, x, the following inequality holds*

$$\mu_r \geq g(b(x)). \tag{20}$$

Proof. In a full disclosure equilibrium, the manager always discloses, the firm equity is correctly priced and the firm invests in the positive NPV project. Hence, the firm's expected payoff is $x + \mu_r - g(b(x))$. We need to show that none of the types has incentives to deviate. One potential deviation in a full disclosure equilibrium is for a type not to disclose and not to invest which yields a payoff of x. A necessary condition to preclude such deviation is that the disclosure cost of all types are lower than their expected return on the investment, μ_r. So, a necessary condition for the existence of a full disclosure equilibrium is that condition (20) holds for any x, or equivalently, $g^{-1}(\mu_r) > b(x)$.

Condition (20) is not only necessary, but also sufficient for existence of a full disclosure equilibrium. To show the sufficiency of this condition one needs to preclude any form of deviation. There are two other types of potential deviations: (i) disclosure of a report other than the type's equilibrium report and investment and (ii) non-disclosure and investment. By construction of the bias function

$b(x)$ in Lemma 1, any deviation to a report of another type decreases the manager's expected payoff and therefore is precluded. To preclude deviation to non-disclosure, we need to assign off-equilibrium beliefs that guarantee that no type wants to deviate to a non-disclosure and investment strategy. The only off-equilibrium beliefs that preclude such deviation by any type are the ones under which investors believe that the manager is of the lowest type if he raises capital without issuing a report. ■

Lemma 15 *A necessary and sufficient condition for existence of an equilibrium with intermediate news undisclosed is that there exists a value of assets in place, x, such that*

$$\mu_r < g(b(x)). \tag{21}$$

Proof. Condition (21) implies that there are exactly two values of assets in place, x_1^D and x_2^D, for which $b(x_1^D) = b(x_2^D) = g^{-1}(\mu_r)$. We want to show that the following equilibrium exists if condition (21) holds: For all $x \in [0, x_1^D)$ and $x \in [x_2^D, \infty)$, the manager invests and discloses according to Lemma 1 and for all $x \in [x_1^D, x_2^D)$ the manager does not invest and does not disclose. If investors observe an off-equilibrium report they believe that the manager reported according to Lemma 1. These off-equilibrium beliefs together with the manager's reporting strategy guarantee that any type that issues a report in equilibrium does not deviate and issue a different report (either on or off the equilibrium path). As before, we assume that if a firm invests without issuing a report investors believe that the firm is of the lowest type, i.e., the value of the firm's assets in place is zero. Together, this shows that condition (21) is sufficient for an equilibrium with intermediate news undisclosed to exist. To show that condition (21) is also necessary, note that if condition (21) does not hold then any type that is supposed not to disclose and not to invest would deviate and issue a report according to Lemma 1 and invest.[34] ■

[34] This arguments relies on off-equilibrium beliefs that equal the full disclosure beliefs, however, it is easy to show that if condition (21) does not hold an equilibrium with intermediate news undisclosed cannot exist for any off-equilibrium beliefs.

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